Design of Mechanical Transmission Systems

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Week – 11

Lecture - 30

Lecture 30_Clutch: Torque Transmitting Capacity – Uniform Pressure & Wear Theories

Good morning. So, we will continue with the clutch design. Yesterday we have discussed the basic things clutch types and also the working method of clutch. And today we are going to learn about the torque transmitting capacity, how much torque when clutch is engaged transmitting during the engagement. Within that we will discuss about uniform pressure theory using that what would be the torque and also uniform wear theory. As I said the difference between brake and clutch are similar they are not very different ok particularly with respect to clutch within the brake I am talking about disc brake.

In the disc brake you have a annular pad either it will be annular pad or circular pad through caliper attachment. The same thing if you extend it as a full size right that become your clutch that is the difference. So, the functional advantage also same the purpose is to hold in clutch aspect whereas, to stop in brake aspect that is a difference yeah. So, now we will try to find out how the task torque going to be transmitted during the engagement ok.

So, I have just taken the simple single plate clutch this is single plate. The single plate clutch where you can see this is the inner diameter and this is your outer diameter in between you have a friction lining. So, this entire thing is friction lining is exaggerated, but for our understanding we made it as a this is your surface contact area or as a friction area entire thing on the pad ok. And this is the side view you could see that and then you would have seen the video and also I have shown the model you have diaphragm right when you engage they will give you a force right, the force will give you to make a contact. So, through the pressure plate you will have reaction forces right the as a pressure along the this entire area the pressure is developed it has to be balanced ok this is the one.

So, we are going to take a small strip as you see the the the complete this is the small strip with the dr thickness from the radius r ok from the centre and we will see what exactly is going to be happen. And where your P is your force small p is the pressure intensity ok acting along the friction lining surface. And if you look at so, we need to understand how the torque will be transferred before that I try to get the basic equation for the force aspect. So, in the elemental area the force normally would be the intensity pressure with the circumference ok.

$$Elemental\ axial\ force = p(2\pi rdr)$$

$$Elemental\ friction\ force = 2\pi\mu(prdr)$$

So, you can see p into 2Πrdr that is elemental axial force same thing if it is a friction force you have to multiply with your coefficient of friction.

So, μ is your coefficient of friction lining yeah. So, that is the one will is helping to transfer the torque through frictional arrangement. Now, the frictional torque would be obviously, the frictional force into distance which is r smaller ok.

$$Elemental\ friction\ torque = 2\pi\mu(pr^2dr)$$

So, and in this we assuming that the coefficient of friction is constant it is not variable with the temperature and Π is a constant only with the variable would be pr^2 and dr element things ok. Now, we want to see the basic equation for the load aspect between the limits

$$The\ total\ load, \qquad P = 2\pi \int_{\frac{d}{2}}^{\frac{D}{2}} prdr$$

similar way for torque basic equation right frictional torque will be

$$The\ total\ friction\ torque, \qquad T = 2\pi\mu \int_{\frac{d}{2}}^{\frac{D}{2}} pr^2dr$$

2Π into μ the same thing except you have a one more r that is it ok this two equation. This is the basic equation before going to uniform pressure theory and uniform wear theory aspect ok yeah. Just note down these two equations please for the total load acting on the clutch and the corresponding torque also yeah right. The descent parameter as I said a capital D is the outer diameter of the friction disc, d is the inner diameter of the friction disc, p is the intensity of pressure at radius r, and P is a total operating force, T is the torque transmitted by the clutch and μ is a coefficient of friction of frictional lining material ok. Now, we will discuss as we did in a disc brake. So, there are two theories used in clutch one is uniform pressure theory other one is uniform wear theory.

So, when the clutch is brand new ok. So, when they made contact right the contact is uniform when you have a contact uniform contact the pressure will be uniform ok that is why it is called uniform pressure theory. You may ask why we have to follow two different theories right I will explain as we proceed in the lecture. And when a new clutch is employing a number of springs the pressure remain constant over the entire surface area of the friction force. So, ok then what is happening to the torque things ok.

