Course Name: Turbulence Modelling

Professor Name: Dr. Vagesh D. Narasimhamurthy

Department Name: Department of Applied Mechanics

Institute Name: Indian Institute of Technology, Madras

Week - 3

Lecture – Lec15

15. Statistical Stationarity and Homogenity in Plane Couette Flows - II

So, let us go to the equations. So, let us start with Reynolds stress equations, Reynolds stress equations. So, the left hand side I will not focus so much now, the left hand side sometimes will drop sometimes remains, let it be. So, I am going to look at first the for i equal to 1 and k equal to 1. So, I am going to substitute the value i equal to 1 k equal to 1 to get an equation for this will be u1 prime u1 prime bar for this particular normal Reynolds stresses, an equation for this.

So, on the left-hand side, I am not focusing so much. I would like to reveal to you what of these particular terms are there on the right hand side right. So, we are focusing on the pressure strain rate term right now. So, I have four other terms.

So, symbolically one can write before this I can write this Reynolds stress equation here. So, symbolically, one can write Reynolds stress equation as you are if you are familiar with this substantial or material derivative form right one can write like this right $\frac{u_i'u_k'}{\mathrm{dt}}$, you are familiar with this I think all of you ok symbolically. Then we had the D_{ik} term, D_{ik}, not the j, j is the divergence term dik is the diffusion rate I am using here diffusion rate plus we had production rate right, or the pressure strain rate term let me call it ϕ_{ij}. I will use ϕ ij ik, sorry, and then I will use the production ik minus epsilon ik. So, names are this is your diffusion rate, this is the diffusion rate term, the pressure strain rate term, the production rate and the dissipation rate.

you will see in many textbooks and reference notes as symbolically written like this without giving out all these terms. But you already know what each of these terms looks like. So, now with this I am going to take equation $\overline{u_1'u_1'}$. So, I would get the first it will be $\frac{\mathrm{D}u_1'u_1'}{\mathrm{Dt}}$. that is the left hand side part, this is the LHS.

So, not worried about this, not worried about the diffusion rate also because I will tell you why I call diffusion rate later. We are only focusing on this particular term here now,

$$\overline{\frac{p'}{\rho}\left(\frac{\partial u_1'}{\partial x_1} + \frac{\partial u_1'}{\partial x_1}\right)}$$

pressure strain rate. So, this will be D_{11}. plus I have this ϕ_{11}. So, let us write it out what it will be.

So, it will be .So, i equal to 1 k equal to 1 will be($\frac{\partial u_1'}{\partial x_1} + \frac{\partial u_1'}{\partial x_1}$). This was the ϕ_{11} term. So, now I have the production rate term, this I would like to expand. So, the production rate term was we had this minus ui prime uk prime.

So, I have this minus of; I can write minus of the production rate, which $u_1' u_j'$. $u_1' u_j' \frac{\partial \overline{u_1}}{\partial x_j}$

plus, again i equal to 1, it is $u_1' u_j'$, k is also 1, $\frac{\partial \overline{u_1}}{\partial x_j}$. So, I basically get minus 2 of this. I am only expanding from the previous equation, setting i equal to 1, k equal to 1. So, the production rate term looks like this and I have the dissipation rate term.

Dissipation rate is $-\epsilon_{ik}$ which is $2\nu \overline{\frac{\partial u_1'}{\partial x_j}\frac{\partial u_1'}{\partial x_j}}$. So, this is the equation for your u1', u1' bar Reynolds stresses. Diffusion rate I did not expand, you can one can expand it and see not a problem. So, now here the important to see here is that this will be of course 2 times S. So, if I take this as LHS equal to

$$LHS = D_{11} + 2\overline{\frac{p'}{\rho}\frac{\partial u_1'}{\partial x_1}} - 2\left\{ \overline{u_1'u_1'}\frac{\partial \bar{u}_1}{\partial x_1} + \overline{u_1'u_2'}\frac{\partial \bar{u}_1}{\partial x_2} + \overline{u_1'u_3'}\frac{\partial \bar{u}_1}{\partial x_3} \right\} - 2\nu\overline{\left(\frac{\partial u_1'}{\partial x_j}\right)^2}$$

It is straight away to see here that this last term must be greater than or equal to 0 and there is a negative sign.

