13. Reynold's stress: governing equations - II

So this the one in this red bracket that is the divergence term $\frac{\partial}{\partial x_j}$ of I can write this entire part as $\frac{\partial}{\partial x_j}$ of all the three terms together. So this particular term is what we call the transport term. I will explain to you why we call that. Let us just name them as the so called divergence term or the transport term. or even the diffusion term.

So, it is basically going to diffuse Reynold stresses inside your system. How does it do? So, this particular term here is the transport of your Reynold stresses due to turbulence itself. So, turbulent stresses are being diffused by turbulence itself. I already told you in the very beginning how turbulence can be and what are the characteristics of turbulence right.

We discussed that it is random that I showed you what we mean by a random process and then I said it is diffusive right. So, this is this nabla dot or dou by dou xj of this triple velocity correlation indicates that turbulence transports the field any scalar field in this particular way in this particular equation it is a Reynold stresses. Turbulence itself is transporting the Reynold stresses. So, this particular term we call it turbulent diffusion. It is called turbulent diffusion rate.

Each of this is a rate term here the rate of change of either production transport and so on. So, now we have the another bracketed term red bracketed where viscosity is coming into play right it is $\nu \frac{\partial}{\partial x_j}$ of this particular term. So, this is the viscous diffusion rate of your Reynold stresses turbulent diffusion rate viscous diffusion rate.ok Let me just take out this So, this particular term is called viscous diffusion rate. So, this particular term is viscous diffusion rate diffusion meaning transport.

So, the rate of diffusion of Reynolds stresses due to viscosity, the rate of diffusion of Reynolds stresses due to turbulence and the first bracketed term is the rate of diffusion of Reynolds stresses due to pressure. So, it is a pressure diffusion rate, you have a pressure and velocity correlation. So, this is the pressure diffusion rate. So, there are three transport terms, three diffusion rates. And so, we have this another particular term here, this pressure strain rate a unique term.

I would like to let us say use a different color here. me say use green here. So, this particular term is actually a correlation of pressure fluctuation and the strain rate.ok So, this is a pressure strain rate term, pressure strain rate term. What it does? I will explain.

It is a very unique term, very special term. I told that turbulence diffuses dissipates in addition to that it also redistributes this I mentioned this redistribution is a very unique property in turbulent flows. It dissipates due to viscosity also, but it also redistributes I will come to that what we mean by a redistribution. So, this pressure strain rate term is also called redistribution term or redistribution Right So, I have two more terms here. So, this particular one here let us say I use this particular part as I already said is the production rate that is already mentioned.

Right Production rate term this particular thing. It is a production rate term and this particular term here is the dissipation term, dissipation rate, the rate at which the turbulence or Reynolds stresses are dissipating inside the system. so this was not obvious when we just looked at navier stokes equation right what it was doing navier stokes obviously is computing all this we started from navier stokes to arrive at these equations without making any assumptions right we just consider constant density and every term that has popped out has been placed in front of you but now we see that it is doing different things and i will show it to you or rather give a proof why we call each of these terms what they are So, at least now we know that when we have a transport equation for Reynold stresses, you will have the familiar unsteady rate and advection rates on the left hand side. And on the right hand side you have four particular terms, one is the production rate okay the production rate of the Reynold stresses and the next one is the dissipation rate. So, production and dissipation, production implying something the rate at which it is being generated and the rate at which it is being destroyed is the destruction rate or dissipation rate and that is happening due to viscosity, you see viscous action right.

The viscous action is so even without doing any experiment simulations already the equation is telling lot of information. So, the viscous action is dissipating the turbulence and the production is carried out by mean strain. So, you must have a mean strain. If you do not have mean strain inside, if you do not have velocity gradients, you will not be able to get the turbulence generation. and the diffusion or the transport is done by three particular terms, viscosity is transporting, turbulence itself is transporting and pressure is also transporting and finally a very special term called pressure strain rate or redistribution.