So, that is what we are going to do that. So, we need to find out the load right. So, the basic equation we know that load

$$P = 2\pi \int_{\frac{d}{2}}^{\frac{D}{2}} prdr$$

$$P = 2\pi p \left(\frac{r^2}{2}\right)_{\frac{d}{2}}^{\frac{D}{2}}$$

So, then we substitute limit right. I will straight away go for the equation

$$P = \frac{\Pi p}{4}[D^2 - d^2] \qquad (1)$$

then P would be the acting load. So, this is the P is the load please and this p is the pressure intensity yeah please understand that this is the equation for the acting load maybe we can have equation number 1. Now, we need to find out for the torque also ok. Torque equation we know right torque equation

$$The\ total\ friction\ torque, \qquad T = 2\pi\mu \int_{\frac{d}{2}}^{\frac{D}{2}} pr^2dr$$

So, when you again integrate and substitute the limits finally, I am going to give an answer the torque would be

$$T = \frac{\pi\mu p}{12}(D^3 - d^3) \qquad (2)$$

Πμp by 12 ok. So, we will have a D cube and d cube. So, this is the equation torque right maybe we can take it as equation 2, but already we know that the P equal to what from equation 1 the activation force P equal to this is the equation right.

$$P = \frac{\Pi p}{4}[D^2 - d^2] \qquad (1)$$

So, you can substitute just I want to replace the pressure intensity right and try to bring everything with respect to load ok. So, when you do that finally, you will expect your torque equation

$$T = \frac{\mu P}{3} \frac{(D^3 - d^3)}{(D^2 - d^2)} = \mu P R_f \quad (3)$$

So, this is the equation we are expecting. So, in fact, the same we did the same equation we arrived at when we did for the disc brake system right. So, μ right P is the load as the entire thing I am talking about this entire thing we taken as a R_f, R_f is nothing, but frictional radius ok. This is R_f equal to frictional radius which is nothing,

$$R_f = frictional\ radius = \frac{(D^3 - d^3)}{3(D^2 - d^2)}$$

So, this is the value. So, this would be vary again for uniform wear theory also try to understand that ok.

So, this is for the uniform pressure theory already we got equation. Now, uniform wear theory. So, this is for the uniform pressure theory ok. So, what do you understand uniform wear theory why there is a uniform wear theory concept in clutch thing ok.

The same concept you have a we take the d is a inner diameter and D is the outer diameter right this entire portion would be frictional lining force right the surface area. So, the wear is uniform when the pressure is not uniform this is usually applied to worn out surface ok worn out surfaces ok on the clutches. When do you expect the uniform wear that is the question. So, when the initially when the clutch is brand new. So, the surface is very smooth and nice. So, when they make contact the pressure is uniform. So, over the time period. So, they have to stick and again they will rub each other because the frictional thing right they want to hold it. So, because of that there will be a wear is happening. So, when the wear is happening.

So, some places we have a wear other places we may not have it. So, what happen to the contact? The contact would not be uniform ok. So, when the contact would not be uniform the pressure cannot be constant. So, the wear is taking place right that is why and again this will continue until the surface is wearing out through the the frictional surfaces to make it as a uniform wear theory we will go one by one. And the same similar to what we did uniform pressure the same way you can do that only difference is what is the difference here what do you expect the pressure aspect? There uniform pressure theory the pressure is uniform, but here it cannot be ok the pressure would be more at the inner radius.

Obviously, this smaller radius pressure will be things. So, you will have nice parabolic you can see that. So, this is a pressure pressure intensity yeah. So, at the inner you will have a more pressure the outer you will have a less pressure. So, that is the thing and generally we say as a p_a maximum allowable pressure intensity ok.

p_a is the probably you can put this is p_a is the maximum allowable pressure intensity. So, already we have basic equation from there we are going to proceed before that I try to understand ok. So, wear is proposed to the friction force have you heard about arched equation anybody arched equation right. So, the higher the velocity there right the wearing of the more will be right the more the force again the wear will be more right. So, obviously, the wear is proportional to your friction force and the rubbing velocity right.