And at least it is revealing, giving a clue why we are calling this a destruction rate because it is minus of a positive value. It is taking away something from the system. I will come to that later. So, I have this term. So, now we apply what we learnt about this plane Couette flow when it has reached statistical stationarity and it has exhibiting homogeneous in the two direction.

So, any terms which will go away in the terms that we have here. what about the first term here before that does this term goes away or that it remains? Somebody said 0, why it is 0? fluctuation mean. Do you have pressure fluctuation also is it 0? This is not 0. It is a correlation of pressure fluctuation and strain rate fluctuation. So, that is why the name came pressure strain rate.

This is not 0, it is a correlation term. And we cannot apply any other rules here, the statistically homogeneous thing because that is not for, that is for a gradient of a statistical term, anything in this over bar and there is no gradient that we are looking into. But what about this particular term here? dou by dou x1 of any statistical quantity, right? So, this is 0, homogeneous along x1, right? Statistically, statistical homogeneity along x1 for the plane Couette flow, ok. So, we are obviously only looking into plane Couette flow, as I already said. Flow is what we are discussing, applying the equation for that particular flow.

What about the second one? This is non-zero because this is the main gradient term that you have, mean strain gradient; x2 is the wall-normal direction, right? So, this term remains. What about the third term? 0, $\frac{\partial}{\partial x_3}$. So, this is 0. Same thing here, statistical, statistical homogeneity along x3 as we discussed. So, this particular P11, so what is this? This is P11 term.

P11 is non-zero here. So, this, so there is a production rate term existing for your one of the Reynolds stress term, ok? So, that is fine, and this is also surviving here this term is a correlation we need to retain. So, now let us see the what is the what will happen to the equation for i equal to 2 k equal to 2. So, i equal to 2, k equal to 2, which will give you $\overline{u_2' u_2'}$. So, the LHS will be $\frac{Du_2'u_2'}{Dt}$ equal to D22, plus I have the pressure strain rate term, which is $\frac{p'}{\rho}$. This is i and k, i equal to k here.

So, it is $\partial u_2'$ by or I can simply write since you are already experts of all this, it is easy to see that it will become $\frac{\partial u_2'}{\partial x_2}$. right minus this will also have two of this because it is ik i prime k prime i prime sorry ui prime uk prime and then you will also get uk prime uj prime and ui prime uj prime. So, i equal to k will give you u2 prime uj prime. So, I get minus 2 for this, I get u2 prime uj prime, and then I get dou u2 bar by dou xj minus 2 of this.

That is correct, yes. So, I would get this as the production rate term minus 2 nu again it is ik so that means it will be u2 prime by dou xj term. So, this is the epsilon 22 term here. If I write it out, this will be the ϵ_{22}, this is my P_{22} term, this is the ϕ_{22}. So, now let us expand the production rate this plus this. We cannot say anything about the ϕ_{22}, the pressure strain rate, because it is a correlation; let it be as it is, only the production rate we are focusing on.

$$\underset{\overline{u_2'u_2'}}{i=2,\ k=2} \Rightarrow \frac{D\overline{u_2'u_2'}}{Dt} = D_{22} + \overbrace{2\,\overline{\frac{p'}{\rho}\frac{\partial u_2'}{\partial x_2}}}^{\phi_{22}} - 2\left\{\overline{u_2'u_j'}\,\frac{\partial \bar{u}_2}{\partial x_j}\right\} - 2\nu\,\overline{\left(\frac{\partial u_2'}{\partial x_j}\right)^2}$$

So, minus 2 of again u2 prime u1 prime dou u2 bar by dou x1 plus j is now 2, repeated index you have to sum it over. So, it becomes u2 prime u2 prime dou u2 bar by dou x2 plus u2 prime u3 prime dou u2 bar by dou x3 minus of your ϵ_{33}. So, now we again see the production rate term P_{22} what will happen to that. So, you see dou by dou x1 homogeneous this term is gone it is also $\overline{u_2}$ is 0 here that is also there $\overline{u_2}$ itself is 0 when the flow is fully developed correct. So, this entire term will go away both because the $\overline{u_2}$ is going away.

