Course Name: Turbulence Modelling

Professor Name: Dr. Vagesh D. Narasimhamurthy

Department Name: Department of Applied Mechanics

Institute Name: Indian Institute of Technology, Madras

Week - 3

Lecture – Lec14

14. Statistical Stationarity and Homogenity in Plane Couette Flows - I

Let us get started. So, in the last class, we looked at the Reynolds stress equations. And equations were also not closed, it had far too many unknowns, right. And so therefore, there is no point continuing in that direction that is again trying to get an equation for a triple velocity correlation and so on. More and more you go in that direction more unknowns come.

So, that equations always remain not closed. Therefore, modeling is the right direction for that. So, before we visit the modeling part of course, we need to first understand each of these terms right. I mentioned that there are particularly four terms on the right hand side.

The rate at which the production occurs of renal stresses, the rate at which it diffuses or transports due to pressure, due to viscosity and due to turbulence itself and also the rate at which it gets destroyed or dissipates and the rate at which it redistributes, where turbulence is redistributing. We will see what that we will get started with that part the redistribution. So, before that I would like to introduce some concepts in the statistical analysis because we will kind of take help from a flow problem to understand its behavior. So, let us revisit the statistical analysis. So, one I just call it revisit statistical analysis ok.

So, in this I would like to introduce two new concepts to you. One is called what is called a statistical stationarity, statistical stationarity. So, what this means is when we say the flow is statistically stationary. So, we have to understand I am not talking about a stationary flow. turbulent flows are not stationary flows right.

The turbulent eddies or vertices are always moving around. So, turbulence we already defined is a three dimensional unsteady process ok. So, flow is, a turbulent flow is not stationary. So, I am talking about statistical stationarity, a flow where it comes to a stage where it is statistically stationary. This is important for especially when you do experiments or if you are doing the simulations, then you need to know from where I need to start to take my measurements, from where I should start to record the data for that part.

So, what it means is if I explain to you as if you are doing some simulation or experiment is let us take you have a flow, ok. So, let us say there is a flow here between two different two parallel plates. So, let us say this top plate is moving to the right direction, the bottom plate is fixed. Top plate is moving to the right direction. Are you familiar with this type of a flow? What is it called? A Couette flow.

This is a plane Couette flow. You also get a Taylor Couette flow or a circular Couette flow that is the flow between two concentric cylinders rotating in relative motion. So, this is a plane Couette flow. This is just an example. It can be applied to any flow.

Just taking an example, a plane Couette flow, turbulent of course. So, in this particular flow problem, so the top plate moves to the right in relative motion to the bottom one, which is fixed. So, you get some turbulent motion in between. So, let us say you are going to record a data at let us say at this position. Let us say a student x wants to record data at this position, x1, y1, z1.

x1, y1, z1, t at a particular t. Now, for some reason, he gets sick and cannot continue the experiment, but today he has recorded the data, and he will come back and he will take the data again at the same location. If these two statistics are matching, then what does that mean? He has taken many samples. and he has done the statistical analysis he has calculated the mean velocity rms all these things all the statistics that we discussed variance standard deviation third-order statistics all this he has done all the statistics by measuring data at a certain location x1, y1, z1 so this student let's call A has done the measurement and then he has come back another time maybe the same day or the next day, and he gets new data Let's say the tunnel is still working. Let us say this is a wind tunnel experiment where it is still working, and he will come back and measure if he is going to get the same statistical data that means the flow has come to a statistical stationarity the statistics are not changing in time ok.

So, statistics are independent of origin in time statistics are independent of origin in time. That means at what time t you started does not matter anymore, ok? So, this can be illustrated, for example, if I just take the same case and see how the data would have looked. So, let us say I am plotting a graph here ok, where I am looking into let us say u1 velocity that student A is taking many samples. So, the flow let us say goes from in the beginning there is no flow, he starts the blower ok, he starts the wind tunnel and the flow starts to come and then it will go on and then it starts to go like this let us say. Can you see oscillations or it looks like a straight line? So, maybe I will make it more oscillating.

Let it states goes up. So, let us say he starts to get data like this. Now, he has taken data. ah the student is smart. So, he will look at this data and decides to remove this transients.

Obviously, you should not use all this initial part to compute your statistics. So, he will decide to take from this particular time instant let us say and to up to here and he gets some average let us say u1 average. taken from, we can make it t here instead of samples, we can make it as time itself. So, this is between t1 to t2, he has taken all the samples here is time average to get u1 bar. Now, he comes back later and then takes data.

at another position it does not matter like I mean he can continue to take data here or another student can come it does not matter maybe he will tell his friend you continue I am sick. So, that the student continues from t2 and then he will take data up to t3 and then he will take u1 bar he computes it ok. So, if these two matches, Let us call this u_n bar as the sample 1. Let us say this is let us call it S1 and this is S2 from two different samples. So, if u_n bar S1 is equal to u_n bar S2 both of course, same number of samples are taken.

Let us say both involves 1000 samples right. The number of samples has to be the same. He is not checking for whether statistics are converging if I increase the number of samples that is the later part. The first part is to check whether your flow has come to a statistically stationary state that your statistics are no longer changing in time. If it is come to that part then this process is called statistics are independent of origin in time.

So, it does not matter when you come to take your measurements your flow has come to a statistical stationary state, but you need to verify this ok before you do any simulation or experiments before you take the data to process for statistics this must be done ok. So, another concept is this is clear statistical stationarity I am not talking about you know a stationary flow. Flow is not stationary here. Turbulence is completely unsteady three-dimensional flow.

