1. Introduction to turbulence

Okay, let's get started. I welcome you all to the Turbulence modelling course. My name is Vagesh, and I'm a faculty at the Department of Applied Mechanics and Biomedical Engineering at IIT Madras. And if you have any queries, you can always reach out to me on the email address in the slide below. So, before we begin, I would like to highlight why we have to study turbulence or turbulence modelling.

So, we all know that practically any flow that we see in nature or in industry is turbulent and this could be either atmospheric turbulence or it could be aerodynamics or turbulent convection and so on. So, these are flows that we come across on a daily basis, and it is very important to understand them. And it need not be a single-phase turbulence. It is also important when or how turbulence will interact with, for example, when it is subjected to two-phase or multi-phase flows, for example, bubbles, droplets or even particulate flows.

So, here there is an example of how particles are being deposited or dispersed in a background turbulent flow. and such applications are very important. So, turbulence plays a very important role here and also turbulent reactive flows. This could be aerospace propulsion or internal combustion in an engine or fire and explosion safety, and so on. So, how does turbulence interact with chemical reactions becomes very important for application point of view.

So, there are wide range of application, this is just a few studies that we have been looking into. So, there are obviously lot of applications where turbulence plays a very important role, and it is important to study turbulence. So, some practical information that I think most of you know is that to study turbulence modelling, you must know Navier-Stokes equations and how it is derived and what is the physical meaning of each of these terms in the Navier-Stokes equations are important. And you must also have a basic understanding of computational fluid dynamics, be it finite element method, finite volume or finite difference. some methods and its basic foundations are important.

Additionally, if you know Cartesian tensors, it is good. If not, we are going to revise in the course, which is not a problem. And if you already know turbulence theory, it helps. But if not, that is not an issue. We are going to cover some turbulence theory in the beginning so that you can build on to modelling from the knowledge gained.

And there will be handouts or transcripts given from this course that will be useful to you. In addition, if you want to have a textbook, I think Stephen B. Pope's Turbulent Flows is a good book to look at. And, of course, you can also have some reference books like Versteeg and Malalasekera. This covers primarily from a finite volume method point of view and it is a more CFD book.

But if you are interested in Reynolds average Navier-Stokes equations and modelling RANS is a good reference book for that. And if you are interested in Reynolds stress modelling, then the book by Hanjalic and Launder is a good reference book. And also DC Wilcox book is also a good reference for RANS modelling. So, essentially, the reference books what I have given is mostly covering the RANS aspects, while Stephen B. Pope also looks into LES and other techniques.

There is also, of course, a lot of information on the web, and so there is this ERCOFTAC community, which is the European research community on flow turbulence and combustion. So, there is good information on turbulence modelling as well as applications of turbulence modelling to various class of flows. So one can take a look at their website as well as there is some information on specifically to RANS modelling on this site at the Langley Research Center. This is also a good reference site to look at. And also, there are, of course many other sources where you can learn turbulence from.

So, some of the topics that will be covered in this course is of course, I will start with turbulence theory because without knowing the theory, you cannot start to model and appreciate why we are modelling a certain class of flow the way it is. And once we have the theoretical knowledge of course, we look into what is called direct numerical simulation. That means here we do not do any turbulence modelling. So, turbulence will be resolved rather than modelled here, but still, it is important to know about this because many of the DNS datasets are used to actually validate or even build upon certain class of turbulence models. And then we go into this next group, which is the RANS model, which is you can say it is a statistical model.

So, it is more numerical than physics-based. And here we look into all the class of RANS models, be it the eddy viscosity model or a Reynolds stress model. And also, these models have some deficiencies, and we look at certain ways of how you can overcome. In addition to this, of course, there is the large eddy simulations or LES. So, here we look into various filters and its impact on LES as well as the subgrid-scale modelling various options that we have.