$$Wear \propto (\mu p)(2\pi r n)$$

where, n is speed in rpm

So, this is your friction force this is your velocity rubbing velocity usually we say sliding velocity V equal to rω right. So, you can have a r 2 Π n, n you can take it as a rpm or rps either way it is fine. So, we are take it as a rpm then when the coefficient of friction speeds are constant what happened to your wear my friction is constant ok and speed are constant. So, what do you expect the wear will be wear will be constant right there were because of no variables the p is constant your μ also constant. So, there will be proportional wear is proportional to the μr, but this is when the wear is uniform this is this is what happened, but in actual contact the pressure variation will be there right when the pressure variation is there. So, the pressure intensity is maximum at the inner diameter. So, obviously, what do you expect this

$$pr = p_a\left(\frac{d}{2}\right)$$

pr would be proportional to your p_a into d by 2 at the inner radius yeah at inner radius ok. Please understand this difference between the uniform wear theory and pressure theory. Now, we will do the as usual the load aspect the only difference is p_a will come into the picture

$$P = 2\pi \int_{\frac{d}{2}}^{\frac{D}{2}} pr dr = 2\pi \int_{\frac{d}{2}}^{\frac{D}{2}} p_a\left(\frac{d}{2}\right) dr$$

So, ok. So, once substitute and do the integration finally, I am going to give the equation straight away your activation force will be

$$P = \frac{\pi p_a d}{2}(D - d) \quad (4)$$

So, this is the equation may be you can take it as a equation number 4. Similarly for torque for torque

$$T = 2\pi\mu \int_{\frac{d}{2}}^{\frac{D}{2}} pr^2 dr = 2\pi\mu \int_{\frac{d}{2}}^{\frac{D}{2}} \left(p_a \times \frac{d}{2}\right) rdr$$

T equal to 2μΠ this is your standard equation right yes. So, substitute the this here 2μΠ is the constant whereas, the variable would be p_a , I am just replacing p right p_a into d by 2 then you have a r square right yeah I think no I should be r only not r d r. So, when you substitute you would expect the equation

$$T = \frac{\pi\mu p_a d}{8}(D^2 - d^2) \quad (5)$$

So, already we know equation number 4 from there if you take it right and substitute for the P a aspect you would expect the equation torque equal to

$$T = \frac{\mu P}{4}(D + d) \quad (6)$$

So, this is the equation expect equation number 6 ok. If you standardize

$$T = \mu P R_f$$

Where, $R_f = \frac{(D+d)}{4}$

ok remember there in uniform pressure theory you have

$$R_f = frictional\ radius = \frac{(D^3 - d^3)}{3(D^2 - d^2)}$$

So, that is the difference frictional radius will be different yeah. So, I think again this is same equation what I have written is coming here yeah that is ok right. So, now, I am going to give a concept little bit concept since the various proposed to the radius where at the outer radius will be a more right pr right. So, obviously, where will be more. So, the edge will be the where will release the pressure at the outer radius this will continue until you remove this the wear material. So, what happen because of that again the contact will be uneven right the pressure will shift to the inner radius right, the inner radius will start to varying out.

So, no further wear at outer edge, but wear will now take place at inner edge due to the contact pressure of the plate. So, this will release the pressure and stop further wear at the inner edge this is what going to happen. So, this will continue until you will have a uniform wear through the surface a contact surfaces that is why this is called as a uniform wear theory this process of wear mechanism continues alternatively at inner and outer edges until pr is a constant resulting uniform wear theory at any given radius. So, this is what happen ok. In fact, what are the factors will affect the torque capacity.

So, decreasing the diameter reduce the torque capacity that is one aspect, the second is a smaller diameter clutch as a smaller diaphragm which will give lower clamping force. So, when you have a clamping force again you cannot do the desired function ok that is a another disadvantage if you have a smaller. So, adding a second clutch disc double the torque capacity obviously, your number of rubbing surfaces will be increased right. So, same concept is applicable multi disc clutch aspect, increasing the coefficient of friction obviously, of the clutch disc increase the torque capacity. If the multi disc clutch is correctly designed and engineered the losses from the smaller diameter and lower clamp force are much less than the gain in coefficient of friction and extra disc giving the torque increases understand please ok.

