Design of Mechanical Transmission Systems

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Week – 08

Lecture – 22

Lecture 22_Brake: Problem Solving

Good afternoon. So, we have discussed so many things in design of brake system aspect. So, as you see all these topics in red mark are covered. In fact, we have discussed the brake distribution, the brake force distribution also we have discussed about the optimization. In fact, we will understand more optimization aspect of the for a brake force distribution by solving problem. Today we will focus today may be another couple of lectures we will focus on problems. There you will understand many things. Yeah, this is the problem number 6. It is a big problem.

The Tata venture GX model van as a length, width, height, dimensions are 3950, 1565, 1878 in mm. The rated maximum gross weight GVW of the van is 19.52 kN. The wheel base is 2540 mm and the ground clearance is 160 mm. The height of the CG nothing but center of gravity is 650 mm above the ground and 1000 mm in front of the rear axle. The wheel size 650 mm dia 150 mm wide. The coefficient of rolling friction is 0.02. The aerodynamic drag coefficient is 0.3 and the coefficient of road addition is 0.85. Determine:

(a)what should be the braking proportion in the front and rear axle to get optimum braking effort, that is a question number 1. Then also find the same braking proportion that is means solving the braking proportion for Ice surface where μ_s is equal to 0.1 if the vehicle travel on the same surface.

(b) if the van with the above braking proportion traveling at 70kmph on a level road having the coefficient of road addition is 0.7 and the coefficient of rolling friction is 0.015 is braked. In that case which wheels which wheel will skid first? what is the maximum deceleration rate achievable. This is the question okay. Maybe I should give some more clarity if the van braking above braking that is what is clearly given right. This above braking means its meaning about the above talking about concrete surface not the Ice one okay.

It is talking about the concrete surface. You could see what are the data do we need. We need to know about the weight of the vehicle. This is your wheel base okay and this is your h height right center of gravity from the ground level and what is it mean this one what is it 100 mm in front of the rear axle and the center of gravity it is a l_1 or l_2 it should be l_2 excellent correct. This is l_2 and this is your μ_r 0.02, and this is your μ_s 0.85 all those information is given. So we need to find out what would be the optimum braking effort that is the thing okay. So data were given I will rewrite again data were given:

W is 19.52 kN yeah, that is clearly given and the wheel base L is given 2540 mm okay. From the problem l_2 given 1 mm sorry 1 m, okay 1000 mm obviously l_1 will be 1.54 m yeah that is clearly given okay. Then the μ_r equal to 0.02 and h is given 0.6 m and what else is missing we are okay radius given right. The diameter of the tire is given right yeah diameter of the wheel is given as a 0.65 m okay that is given yeah. The case 1 right (a) we want to take concrete surface when you say concrete surface the typical is 0.8 to 0.9 but specifically is given in the problem your road addition factor is 0.85 right. So we have to find out what is the optimum your K_b front axle divided by K_b rear axle this is what we need to know that okay. So if you want to write the equation

$$\frac{(K_b)_{FA}}{(K_b)_{RA}}$$

$$\frac{(K_b)_{FA}}{(K_b)_{RA}} = \frac{l_2 + h(\mu_s + \mu_R)}{l_1 - h(\mu_s + \mu_R)}$$

$$\frac{(K_b)_{FA}}{(K_b)_{RA}} = \frac{1 + 0.65(0.85 + 0.02)}{1.54 - 0.65(0.85 + 0.02)}$$

$$\frac{(K_b)_{FA}}{(K_b)_{RA}} = \frac{1 + 0.65(0.85 + 0.02)}{1.54 - 0.65(0.85 + 0.02)}$$

$$\frac{(K_b)_{FA}}{(K_b)_{RA}} = \frac{1.566}{0.9745}$$

$$\frac{(K_b)_{FA}}{(K_b)_{RA}} = 1.606$$

K_b front axle to K_b rear axle okay l_2 plus h μ_s and μ_r divided by l_1 minus h μ_s plus μ_r okay. So can you substitute the values please in this equation

this is nothing but your K_b front axle to K_b rear axle, the ratio only this is the ratio 1.606 but already we know that

$$(K_b)_{FA} + (K_b)_{FA} = 1$$

$$(K_b)_{FA} = 0.62 \rightarrow 62$$

$$(K_b)_{RA} = 0.38 \rightarrow 38$$

your K_b front axle and K_b rear axle alright the summation always should be 1. So if you using these two and substitute finally you would expert the K_b front axle okay front axle 0.62 or else we can write a 62, the K_b rear axle right. So 0.38 is nothing but your 38.

So this is a proportion yeah this is your distribution proportion at the front axle as well as the rear axle okay that is fine that is fine. So now given this condition we know this the surface is concrete which is road adhesion factor 0.85 already we have find we got the distribution the brake distribution force for front axle as well as the rear axle also we got it. Can we find out what happen if this road adhesion and this is the distribution how does the which we lock first we will find out.

