Design of Mechanical Transmission Systems

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Week – 08

Lecture – 23

Lecture 23_ Brake: Braking Efficiency & Distance and Brake Factor

Good morning to all. So, we will continue our design of brake system. Today learning outcome would be, we are going to discuss about braking efficiency and braking distance or stopping distance and also, we are going to learn about brake factor. So, these are the two topics we are going to cover or discover okay and understand in today's lecture.

So, braking efficiency is the ratio between your maximum deceleration rate to the road adhesion coefficient. Yeah, so the breaking efficiency you can have η_B equal to the maximum deceleration rate depends on is a front axle or rear axle and with respect to road adhesion coefficient μ_S at ground and tire interface. So, that is how it happens.

$$\eta_B = \frac{\left(\frac{a}{g}\right)_{Max}}{\mu_S}$$

So, what do you expect okay. When your deceleration rate right is matching with your road adhesion coefficient. What do you expect? The brake efficiency will be 100 right. The braking efficiency would be 100. That's way where you will get your minimum stopping distance S is a minimum stopping distance. You may ask question, so what happen my deceleration rate $(a/g)_{Max}$ is less than μ_S, What do you expect. The braking efficiency should be less than 100 percent right. So, what will be the consequence? Your deceleration is not matching the road adhesion coefficient. So, what happen? It will struggle to reach the stopping distance. That means you will have a more distance, the stopping distance would be more. Stopping distance is not minimum will be more okay. So, another question is, do you think is it possible to have braking efficiency more than 100? Is it possible, yes or no, no right. You can meet, you can meet. Remember, so you can able to meet exactly 100 percent, not beyond that. Beyond that what happen right. So, Assuming, it is your reaction force. Then, this is your breaking force right. This is what happening and this is moving in this direction assuming that and of course, you have all the other forces, I am not going to show right this is rolling. So, when you say self-locking, the self-locking supposed to be equal to

$$P_B = \mu_S N$$

This is what happen right. This is the way. This is the condition you will expect the self-locking. Then, the wheel start keeps rotating right when the moment reach it will stop. What happen further. It should drag, it should start skid or should slide. When do you expect sliding or skidding. This should be more right, your braking force should be more. Then only, that can be achieved, the skidding or sliding can be achieved okay. So, please understand this is about the braking efficiency. Now, very important parameters okay whatever the dimensions, whatever the engine power torque or size of the braking, your brake system either it could be disc brake or drum brake or combination of both, does not matter we have to maintain the minimum stopping distance. That is very critical, okay.

Now, we will discuss about the braking distance aspect. So, for a braking distance or stopping distance okay, this depends on two things right. Your deceleration rate, the stopping distance in differential equation form. What do you expect? The deceleration ds right. The deceleration again depends on your breaking effort $P_B + \Sigma R$, all resistances. Remember this we discussed earlier. Then, when vehicle moves right, when a vehicle moves so you have energy, kinetic energy is there but again there are a certain component. I am talking about certain components your clutch, gears, universal gears, all those rotating will have initial effect also right. That has to be taken care of that okay. The translation masses which give another energy that is also you have to take care of that is $K_m(W/g)$. So, this is what happen.

$$ads = \left(\frac{P_b + \Sigma R}{\frac{K_m W}{g}}\right) ds = vdv$$

dv is your equivalent to your acceleration aspect. So, the K_m is the correction factor for rotating masses in the vehicle. Usually, it varies from 1.03 to 1.05. We will discuss more when we go for a thermal analysis. In the thermal analysis, this term will come into the picture okay. Then, determine the stopping distance. To determine from initial velocity. What is the initial velocity v_1 to a final velocity v_2 right. So, when you want to do that, this would be. Just I am rearranging,

$$ds = \int_{v_1}^{v_2} \frac{K_m W}{g} * \frac{vdv}{P_b + \Sigma R}$$

Maybe, we can say this is equation number 1 okay. What is R. It is a tractive resistance right. Hope you remember $\mu_R W \cos\theta$, that is one term. Another term would be plus or minus your gradient $W \sin\theta$, okay. The aerodynamics R_a and transmission resistance also there, all these things are ΣR, total resistance when the vehicle is move right.

