Design of Mechanical Transmission Systems

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Week – 12

Lecture – 34

Lecture 34_ Clutch: Dynamic Analysis Problem Solving

So, good morning. So, we will continue the lecture where we stopped in the last lecture aspect and this is about the clutch system design. We have been covered; these are the topics. Currently, we are doing with the dynamic analysis and in fact, we are doing the problem right. So, I am sure you would remember, we have done half way through. The torque is given 220 Nm and the power 138 kW and operates between 4400 rpm to 7000 rpm. These are the gear ratios, and final drive ratio. It is asking to find out the right for a 10 up and down shifting cycles of a clutch and it is a single disc clutch. The only information as a clutch design engineer we given the freedom for you to choose a bronze base sintered with the cast iron friction lining aspect. And, also the tire diameter is given.

In fact, in this table what we have done, we have chosen the pressure this is the bronze sintered. So, this is the given pressure and the temperature and the corresponding dry lubrication because this is a single disc plate right. So, it has to be dry if it is a multi-disc clutch in the sense you can have a oil lubricated. So, since it is a single disc dry would be good and we have chosen $\mu = 0.4$, you can choose depends on how do you want to and pressure we have chosen as a $1.7 * 10^6 N/m^2$. This is the value I have chosen and also based on this value we are able to find out the dimensions hope you remember dimension $d = 100mm$, inner diameter of the clutch and the outer will be $170mm$. So, this is what we have done that and we have tried to carried out the velocity table right to find out the velocity for the vehicle for different gear speeds that is where we stopped in the last lecture. So, we will continue.

In fact, I ask you a question what speed should we choose, should we choose the maximum speed 7000 rpm or 4400 rpm which one we have to choose that is what the question where I asked you. Did you find the answer for that? Any answer? Is it for the maximum power condition or maximum torque condition that was the question. So, you can go with the maximum torque condition aspect. You can have two approach one is this the torque aspect otherwise if you want understand the maximum possible heat generation in that case you can go for a maximum speed also maximum power condition, two way you can solve. So, I am going to solve with the maximum torque. So, $N = 4400\ RPM$, that is the maximum torque condition, it is an engine speed. So, $T_{Max} = 220\ Nm$. I am sure you know this equation, which we did similar to the gear box.

Gear	Gear Ratio (i_g)	Overall Speed (i_o)	Vehicle Speed ($V_i = \frac{\pi D_W N_e}{60 * i_o}$)
I	2.786	12.370	13.037
II	1.614	7.166	22.505
III	1.082	4.761	33.874
IV	0.773	3.432	46.990
V	0.566	2.513	64.174

So, this is the table for the clutch aspect ok. Now, ultimately see what exactly happening when the clutch is engage and disengage during the time the heat is generated. So, we need to find out what will be the maximum heat ok whether that is within the permissible limit from the table based on that we are going to do the validation of the clutch design ok. I just wanted to make sure that you would remember certain equations, which already I have explained at the beginning right three equations. Remember the moment of inertia is,

$$I_Z = K_M m \frac{v^2}{\omega^2}$$

K_M we can choose as a 1.2 for a taking 20 percent additional initial effect, K_M we choose as a 1.2 what is the mass value is a 15.7 kN. So, it is a 1570 kg. So, that is your mass. So, this is the equation we are going to use it. Then another equation which is talking about the torque what is the torque we need to find out the load torque right T_a or acceleration torque we need to find out. So it is,

$$T_a = T_E - T_L = \frac{I_Z(\omega_2 - \omega_1)}{t}$$

t is the slipping time. So, this is the acceleration torque we need to find out or else acceleration time both way is there. Then another equation, we need to find out slipping time t_s,

$$t_s = \frac{I_z(\omega_2 - \omega_1)}{T_E - T_L}$$

So, this is the equation we have. Similarly, heat generation is,

$$Q_s = \left(\frac{I_z * (\omega_2 - \omega_1)^2}{2}\right) * \left(\frac{T_E}{T_E - T_L}\right)$$

So, this is these are the equation. Already, I have explained this is for the heat generation per engagement. Based on this we are going to calculate for each gear what will be the heat generation that is what we are going to do that and then there is one more load torque we need to find out load torque T_L,

$$T_L = \mu_R m g * \left(\frac{V_i}{\omega}\right)$$

Where, μ_R equal to coefficient of rolling resistance. Generally, its value is not given it vary to 0.2 to 0.3. So, in our calculation we will choose μ_R equal to as 0.02. So, this is the value we are going to use it and try to solve the problem ok yeah.

