Design of Mechanical Transmission Systems

Prof. Ramkumar

Department of Mechanical Engineering

Indian Institute of Technology Madras

Week – 07

Lecture – 21

Lecture 21_ Brake: Dynamic Analysis – Brake Force Distribution and Optimum

So, good morning to all. So, we will continue where we left in the last lecture. So, so far within the brake system design we have discussed about brake working methods, brake types, brake torque requirement within that drum brake system, disc brake system all these things we have discussed. In dynamic analysis we have discussed about the mass transfer during the braking aspect and also, we have discussed the brake distribution between the front axle and the rear axle as well. Now, we will continue with the brake force distribution and also, we are going to learn about the optimum aspect as well ok.

So, before going to things like to recap what we have discussed about the brake distribution. First please understand there are three different coefficient of friction we do have in this brake system. One is μ or μ_L is nothing but coefficient of friction of brake line right brake lining. The other one is μ_R coefficient of friction in rolling, that is between tire and ground ok, this is the one. The third one is a μ_S is called road adhesion ok. Road adhesion coefficient or coefficient of friction of road ok, particularly we talking about the road and tire interface. This is what we have discussed. And also, we found the front axle reaction forces N_{FA} right,

$$N_{FA} = \frac{Wl_2 + h(P_b + \mu_R W)}{L}$$

Similarly, the reaction force from the rear axle N_{RA},

$$N_{RA} = \frac{Wl_1 - h(P_b + \mu_R W)}{L}$$

Please understand, this WL_2 or l_2/L, right it is a static load for front axle. Whereas, l_1/L is a static load again for your rear axle, obviously right. l_2/L that is what is happened ok. Then we found the maximum braking force right which developed at the interface when I talk about. So, when the vehicle is moving right, so when I apply the brake the brake force is developed between the wheel and tire. $(Pb_{Max})_{FA}$ for front axle ok.

$$(Pb_{Max})_{FA} = \mu_S \left[\frac{Wl_2 + h(P_b + \mu_R W)}{L}\right]$$

This is the one we have. Similarly,

$$(Pb_{Max})_{RA} = \mu_S \left[\frac{Wl_1 - h(P_b + \mu_R W)}{L}\right]$$

So, this is what we found. And also, what we have taken we have taken the proportion of these two equations right. We have taken the proportion of these two equations. And finally, we arrived at it is,

$$\frac{(K_b)_{FA}}{(K_b)_{RA}} = \frac{l_2 + h(\mu_S + \mu_R)}{l_1 - h(\mu_S + \mu_R)} = Brake\ Distribution$$

This is what we have right. Again, we will clarify one more thing is $(K_b)_{FA}$ is a brake distribution at front axle right. $(K_b)_{RA}$ is brake distribution at rear axle right. For given that,

$$(K_b)_{FA} + (K_b)_{RA} = 1$$

This is what we said, am I right ok that is what we did. And also, we have discussed what we have said. So, the entire thing right the braking distribution is depending upon the hydraulic pressure of your brake fluid as well as wheel cylinder. Based on that the proportion is determined. In fact, whatever the braking force and normal force generated during the braking must be balanced with the distribution, which is happening in the hydraulic pressure as well as wheel cylinder ok that is what we have discussed. And also, we have discussed one more important thing. So, this is the proportion we need to have it ok. And the brake forces what are the brake forces, when you talk about brake force, please remember this is your braking distribution force. I am talking about your $(P_b)_{FA}$ and $(P_b)_{RA}$. The brake forces should be developed at the same time in all four wheels ok. They have to develop the braking force effect at the same time in all four wheels, that is supposed to be happened. But generally, it would not happen. Either front axle lock first or else the rear axle lock first. So, what do you mean by lock first? what do you mean by lock first thing right? what do you understand about that? So, this is I am just showing the only one wheel ok. The only one wheel I am showing that. Assuming that the vehicle is moving in direction. Then, obviously the rotation would be anticlockwise. So, the moment right is going this direction you would expect, this is your *N* right this is *N* assuming for front axle does not matter ok. So, the moment you apply the brake right. So, this already the load is acting the load is there I am not showing the load, but assuming it is a load is there on the wheel. So, this is the normal force and this is the where you are expecting your the braking force. The question is when do you expect the self-locking? When do you expect the self-locking? So, this one is indicated about your μ_S right this is indicating about the road adhesion factor or coefficient of road that is what is happening. So, the moment the braking force whatever is developed is must be equivalent to $\mu_S W$. So, if that condition what do you expect. So, this has to be balanced right. So, what are the braking force is developed must be equal to your $\mu_S W$. In this condition, you are going to have a self-locking right self-locking. So, until then the tire keeps rotating or rolling. So, at that point when you have a brake force right. So, the rolling is stopped. So, that is why this is called self-locking ok. This condition is called self-locking yeah. So, in fact we will go ahead to discuss in detail further more.