$$\Sigma R = \mu_R W \cos\theta \pm W \sin\theta + R_a + R_t$$

R_t is too small. So, we can neglect it. That is absolutely fine right. Now, I want to understand the R_a aspect. This R_a is an aerodynamic resistance or air resistance right. What is R_a? It is

$$R_a = \frac{1}{2}\rho C_D A_f v^2$$

This is the equation. I am sure you would know that right. Where ρ is the air density, C_D is the drag coefficient and, A_f is frontal area of the vehicle okay. I will give a problem with this aspect, vehicle frontal area aspect. When you change the vehicle frontal area, the R_a will change. Then, further how R_a would affect your braking. So, I will give one problem in an assignment. Of course, v is the vehicle velocity right. So, among this term the density is constant, the drag coefficient is also constant, for given one shape your frontal area is also constant. So, finally, it is a v^2 that is the variable right for braking. So, what I am going to do

I will combine all this term as a C_{ae} coefficient of aerodynamic resistance with the v^2 with respect to R_a, yeah.

$$R_a = C_{ae}v^2$$

If, I rewrite then,

$$\Sigma R = \mu_R W \cos\theta \pm W \sin\theta + C_{ae}v^2$$

So, I am going to substitute this one into the main equation 1. If I do that then,

$$ds = \int_{v_1}^{v_2} \frac{K_m W}{g} * \frac{vdv}{(P_b + \mu_R \cos\theta \pm W \sin\theta + C_{ae}v^2)}$$

If you do integrate finally, you would expect the equation,

$$= \frac{K_m W}{2g} l_m \left[\frac{P_b + \mu_R W \cos\theta \pm W \sin\theta + C_{ae}v_1^2}{P_b + \mu_R W \cos\theta \pm W \sin\theta + C_{ae}v_2^2}\right]$$

Again, if I do the simplification, we will come back to simplification later. So, the v_1 is the vehicle initial velocity, v_2 is the vehicle final velocity. What is v_2 generally? v_2 will be 0. So, if we substitute that one in this equation number 2, you expect the stopping distance *S,*

$$S = K_m * \frac{W}{2gC_{ae}} l_n \left(1 + \frac{C_{ae}V_1^2}{P_b + \mu_R W \cos\theta \pm \mathrm{W}\sin\theta}\right)$$

So, this is the equation number 3 and this is your stopping distance of the vehicle. So, you look at this, it is related with the K_m the vehicle weight, then aerodynamic coefficient right. Finally, the braking effort P_b, your μ_R nothing but a rolling coefficient of friction, then the initial velocity v_1. This is what happens, fine. You know that when do you expect the braking efficiency will be 100 percent, when do you expect? When P_b equal to $\mu_S W$. That is the way braking efficiency will be 100 percent. So, I will move now.

So, when the total braking effort P_b equivalent to $\mu_S W$. Other time your braking efficiency will be 100 percent. Not only that the rotational of the masses also will be equivalent to 1. So, from equation 3,

$$S_{Min} = \frac{W}{2g} l_n \left[1 + \frac{C_{ae}v_1^2}{\mu_S W + \mu_R W \cos\theta \pm W \sin\theta}\right]$$

This is the equation. The braking efficiency is 100 percent, but generally the braking efficiency would not be 100 percent. It is always less than 100. $\eta_b < 100\%$. In this case the S_{Min} the distance would be

$$S_{Min} = \frac{W}{2g} l_n \left[1 + \frac{C_{ae}v_1^2}{\eta_b \mu_S W + \mu_R W \cos\theta \pm W \sin\theta}\right]$$