So, better if you have a higher coefficient of friction and more disc or that means, more friction surfaces that will contribute increasing your torque capacity right. So, now, I am going to give a little bit scenario. So, this is the general equation right where you have this is already we know that and this is for the uniform wear theory both I have given. Now, I am giving smaller values ok. So, just give you capital D 140 and 80 and try to find out what will be R_f using uniform pressure theory and this is uniform wear theory and this is 56.36 and 55 and 200 outer dia 100 is the inner diameter my R_f 77.78 and 75 for the uniform wear theory. So, on if you do for 200 and 180, I am reducing the gap also 95.09 under uniform pressure theory 95 for under uniform wear theory ok. What is your observation what you see do you find any difference between uniform wear theory and uniform pressure theory with respect to friction radius or effective radius this is a small increase for uniform pressure theory ok.

The small incremental things ok how about torque which one you would expect more torque is it under uniform pressure theory or uniform wear theory that is a question uniform pressure theory will give you maximum torque ok. So, obviously, uniform wear theory will have a lesser torque ok. So, as a decision engineer which is the safest one is it uniform pressure theory or uniform wear theory. So, look at uniform pressure theory as a maximum torque while uniform wear theory have a less torque ok. So, already we fix the maximum torque for when the brand new as progressing due to where the torque is reduced when you have a smaller torque will be able to perform the desired function that is the question.

Is it possible may require torque is assumed at 50 Nm, but the that is the that is the design torque operating torque whereas, generating in the contact is a 45 Nm. So, in that case do you able to do you expect the clutch would perform its duty yes or no, no it is not possible ok. So, what is that way? So, as a decision engineer we always design a clutch based on the uniform wear theory not the uniform pressure theory that is why that is how that is how we do that ok. And one more thing is so now, we have seen by varying the inner diameter and outer diameter ok, but there is a relation when you want to have a good or maximum torque transfer.

So, there is a way to do that. So, normally better to follow the $\frac{D}{d}$ ratio between 1.2 to 2 times if you do that. So, you are able to achieve in fact, a precisely the $\frac{D}{d}$ ratio would come for maximum torque is 1.77, but you can vary 1.5 to 2 right. So, in fact, we are going to use this concept while designing a clutch ok we are going to do that. So, one more thing is so this is what I was when the friction lining new uniform pressure is applicable wear is proposal to the radius of clutch ok. The life of friction lining comes under uniform wear theory right based on the friction lining only other than that it nothing can be done. So, obviously, for designing clutches in terms of safe and logical aspect uniform wear theory is a more reliable than the uniform pressure theory. So, one more information I would like give is about the rated aspect ok.When the design torque is given by

$$T_d = K_s T_r$$

T_d by K_s and T_r where K_s is service factor or friction margin factor it accounts for any slippage during the transmission and T_r is the rated torque to be transmitted and T_d is the torque capacity of the design purpose ok. So, always better to in the operation normally this is what you are going to have it ok better to multiply certain factors such way that you can have a safe torque. So, that is the practice we have to follow during the clutch design ok. Now, we will do a problem please yeah please take down the problem.

An automotive plate clutch consists of two pair of contacting surfaces with asbestos friction lining. The maximum engine torque is 250 Nm and the coefficient of friction is 0.35. The inner and outer diameters of friction lining are 175 and 250 mm respectively. The clamping force is provided by the 9 springs each compressed by 5 mm to give a force of 800 N which is the P ok when the clutch is a brand new. So, what should we do? What is the factor of safety with the respect to slippage when the clutch is a brand new that is a question number (a). (b) The same thing what is the factor of safety with the respect to slippage after initial wear is occurred which is a for uniform wear theory aspect. Then in (c) how much of wear friction wear of friction lining can take place before the clutch will slip ok. What do you mean by understand slip? What do you mean by slip? What is the meaning of clutch first of all? Little English meaning clutch means hold together right clutch means to hold to that is a thing.