So, now we will talk about the road adhesion coefficient μ_S. Look at this. It is given for the various condition. This is for the asphalt and concrete is 0.8 to 0.9 is clearly given. The moment you go for wet surface, is the drastically reduces, your road addition factor ok. I am sure you would have known or you would have seen during the rainy season right. It is always skidding. Why skidding is happening? Skidding is happening because of your friction is drastically reduced during the wet ok. But, look at if you have a concrete. When you have a concrete surface what I said earlier asphalt nothing, but your tar road ok. The moment I am talking about concrete even though it is in wet condition, the coefficient of road adhesion is almost same. Now, you know that why concrete roads are preferred than your asphalt right. So, that you will you will understand that ok. And, that is the highest value you see right and you can see this is the lowest value. And also, when you have a snowfall or rising thing, your coefficient of friction is drastically reduced. So, there again controlling a break is very tough and difficult also. In fact, they would see when the moment they have what you call it snow falling, they try to do the because of the snow is very loose. So, they make it as a packed to improve the coefficient of friction. On top of that they will pour what you call gritty kind of granules they will pour such a way that will improve the friction things. So, by doing that it will be avoidable skidding aspect ok.

So, we will see the two different conditions. So, when we have a coefficient of friction for concrete ok, I will use for concrete. Try to find out what happened to the your K_b proportions for concrete surface. Just understand! Concrete surface or road. Usually that is μ_S equal to 0.85 we have taken because this is vary between 0.8 to 0.9. So, I choose μ_S equal to 0.85. And, your μ_R is the rolling resistance equal to 0.01 that is another thing, you can choose it. Then, $\frac{h}{L} = 0.18$, the value this is for the light truck. For light truck, I am talking about ok. So, then the static load at the rear axle is $\frac{l_1}{L} = 0.68$. And, $\frac{l_2}{L} = 0.32$ for front axle. So, can you tell me what is that proportion,

$$\frac{(K_b)_{FA}}{(K_b)_{RA}} = \frac{l_2 + h(\mu_S + \mu_R)}{l_1 - h(\mu_S + \mu_R)}$$

If you substitute the values,

$$\frac{(K_b)_{FA}}{(K_b)_{RA}} = \frac{0.32 + 0.18(0.85 + 0.01)}{0.68 - 0.18(0.85 + 0.01)} = \frac{0.47}{0.53}$$

So, you would expect this. In terms of fraction, it is $47/53$. This is the proportion. I think yesterday someone asked question ok. What they asked, in vehicle the front we have a disc brake, at the rear we have a drum brake ok. So, how does the proportion going to be work? That is a question was raised right. See, as I said earlier the proportions already taken care of by your wheel cylinder, and the hydraulic pressure right. So, whatever the x proportion will go to the front axle, whatever the remaining portion may be say as a y proportion will go to the rear axle ok. So, that is about the proportion only. So, to meet the dynamics, right we use different brake system. For rear, we use a drum brake system and for front, we use a disc brake system. So, as long as you meet the proportion how does it matter right, is it clear to you. So, the either disc brake, drum brake must be equivalent to whatever the proportion of you are getting that is what you have to satisfy dynamically right. So, it does not matter whether you do it with the drum brake system or disc brake system right. So, this is for the concrete surface.