So, now we look at the objective is to solve this equation. I mean we wanted to solve Reynolds average Navier-Stokes equation with 6 unknowns and we said we do not want to model it, we will have an equation for it, we achieved an equation but can we solve this is the next question. So, we will see how many unknowns and equations this is coming with. So, now I have on the left hand side, so I have equations, a familiar exercise we do it in this exercise. course throughout.

So, I have 6 extra equations, which is great. I have 6 extra equations to close RANS equation. So, I have 6 equations here. Good. Let us look at the unknowns.

So, on the left hand side it is not a problem. If I go back to the previous slide here, left hand side is not a problem. This unsteady rate of Reynolds stresses and the diffusion or the advection or convection rate of Reynolds stresses. uj bar is an unknown here, but anyway you are solving for RANS equation to get the mean velocity. So, left hand side is taken care of, we do not have to worry.

The right hand side we will see here. okay the first thing is p'u'_kor p' u'_k i and k is free index so if i give value for example if I want to know the Reynolds stress for when i equal to 1, k equal to 1, that means it is u'_1 u'_1 . For that particular term, this becomes p'u'_1, p'u1'basically. So, it is a i and k are free indices and i and k can take numbers 1, 2 or 3. So, what is this? A vector or a second order tensor or a third order, what is it? p'u'_{k}or p'u'_{i}.

It is a vector right. So, you have p' $\overline{u'_1}$, p' $\overline{u'_2}$, p' $\overline{u'_3}$ there are three right. So, the pressure diffusion is different probably same we do not know you can already see the complexity in any flow the diffusion rate in each direction can be different or same so this talks about the another property of turbulence which is anisotropy i told you right so the somebody asked a question in the last class why are we looking into this ui'ui'is essentially nothing but your u'_1 u'_2 u'_3or colloquially you can say u' v' w' So this is a fluctuating vector. So you would like to see if I have a coordinate axis like this, how much of this velocity is oscillating, exhibiting its randomness in x direction, y direction and z direction. If it is oscillating at the same extent, then it is probably exhibiting isotropic behavior. but if it is having oscillations are different magnitude in one direction compared to the other then this brings the anisotropic nature of turbulence that is turbulence is directional dependent in one direction turbulence can be large in the other direction it cannot be that much ok.

So, p' u'_{k} so let us go to this slide. So, I have the first, of course these unknowns are all, this is also an unknown but this is taken care of, no problem we have 6 equations for it. Now I have this p'u'_{k} or p'u'_{i} , this is essentially nothing but your p'u'_1 . p' u'_2, p' u'_3, 3 terms of pressure and velocity fluctuations. Therefore, this is giving me 3, 6 equations.

So, this is also 6 which is not a problem. Already you see I have 3 extra unknowns that has come. we wanted to solve this 6 unknowns in the rance equation got closed, but I need 3 extra unknowns. So, now what is the other term that we have? So, the pressure diffusion rate is fine. So, I have the viscous diffusion rate.

Luckily here the viscous diffusion rate is depending on Reynold stresses itself. So, that means that is no longer an unknown I am going to compute for it and in CFD techniques when you see that that you are solving for Reynold stresses and then it is depending on Reynold stresses you need to solve this what we call an iteratively an iterative solution must be achieved. those things CFD those who have taken I think almost everybody here would know this. Anyway, that is another part. So, at least Reynold stresses is depending on itself.

So, perfectly fine no extra unknowns in the viscous diffusion rate. And I did mention once that If you derive this Reynolds stress equation starting from a different point not the way we derived. Here we took the equation for fluctuating velocity transport equation and multiplied by another fluctuation average. If I start from a different point this will this particular viscous diffusion rate looks completely different.