And also we will briefly touch upon the hybrid LES RANS models or the detached dissimulations and so on. So when it comes to turbulence modelling, this attracts a wide range of people, be it engineers, for obvious reasons, right? I mean, you need to design cars and your rockets or aircrafts. So engineers are obviously interested in it. And of course, physicists are interested to study turbulence. It is a classical mechanics problem.

And also mathematicians are interested in it because of the underlining governing equations being partial differential equations and non-linearity in the equations. So, it attracts different streams in science and engineering and technology and obviously a mix of this will also be for example, those who are doing computations, it is a mix of mathematics and engineering or mathematics and physics or it could be experimentalists where they have some experimental techniques used to understand the physics. So this is more added to humour. So I don't really mean that what is being predicted here by an octopus or a parakeet is actually turbulence modelling. But if you actually look at the underlining aspect here that these two are actually making some prediction.

So, at the end of the day when you are using turbulence models, you are actually predicting. Of course, here the predictions are based on sound theory unlike the arbitrariness involved with the way octopus or a parakeet will predict. So, this aspect has to be keep in mind. that when we are talking about turbulence modelling, we are actually predicting the flow. And when we have a prediction, obviously it comes with an error.

So model accuracy and model uncertainty must be considered while making such studies. And since I talked about model uncertainty, I have just added here a graph to just to illustrate the wide range of uncertainties that you can see. So, here you have the symbols represent experimental data. And if you can imagine that there is no experimental data here, then you can see that the three lines are from three different turbulence models and they have sometimes they are predicting very similar results and sometimes they have wide range of results. And therefore, it is difficult to know in the absence of experimental data or in the absence of a reference data, it is difficult to know which model predicts better and we have so many options of turbulence model.

So, this course will hopefully help you in judging or picking up the right model for your flow problem. And there are, of course, other uncertainties that one must keep in mind. So, it is not just the model uncertainty, you can have the uncertainty due to the application of a boundary condition in a CFD problem or it could be the programming uncertainties, the way people code is different or it could be the uncertainties due to the numerical methods that are used, be it the finite difference or a finite volume or the order of numerical methods accuracy and so on. And here there is a snapshot of the same problem being simulated using two different solvers. One is an Ansys fluent, and the other one is openfoam.

It is an exact same flow problem, the same turbulence model is used and the same numerical methods are used, same boundary condition is applied. The only difference is these are two different flow solvers and you can see that it is completely predicting very different result and in the absence of experimental data we actually do not know which one is correct. So, this kind of uncertainties also play a role and this model accuracy comes at a cost that means essentially you are looking into the error that is coming from the model predictions. And I briefly touched upon these three class of models that is a direct numerical simulation or a large eddy simulation or a Reynolds-averaged Navier-Stokes simulation. And so the higher the computational cost that we are prepared to pay, we can get better and better results.

So it always comes at a computational cost. And this is just to highlight that if you want to capture more physics, that means if you want more accuracy, then you need to go into DNS. and of course, that comes at the highest computational cost and in many cases, it is not even feasible to do a DNS for a particular flow problem. So, in industry, usually, they go into what is called a RANS, which is a statistical approach, a numerical approach rather than physics-based or something in between which is the LES or hybrid RANS LES. And the top three techniques here, that is the DNS, LES, and hybrid RANS-LES, they belong to what is called eddy-resolving techniques.

And the RANS comes under the statistical approach. At the end of this course itself, you can say that what is the main takeaway. Or what do we know for sure at the end is that, as of today, the question we ask is, is there a universal model for turbulence? That is, is there a model that I can use for any type of problem, be it combustion, be it multi-phase flow, biological flows, or any such class of flows? And the answer is no. That is because each of these flows is different, and it demands custom modelling. And that is because there is no universal turbulence theory yet, right? So, the turbulent flows are just different types of flows and the way turbulence behaves in a reactive flow is very different compared to how it would behave in a two-phase flow.

