Design of Mechanical Transmission Systems

Prof. Ramkumar

Department of Mechanical Engineering

Indian Institute of Technology Madras

Week – 01

Lecture – 03

Machine Tool Gearbox: Ray Diagram Construction

So, very good morning to all. So, we will continue machine tool gearbox aspect particularly the ray diagram aspect. So, this another diagram you can see the ray diagram. So, by looking at it one is the initial constructions given this is the initial construction ray diagram the other one is a complete diagram. If you look at from the ray diagram what information we can get the first thing is the structural formula. If, you look at very carefully. So, we have from here from the observation this is your input speed right it goes to the up to this position. Then after that this you will have a this is the beginning of a first stage from there the speed split into two then you have two outputs in this point. Then you have three speeds you can see that three speeds you have output speed to the second stage from there again you go to two outputs speeds in third stage. So, maybe I will rewrite here this is the first stage, second stage and third stage.

So, usually when you do the ray diagram, we always focus only about the gears arrangement we are not considering about the input aspect. So, we can ignore this aspect. So, from here if you look at, I can straight away write two output speed two of one then three output speed in the second stage. So, three of two inputs from first stage then finally, right. So, we have two outputs two should be six this is the structural formula you can see that yeah.

So, that is observation you can make it out. So, and also as I said earlier it has to follow the optimization aspect particularly the i_{Max}, and $i_{Minimum}$ aspect. So, for except first stage all other stage should have the ratio gear ratio

$$i_{Max} = \frac{N_{Max}}{N_{Input}} \leq 2$$

speed that is one thing and another thing is gear ratio for minimum gear ratio

$$i_{Min} = \frac{N_{Min}}{N_{Input}} > \frac{1}{4}$$

These two-thumb rule should follow apart from always your $N_{Max} \geq N_{Input} > N_{Min}$. So, we have to follow these three steps.

Thumb rules during construction of ray diagram this is applicable all the stages except first stage except first stage it would not be applied except first stage yeah that is what happened and also, I need to give one more information and if you look at as ever of the each the vertical position indicate about the step ratio in logarithmic format $\log \phi$ right and in fact, if you look at this vertical thing its indicate about the shaft. So, in fact this should be an input shaft and this

should be your output shaft in between you have an intermediate shaft. So, in a three-stage gear box you will have four shafts because of it has to maintain the center distance based right one stage means at least two shafts should be there second stage three shafts third stage four shafts that is the arrangement. Interestingly you can see that the speeds are arranged in a step ratio manner and also you are aware of step ratios nothing, but the how much you want increment or decrement speed from one stage to another stage right irrespective of the arrangement that has to be maintained and clearly if you look at the horizontal lines ϕ is given. So, 45 to take the ratio between 45 to 31.5 rpm should be step ratio of 5. So, that has to be maintained not only that even if the lines are connected as a parallel look at this I have just I will make you one example. So, if this position and this position. So, this you say 31.5 speed this is a minimum speed and if you draw the line and parallelly,

So, what is your observation do you think these adjacent speeds should or the maintaining step ratio right. So, and not only that and if you look at between 355 and 250 and we aware of that 355/250 as a step ratio, but if they connected parallelly still the step ratio is maintained. So, the clearly you can understand whether the lines horizontal wise vertical wise or inclined wise if they are parallel the step ratio will remain same. So, that is how the ray diagram is constructed in fact by doing the problem we will understand even more better.

So, shafts are shown by vertical equidistance and parallel lines the speeds are plotted vertical on a log scale with the $\log 5$ as a unit the transmission engaged at different speeds of driving and driven shaft are shown on the diagram by rays connecting the points and the shaft lines representing the speed that is how the ray diagram is constructed. So, let us move on to the problem we will solve one problem it is an interesting problem. 12 speed gear box the speeds available at the spindles are given 31.5, 45, 63, 90, 125, 180, 250, 355, 500 and so on a maximum of 1410 rpm. The preferred structural formula is given 2(1) 3(2) 2(6) using the ray diagram. Determine the specific value of all transmission ratio and the speeds of all the shafts in drive. Also construct a kinematic diagram and indicating the number of speeds number of sorry the also construct kinematic diagram and indicating the number of teeth on each gear.

