**Design of Mechanical Transmission Systems**

**Prof. Ramkumar**

**Department of Mechanical Engineering**

**Indian Institute of Technology Madras**

**Week – 12**

**Lecture - 33**

Lecture 33_Clutch: Dynamic Analysis

Good morning. So we will continue with the clutch design aspects. We have been covered these topics and also in last lectures, we have discussed about the centrifugal clutches aspect and today we are going to learn about the dynamic analysis for clutch. Before going to dynamic analysis just wanted to give a small a brief discussion the difference between clutch as well as brake. The brake system is very complicated right it is very complicated because it is involved with the various aspects it is not only about the vehicle dimension right the vehicle dimension, the vehicle weight, the speed right and also there are three different coefficient of friction are involved when you talk about three different coefficient of friction one is your friction lining, that your friction the other one is the road surface road adhesion coefficient of friction the third one is your rolling friction all those things are involved. Of course the application of the brake is to stop the vehicle right we stop the vehicle in different condition the condition would be single stop condition or repeated braking condition as well as the drag or continued braking condition so you would have seen that right again as the size of the vehicle increases the brake thing also is increased okay however if you come compare clutch with the brake clutch is very simple right and it won't undergo as similar to brake aspect the reason is the purpose of a clutch is just to engage and disengage during the start of the vehicle or changing the speed or what to stop the vehicle okay these are the three condition you are using the clutch system okay so in that case would you expect heat generation

That is the question right you would expect this generation the generation would be more in brake system whereas heat generation will be there in the clutch only this is a rotating your assuming that your basic cover plate from the engine side the other one is the friction lining or friction disc is going to engage so both are different speed so both are different speed and try to make a contact during the contact the brief time right so they will rub each other right, when they rub each other the heat is generated other than that is not much the the dynamic aspect only is difference would be as you change the speed of the vehicle right the rotation of the driven member that will change right so what I want to say that it's it is during the engagement of the particular gear the heat generation will depends right okay so we will go for the dynamic analysis here I am going to give setaway equations and the concept I'm not going to derive the equations this is very basic simple physics based equations okay so dynamic analysis of clutch the $T_E$ is the engine torque or dynamic torque that's our source then you have a acceleration torque $T_a$ right acceleration torque $T_L$ is the load torque $T_I$ is the idle torque during the idle time that's also there will be one torque then there is a $T_S$ is the static torque okay so you have a five different torques engine torque, acceleration torque, load torque, as well as ideal as and static torque okay

I will give explanation about the what is the engine torque load torque and so on $T_E$ is coming from the engine or else we can say the dynamic torque so this is the torque, this torque is the engagement torque is the effective torque on the shaft while the clutch slipping, clutch slipping okay in the sense the clutch making contact right so that is the effective torque during acceleration or deceleration up to the point of synchronization between the driving and driven members.

So that's your engine torque as well as or else a dynamic torque okay the load torque $T_L$ okay which acts on the output shaft or output side I would say okay of the clutch as the result of load that's your load torque now, the acceleration torque $T_a$ the acceleration torque accelerates the given masses right from $n_1$ speed to $n_2$ speed within the given time okay within the given time, so we have one member is rotating from the engine so another member rotating right I am talking about the friction plate or disc plate right that will be rotating so when they try to contact each other right when the tortoise again the load depends on the the speed of the driven member from the clutch right when the making engagement so then we will have acceleration torque okay so normally the acceleration torque right equal to $T_a$ what do you expect we have the torque coming from the engine right there is a one torque because of the rotator one speed similarly would you expect the clutch also will rotate or will be zero velocity it depends on the what gear the vehicle is move is it first gear or second gear or third gear based on that there will be a certain velocity so that's also will have a torque right