Since the turbulence behaviour itself is different, it requires a different type of model. So, that is what we are going to learn in this course. So, we will look into turbulence theory before we start to model, and we will briefly look into some aspects of what we call turbulence first. So we all know about the turbulence problem, and then we do say turbulence itself is a problem. So why is that? And then, if we look around and see what are the statements made by some of the popular scientists, we see that the first statement is coming from Richard Feynman, where he says turbulence is the most important unsolved problem of classical physics.

This is coming from a physicist, and even today, the statement is true. It is still a problem that is not solved. And the second statement is, of course, this is debatable, whether Horace Lamb said this or not is still not clear, but it is still a very humorous statement. And as you see, he says, I am an old man now, and when I die and go to heaven, there are two matters on which I hope for enlightenment. One is quantum electrodynamics, and the other is the turbulent motion of fluids.

and he says about the former, that is, the quantum electrodynamics. He says he is rather optimistic, so that means he is kind of implying that even god would not have an answer to turbulent flows. And the third statement is from an experimentalist, Stanley Cossin, who's saying that experiment and theory are unforgiving. For a serious researcher, it's both fun and it's also a living. So, he's taking a more practical viewpoint. That is, since turbulent flows are a challenging problem since it is still not solved.

And therefore, there is a demand for those who know turbulence physics and those who know turbulence modelling. So it is giving a living, it is offering us jobs. So these are different viewpoints coming from physicists to experimentalists to mathematicians. And, of course, we have more such statements, one by Richardson. There is also a popular name, Richardson Cascade in Turbulence.

and he was a meteorologist and he says he is kind of giving you a kind of a qualitative view of what is turbulent flow right. So he is saying that big whorls have little whorls, that is, there are bigger eddies or vortices, which have little smaller vortices which feed on their velocity, and little whorls have lesser whorls and so on to viscosity. So, he is kind of giving a hint that turbulent flows are made up of bigger eddies and smaller eddies. So there are a lot of vortices of different scale. And finally, he's saying that at the end, the smaller and smaller vortices will go on, and then they will have a viscous action at that scale.

And then we have a statement by Peter Bradshaw, who was a computationalist and also an experimentalist. And he has a more pragmatic view of how we can solve this. He's giving a solution. So he's saying that when people regret that we do not understand turbulence, they are really regretting that we are not able to integrate, that is, numerically integrate the Navier-Stokes equations in our heads. When we say in our heads, what he means is that he just needs a big supercomputer.

And this statement is probably three decades old. And Cray was a big supercomputer in those times. That is what he says. We need a cray in a cranium. So he is saying that, he is implying that if we just have an extremely large supercomputer, then one could just solve Navier-Stokes numerically and we have all the solution.

This is true. The question is, do we have such a large supercomputer which can solve all the flow problems, right? That we will see later. and if you go on to look into this what is called a clay mathematics institute there are seven million dollar problems that means if you solve any of this mathematics problem you will get one million dollar and this is this these problems are open from more than a century now and one of the seven problems you will see is what is called Navier-Stokes equation. So, the question is, what is the problem? I mean, people are looking into Navier-Stokes equations and even try to get some analytical solutions of simplified form of Navier-Stokes equation or even, you know, numerically, we solve using CFD techniques. So the question that has to be answered to get a million dollar is that there is no proof for the most basic questions one can ask here. That is, do solutions exist for Navier-Stokes equation? So that means they are not asking for numerical solutions.

or experimental solutions, what they are asking for is a mathematical or an analytical solution to the full Navier-Stokes equations. That means which involves all the terms in the Navier-Stokes equations without assuming that or without neglecting any of the terms. You need to take the full Navier-Stokes equation. the unsteady, convective part, the diffusive part and also the all the other forces that may be there in the problem. So, and we have to answer whether it has a solution, whether it can give a solution and if they are unique that means the solution coming out is it unique or is it non-unique.

So, this is the open question to be solved. So now we can see the challenge or the complexity of turbulent flows. Because obviously, the turbulent flows relies on Navier-Stokes equations. Navier-Stokes equations knows about turbulence. And then there is a challenge that we don't know whether there is a unique or a non-unique solution to Navier-Stokes equation. Okay, this is a take on, for example, now we go into like, what is turbulent? So before that, I just want you to think which of this signal is turbulent, right? I have put two graphs here.