So, this is the equation for stopping distance for brake. Let me ask question this is in actual condition. In reality, what happened? What do you expect? Should we not consider the reaction time of the driver, right. So, the reaction time of the driver also should be considered. By that time, you will cover certain distance right that is also there. So, some distance already covered in the reaction time. Normally, the reaction time vary with the fraction of 0.5 seconds to 2 seconds maximum right, then you apply a brake suddenly. So, by the time you would have cross certain distance also. So, that is also you have to consider when you calculating for stopping distance right. This is absolute condition the breaking distance whatever we find now the minimum distance is absolute, but in reality, we have to include the driver reaction and the consequently the distance crossed during the time ok yeah. So, this is for your braking distance aspect.

Now, we will move on to the brake factor. I just want to recap again what exactly happening right. We know that we got μ_S is your road adhesion coefficient, that is already known to us fine. μ_R is the rolling resistance. So, μ_L is a coefficient of friction of brake lining. We know that the brake line is a now that the vehicle is move right on the road surface yeah. it is moving in this direction right Wa/g. Then, your deceleration rate also will be there. Then, I am just going to show this is W it is happening right. So, when it is happening this P_b will come here your reaction force will come into the picture. So, we know that this is entire kinetic energy right. Whatever you see, we are talking about the entire kinetic energy of the vehicle. So, you will have a brake torque, the brake torque coming from the vehicle. That is fine ok. This is one aspect. So, when I want to stop the vehicle, the entire kinetic energy should be matched with the frictional energy right. Then only the dissipated happen the vehicle can stop. Until then vehicle cannot stop ok. So, entire kinetic energy must be equivalent to my friction energy ok. So, it has to be equal right. So, we will have a brake torque from your vehicle then there is a brake torque from your brake, this is from brake system. And, this is from your vehicle system. Both are different right. So, now, I try to understand this is coming from where this is come your pedal force right. From pedal force to vacuum booster, vacuum booster to master cylinder right, the from master cylinder to a brake circuit right. From brake circuit to it goes two direction one is your disc brake system. Other one is your drum brake system right. Can you see that? One is coming from the pedal force to where it is going to be brake. whether it is a disc brake or drum brake that is a one thing is there. So, that is one aspect. The other aspect the vehicle is moving when a vehicle is moving with the mass right with the kinetic energy. So, we are talking about the vehicle weight, the vehicle dimension we are talking about a vehicle dimension including wheel base, tire diameter, the frontal area all those things come into the picture. So, it has to be match. Now I wanted to interlink them. So, how this is exactly will be balance based on the which component or which parameter. That is based on your friction lining. That is the one generating the friction right. Not no other component right. So, an entire energy, we are talking about the vehicle then entire brake or frictional energy should come the brake system through friction lining. So, the question is we never discuss anything about the wheel right. Did we discuss anything? And also, how do we know, what is the value do we need to match this kinetic energy. So, that is where the brake factor will come into the picture. Now, can you see interconnection? What is happening right? When you applying the brake right to the brake system right to activate then the kinetic energy happening through vehicle through your wheel and road adhesion factor need to be balanced through your friction lining with the corresponding friction value. So, that we have to find out what would be the μ typically for given vehicles. This one it depends upon the what type of brake system are you going to use it that is what we are going to discuss about the brake factor aspect.

So, the braking wheel and the drum brake torques. The maximum braking wheel torque is limited by your wheel slip. Wheel slip in the sense the wheel rotates the moment you reach to the maximum braking torque, the wheels try to skid or slide that is what it means to say the slip aspect. The second thing is the torque produce at the brake drum caused by frictional force between the brake lining and the drum to bring the wheel to stand still position. That is one aspect right, the two things are there yeah. Both wheel and drum torque must be equal at the point of wheel slip. You understand now? So, it has to be equal, but should be in opposite direction. Then, only the vehicle can stop yeah. So that the both torques are equal and vehicle will stop ok. Now, we will move on to what exactly going to happen we will understand the shoe factor and brake factors ok.