So when the joint together right then there will be a net torque there will be a net torque which is making as a acceleration torque okay so the $T_a$ nothing but your difference between the engine torque and load torque normally you can rewrite because everything happening due to the rotation of the masses so everything we want to convert with respect to the torque aspect so

$$T_a = T_E - T_L = \frac{I_z(\omega_2 - \omega_1)}{t}$$

okay so where I is it I'll explain t is the acceleration time or slipping time, $I_z$ is it okay dynamic moment of inertia so normal initial moment of inertia is different dynamic in the sense it's rotating right it is rotating yeah $N_1$ is the speed before acceleration whereas $N_2$ speed after acceleration okay so that's the fine so just we wanted to understand about the dynamic of the moment of inertia remember moment of inertia you know for an area moment of inertia know so you have the things okay like to find out the mass thing right you mass into a radius of gyration that is what is given but here the vehicle is moving in a straight line whereas you are rotating component when you talk about rotating components, we are talking about the clutch things the clutch things so that has to be taken care of right because of when rotating so you will have a initial effect also that has to be considered in fact remember in the brake we derived one term called $K_m$ right when the vehicle is moving right you have to take the initial effect of the rotating masses, when you say initial effect of rotating masses we have taken care of the not only the wheel the gearbox and the clutch right that's what we have discussed remember similar way now, we are focusing only on the clutch aspect right clutch aspect that has to be done, the same thing we are giving to you in a different aspect so assuming that you have a clutch here, this is the just clutch so there is a input this is coming from the engine the shaft then it attached to the one gear okay assuming this is your gear so this is gear one then we have a another gear two this is your shaft okay just understand just schematically purposely you have one more gear and this is a shaft so this is gear two and this is a gear three and this is the input shaft you can say that maybe I will say that is input shaft say that I made $S_{i/p}$ okay and this is your counter shaft and I will take it as a this is the $K_i$ okay the total aspect the $K_i$, if I want to find out the moment of inertia of the gear one right I want how do I do so I have to take the consideration of the shaft right the shaft, clutch, and gearbox all three together right I have to do that so that would be the

$$I_1 = J_{K_i} + J_{Z_1} + J_{S_{i/p}}$$

$$I_2 = J_{c_s} + J_{Z_2} + J_{z_3}$$

so this is what we have to do that yeah so the moment of inertia $I_2$, we need to find out that is the moment of inertia $I_2$ is reduced to clutch input shaft where that rotating $\omega_1$ speed yeah, so if I want to find out $I_2$ to reduced with the initial speed of the clutch

$$I_{2\,reduced\,\omega_1} = I_2\left(\frac{\omega_2}{\omega_1}\right)^2$$

it should be what, proportional to the velocity right yeah that is square of the velocity in fact the square speed ratio therefore the total $I_z$ to be accelerated right by the clutch on $\omega_1$ speed is given

$$I_z = I_1 + I_{2\,reduced\,\omega_1} \ \ kgm^2$$

so if you simplify

$$I_z = K_m m \left(\frac{V}{\omega}\right)^2 \quad kgm^2$$

Where,

m equal to mass of the body moving in straight line

V is the velocity of the mass moving in a straight line

ω is the angular speed of the shaft

the $I_z$ what happened is equivalent to $k_m$ right I just put this is the m and this is smart okay so now understand that everything they are rotating different different speeds right the vehicle is moving in a translation direction right I had to accommodate both so how do I do that here v square ω square so I have taken everything, I have taken to care of the your translation as well as rotational things right so by this is by converting the effective mass moving in a straight line to the moment of inertia right the $I_z$ would be equal to $k_m$ into m into v square by ω square where yeah so now ultimately we need to find out the slipping time so my question is so you want to engage the clutch by changing the speed what is the typical duration would you expect how much time for slipping aspect the moment I say is slipping aspect, the clutch making release and make a contact achieve the same relative speed that is the purpose, so what do you expect the typical slipping time for a clutch is it in seconds a few seconds or minutes what do you expect in seconds only so in seconds yeah so the slipping time $t_s$ right the time during the friction faces of the clutch more relate to another right under contact pressure so that defined as

$$t_s = \frac{I_z(\omega_2 - \omega_1)}{T_E - T_L} = \frac{I_z(\omega_i - \omega_{i+1})}{T_E - T_L}$$

so this is the thing right so we got $I_z$ converting into translational rotation that's got and we understood about the the the slipping time right so when the slipping time happened what do you expect the heat generation will be there right again heat generation depends upon the the initial velocity and the final velocity and the $I_z$ so the heat is generated by individual or repeated clutch engagement clutch engagement operation okay can be calculated

$$Q_s = \frac{I_z(\omega_E - \omega_{i+1})^2}{2} \times loadfactor\ per\ engagement$$

with respect to load factor this is everything happened per engagement we are talking about per engagement yeah so the load factor nothing but how much is torque net torque is transferred during the engagement time so that is nothing but

$$Q_s = \frac{I_z(\omega_E - \omega_{i+1})^2}{2} \times \frac{T_E}{T_E - T_L}$$

this is nothing but your load factor right.