Then we can go for the next one, what is the next one they are asking about using the same braking proportion if μ_s is changed right. The ultimately what we want to understand how if you have a same road adhesion if you change the brake distribution what is going to be happen or else if you have this same distribution by changing the road adhesion factor both way we are trying to understand that okay. Then that will give us the role of critical role of the distribution as well as the road adhesion factor or coefficient of road adhesion μ_s also okay. This is we got it now okay this is with the proportion aspect okay we will do it for the ice later. Now we need to identify which will lock first right we need to identify which will lock first.

$$\left(\frac{a}{g}\right)_{FA} = \frac{\mu_s.\frac{l_2}{L} + (K_b)_{FA} \times \mu_r}{(K_b)_{FA} - \mu_s.\frac{h}{L}}$$

$$\left(\frac{a}{g}\right)_{FA} = \frac{0.85 \times \frac{1}{2.54} + 0.62 \times 0.02}{0.62 - 0.85 \times \frac{0.65}{2.54}}$$

$$\left(\frac{a}{g}\right)_{FA} = 0.86$$

So your a by g the front axle okay μ_s remember this equation. If you substitute tell me the what is the value you are getting. This should be 0.86 this is the value you should expect your a by g the front axle 0.86. So already we got the one value right. a by g rear axle now, the deceleration rate this is nothing but your deceleration rate yeah.

$$\left(\frac{a}{g}\right)_{RA} = \frac{\mu_s.\frac{l_1}{L} + (1-(K_b)_{FA}) \times \mu_r}{(1-(K_b)_{FA}) + \mu_s.\frac{h}{L}}$$

$$\left(\frac{a}{g}\right)_{RA} = \frac{0.85 \times \frac{1.54}{2.54} + 0.38 \times 0.02}{0.38 + 0.85 \times \frac{0.65}{2.54}}$$

$$\left(\frac{a}{g}\right)_{RA} = 0.87$$

μ_s l_1 by L this 1 minus K_b front axle okay into μ_r 1 minus K_b front axle plus μ_s h by L substitute the value please.

So now the a by g rear axle is coming as a 0.87 okay. Let me ask question. So already we know the a by b right the front axle came 0.86. Tell me which will lock first as almost same value or different values right. Can we consider they are almost same yes or no? Yes we can consider. So for one particular road addition 0.85 we got the optimum K_b front axle and K_b rear axle and try to find out which will lock first. So we got the deceleration rate, deceleration rate looks are equal. So they will lock at the same time both axle will lock same time both axles lock at the same time okay. Now look at the problem now saying that using the same proportion we are changing the surface μ_s equal to 0.7 and μ_r equal to 0.015 I think okay. So in that case what happened to your deceleration rate? In that case do you expect the same behaviour or same phenomena or would change? That is what we are going to understand in the problem statement okay yeah. For keeping the same braking proportions proportion we will try to find it which will lock first for given μ_s equal to 0.7 and μ_r equal to what is the μ_r equal to it is a 0.015 yeah right. So substitute please a by b front axle I will straight away put the values rather than the writing re-writing the formulae

$$\left(\frac{a}{g}\right)_{FA} = \frac{0.7 \times \frac{1}{2.54} + 0.62 \times 0.015}{0.62 - 0.7 \times \frac{0.65}{2.54}}$$

$$\left(\frac{a}{g}\right)_{FA} = 0.646 \cong 0.65$$

this is my front axle yeah. Similarly, I have for the rear axle

$$\left(\frac{a}{g}\right)_{RA} = \frac{0.7 \times \frac{1.54}{2.54} + 0.38 \times 0.015}{0.38 + 0.7 \times \frac{0.65}{2.54}}$$

$$\left(\frac{a}{g}\right)_{RA} = 0.77$$

this is 0.38, 0.015 just give me 0.77. So tell me what is happening from this value right your right your front axle deceleration rate front axle is lesser than the rear axle right that is what happening. So in this condition which will lock first? Front axle lock first yeah. Now, let me ask question so what do you understand now? We are using the same proportions we are using the same proportion if you change your μ_s obviously the scenario completely changed. Remember when we derive the equation we said this is valid at one particular road adhesion factor. If the road addition factor or value change obviously the scenario locking of the wheel will change accordingly okay.

So there is one more scenario will keep the road addition same but will change the proportions that later stage I will give you a problem right. So I believe the front axle or the wheels at the front side will skid first this is what happening here yeah. So fine so the one more question is clearly given okay. One more question is clearly given find out if road adhesion factor 0.1 for ice surface what would be the brake proportions right yeah.