So, let us see what happen if we have ice surface ok. Same thing I am just wanted to see the proportion, how is going to be change. For ice condition or ice surface, for ice surface μ_S usually is 0.1 right, very low coefficient of friction ok. So, similarly if you find out,

$$\frac{(K_b)_{FA}}{(K_b)_{RA}} = \frac{0.32 + 0.18(0.1 + 0.01)}{0.32 - 0.18(0.1 + 0.01)} = \frac{0.34}{0.66}$$

In other words, it is $34/66$. So, at any given time yours, $(K_b)_{FA} + (K_b)_{RA} = 1$. So, that is need to be satisfied. So, look at what happens when your coefficient of friction is very low. Look at, what is happening to your rear axle. The proportion is changing right. So, this is where we need to understand how to optimize aspect also ok. So, the one thing is, I would like to say as we aware of that the braking effect is happening, at the same time on the both axles or 4 wheels that is not happening. So, what is the consequence you expect. So, assuming that my front axle lock first. What do you think, if the front axle locks first when you apply a brake. Is it good or bad. So, loss of directional control.

So, the first thing is loss of directional control causing the driver cannot steer to the right or left. The moment you lock right, the front axle lock. So, the driver cannot move the vehicle either to the right or to the left that is a problem. So, that is why it is called loss of directional control when you have a front wheel lock first. But good thing is there is no directional instability. I will tell you what is it mean ok. So, because of the lateral movement of the front tire occurs, there is a self-correcting movement due to the initial force will try to push the vehicle to move into straight path ok. So, it allows when the vehicle is lock at the front axle. It allows the vehicles to move straight away to the direct path or straight line, but it would not allow to the right or left that is what happening right. So, driver may deduct the loss of steering and control by regaining release either fully or partially to release of the brake. So, again this is again depending upon the maneuvering of the driver skills is very important. So, you have to play very smartly to operate your brake in such a way that either to release a slowly or a partially such a way that to regain the control of the vehicle right. This is when both front wheel lock first.

Now, the second thing is if both rear wheel lock first right. So, the front wheel track straight ahead right. That is easier because of the front wheel keep rotating it. It is moving straight right. Now, the rear wheel lock first, what do you expect? Yes, still you can change the direction, but what happen if they locked. So, they see the moment when I say lock the wheels cannot rotate anymore. Please understand that the wheels cannot rotate. So, rather they start sliding, they start sliding. My front wheels are rotating in the front axle whereas, rear wheels are not rotating they are sliding. So, that will create imbalance, ok. And, remember during the braking certain mass also added to the front axle. Whereas, the mass is detected at the rear axle. So, what do you expect now? Already, I had given the clue. The front axle is moving straight away with the heavy load whereas, the rear axle become lighter, but they are locked. So, what do you expect now. Then, they will start that is why it is called directional stability ok. Then, the rear wheels deviate to the side. So, until the vehicles cannot track straight any longer than the rear starts to spin around the front that is what's happening. It is called yaw action ok. So, it is more critical and difficult to control direction stability. Finally, you can see that what is happening now. So, the vehicle will spin out. So, spinning out is good do you think? It is not good. Not at all good. Can you see the difference, when the front wheel lock first and the rear wheel lock first ok. So, which is more important. So, as a design engineer you want to preferred at least the front wheel need to be lock first for safety aspect. So, what do you understand? So, in the front wheel locks, you will have a loss of directional control ok. Whereas, direction instability not there ok. The moment you have rear wheel locks, you will have a directional instability that means it start spinning around which is not good ok. So, the question is how are we going to do optimize it right, that is why we talking about the brake force distribution optimizations. The optimization means it is necessary to understand the condition at which the front and rear tires will lock first at what condition. How do we know that? Further, how to quantify it right. Further, it is important to determine quantitatively which wheel will is going to lock first. That is also we need to do that ok yeah. So, now we will try to go move around try to find out the which wheel lock first aspect.

So, as I said earlier remember, we have a surface here right the road surface, the wheel is there, there is a heavy load, this is the vehicle weight and moving in in this direction ok. So, obviously, the rotation will be anticlockwise. So, this is your the reaction force and this is your braking force, this is what happen right. So, $P_b = \mu_S W$, I will say that. So, everything we are talking about I need to understand and quantify which wheel lock first. That means, I am talking about the deceleration rate aspect I have to talk about ok. So, obviously, we know that

$$(P_b)_{Max} = \mu_S N_{FA}$$

$$(P_b)_{Max} = \mu_S N_{RA}$$

So, we have this this equation, ok. Maybe you can say as a C and D. In these terms, $(P_b)_{Max}$ we have and N_{FA} we have right. N_{FA} we have. Both we need to bring it with respect to deceleration aspect. Then, then we will solve it and try to find out what exactly happening during the first front axle lock first or if rear axle lock first ok.