The shoe factor is the ratio of the friction force at the drum radius to the actuation force at the show tip. Now, we are talking about with respect to a drum brake system. We will discuss for a disc brake also as we move on. The shoe factor is the ratio of the friction force at the drum radius to the activation force P at the shoe tip that is one thing is there. So, brake factor is the combined effect of two shoes. Normally, when you say drum brake, drum brake is four types right. One is leading and trailing right. So, that is why two shoes are coming. Another one is leading-leading yeah. What is the third one? S cam, right. Then other one would be servo duo right. So, the four types are there. But for understanding we will choose only one drum brake system, which is the leading and trailing aspect. With that we will try to find out what would be the shoe factor and the brake factor. From there how to select the corresponding friction lining for the given vehicle yeah.

So, I am going to choose a simple one leading-trailing brake drum. Very simplest way, I am not going to do very complex. So, but I will give a problems very complex way in an assignment this is your drum right. You take it as a radius r, drum radius r. I am sure you know this is the pivot. From the pivot you will have a like this ok. From the pivot, this is shoe arrangement. So, the distance from the pivot to the way the tip the force going to activate is a C, which already known to you that is there. So, I am just showing only that small amount of friction lining with entire thing is a friction lined, but for our understanding, we are going to choose only small area. So, this is the friction lining and this is the direction. So, just for schematic purpose. So, assuming that this is moving in this direction ok. But I want to give you information from the pivot to the center of the drum, the dimension is given as a n. I will give you as n ok. Then, from the pivot to center of the thing. Again, from the pivot to this vertical distance. From pivot to the edge of the drum right this distance is given as m right. So, when you apply the force the tip force right you would expect obviously, two forces you know that. One is your normal force F_n where we have taken as m, n there. Your frictional force F_f is coming here. This is what two things are there ok. So, what we are going to do we will take the moment, take the moment around similar to what we did earlier the shoe point pivot. We will take it as a pivot A. We are talking about what are shown is only for the leading shoe aspect, not the trailing shoe. When you talk about A right. $P * C + F_f * m - n * F_n = 0$. So, just I try to balance the forces momentum aspect. So, finally $P * C + F_f * m = n * F_n$. Whereas my friction line μ_L depends on your friction force to the normal force. $\mu_L = F_f / F_n$, which already known to you. Please understand that ok. μ_L equal to coefficient of friction of brake lining yeah that is there. So, this is the equation. $PC + F_f m = nF_n$, but already know μ_L equal to F_f / F_n because we want to find out the μ_L. If you substitute you will expect, $PC + F_f m = nF_f / \mu_L$. I want everything bring in terms of my shoe factor (SF), nothing but ratio of the friction force to the activation force.

$$SF = \frac{F_f}{P}$$

Maybe, I can even more simplify that.

$$SF = \frac{F_f}{P} = \frac{\mu_L C}{n - m\mu_L}$$

I am simplifying that. Of course, one more term I forgot to mention, the activation force (*P*). It is nothing but your F_n. So, you would expect the shoe factor SF for the leading,

$$(SF)_{leading} = \frac{\mu_L C}{n - m\mu_L}$$

This is for the leading shoe factor. My question is what will happen to trailing. What do you expect trailing. The same term will be there, but instead of minus that will be positive.

$$(SF)_{trailing} = \frac{\mu_L C}{n + m\mu_L}$$

So, this one you would expect for the trialing. So, when you talk about a brake factor is nothing but summation of leading shoe factor and the summation of trialing shoe factor.