For ice surface μ_s equal to 0.1 right. Can you find out please K_b front axle.

$$\frac{(K_b)_{FA}}{(K_b)_{RA}} = \frac{l_2 + h(\mu_s + \mu_R)}{l_1 - h(\mu_s + \mu_R)}$$

$$\frac{(K_b)_{FA}}{(K_b)_{RA}} = \frac{1 + 0.65(0.1 + 0.02)}{1.54 - 0.65(0.1 + 0.02)}$$

$$\frac{(K_b)_{FA}}{(K_b)_{RA}} = 0.737\ (a)$$

$$(K_b)_{FA} + (K_b)_{RA} = 1\ (b)$$

$$(K_b)_{FA} = 0.42 \rightarrow 42$$

$$(K_b)_{RA} = 0.58 \rightarrow 58$$

Just substitute the values, this is your K_b front axle and K_b rear axle the proportions I am sure we know that again the summation of both proportion should come as a 1 right. So if you solve these two thing a and this is the b. So finally we will expect your K_b front axle equal to 42 the K_b rear axle equal to 58 okay. So earlier what was the value we got when you have a concrete surface μ_s when 0.85 there your distribution of the K_b front axle was 62 right rear axle was 38. Now the moment you change the surface condition what happened your the road addition the moment you change the road adhesion value the distribution also will change. So when the distribution change what do you expect the deceleration rate do you think the both the front and rear will have same deceleration rate at different decelerate will be will be different. So the gain so that is how we have to find out somewhere to optimize it okay. Let me ask one more question see ultimately in this problem given (GVW) gross vehicle weight right.

What happened this is the vehicle is fully loaded and if you have the vehicle without any load that means only the vehicle weight and one passenger which is a driver what do you expect that is also will alter right. So we need to make attention in this aspect also. So we will discuss in subsequent lectures in this the brake distribution optimization aspect and we can go for one more problem. So now we will do another problem okay. Please note down another problem number 7

A passenger car weighs 20 kN as a wheelbase of 2794 mm the center of gravity is 1270 millimeter behind the front axle and 508 millimeter above the ground level. In practice the vehicle encounters a variety of surfaces with the coefficient of road adhesion ranging from 0.2 to 0.08 and the coefficient of rolling resistance of 0.015 with their view of avoiding the loss of directional stability on surfaces with the low coefficient of adhesion under emergency braking condition what would you recommend regarding the braking effort distribution between the front and rear axles okay. So this is what a problem yes this is similar to the previous problem but there is a slight change right, this is slight change is given.

So we have the vehicle weight which is 20 kN you can see that and the wheelbase is given 2794 mm and this is actually that is a wheelbase and this is the center of gravity is 1270 mm behind the front axle what do you mean by behind the front axle l_1 it is not l_2 okay remember this is your wheel right and this is the center of gravity it says behind means something backward in this direction so obviously that means l_1 yeah that means l_1 which clearly given 1270 mm and this is your h that is from the above ground level clear this h is given and we have to change the μ_s by from a 0.2 to 0.8 with the μ_r is clearly given as a 0.015 okay with the view of avoiding the loss of directional stability on surfaces with the low coefficient of addition under emergency braking condition what would you recommend regarding the braking effort distribution between the front and rear axle. That is the question. So what do you understand that yeah what do you mean by avoid loss of directional stability there are two things when the front wheel locks what happened the one what is that there is a directional instability is different, direction instability one is there and loss of direction one is there right so they are both different loss of direction means where you can't steer or move the vehicle either to the right side or the left side, that is loss of direction okay, that will happen when you have a front wheel lock first front wheel lock first that will happen okay, if the rear wheel lock first what happen because of that you will have yaw effect so we will have a spin so that is the loss of directional stability both are different, direction stability different loss of direction is different okay so they are asking they are expecting you want to avoid rear wheel lock first that means you want to aim to lock front wheel that's how you have to design okay so that's a problem yeah so W is clearly given 20 kN L is 2.794 m, l_1 is 1.27 m, l_2 obviously L equal l_1 plus l_2 so from this you will find out l_2 equal to 1.52 m is clearly given okay h is given 0.508 m and μ_r is 0.015 so what we have to do μ_s is ranging from 0.2 to 0.8 that's what the problem so we will take away straightaway two things the lower limit and upper limit and try to find out what is happening okay so first this is lower l and this is upper limit U okay