So, now first thing I will explain how to find out the moment of inertia. I am going to give you only one step then we will go for a table the first thing for first gear and all I am going to do for the respect to the first gear aspect right. We have to find out the I_z, the moment of inertia $I_Z = 1.2m\frac{v_i^2}{\omega_e^2}$. So, this is the thing. So, engine speed is $\frac{2\pi N_e}{60} = \frac{2\pi 440}{60}$. That should give me 460.77 radian per second. So, this is the value we got it right this is the we got it and $I_z = 1.2 * 1570 * \frac{13.037^2}{460.77^2}$. What is the value you are getting? Are you getting 1.5375? Yeah 1.5375. So, that is your I_z for the first gear. everything we are doing with respect to the first gear. The second one we need to find out the load torque, load torque T_L is $= 0.02 * 15700 * (\frac{13.037}{460.77})$ So, tell me what is the value you are getting? Are you getting 8.885? 8.885. So, much Newton meter that is your load torque that is your load torque. Now, this fine. So, we got load torque. Now, next thing is we need to find out the time slipping time where when the engage right. So, how much time is taking to attain the relative speed equal to 0. So, the engine running at the one speed. So, the gear box run at one speed right you then you are disconnected it is disengaged and you want to engage. So, both have to have a relative speed 0. That means, what are the driven member right will attain the drive member speed that is what it meaning. So, we need to find out what will be the timing.

Again, I rewrite the equation for a better understanding right. It is $t_s = \frac{I_Z(\omega_2-\omega_1)}{T_E-T_L}$. So, this is the equation. So, in fact, I will make it very better thing. So, this is $t_s = \frac{I_Z(\omega_e-\omega_i)}{T_a}$, T_a is nothing, but your acceleration torque right. So, $t_s = 1.537 * \frac{(460.77-?)}{220-8.885}$. So, just want to give one more concept. So, I need to find out the secondary angular velocity with respect to the engine speed when the gear is operating at first speed assuming that. So, the equation will be normally the initial velocity the change in velocity whatever the corresponding then your engine speed this is the equation. The question is in the first gear I do not know what is the initial velocity right, we do not know that. So, obviously, the ω_2 will be 0. So, what you have to do you put 0 here and tell me what is the value you are getting? Are you getting 3.356, you should get 3.356 seconds. So, this is the slipping time would require in the first gear engagement are you getting the value 3.35 yes that is one thing is there. Now, this is done. So, we have done the time aspect we have we have got the load torque also and we know acceleration torque from the engine when you subtracting from the engine torque.

Finally, we need to find out how much heat generated with respect to the first gear right heat generated. So, $Q_S = \left(\frac{I_Z*(\omega_2-\omega_1)^2}{2}\right) * \left(\frac{T_E}{T_E-T_L}\right)$. It is equal to $Q_S = \left(\frac{1.5375*(460.77-0)^2}{2}\right) * \left(\frac{220}{220-8..85}\right)$. So, can you give me what is your heat generation it is happening for first gear engagement this is should be in joules. So, are we getting 170085 joules. This many joules you should expect in the first gear engagement aspect yeah are you getting this value right. So, this is the way you do. So, initially find out I_z and loading torque corresponding the speed then the time for slipping aspect. Then finally, you will have a heat generation for the first gear engagement. So, this is the way you need to do that. In fact, I will do with respect to the entire gears' aspect.

Gear	I_z (kg/m^2)	T_L (Nm)	$\omega_2 = \frac{V_i}{V_{i+1}}\omega_e$ (rad/sec)	t (sec)	Q (J)
I	1.5375	8.885	0	3.356	170082
II	4.5814	15.337	266.921	4.339	95529
III	10.380	23.085	306.123	8.152	138674
IV	19.973	32.024	332.158	13.665	100473
V	37.253	43.735	337.389	26.076	353903
ΣQ					858661