So in typically in clutch the heat is calculated the number of upward cycle and downward cycles for design aspect when I say upward cycle and downward cycles so you will use the clutch for engagement first gear right that's one, second gear, third gear, and fourth gear, so that is entire one thing is upward cycle then again downward means so fourth gear, third gear, and second gear, and so on so that's how the heat is accumulated right that's why it is accumulated so then the total heat generated total energy rate per shifting cycle this is nothing but $Q_s$ that's already done so we want to know that the total heat energy generated per hour okay

$$Q_{TH} = Q_s * N(J/hr)$$

$Q_{TH}$ equal to the Qs into number of cycles so much joule per hour where N is the number of shift cycles or engagement both are same engagement per hour so this is the your thermal equation $Q_s$ equal to the how much is it is going to be generated okay just a physics nothing else is a simple equations right now, we are going to do a problem okay what is that we are going to verify or find out in the clutch dynamic analysis okay

We have designed the clutch using uniform wear theory assuming that you have designed the clutch uniform wear theory how do you know that is safe how will you identify how will you prove the clutch is safe under the design condition, what are the possible way to to showcase the clutch is safe any idea, one is stress aspect that is purely design how about in a thermal aspect, you have to make sure that the temperature whatever the temperature or heat is accumulated during the operation that should not be exceed from the design value that's a one way where you can verify the the design aspect for clutch in fact we'll do a problem then we will understand what exactly is happening please take down the problem

An automotive vehicle develops a maximum torque 220 Nm at 4,400 rpm and the corresponding power 138 kW at 7,000 rpm the vehicle weight is given 57.7 kN the gear ratios are 2.786 1.614 1.082 and 0.773 and 0.566 okay fine the final time ratio is 0.44 is to 1 assuming that the vehicle is traveling in motorway needs at least 10 up and down shifting cycles of the above nature per hour per hour a single disc clutch lined with the bronze base centered with the Cast iron here under this dry sliding condition is provided take the tire diameter is 700 mm check whether the design is satisfactory or not if not please give your comment okay

So nothing is given right you the vehicle maximum torque is given and the maximum power is given and the gear ratio is given and we need to design a clutch for a 10 up and down cycles per hour that's what is given okay of course you have a tire diameter 700 mm is given, one design input is given what is that? The friction lining material right that is given you can choose a right bronze base sintered cast iron that is the only one information is given other than that nothing, no other  information yeah so how are we going to do this that's a question

Okay you can see so these are the friction lining material coefficient of friction in fact this already I have shared in the beginning of the lecture so these are the four possible so what material is given for our problem bronze sintered metal this is what is given right this is given so this is a dynamic coefficient of friction is given 0.1 to 0.4 the corresponding pressure is given 1030 to 2070 kPa and the maximum operating temperature between 232°C to 677°C, this is given okay so this is the only input we have so using this input we are going to design a clutch and also check the validation all right so it's going to be a very elaborate problem maybe need a three lectures to complete this okay so now we know that this is the material friction material we are going to choose yeah so the maximum pressure is clearly given okay as a design engineer you have freedom how much pressure you wanted to have  how much coefficient of friction you can take so what we could do we can choose our the from here

We can go for a maximum the friction is 0.4 so that's one thing is there and the pressure you can choose either way okay whatever the pressure so here we are going to choose the maximum the $p_a$ allowable pressure 1.7 ten power six $N/mm^2$ so these are two inputs we have taken for a bronze sintered a metal cast iron friction lining aspect okay other than that we don't have any other  information do we know right okay so how are we going to proceed remember there is one more relation if you want to have a maximum torque what is the relation between the inner diameter and outer diameter that relationship we can use it okay are we going to  use the uniform pressure theory or uniform wear theory, unifoem wear theory okay so use uniform wear theory for clutch design and also with the maximum torque condition when you say maximum condition with

$$\frac{d}{D} = 0.57 \cong 0.6 \rightarrow d = 0.6D$$

d by D right supposed to be 0.7 or approximately we take it as a 0.6 so that's a clearly given yeah right so using a uniform wear theory