For μ_s equal to 0.2

$$\frac{(K_b)_{FA}}{(K_b)_{RA}} = \frac{1.52 + 0.508(0.2 + 0.015)}{1.27 - 0.508(0.2 + 0.015)}$$

$$\frac{(K_b)_{FA}}{(K_b)_{RA}} = 1.4035\ (a)$$

$$(K_b)_{FA} + (K_b)_{RA} = 1\ (b)$$

$$(K_b)_{FA} = 0.58 \rightarrow 58$$

$$(K_b)_{RA} = 0.42 \rightarrow 42$$

I'm not writing equation anymore that just substitute the values and give the answer straightaway this is the K_b front axle and the rear axle so if you substitute individually what I what we get I'm giving answer straightaway front axle equal to 58 the K_b rear axle 42 so this is the value you are getting, so this is for μ_s equal to 0.2 okay μ_s equal to 0.2 similarly for μ_s equal to 0.8 so we have to find out the values also can you give me the values piece what is that value you are getting, yeah

$$\frac{(K_b)_{FA}}{(K_b)_{RA}} = \frac{1.52 + 0.508(0.8 + 0.015)}{1.27 - 0.508(0.8 + 0.015)}$$

$$\frac{(K_b)_{FA}}{(K_b)_{RA}} = 2.259$$

$$\frac{(K_b)_{FA}}{(K_b)_{RA}} = \frac{69}{31}\ for\ \mu_s = 0.8$$

1.52 right 0.8 plus 0.015 1.27 yeah so finally the values coming as a 2.259 separately right the K_b front axle rear axle 69 by 31 okay this is for when you have a μ_s equal to 0.8 so individually we got the proportions right so front axle and rear axle proportion we already got it. So what is the next step is we need to find out the deceleration rate for both surfaces yeah we need to find out the deceleration rate for both surfaces for μ_s equal to 0.2 that's what we are going to do now

$$\left(\frac{a}{g}\right)_{FA} = \frac{0.2 \times \frac{1.52}{2.79} + 0.58 \times 0.015}{0.58 - 0.2 \times \frac{0.508}{2.79}}$$

$$\left(\frac{a}{g}\right)_{FA} = 0.216$$

a by g front axle, I think one minute this there is a mistake this is distribution right the distribution μ_s is what we are getting for I surface I'm sorry yeah 0.58 okay 0.58 and this should be also 0.58 yeah that was mistake then 0.08 by 2.79 expect 2.216 right this is a by g front axle 0.216 when you have μ_s equal to 0.2, then similarly for the rear axle substitute the values again

$$\left(\frac{a}{g}\right)_{RA} = \frac{0.2 \times \frac{1.27}{2.79} + 0.42 \times 0.015}{0.42 + 0.2 \times \frac{0.508}{2.54}} = 0.28$$

$$\left(\frac{a}{g}\right)_{RA} = 0.28$$

$$\left(\frac{a}{g}\right)_{FA} < \left(\frac{a}{g}\right)_{RA}$$

when you do that this should be 0.42 K_b right the entire board went to the very slack that what is the value you are getting here supposed to be 0.42 right yeah so getting I getting I think 0.28 I'm getting this one yeah so there is a huge difference so from here what do you expect this is the front axle and this is a by g the rear axle so the front axle has a 0.216 right 0.216 which is lesser than the your rear axle so what happened in this condition when μ_s equal to 0.2, so rear wheel lock first or front wheel lock first, right front wheel lock first yeah lock or skid first, so similarly already we know that the ratio also we found the ratio for point μ_s equal to 0.8 as a 69 by 31 right already we have what is the value for μ_s equal to 0.8 right I think we just now did 69 by 31 right yeah so now we will go for the a by g

$$\left(\frac{a}{g}\right)_{FA} = \frac{0.8 \times \frac{1.52}{2.79} + 0.69 \times 0.015}{0.69 - 0.8 \times \frac{0.508}{2.79}}$$

$$\left(\frac{a}{g}\right)_{FA} = 0.8194$$

so what is the value you are getting I'm straightaway giving the value so what is the value you are getting now front axle yes so you should expect I'm straightaway giving the answer 0.8194 and 0.8094 so we got so a by g the front axle, a by g rear axle respectively okay, front axle greater than the rear axle right so here the rear wheel lock first okay the question is you have chosen the two different surfaces one is point μ_s equal to 0.2 the other one is 0.8 in the first case front wheel lock first where as other case rear wheels lock, so any recommendation you want to give because it is clearly mentioned loss of directional stability, it is satisfied for μ_s 0.2, right nothing to you know need to be give any recommendation so this proportion is fine okay I think I will stop now I think this I think we have done the two problems to understand about in next class we are going to discuss various aspects if the today we discuss μ_s if you have a having one μ_s at different proportions or having one proportion and different, μ_s how is going to be where is the way to you draw the optimum proportion how will you achieve it those things we will discuss in next class and also not only that see the braking so far we never discuss about braking distance right how to achieve the braking distance that is also we are going to discuss okay and one more critical thing so far, we discuss when you apply a through brake pedal the force is go goes there and activated through either in the disc brake or drum brake activation force you are having a torque that is one side the other side the vehicle is moving certain velocity you are applying a brake again the distribution coming from your wheel and road surface that is another torque we are there but how to connect them where is that connection that connection also, we are going to discuss in forthcoming lectures okay thank you.