The gear box receives 5 kW from an electric motor running at 1500 rpm that is the problem. So, in this problem what we have to do we have to construct a ray diagram that is the first part of the problem later we need to base on the ray diagram we need to construct kinematic diagram that is the second part. The third part is we need to identify the number of teeth of each gear in the gear box right the three stages we are going to solve. The first will focus about construction of ray diagram right and prior to that I need to give information again for your better understanding. Remember I gave three ratios right the three thumb rules

$$\frac{N_{Max}}{N_{Input}} \leq 2 \text{ and one more } \frac{N_{Min}}{N_{Input}} > \frac{1}{4}$$

$N_{Max} \geq N_{Input} > N_{Min}$, this needs to be followed all stages except the first stage ok.

That clearly says in the first stage in the first stage in the first stage ok already you know the ray diagram. So, I am just going to tell you how to do that ok this is my smaller ray smaller portion I am taking assuming that this is input speed it is a input speed N_{Input} ok yeah this is the input speed. So, the first stage you can have gear reduction in this manner ok and another possibility again you take this is your input and speed N_{Input} then you can have reduction of the same speed from input to output fine and also you can do the arrangement in this manner

ok. Then there is a third option is your input speed right this is input speed and what you could do you can have a same speed of your input speed, but you do not want to have a higher speed the purpose of the gear box itself is speed reduction right. At the beginning itself you do not want to have this type of a combination this is possible you can do that this also, but not this one that is was the reason we have clearly mentioned these one thumb rule 2 3 are applicable all the stages except first stage ok.

And for other stages so the combination can be many ways I just giving you a few important so this is your input speed and these are the output speed so you can do this way another way also you can have same speed as input speed for the output and also even higher than the input speed. So, these things possible in other stages, but not applicable to the first stage aspect yeah. So now we will take the problem so we can see that the instruction clearly given the data from the data we are able to understand that is a 12-speed gear box and the preferred structural formula is given 2(1) 3(2) 2(6) ok and also the we have 31.5 rpm is the minimum speed and 1410 rpm as the maximum speed let us solve the problem. So, data given the data given is a 12-speed gear box ok then the structural formula it is already they are given so we are not going to use the optimum one structural formula given 2(1) 3(2) 2(6).

So, this is your third stage this is your second stage and this is your first stage these stages already is given N_{Min} from the information what we got is 31.5 rpm and N_{Max} is 1410 rpm and the motor speed is given and the motor speed is 1500 rpm. So, tell us what is the step ratio step ratio we will take the from the ray diagram not only from the ray diagram from the given problem maximum speed is 1410 the next adjustment speed is given as a 1000 rpm if you look at this is the step ratio comes as a 1.41 do you think the step ratio is standard one it is not standard one ok and if you refer the preferred series aspect right this falls,

$$R_{10} \times R_{20} = 1.26 \times 1.12 = 1.41$$

ok you would expect 1.41 let us say for R10 is a please refer the table 1.26 step ratio for R20 obviously as a series number increasing a step ratio it decreases that you know that 1.12 if you multiply that you would expect 1.41. So, this gear box is the combination of 2 series R10 and R20 ok. Now we will solve these given the speeds based on the speed for constructing the ray diagram aspect ok.

So as you aware of that this is the ray diagram it has 12 speeds clearly given this is your minimum speed 31.5 rpm and this is your maximum speed 1410 rpm ok that is clearly given and also, we have to rewrite the step ratio structure formula 2(1) 3(2) 2(6). So, the important thing is when you consider ray diagram it is good practice always start with the minimum speed if we do that then it is easier to go up to the other stages from third to second to first stage if you start with the first stage what happen it will be cumbersome it will be lot of overlapping and unnecessary time consume also. So, to avoid that right what we should do that we will always start with that the lowest speed is given in the gear box start with the lowest speed then move forward to the first stage if you do that it will be much easier ok.