Slip means you cannot hold it is opposite of that. So, the moment remember in the video we have mentioned that. So, during the engagement the diaphragm will give you maximum of the clearance is 1mm right that is what is given the gap between during the engagement disengagement between the friction lining and your pressure plate gap is 1 mm ok. If friction lining is wearing out more what happen to your clamping force right will decrease ok. When the decrease is though the stiffness is constant because of the force decreases you cannot extend, already the some portion is worn out.

So, would you expect make a contact? No, then you will have slip. So, at what condition you are going to have a slip that is what the clear given. So, how much need to be wear out and yesterday I have shown the model also right what is the typical thickness of the friction lining did you observe? Is it in few mm 5 mm or 6 or 7 that is it not more than that. So, very small thickness, but of course, it will be there on the both sides yeah ok.

So, now, we will solve the problem ok. Remember it is a two pair of contact surface not the one there are two contact surfaces given which gives 250 Nm yeah. So, torque is given 250 Nm ok. What is the coefficient of friction? μ equal to 0.35 yeah and the outer diameter 250 mm or as 0.25 m you can say that inner diameter ok 175 mm same thing 0.175 m ok. Number of springs are 9 and the force is given P equal to 800 N. So, this is given ok. So, we want to the two pair right this is the two pair. So, torque capacity for single pair, what is the value you expect? 250 by 2 right because the two pair is given.

Torque for single pair $T = \frac{250}{2} = 125\ Nm$

So, this should be 125 Nm. So, that is clear ok. So, now, we need to understand the total spring force clamping force right total clamping force P. So, what is the value you are getting right should be number of springs right right number of springs into force that is the total force will be 9 into 800.

Total force $P = 9 * 800 = 7200N$

So, that should give me 7200 N this will be the total acting force or clamping force acting on the clutch this is the N ok yeah. So, now, so we have got all the information the case (a) is what will be the factor of safety when the clutch is brand new right that is our question. When the clutch is new so when the clutch is new what will be torque equation remember

$$T = \frac{\mu P}{3} \frac{(D^3 - d^3)}{(D^2 - d^2)}$$

$$T = \frac{0.35 * 7200}{3} \frac{(0.25^3 - 0.175^3)}{(0.25^2 - 0.175^2)}$$

$$T = 270.529\, Nm$$

this is the value ok. The question is they are asking factor of safety what is the definition of factor of safety designed torque and allowable torque right designed torque maximum allowable torque right.

$$FOS = \frac{\max allowable\ torque}{Design\ torque} = \frac{270.529}{125} = 2.16$$

So, what is so we have 125 Nm that is your design it are supposed to be right. So, 125 is designed and maximum is 270.529. So, so what is the factor of safety you are expecting now 2.16. So, that is a factor of safety when clutch is a brand new ok when clutch is brand new. Now, I will do for the when the clutch is old right or worn out. So, what is the torque

$$T = \frac{\mu P}{4}(D + d)$$

$$T = \frac{0.35 * 7200}{4}(0.2 + 0.175) = 265.75\, Nm$$

So, what is the value you are getting this should be lesser more than the will less right will be less yeah 267.75 Nm. So, this is your torque right it is a maximum allowable torque maximum allowable torque you are getting. So, the factor of safety is T_{max} by T_{design} right yeah. So, this is your torque maximum allowable torque.

$$FOS = \frac{T_{max}}{T_{design}} = \frac{265.75}{125} = 2.14$$

So, 265.75 divided by 125 right. So, let 2.14, 2.16 for the new clutch whereas, the factor of safety is reduced if the clutch is worn out right worn not worn out worn ok. So, we got two things. So, what is the case number (c)what is the problem number when at what thickness is worn out. So, that clutch cannot perform that means, where clutch will start slip that is what the question right ok. So, how are you going to do that one yeah right how will you do that, what is the way to do that yeah tell me please already I gave information even though it is wearing out right the stiffness would be remains constant right clamping force will be reduced ok.