So, there is no gradient for the $\overline{u_2}$ because the fully developed. $\overline{u_2}$ is 0 here for a plane Couette flow. Also, the first and last one is also because of the homogeneity. You can also say dou by dou x1 dou by dou x3 of that. So, what what did this happen? So, now what is happening here is that your $P_{22}$is 0. ok poor fellow this particular Reynolds stresses has no production rate P_{22} is 0 for it and ϵ_{33} there is a sink term, this is a square.

So, the dissipation rate is occurring here ok nobody is to produce any turbulence for it, but there is somebody distracting whatever is left ok. So, we will see what happens to this particular term right now we do not know what these stresses look like for this. We do not have, we have not applied any data. We have only considered a flow problem and then applying conditions. So, P_{22} is 0 and we will see next what happens to the next which is i equal to 3, k equal to 3 which is u3 prime, u3 prime average.

$$i=3,\ k=3 \qquad \overline{u_3'u_3'} \Rightarrow \frac{D\overline{u_3'u_3'}}{Dt} = D_{33} + 2\overline{\frac{p'}{\rho}\frac{\partial u_3'}{\partial x_3}} - 2\left\{\overline{u_3'u_j'}\,\frac{\partial \overline{u_3}}{\partial x_j}\right\} - 2\nu\underbrace{\overline{\left(\frac{\partial u_3'}{\partial x_j}\right)^2}}_{\epsilon_{33}}$$

I get d u3 prime, u3 prime. By dt, equal to D_{33}, the diffusion rate plus the ϕ_{33}, which is 2 p prime by rho dou u3 prime by dou x3 average minus 2 of I have now u3 prime uj prime dou u3 bar by dou x j minus 2 nu dou u3 prime, dou u3 prime by dou xj squared average. This is your ϵ_{33}. Again, there is a destruction rate here and we also have the pressure strain rate term available, but $\overline{u_3}$ is 0. I do not have to expand here; $\overline{u_3}$ is 0 for a fully developed again fully developed turbulent plane Couette flow $\overline{u_3}$is 0, not yeah yes no.

Now, that is what I said. So, I will show you the data. This data that $\overline{u_2}$ and $\overline{u_3}$ are 0, I will show later. From the data, I know that these two velocities are 0. We can only say that the flow is homogeneous along the x1 and x3.

That is also from the data. We can see that. That is no problem. You can go ahead and do it. All I am trying to do now is to tell you what is the role of pressure strain rate.

I called it redistribution. I want to prove it to you that it is redistributing. For that, I am taking help of a particular flow problem and I have the direct numerical simulation data done by myself. So, I am going to show you those data to prove what it does. In a given problem, you can go ahead and see what it does. It does what it does, what we are going to reveal to you.

So, here what happened now? ϕ_{33} is 0 again. So, there is no production rate term for this, and the dissipation rate is working very hard; it is removing the turbulence from the system. So, and then we will see the data of these 3 terms to see what exactly happens to these three terms. So, I can show you one graph.

So, I am showing you the RMS data. I will come to that detailed discussion in the next class of this flow problem. But just to give you a clue. So, I have plotted the RMS data taken from my own direct numerical simulation. So, this is instead of this let us say this is the your wall normal coordinate x2. And I have the three rms component here u v w, which is basically your u1 rms.

This v rms is the wall-normal, which is our u2 rms. This is the u3 rms or the stresses, or you can think this off as you already know, rms means if you square this, I will get the stress. Upon squaring I would get this one, this, is this 0, all the terms? Who is producing them? u1 prime obviously there is a production rate. That term is working very hard, the production P_{11}, the most hardworking, ok?