So, our general equation you know that,

$$P_b + \mu_R = \frac{W}{g}a - R_a - W\sin\theta$$

This is what our general equation right. So, here we will be having, this is your rolling resistance or aerodynamic resistance. Aerodynamic resistance right, it is aerodynamic resistance, and this is your gradient. For simplification, what we are going to say assuming that the vehicle is moving in the plane road right. We will assuming that the vehicle is moving in the plane road. So, moment you say in this direction this will be zero and this also we neglect. So, finally, we will have a general equation,

$$P_b + \mu_R W = \frac{W}{g}a$$

So, this is the equation general equation right. We are going to use this equation ok. But we know already,

$$N_{FA} = \frac{Wl_2 + h(P_b + \mu_R W)}{L}$$

Similarly, your N_{RA}

$$N_{RA} = \frac{Wl_1 - h(P_b + \mu_R W)}{L}$$

So, these things we know. As I said we want everything N_{FA}, N_{RA} with respect to deceleration aspect ok. For that what we are going to do is. Your $N_{FA} = \frac{Wl_2 + h(P_b + \mu_R W)}{L}$. So, that is we know that. But we know from the general equation that, $P_b + \mu_R W = \frac{W}{g}a$. This is already known general equation. So, everything I am going to replace this term by putting this. Then, the equation

$$N_{FA} = \frac{W}{L}\left[l_2 + \frac{a}{g}h\right]$$

We will take this as equation number 1. Similarly,

$$N_{RA} = \frac{W}{L}\left[l_1 - \frac{a}{g}h\right]$$

This is equation number 2. So, this is the way we do that yeah. And, we know that the braking force which is the general one the braking force must be summation of the braking force front axle plus braking force at the rear axle $P_b = (P_b)_{FA} + (P_b)_{RA}$ that is one term we know that. Similarly, the braking proportion is $K_b = (K_b)_{FA} + (K_b)_{RA}$. Remember this is nothing but the braking force developed at the interface wheel and tire that is the braking force. Whereas, K_b is the distribution of your brake. So, both are different both are very different. So, I need to find out individually because I had to find out what exactly, when the front axle is going to be locked or rear axle going to be locked. So, that means I need to understand what would be the real braking force at the front or rear alone that is what I need to find out. So, the braking force P_b is,

$$(P_b)_{FA} = P_b * (K_b)_{FA}$$

So, already we know the P_b, what is the P_b now? We will keep the $(K_b)_{FA}$, instead of P_b, what we can do that. From the general equation, $P_b + \mu_R W = \frac{W}{g}a$, right. So, I want to know only the P_b right. This is what is going to come. So, I am going to replace that. Then your $(P_b)_{FA} = (K_b)_{FA} * \left(\frac{W}{g}a - \mu_R W\right)$. So, if I take the W alone the weight separately, it is $(P_b)_{FA} = (K_b)_{FA} W\left[\frac{a}{g} - \mu_R\right]$, this is what your expression. So, this is we can take it as equation number 3. This is braking force at the front axle. Similarly, the braking force rear axle what you expect now right. Your $(K_b)_{FA} + (K_b)_{RA} = 1$, right. So, already I know $(K_b)_{FA}$. So, $(K_b)_{RA} = 1 - (K_b)_{FA}$, right yeah. That's it. So, by doing that I can straight away substitute,

$$(P_b)_{RA} = [1 - (K_b)_{FA}] * \left(\frac{a}{g} - \mu_R\right) W$$

This is equation number 4. So, already we got it here right. So, we know the equation. So, we got $N_{FA}, N_{RA}, (P_b)_{FA}, (P_b)_{RA}$ all 4 equations. So, with respect to deceleration rate a/g that is what you got it right. So, now we will find out the front tire approach lock first. So, what do you expect? What do you expect, when the front wheels need to be locked first. What do you expect? Your $(P_b)_{FA} = \mu_S N_{FA}$, right yeah. So, already you know that from previous equation you know that. From equation 3, you can substitute for this. From equation 1, you substitute for N_{FA} when you substitute it right, we will see that what is going to happen, maybe I will move to the next slide. It will be much better your $(P_b)_{FA} = \mu_S (N)_{FA}$, this is supposed to be happened. So, I know this should be,

$$(K_b)_{FA} W\left(\frac{a}{g} - \mu_R\right) = \mu_S\left[\frac{W}{L}\left(l_2 + \frac{a}{g}h\right)\right]$$