Now we will take the ray diagram ok and look at we have to follow the same the thumb rules the ratio gear ratio the maximum gear ratio and the minimum gear ratio this 2 we have to follow ok.

$$i_{Max} = \frac{N_{Max}}{N_{Input}} \leq 2, \; i_{Min} = \frac{N_{Min}}{N_{Input}} \geq \frac{1}{4}$$

And from here what we could see ok the right already we have the minimum speed 31.5 rpm clearly given and 1400 is a maximum speed is given. So, what we need to do from there we need to construct the ray diagram aspect ok. So, if you look at from the given the structural formula, we keep using multiple times 2(1) 3(2) 2(6).

So, this is the one we are going to focus now this is third stage. So, what is the indicate 2(6), 2(6) indicate you have right 6 output speed is not it 6 output speed and from the formula you have a 2-input speed. ok that is fine right for 1 input speed how many outputs speed you would expect for 1 input speed you should get the 2-output speed I am not saying which speed, but let us say 1 output speed sorry 1 input speed should give should have 2 output speed based on the structural formula ok. So, if you follow that. So, I fix it this is the one point right this is the one point what will be the next output.

So, now, we are fixing the output we are moving from the output of the last stage to the input of the same stage then move on ok I fix the lowest one ok what is the next output speed it will be remember the 2 of 6 right. So, another information what you get the output speed of this 1 given stage should be 6 indicate 6 steps away 6 steps away that means, if you look at 45, 63, 90, 125, 80 and 150, 250. So, we will have 6 speeds away from the lowest speed ok. So, if I look at the 250 will be the thing these are the 2 points ok clearly, I have mentioned this formula is given 2(6). So, 1 output should place right.

So, when you have 1 input you will get 2 output the 2 outputs should be place apart 6 speeds that is another information you are getting from the structural formula ok that you can see this is 1 point this is another point ok that indicate right that indicate. So, I have my minimum speed is N_{Min}, N_{Min} 31.5 rpm. N_{Max} it indicates 250 rpm. So, I need to find out what would be the input speed for this third stage right. So, what are the possible you would expect the input speed what are the possible ways ok. So, when you talk about what are the possible ways. So, we can do the trial-and-error method because we do not know we have we do not know. So, we will start with the first trial we start with the first trial the trial 1 trial 1 I will choose my N_{Input} speed N_{Input} let use N_{Input} as a 180-rpm ok.

I will choose N_{Input} as a 180 rpm. So, can we solve it right can we solve can we solve it please yeah. So, follow the $\frac{N_{Max}}{N_{Min}}$ what is the max 250 your input will be 180. So, that should give how much I should get 1.38 which is less than 2 right which is less than 2 that is correct fine. Now, if I do then now, I had to go for the minimum aspect is not it yeah. So, when you do the minimum aspect 31.5 is equal to 31.5 divided by 180 is gives me 0.175 which is less than 1 less than $\frac{1}{4}$, $\frac{1}{4}$ which is not correct right. So, that is not satisfying that is not satisfying ok. So, what should we do we need to do the trial 2 we need to go for the trial 2. So, when you do we need to do the trial 2 perhaps I will use the same space ok. Trial 2 we did trial 1 we did trial 1 is not matching now let us use what is speed we are going to use it 125 rpm can you choose 125 rpm.