$$Brake\ factor = (SF)_{leading} + (SF)_{trailing}$$

May be, I would have rewritten here also. You can see that. That is fine. I think this is clear to you. The brake factor equal to the shoe factor of trailing plus shoe factor of leading. Both need to be added. Similarly, if you have leading-leading, will be two times right and so does for a S-Cam as well as Servo duo also. In fact, we can see the comparison. See, I have taken this from brake design and safety by Rudolf Limpert. You can look at this, it is μ_L and corresponding brake factor, you can see that. So, we will look at the first of the trialing and leading right. For one coefficient of friction, may be will choose 0.5, if you choose 0.5 you would expect in the range. Any reason for this? Why the brake factor will vary from upper value to lower value. Any reason for this? What is the reason for a leading and trailing, you can think about that. So, similarly if you look at for the two leading again 0.5 right, if you use the μ_L, you will expect the wide range. Within among that the servo duo is has a better brake factor, you can see that the servo duo has a better brake factor, then leading-leading, and leading-trialing. So, this is the arrangement is clearly given. For disc brake you can see that the maximum is given 1.2 that is a value and the single value any reason. The arrangement is disc brake is the pad calipers right. Either it is coming as a circular pad or annular pad. There is no leading and trailing. So, then you do not get a range right. So, straight away it is a fixed value. And, one more thing is, is there any self-locking in disc brake system. No right, there is no self-locking ok, but you have self-locking in a drum brake system when M_n equal to M_f, remember the one initially we have derived the equation. So, you will have self locking which is not advisable ok. The question is, the break factor is good for a leading and trailing whereas, disc brake will have a lesser break factor, but as a no self-locking. Then what is the reason we still using a trailing and leading drum brake system that is question will come right. So, there is one more term is called stability you have to look at stability also right. In fact, again this is the reference taken from brake design and safety by Rudolf Limpert. Look at that, we are talking about disc and pad the maximum shoe factor you would expect 1.2 right. The stability is good.

Whereas, if you have leading and trailing shoe the brake factor is good, but the stability is not enough right. Again, if you look at the two leading shoes the brake factor is really good right, but the stability very low. The duo servo has also very high brake factor right, but it has a very low thing ok. Now, do you know the reason, why the drum brakes are fitted at the rear and the disc brake are fitted at the front. Remember, in one of the dynamic analyses, we have recommended to design the brake system always for front axle lock first ok. Think about the scenario, instead of disc brake if you expect the drum brake at the front, what do you expect? Instead of disc brake, the drum brakes are fitted at the front axle what do you expect? First of all, they have self-locking right. So, when they have self-locking even then if it is front axle lock, do you think that will be better? Do you think that will be better? You can replace the drum brake with the disc brake that is allowable because even then you have a less back factor, but the stability is better. You always look for the stability that is very critical because it is a reliability component, brake system is a reliability, it is a life of death and catastrophic thing right. So that has to be taken care of while designing. That should be addressed. So, you can always replace drum brake with the disc brake, if you do the vice versa that is going to affect the stability and leads to catastrophic failure. Now you know that ok. In fact, the modern vehicles now a days come with all four wheels with the disc brake right. So, that is how the brake needs to be taken care of that ok. I think I will give you a problem.

Now, you can see that the friction lining clearly mentioned we have from the analysis. So based on the SAE coefficient of friction code, we have given the various category C, D, E, F, G, H that is like that. Now, you can see that the C has the lowest coefficient of friction range. As a grade is improving the friction range is increasing ok. Again, I am asking a question instead of giving one single value, why the friction lining is given as a range? What is the reason for that. Why cannot have one particular value? What is the reason behind it? Then, that is where we talk about thermal analysis ok. The brake can be applied to various aspect either you can have a single stop braking, repetitive braking or drag braking, based on that your coefficient of friction will change right. So, if I apply sudden brake, it is a single brake right that is one way of applying a brake. Otherwise, if you keep apply release, apply release what happen, you are allowing the system to cooling down. There your friction requirement will be different ok. Similarly, if you move on to the downhill, brake requirement will be very different. So, when you talk about the vehicle will undergoes different condition, when you have different condition, you need to have different coefficient of friction. So rather having one single coefficient of friction, you should have a range such a way that to cater all the situations yeah ok so this is the way you have to select in fact we are going to do a problem. I think the time is very limited so I am going to stop now but we will continue tomorrow.