Yes. Look at the next topic. This is called homogeneity. Homogeneity or you can say this, this is statistical stationarity. The other one is statistical homogeneity you can say. That is the correct statistical homogeneity.

a homogeneous random process. So, when you have a statistical homogeneity, what it means is the same experiment if I take. So, the same have this I can write in the same place. So, now I have So, at location x1, y1, z1, let us say both students A and B are in a hurry to graduate. They want to do simultaneous measurements.

They do not have time they want to do it today at the same time and the tunnel is very long. So, student A decides I measure at x1, y1, z1, and student B decides he will measure at x2, y2, z2, and they will do the same process. ok they say ok i have got my mean velocity rms all these things then they compare if the statistics from location x1, y1, z1 is matching x2, y2, z2 or any other location ok that means as you go in the spatial direction your statistics are matching that means the flow has come to a statistical homogeneity. So, this is here, of course; it is not x2, y2, z2. Let me write this should be x2, y1, z1.

Ok, only in x they have moved, but y and z, it is the same position. So, here, the flow has come to a statistically homogenous flow is showing not come to the flow is showing that there is a statistical homogeneity in x direction. Every flow need not display this whether you can get completely inhomogeneous turbulence in all the three directions, but there can be flows where the flow can exhibit a statistical homogeneity in one direction or two directions or all the three directions. Let us say here it is only the x direction where I measure flow at this location and then I measure flow here and the two mean velocities or urms or vrms any of these quantities are matching between this location and this location or any location as I walk along the x direction. Then, I can say the turbulence is homogeneous along x1.

So, this statistical homogeneity implies that statistics are independent of origin in space. So, statistics are independent of origin in space that is x1. and or x2 and or x3 depends on the flow. For a plane Couette flow, I can tell that you will get homogeneity both along the x1 direction. Let us say if I am going to call this, this is x1, and if I call this, this is anyway x, y, z coordinates.

So, this is x, this is y and the out of plane is z. So, both x and z will exhibit statistical homogeneity in a plane turbulent Couette flow. In the y direction, you will not; that is obvious. or the direction wall normal direction you must get gradients right so your statistics if i measure here one student will go very close to this wall the other will go to the close to the other wall they will measure okay that some quantities can become similar if the profile is symmetric some quantities but not all quantities some can exhibit anti-symmetric behavior So, there are gradients exist. So, the flow along the y direction for a plane Couette flow is inhomogeneous.

And we define this mathematically also when I say this statistical homogeneity. So, if I say homogeneous in, homogeneous I can say along x1 direction implies dou by dou x1 of all the average quantities is 0. So, this is the bracket term here. So, this is just any quantity that you can put here, let us call it phi. You can put your velocity, standard deviation, any other quantity.

Any statistical quantity you are looking into, the gradient along x1 should be 0. Similarly, for x2, the same thing, your $\frac{\partial}{\partial x_2}$ of or I can say just $\frac{\partial \bar{\Phi}}{\partial x_2}$ is 0 and similarly for x3 and so on. And we can use the same concept also for the statistical stationarity also because your statistics are no longer changing in time. So, the time derivative should be, time derivative of what? Instantaneous or the statistical quantity? Statistical quantity, ok? $\frac{\partial \bar{\Phi}}{\partial t}$ has to be 0.

So, you should be very clear here. I am not talking about this where ϕ is instantaneous or I am not talking about, I am not talking about the random component or instantaneous which has the random component, only talking about the true mean, $\frac{\partial \bar{\Phi}}{\partial t}$. Similarly, here also the same thing, here also the same thing your dou phi by dou x1 is not 0, dou phi prime by dou x1 here is non-zero, only the statistical value. over bar. So, then we say, so this kind of flows exist where you have a statistical stationary flow as well as a statistical homogeneous in a particular direction or more than one direction. Is this clear? So, now, apart from the statistics, we are going to apply this knowledge to a particular flow, the same plane Couette flow and revisit the Reynolds stress equation.

So, let us just look at this particular flow now: plane turbulent Couette flow. I am just taking this as an example. So, if I take this particular example, I am going to consider what is called fully developed stage. So, consider the flow is fully developed and you have also make sure that the flow is statistically stationary and all these things.

So, that you have done. So, in this particular case now, so what will happen when all these conditions are there? I have checked for statistical stationary, right? So, statistical stationarity has achieved. So, flow has come to, flow has achieved statistical stationarity. So, here my dou by dou t of all the quantities statistical quantities are 0 in this flow now. And then I also have inhomogeneous sorry inhomogeneity only in the one direction right. So, if I take instead of this x, y, z direction if I simply take this problem as in the coordinates that we are using.

So, we have x1, x2 and x3 is the out-of-plane component, right? x3 is the out of plane component. This is the fixed wall and this is the moving wall or u infinity, let us call it some u reference, reference velocity. So, if I have this then here we have homogeneity along homogeneous along x1 comma x3 which implies dou by dou x1 of any statistical quantity 0 dou by dou x3 of any statistical quantity is also 0. I will also show the data later from my own direct numerical simulation. But let us consider that it has come to a stationary state, statistical stationary state and it is also statistically homogeneous along x1 and x3.

And I also mentioned that the flow is fully developed. When the flow is fully developed, turbulent plane fluid flow, we also get what is that as the second and the third velocity components. When this happens, your u2 bar equal to u3 bar equal to 0. in a laminar plane squared flow that is easier to see, one can prove this analytically. In a turbulent squared flow also, I will show you the data u2 bar will be equal to u3 bar equal to 0, only the u1 bar exists ok.

So, that will look like this: it will have a profile like this; this will be your u 1, not linear as it should have been if the flow was laminar, right? So, a turbulent plane squared flow will have u 1 bar looking like this. So, I have this condition, right. So, with this we will go back to our equations and see what happens to each of the term,