So, N_{Input}125 rpm again we have to check this i_{Max} and i_{Min}. So, when you want a i_{Max} right. So, I have 250/125 which give me 2 is less than or equal to 2 which is correct i_{Min} the ratio 31.5/125. So, when you have 31.5 and 125, I should get greater than or equal to one-fourth I am getting 0.252 which is greater than one-fourth. So, can we choose 125 as an input speed ok we can choose yeah. So, from here so the input speed of third stage N_{Input} of third stage is 125

rpm 125 rpm that is fixed yeah. So, in that case we can mark one point here right. So, that is done that is done ok. Yes. So, you can see that we have marked 31.5 and 250 rpm and 125 rpm ok fine. So, this is enough now what is the structural formula again 2(1) 3(2) 2(6) ok. Now so we since we are starting from the last stage move on to the first stage let us find out what ok. So, now we know that, but prior to the second stage before going to second stage my third stage should have how many speeds tell me how many speeds, I should have an input speed. 6 input speed right that is given a 6-input speed ok fine. What is the difference between the second stage and the third stage yeah right.

So, I need to have in the second in the second this is my third stage this is my third stage this is my second stage and this is the first stage ok yeah it is ok yeah. So, how do I move on now how do I move on right remember. So, there is an overlap even though this is third stage right third stage input will be output of your second stage right. So, if you look at 3(2) what is the indicate 3(2) what is the indicate you have a 2-input speed right you need to have a 2-input speed and 3 output speeds. That means, I need to have a by end of the second stage I need to have 3 output speed is not it 3 outputs.

So, again the one more information I would like to say 3(2) right this is your input and this is your output. So, another information what I said for given one input right the output will have 3 and that should place with the difference of 2 step ratio the difference of 2 step ratio. So, tell me what is 2 step ratios. So, this is my lowest one. So, can I is it possible for me to move down now because already I fixed N_{Min} 31.5 of the gear box. So, I cannot only way is I have to move up. So, then if I follow these 2 steps away the next speed would be like this 250 rpm. the other one would be 500 rpm. So, now we will go for a stage 2 aspect this is what happen now you know that right.

So, now we when you go for stage 2 what information do you have from stage 2 stage 2 again 2(1) 3(2) 2(6) right, we will choose this one stage 2. So, from this right I can only move up I cannot go down because I already fixed the lowest speed or the minimum speed. Tell me what is the minimum speed from ray diagram I am getting for second stage the minimum speed and the maximum speed the minimum should be 125 rpm right the minimum is 125 rpm and the maximum is 500 rpm. So, we have to repeat the procedure which we did it for the third stage to find out what is the appropriate input speed for the given N_{Min}, N_{Max} in the second stage aspect.

So, we have to find out input speed for this ok. So, again we can the variations given starting from 125 rpm to 500 rpm can be used as an input speed for stage 2. So, we will do similar to the trial 1 trial 2 like that what we did. So, can we choose 355 rpm let us choose let use N_{Input} as a 355 rpm I will start from the top.

Now, I need to find out this is trial 1 trial 1. So,

$$i_{Max} = \frac{N_{Max}}{N_{Input}}$$

what is N max here 500 rpm by 355 right So, that should give me 1.4 which is less than 2 then your gear ratio minimum gear ratio,

$$i_{Min} = \frac{N_{Min}}{N_{Input}}.$$

So, minimum is 125 rpm right minimum is 125 rpm and this should be 355. So, we got 355. So, it is giving you a 0.35 which is greater than one fourth. So, I think both are satisfying am I right both are satisfying, but we will do one more trial to understand. So, let choose a trial 2 N_{Input} I will choose as a 500 rpm I can go down also, but let me choose 500 rpm. So, I will choose as a 500 again i_{Max} i_{Max} it is a right i_{Max} 500 by 500 right 500 by 500 that should give me 1 less than or equal to 2 fine i_{Min} will be 125 and 500 that give me precisely 1.4 of given 1.4 both are ok. Now, we have two scenarios in trial 1 we got i_{Max} 1.4 ratio which is less than 2 correct and i_{Min} is 0.35 which is greater than 1.4 correct that is also fine. In trial 2 for given speed 500 rpm thei_{Max} gear ratio is given 1 which is perfectly alright is a less than 2, but i_{Min} right i_{Min} is same exactly one fourth of your things.