So, where do you expect clamping force is it under uniform pressure theory or in uniform wear theory that is the question obviously, it is uniform wear theory that is where your the wear is happening ok. So, when the wear is happening we know that yeah. So, right the torque we know that torque

$$T = \frac{\mu P}{4}(D + d)$$

equal to µP uniform wear theory by right that this is the equation yeah. So, what is the value we should as expect the descent value 125 Nm right that is the value we suppose to expect ok because of uniform wear theory. So, which value would you expect to change your force the force will change right the clamping force will change.

$$125 = \frac{0.35 * P}{4}(0.25 + 0.175)$$

$$P = 3361.34\, N$$

can I have answer what will be the force you expecting this is the total clamping force the total clamping force P should be 3361.34 N. So, this is the clamping force. So, for one spring per spring ok. So, this is the clamping force.

So, for one spring $P\ per\ spring = \frac{3361.34}{9} = 373.48\, N$

So, this is the value ok. So, the moment the force goes below 373.48 Newton that is at that is where the clutch will slip when the force per spring right per spring will be less than 374, I am just rounding off as a 374 N right the clutch will slip. So, this is the clamping force.

So, already we got this what force right, but when. So, when do you expect the force will be reduced at what. So, how much need to be wear out from the friction lining that is what we need to find out. Do we know the friction lining thickness we do not know we do not know the friction lining thickness ok, but we know how much is δ your deflection that is clearly given right δ is given yeah. So, what we could do we will we will make we will assume certain things right. Let x equal to wear of friction lining, this much thickness has to be reduced ok.

Compression of the spring = initial compression – x = 5-x

how much the wearing out that is what we need to know because we do not know how much is going to be wearing out right that is what we need to know. So, what is the initial compression is given 5 mm clearly given right initial compression is given 5 mm. So, from 5 mm certain amount of has to be wear out right then only the slip will occur ok.

Compression of the spring = initial compression – x = 5-x

So, this is what happening. So, spring stiffness will be when that force what we got 374 N right

Now spring stiffness = $\frac{374}{(5-x)}\,Nmm$ (1) ok this is the spring stiffness. So, much Nm assuming that this we can have a equation number 1 ok right. So, initially 5 mm is compressed for how much spring force 800 N right ok right. So, my question is do you think the stiffness of the spring will change, never it will always constant 90 percent always constant ok yeah I am sure you are familiar with the spring aspect I have heard about open length, shut length, workable length all those things they there. So, how all force is there the stiffness will be there only the deflection will change it depends on the how much effort you are giving that is what is happening here.

Initially my spring is compressed 5 mm for 800 N right that is what happening, but my force further reduced when the further reduced with to 374 N it reduced right. So, when it is I have reduced obviously what happen my deflection will be reduced how much is reduced we are going to find out, but both operation my stiffness remain constant it would not change yeah. So, we are going to equate both yeah for initial condition initial condition means we talking about as a new right. So, this spring stiffness 800 by 5 right so much Nmm ok. For initial condition,

$$Spring\ stiffness = \frac{800}{5} \quad (2)$$

So, this equation number at 2. So, equate both right equate 1 and 2 this stiffness should be remain constant when you do that can you tell me what is the answer you are getting

$$\frac{374}{5-x} = \frac{800}{5} \rightarrow x = 2.66\,mm$$

Yes yeah 2.66 so much mm. So, right so what is the what you understand now right we do not have thickness of the friction lining ok, but this this much of the thickness need to be worn out when this much of thickness is worn out right due to drop in the pressure right. So, 374 N right that is the place where the clutch cannot perform anymore until then it will perform ok. So, when the wear of friction lining right is more than it should go cross beyond 2.67 ok.

So, this is the way it should go cross beyond 2.66 mm the spring force will be less than 374 N ok. So, at this condition at this stage the clutch will slip. So, this is the spring force and that is it you understand now. So, why is so now, do you know that why uniform pressure, uniform wear theory is very important right you have to achieve that they say torque also right I think I will stop now. So, then next class I will discuss couple of other type of clutch then I will go then we will discuss about the dynamic analysis ok. Thank you.