They all look turbulent, that means random oscillating types. So the idea is that like not everything that looks like turbulent should be classified as a turbulent flow. Any signal which is oscillatory, we cannot say that it is belonging to a turbulent flow, right? So what this is showing here is that one, the top graph, it is actually coming from a stock index. So this is showing you for many years what is the stock market oscillation of an index. The bottom graph is showing you the temperature and dew point data in a city for over a month period.

Obviously, both of them look turbulent, but only you can say the bottom graph is associated with turbulent flow. So now the question comes that since I said not all turbulent looking signals should be associated with turbulence, then what is turbulence? How do we define a turbulent flow? So these are the largely agreed characteristics of a turbulent flow. That is, the first one, people say it is irregular, it is random, it is chaotic or it is even a stochastic process. So, this is largely agreed. People, even in layman terms, people say turbulent flows are irregular, random.

No extra explanation is required for this point. The second one is. it is characterized by eddies. To an extent, this is also in layman terms people will agree because when they see a turbulent flow, they will see a lot of vertical motion occurring. So, they would say yes, it is characterized by eddies. But the challenge here is that here the scales are ranging from several kilometers or even hundreds or thousands of kilometers to a micrometer.

So, for example, if you see a storm, a cyclone, then you will see the diameter of a cyclone or a storm is probably 100 kilometers in diameter. And within that, there are many other smaller eddies and vortices. And the tiniest eddy is probably few tens of micrometers. So we have all scales of eddies that is inside sitting in a turbulent flow. And if you want to understand the entire storm, you must actually understand even the tiniest vortex that it is sitting inside.

they all communicate with each other, right? And the third point is that turbulence is diffusive. So, we know that even in the, from the Navier-Stokes point of view, we know that there are two types of transport. You have a convective transport and we have a diffusive transport. And we learned that diffusive transport occurs due to viscosity.

So here I am saying turbulence is also diffusive. That means turbulence can also do transport, transport of your mass, momentum, energy, and mixing. So when turbulence is present, it obviously dominates over the viscous transport in general. So turbulence is also giving diffusion. Another characteristic of turbulent flows is that it is dissipative. That means if you turn off the power or the energy supply to your flow system, then entire turbulence decays.

That means let us say if you imagine a wind tunnel and then the wind tunnel is up and running and then you see a turbulent flow developing downstream. And if you just switch it off, switch it off the blower or the fan, then slowly with time, the turbulence will decay. So, in the absence of energy supply, turbulence decays. So, turbulence is also that means it is dissipative. You need to keep supplying energy to the flow system to maintain, let us say, a certain momentum or any other feature that you want to look at.

Otherwise, turbulence is dissipative. It is taking away a certain momentum, let us say, from the system. Another interesting property of turbulence is anisotropy. That means turbulence by nature is not isotropic. So, turbulent fluctuations are directional dependent.

So, if you look at the velocity vector, so it has three components. And then that means these three components will have a different behavior. And that means the oscillation of even, let's say, the velocity vector or the vorticity or any such parameter that you are looking into is changing as you move from one direction to the other direction, right. It has a directional dependency. And the other interesting definition or characteristic of turbulent flow is it is a vortical flow.

Of course, turbulent flows also, they are also viscous flows. All real flows are viscous flows. So, turbulent flows are also viscous flows, but in addition to that it is actually a vortical flow. So, what I mean by vortical flow is that the turbulent flows must have a non-zero three-dimensional vorticity fluctuation. This particular statement is very interesting and important because Let's say if somebody gives you a flow data. They have made measurements or some simulations and they give it to you and tell you that can you identify is this flow laminar or turbulent.