$$p_{max} d = pD$$

I think this is what we derived for uniform wear theory aspect and we know that D is given from here d equal to 0.6 times of your D, D so this is your outer dia and this d is your inner dia of a clutch distance okay so now we need to find out already we know that $p_{max}$ what is the $p_{max}$ 1.7 right did we choose 1.7 or 1.4, 1.7 only right yeah

$$1.7 * 0.6D = pD \rightarrow p = 1.02\ N/mm^2$$

and the P we need to find out so we will have a okay so instead of d we can substitute 0.6 D and P into D so this will cancel out can you tell me what will be the pressure you would expect here what is the pressure you will expect how much 1.02 okay so 1.02 so much $N/mm^2$ so this is what you are getting yeah now we need to find out the total activating load acting on the clutch

$$P = \frac{\pi p_a d}{2}(D-d) = \frac{\pi * 1.7 * d}{2}\left(\frac{d}{0.6} - d\right) = 1.78d^2$$

P equal to so this is that we know this equation right pi into this is 1.7 right so this is D by 2 so we substitute here instead of D we can have a okay point six small d can you tell then what is the value you are getting what is the value with the respect in d square in terms of d square okay and we know torque equal to

$$T = \mu P \frac{(D+d)}{4} = 0.4 * 1.78d^2 * \frac{(1.6d)}{4}$$

but already we know torque equal how much torque is given from the problem what is the torque is given in the problem the torque is given as a 220 Nm right so just equal to 220 Newton meter so tell me what will be the the inner diameter and outer diameter

$$220 = 0.4 * 1.78d^2 * \frac{(1.6d)}{4} \rightarrow d = 91.75\ mm$$

$$d = 91.75\ mm \rightarrow D = 1.6 * 91.75 = 153\ mm \approx 170\ mm$$

$$d = 91.75\ mm \approx 100\ mm$$

that's what you need to do that here what is the value of getting sorry I have taken the different value that's why I am not able to give the answer right I have taken P max equal to 1.4 for that I am doing the problem 91.75 okay 91.75 okay mm and this D would be around 160 something like that right so maybe we will consider d equal to approximately 100 mm so will d equal to approximately 100 mm and D would be 170 mm I am just rounding off D how much value okay so 153 so so much so I am just make it to near about 170 because that's what I did here so I will choose as a 170 mm and 160 mm so do that so this is dimension wise we are talking about this diameter for the friction lining remember please whatever the diameter is for the friction lining dimensions aspect okay now what we need to do, we need to do the velocity table similar to gearbox similar to gearbox because that is going to be useful for the our heat calculation the reason why I am saying that ultimately your $I_z$ equal to right the km into m V by Omega right this is the V of Omega so the V is nothing but your vehicle velocity so that vehicle velocity will change for each gear right so in then in that case we need to have the velocity table also the velocity table so the gear we know that the gear the gear ratio, overall ratio, right finally your vehicle velocity so this one

| Gear | Gear ratio | Overall ratio($i_o$)=$i_g$*$i_f$ | $V_i=\frac{\pi DN}{60*i_o}$ |
|---|---|---|---|
| 1 | 2.786 | 12.370 | |
| 2 | 1.614 | 7.166 | |
| 3 | 1.082 | 4.761 | |
| 4 | 0.773 | 3.432 | |
| 5 | 0.566 | 2.513 | |
| Reverse | 2 | 8.88 | |

what is the overall ratio $I_o$ so overall ratio that's what you need to know that then from there we need to find out the V yeah so what is the final drive ratio in the problem given final drive ratio $I_f$ is given, we are not this is only the forward speed there is a reverse speed also right there is a reverse speed is two in the problem so for that this is given you eight point eight eight now finally we need to find out the vehicle velocity is nothing but pi d n by sixty right into the Io the overall ratio right question what is the n value here are you going to use the engine speed right that's what we did right they are using the engine speed so what is the engine speed you are going to use it it's a 4400 rpm or 7000 rpm right question is will use that is the question that's a question remember in the gearbox we have taken the maximum speed right okay so in clutch what is the speed we are going to use it are we going to use the 4400 rpm or 7000 rpm okay I just wanted to give you one more input information this is for the speed at $T_{max}$ right this is for maximum torque whereas this is for a maximum power 7000 rpm which one which one okay so just think about that so I will stop now that we will continue in next class okay