So, now, you have to find out for the second gear the second gear V_i will change, V_i should be 22.505. So, when you substitute, you will expect the values. $T_L = \mu_R mg * \left(\frac{V_i}{\omega_e}\right)$. So, the corresponding V_i substitute obviously, as the velocity increases your load also will increase right torque load will increase. Now, you know the two ratio the first will be what will be the first speed 13 point the first gear speed. Now, we are changing with the first gear to second gear we are changing from the first gear to second gear. So, from the table if you refer the V_i 13.037 right. Then, $V_i + 1$ would be what is that value 22.505. So, that is what you have to substitute with the engine speed then ω_2 you expect you will find out the ω_2 that should be 266.92. Similarly, if you find out do for your time 4.339 then you will get a heat generation 95.529. So, if you repeat the same format you have to keep changing the velocity of the vehicle velocity for the corresponding gear all right and find out the corresponding change in angular velocity with respect to previous gear. If you do that then finally, you are able to find out the slipping time as well as the load torque and the heat generation. So, I am going to give right away the values since I explained for the first two gears now it is easy for you to follow it up. So, if you do the summation of the total heat generation of all 5 gear shift right 2nd gear 3rd 4th and 5th right that should that should be 858661, so that much heat generated for one operating cycle is called one complete operation. So, usually when you gear engagement right when clutches engagement you will go from 1st gear to maximum gear you are not doing in between right. So, 1st, 2nd, 3rd, 4th, 5th and so on. So, that is the 1st cycle one up and if you want reduce the speed will you straight away go to 1st gear or how will you do in general you have to step down right. So, again 5th 4 3rd 2nd and 1. So, one operation of clutch will be one up cycle one down cycle. In fact, if you look at the problem clearly says for 10 cycles up and down, 10 cycles up and down. So, that means, 10 into 2 times, 10 into 2 times.

Now, we got the complete the table aspect. So, this is the total heat generated per engagement. So, this is the total heat generated 858661 J. But we want to know the total heat generation per hour, the total heat energy generated per hour this should be Q_ss right, we want Q_h, $Q_h = Q_h * No\ of\ cycles$. So, what is the power of the heat generated per hour? So, what is given in problem number of cycle you can take it as a *N* also may be this is $Q_h = 858661 * 10 * 2$. In the problem is given 10 cycles 10 cycles up and down 10 into 2. So, that is what is given. So, if you do that. So, would expect 17713.32 $kJ/hour$. So, what is the total heat generated per hour? So, this is the total heat generated per hour. But we want everything with respect to kilo watt. So, when we do that will be 4.47 kJ/sec, right or else we can have a 4.47 kW. So, this much heat is or power loss happening during the during the engagement this is what happening. So, the question is how are you going to verify it that is the question right now we got it. So, we got the total heat generated for the entire clutch for 1 hour. So, but we do not have dimensions right we do not have still other dimensions. So, that is where we need to understand what exactly is happening.

So, what we are going to do a little bit mathematical thing we know that this is our Q_h this is total heat generated this is total heat generated right. Then we need to this one verify with the heat transfer it is a right. So,

$$Q_h = \alpha A \Delta T$$

T is temperature rise; A is the surface area of the clutch α is the heat transfer coefficient. Surface area of clutch and the temperature rise during the operation. So, we need to find out α what is the α and find out the area then we will from there we will find out the what is the temperature is going to be during the heat generation. If you do that then the problem is solved then the problem is solved. So, just I want to give some guidelines. So, already we know what is the D what is the D 100 mm right clutch in a diameter this is outer diameter and what is the outer diameter 170 mm am I right yeah 170 mm. So, these two dimensions are known to us.

So, now, we will see the guidelines for clutch assembly. This *A* will come through your assembly dimensions α will do later. So, the pressure plate thickness usually you can take how much pressure plate thickness you have seen the dimension right generally pressure plate thickness can vary maximum of 15 mm. You have seen the thick plate right the pressure plate where diaphragm is attached that is will give 10 to 15 mm the pressure plate thickness and the clutch plate thickness clutch plate which is nothing, but your friction plate clutch plate thickness right you can have 10 to 12 mm and the cover thickness is again where the pressure plate and clutch plate all together that can be taken as a 10 mm then then based on that everything will change. Perhaps, I can draw small diagram. So, that you will able to see what exactly you want to do that right. So, assuming that you this is your friction plate right this is your friction plate then this entire thing will be attached with your pressure plate yeah you will have one more pressure plate. Just give you the identify this is your pressure plate then you will have a one assembly cover then you will have an assembly cover. In fact, I should do the proper way this is your friction lining right. So, this is your inner diameter and this is your outer diameter yeah that is given then you will have a pressure plate also, but pressure plate also will be a solid with the hollow thing then on top of that you will have a clutch plate. The clutch plate again there will be a permit there will be an opening hope you have seen diaphragm when you see the diaphragm thing right there is always this is this place is for the heat dissipation that is for dissipation whereas, the heat transfer is happening on this side and the circumference side. So, this surface and circumference surface because ultimately, we need to find out what will be total heat transfer surface area that is the way we are heading now.