And the way to find out is actually search for three-dimensional vorticity fluctuation. if that flow from the data if you can actually see that the three-dimensional vorticity fluctuation is non-zero then yes you can say it is definitely a turbulent flow right so you can actually see the signature of turbulence using a 3d vorticity fluctuation Okay, so now this so-called turbulence process or random process they occur in lab and nature that can also be simulated in computers that we will see later. Okay, so I am just going to do a small tabletop experiment that you can that you also you also can do it. So, here I have, you know, I have taken water in two beakers and then uh, nothing fancy. I just have uh the, you know the the ink dye or black ink dye that I have, which is commonly available in, you know, many shops, so all I'm doing now is that I'm going to take this uh the ink and then drop two blobs of ink into this.

So before I do this, you should think, let us say, instead of dropping two droplets of this black ink dye, if I am going to drop, let us say, a metal ball, let us say, a 5-millimetre metal ball into this, you would easily see that this metal ball would go into it. I mean, it will just go down and then it would not deform. So the metal ball does not deform as it goes inside. But as soon as, let us say, a 5-millimeter droplet is dropped, we would know that the fluid always deforms in contrast to a solid. So therefore, there is a kind of you can say when such a deformation is occurring you must ask a question that what kind of deformation would this fluid develop? So, for example one possibility is that as soon as I drop this black dye, it can lead to, let us say, it can all go straight like a spaghetti down and then showing no oscillations, and that means perhaps it is developing a laminar flow right or it can go completely random you know the it is all going in a random fashion indicating that it can be turbulent.

So let us see what happens if I drop this, and I am going to drop two droplets, one in, I mean one droplet in each, and we will observe how it develops in both of them, right? So, as you see, I have dropped two blobs of this ink, and it is going down, and it is not going in laminar or a spaghetti fashion, it is not going like a column, a straight column down, it is actually going in all kinds of direction. exhibiting strange structures. Another important thing to observe is that it is not showing a symmetrical behaviour from left and right. So you see that one developed in a different way and the other one is developing in a different way. And obviously, you see some similarities, the so called coherent structures we do see, but the surprising thing is, it did not develop like a, you know, the same fashion, right, both are turbulent, but they are not looking similar.

So, you can already see this random process or the stochasticity. So, you can clearly see that by making such a simple tabletop experiment, dropping a blob of dye, you can see that it actually developed differently. So, why did you do that? First, of course, it exhibited a random or a chaotic deformation of the blob as it descended down. And the second thing is, of course, that it did not develop in the same pattern as the other one. The pattern definitely looked different. So what caused this, even though I'm taking the same fluid and it's the same person who is dropping it? Obviously, there could be many scenarios or many causes for this.

One is that there could be a small difference in the size of the blob. After all, I am a human, and then when I drop it, I may be adding a little bit of pressure into it when I actually drop it, and there may be a small pressure difference into it, and it may have been dropped at a different height. I do not know. So, all these small small differences are there between the two cases. So, you can say there is there is probably a difference in the initial condition. Even a small different initial condition is probably resulting in a very different scenario.

So, and that is the challenge. So, even though it looks cool, it is a complicated problem to actually understand. And that means even if you repeat this as you see in the graph, like you can go ahead and do this, you get different patterns. Obviously, there are some similar patterns like for example if you see some of these you know structures, vertical structures that are developing, there are some similarities there in between the cases, but it is altogether developing very differently. It is exhibiting some randomness in its process and this I am talking about temporal evolution that is how turbulence is evolving in time not in a statistical viewpoint. And of course, one can also get this random process captured in a computer, too, because the Navier-Stokes equation knows about turbulence.

Which term knows about it? We will talk later. So here, these two simulations that we did where what we did is like we have taken the same flow case, same boundary conditions. Everything is same. The only difference I did is like we have taken two different initial conditions. When the initial conditions are different and then we do this numerical calculation, we do see that the flow is evolving into completely two different type of flow for the same Reynolds number and for the same problem setup. It is giving a non-unique solution here, right? So, this is the beauty and the challenge of turbulent flow. It is completely, it looks cool, but it has a lot of challenges as you can think of.