So, if you look at closely this is almost at the extreme end 1 fourth, we said 1 fourth greater than 1 fourth, but exactly 1 fourth, but we have option which is slightly more than the 1.4. So, in design you want to use extreme condition you want to be move away from the extreme condition for safer aspect you have to move away from the extreme condition right it is in the border line that is risky right still satisfied. So, design as the multiple solution please understand design you do not get one particular solution you will get multiple solution based on the what is your lead that is a fundamental aspect.

So, we can choose right both ways 0.357 or 0.25 also, but would be nice to have 355 rpm that will be little more even safer when the option is given better to choose. So, I would choose the next input rpm of stage 2 as 355 rpm rather than the 500 rpm. So, this would be your 350 rpm as an input to stage 2 right. So, 355 given. So, again as given the structural formula right how many inputs how many inputs I need for stage 2 and right and how many outputs I need for stage 1 how many outputs I need to need for stage 1 2 right 2. ok at the same time the spacing between two output speed will be one step as a one-step if you look at that is what happened that is what happened ok. So, now, we know that so now, we are move on to the stage 1 2(1) 3(2) 2(6). So, this is one we are focusing now. So, from this if you observe carefully your minimum speed what is that minimum 355 rpm maximum 500 rpm ok. The question is do we need to follow i_{Max} and i_{Min} for the first stage no need do we need to follow you know need because that is valid for the other than the first stage ok.

So, in that case so straight away because it is a speed reduction remember the motor input speed was given 1500 rpm right. So, 1500 rpm. So, we know that speed is given the, but we are only focusing about the what we are constructing that is a thing. So, from here we can either choose either 710 or 1000 for that is does not matter.

So, I choose as a 1000 as an input speed for stage 1. So, now, what we have done we made the basic skeleton this is called skeleton this is skeleton of ray diagram with that we are going to fill the other speeds also. How are we going to fill the other speeds because we know the step ratio as long as you follow these parallel lines connecting one speed to another speed right that will fill automatically the ray diagram you can see that. So, I am naming A, B, C, D, E, F, G and H. So, this is the skeleton of ray diagram right.

Now, from we do how to construct it. So, that is your input speed split into two output speed remember two output one input that is the first stage. Now, second stage right second stage three of two right three of two that means for given one input speed I should have a three output speeds. So, are you getting three output speeds. So, we are. So, far we did a connection now we are connecting them through the rays that is the reason it is called as a ray diagram.

Now, what is that next one 2(6) that means for one input speed I should have a two output speeds it is done now right. So, now, still remaining is there what are the remaining is there this has to be fill and this has to be filled right once this fill then you have another one to branch out this will branch out as a three-output speed your point G will branch out three output speed each output speed again branch out to two output speed in stage three yeah look at this. So, I am doing a parallel line because I am following the step ratio.

So, between. So, look at this is 250 rpm this is 31.5 fine. So, but the output has to be 63 to stage three again it should have a six steps difference you can see that from 63 to 500, 90, 125, 180, 250, 355, 500 which is six step ratios away similarly for E also 125 is my output speed. So, this are the next speed would be six steps away 180, 250, 355, 500, 710 and 1000 So, now, we in fact, we did it now we will move on to the G also I am putting into different colors to differentiate from G I should expect three output speed again as to maintain the step ratio one right look at they are having parallel lines this parallel line and this is the parallel line. Similarly, it extends it all are will follow the structural formula without any right confusion. Now, can you see that can you see did we feel completely the ray diagram did we arrived all the 12 speeds look at 31.5, 45, 63, 90, 125, 180, 250, and 355, 500, 710, 1000 and 1410. So, we have filled completely 12 speeds distributed in a proper structural formula not only that and also using optimum the i_{Max} the gear ratio maximum gear ratio minimum gear ratio and the $N_{Max} \geq N_{Input} > N_{Min}$ with that we are able to construct the ray diagram. I think I will stop now the next class we will discuss about using this ray diagram how are we find out construct a kinematic diagram which talk about the gear arrangement. So, these are the things we are going to discuss we will continue the problem in the next class also. Thank you.