It gives additional unknowns. for that particular reason modeling community does not like that equation, but that is also coming from first principles this is also coming from first principles. Just the way you the start I will tell you when we come there why it is not useful for modeling, but for completeness of the material I will tell you how we can start there. So, at least this is good no additional unknowns in the viscous diffusion rate. So, I have the turbulence diffusion rate u_i', u_j', u_k'. I do not have to tell you here how many unknowns this is leading to.

So, I have u_i', u_j', u_k' over bar. How many it is? 27. 27 new unknowns have come. Of course, one can derive an equation for that, it is possible.

It is theoretically possible.Probably I think it is already done also. One can go ahead and get a transport equation, 27 transport equations. But what do you think will happen? Which higher order term will come there? Fourth order term, right. That is why it is called turbulence closure problem. How much ever you want to close it, it is going to give one extra higher order amount of unknowns.

okay so that complexity is there but we will not and this term is not very small okay the transport due to turbulence because let's say those who are doing heat transfer they actually like turbulence they introduce some rib roughness fins and so on why do you put some obstructions in the flow in a pipe or something because you want to generate vorticity you want to generate turbulence right so you want turbulence to transport your scalar quantity which is essentially temperature it could be something else a passive contaminant also you want to transport it away. So, turbulence transport is very large in a given problem not just the viscous transport turbulence transport is also. So, let us look at any other unknowns we have yes the one term which is called pressure strain rate term this is p' correlated with the strain rate p' $\frac{\partial u_i'}{\partial x_k} + \frac{\partial u_k'}{\partial x_i}$ average. So, let us write this out. So, I have pressure strain rate term which is p' $\frac{\partial u_i'}{\partial x_k} + \frac{\partial u_k'}{\partial x_i}$ average.

What is the rank of this tensor? 2, it is i and k, second rank tensor. So, how many unknowns? nine unknowns in general ok. So, this is introducing nine unknowns and this is a very important term in the turbulence physics. As I said this is doing redistribution not just dissipating or even those who have not done turbulence studies they would as I said those who are doing heat transfer or any other thing they would know the turbulence transports. So, turbulence diffusion is already familiar even though they do not know how it looks mathematically.

right but turbulence redistribute this is not something you would have expected. So, any other term we have. So, the production rate again is depending on the Reynolds stresses which is fine that is great right and the mean strain. So, I have access to the mean velocity and therefore, I can compute the mean strain rate. So, production rate is perfectly fine I it is not introducing any new unknowns.

what about the dissipation rate it is a correlation of two different gradient of fluctuating velocity gradients so this is introducing a unknowns here and j is repeated index so what is the rank here second rank second rank so it is again i, k it is introducing nine unknowns here. So, I have the viscous or the dissipation rate term. It is nu, it is 2 ν, right? Did not I write 2 ν or? 2 ν. Yeah, 2 ν. It is 2 ν $\partial u_i'$ by, what was this? ∂x_{j} and $\frac{\partial u_{\mathrm{k}}'}{\partial x_{\mathrm{j}}}$.

this is giving me 9 unknowns again here just like I mentioned if I start with a different starting point to derive this equation this particular so basically the viscous term wherever it comes so this viscous dissipation rate or dissipation rate term will not look like this so this is not what we call a true rate of dissipation rate this we call it something else but the modeling community likes this lesser the unknowns easier it is to model. It is not easy looking into the amount of unknowns that has come already here, but you certainly do not want more unknowns than what it is already there. But this particular term looks very different if you have what to say if you start deriving the equation in a different way this particular term as well as the viscous diffusion rate. So, basically the viscous term looks different. so obviously I have the turbulence closure problem.

So, equation, so therefore equations are less than number of unknowns, again it is turbulence closure problem we need to model this also if you want to solve for this you have to model this and this particular if you are able to model this.ok If these terms are modeled and the equations are closed by modeling, modeling means approximation then the models is then the particular class of model is called Reynolds stress models, then it is called Reynolds stress models RSM. We will do that. we will try to model them one by one. First the Reynolds average Navier-Stokes then followed by Reynolds stress models.ok