Now the clutch covers the clutch cover what we can have outer dia right outer dia. So, whatever the you are having OD, OD of your clutch plate with that we have to give the clearance we will give the clearance both sides the OD of the clutch plus 15 plus 15 ok this is the allowance this is the allowance. So, this is for a clutch cover outer dia aspect. Then, I will move to another slide clutch cover surface area. So, this is the area of the clutch, the clutch cover surface area what is the circumference what I said the circumference clutch cover diameter right into total thickness clutch what else you have remembered I said like this this is a one is clutch plate this is a pressure plate then you will have a clutch cover like this. So, then we will have a clutch disc cover and then we will have a disc cover. So, this is the pressure plate and then we will have a clutch cover like this. So, then we will have a clutch disc area also that is also all together that will be total surface area that will be total surface area. So, can you find out what will be the clutch diameter now clutch cover outer diameter 170 right 170 is outer dia and we are giving allowance 15, 15 right, 15, 15 outer and inner that will give us a 200 mm that is clearly given 200 mm and how about the thickness what about thickness. So, your clutch you have to think about your friction plate thickness, pressure plate thickness on top of the allowance on top of the allowance clutch plate already you have chosen the thickness you clutch plate 15 mm you have chosen, clutch plate thickness may be 10 to 12 you can choose 10 mm also does not matter, then you have a pressure plate 15 mm then a cover thickness you have another 10 mm. So, how much you are going to get this aspect yeah, the thickness aspect how much you are going to get. See you need to find out the clutch cover dia this is what happening and the clutch disc. So, for me is much easier for I can get straight away $\frac{\pi}{4}(D^2 - d^2)$, that is the area I will get it the d would be right here is it is going to be the clutch aspect. So, D will be 200 mm and d will be 100 mm. So, that is fine, my only concern about the clutch cover diameter aspect clutch cover and total thickness. How much can we have thickness. So, this is $= \pi * 0.2 * 0.07 + \frac{\pi}{4}(D^2 - d^2)$, how much can we have thickness right. So, I am going to take totally, I will take it as a 70 mm total together I will explain may be later. So, let me finish this one. So, I will choose as a 0.07. So, can you tell me how much your surface area getting you should expect 0.06754 m^2. This is your surface area. Then, I need to find out the here as you see that this weight transfers the equation, I said $Q_h = \alpha A \Delta T$, I need to find out the α heat transfer coefficient, heat transfer variable coefficient that is fine this α we need to find out. Here the α is nothing, but there is a numerical equation, $\alpha = 12.8 + 2.9V$. So, straight away take the equation this is taken from the heat transfer aspect. So, we are not going to look at how the equation arrived that is not our point of view you just use the equation yeah. So, this is the equation then the V is the velocity because the clutch curve also is happening which nothing, but $V = \omega_e * R_c$, where R_c is the radius of clutch. So, engine speed we know that ω_e equal to 460.66 rad/sec that we know that R_c already we got 200 right 0.2 meter that is clearly given right from that if you find out the velocity right this is much easier, $\alpha = 12.8 + 2.9(460.66 * 0.2)$, if you do that I would expect the alpha would be 1202 $J/m^2/s/K$. Sorry, R_c is 0.1, then what is the value you are getting? It is 146.3 $J/m^2/s/K$ so, this is given right. So, now $Q_h = \alpha A \Delta T$, we need to find out ΔT, $\Delta T = Q_h/\alpha A$. Q_h already we know, surface area we know, what is the surface area, we got the surface area. So, $\Delta T = 4770/(146.3 * 0.06754)$. So, what is the ΔT, you are getting now so, ΔT is 482.7 $°C$. So, you can assume this ambient temperature right ambient temperature I can choose as a 35$°C$ right. So, then $T_{Max} - T_{oo} = 482.7\ °C$. Then, $T_{Max} = 482.7 + 35 = 617.7\ °C$. The question is so, what is the maximum temperature you are expecting if you use the asbestos sorry not asbestos sintered right sintered based cast iron friction lining what is the maximum temperature expecting. What is the maximum temperature look at the table we have chosen. So, what is the maximum temperature maybe can I go back to the table, what is the maximum temperature range you are getting from the table. So, maybe I will keep it right let me go again look at what is the temperature you are getting $232 - 677$ right. So, what is the maximum temperature you are getting here this is less than your allowable temperature right what is the allowable temperature that is what you see. Allowable is between 232 to 677. So, is it safe or not. Hence, the clutch design is safe. The question is, if the temperature exceeding what is the solution now if the temperature exceeding T_{Max} we got within the allowable limit that is fine if it is not within the allowable limit, it is exceeding then what is the next solution how will you approach either change the material that is one way to do that you can change the friction value. The second aspect how about changing the dimensions there is another approach right that is how we can do that yeah; I think I will stop now.