So, I will cancel out W, that is already done. The remaining will be,

$$(K_b)_{FA}\left[\frac{a}{g}-\mu_R\right]=\frac{\mu_S}{L}\left(l_2+\frac{a}{g}h\right)$$

Now, I am going to separate these terms, because I need to bring it up a/g at one side, other things will be at the other side.

$$(K_b)_{FA}*\left(\frac{a}{g}\right)-\frac{\mu_S}{L}\left(\frac{a}{g}\right)h=\frac{\mu_S l_2}{L}+(K_b)_{FA}*\mu_R$$

So, if I take the common term a/g, separately right. So, I would expect,

$$\left(\frac{a}{g}\right)\left[(K_b)_{FA}-\frac{\mu_S}{L}h\right]=\frac{\mu_S l_2}{L}+(K_b)_{FA}*\mu_R$$

Finally, I would need this a/g alone ok. So, what I am going to do one more term, I will multiply L. So, that I want to cancel out the L also. Shall I move to the next slide right,

$$\left(\frac{a}{g}\right)\left[\frac{(K_b)_{FA}L-\mu_S h}{L}\right]=\frac{\mu_S l_2+(K_b)_{FA}\mu_R}{L}$$

So, this also cancelled out the a/g since we are talking about the front axle just bring it as a,

$$\left(\frac{a}{g}\right)_{FA}=\frac{\mu_S l_2+(K_b)_{FA}\mu_R}{(K_b)_{FA}L-\mu_S h}$$

So, if I rearrange this equation finally, I would arrive at deceleration rate right,

$$\left(\frac{a}{g}\right)_{FA}=\left[\frac{\frac{\mu_S l_2}{L}+(K_b)_{FA}\mu_R}{(K_b)_{FA}-\left(\frac{\mu_S h}{L}\right)}\right]$$

So, this is the equation ok you will get it. This is the deceleration rate of the front axle right. Question is what happened to your rear axle now? What change did you expect now? Instead of l_2 you will have l_1, right. And, $(K_b)_{FA}$ has to be replaced with the $1-(K_b)_{FA}$. Which is nothing, but your distribution at the rear axle that is one thing is there. Do you expect any sign change? Yes, this term will be changed ok this term will change. This is for front axle.

Similarly, for rear axle,

$$\left(\frac{a}{g}\right)_{RA}=\frac{\frac{\mu_S l_1}{L}+[1-(K_b)_{FA}]\mu_R}{[1-(K_b)_{FA}]+\frac{\mu_S h}{L}}$$

So, this is the equation for the rear axle ok. But question is now I got it. I got the deceleration rate for the both front axle as well as the rear axle. Then, how do I know which will lock first? When do you expect which will lock first? When will you know right, when will you know? I have, $(a/g)_{FA}$ front axle, $(a/g)_{RA}$ rear axle. So, if my front will lock first, what do you expect in the deceleration rate. Remember, we are talking about deceleration rate without taking the sign. Normally, deceleration will be negative right, we are not discussing anything negative aspect, we are taking the quantity as it is without any sign. Because when we do the problem there will be confusion. So, before that itself I am just giving a clarification. Let me ask, so, when do you expect the front will lock first. When the deceleration rate of the front axle less than the rear axle, the front will lock first. If the deceleration rate of front axle greater than deceleration rate of rear axle, then, your rear axle or rear wheel lock first, right. So, when you have front wheel lock first, what happen? Loss of direction right. When your rear wheel lock first direction instability. So, these are the thing we are going to happen ok. I think I will stop now. So, we will discuss more on this aspect by solving the problems ok. Thank you.