Design of Mechanical Transmission Systems

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Week – 07

Lecture – 20

Lecture 20_ Brake: Static and Dynamic Analysis

Good morning to all. So, we will continue brake design system aspect. So far, we have discussed automobile brake system design. In that brake working fluid methods, brake types, brake torque requirement, within that we have discussed about drum brake system, disc brake system. Now today we are going to discuss about dynamic analysis. So, within the dynamic analysis we will go for the mass transfer. how the weight is going to transfer that aspect we are going to discuss. And also, the brake distribution particularly for braking optimum right in that aspect also we are going to discuss. Two aspects we are going to discuss mass transfer as well as the brake optimum. Let us move on to the dynamic analysis.

So, in the entire system what we are going to do we are going to focus only two axle vehicles right. So, we have a multiple axle also right. But our focus to understand for automobile vehicle aspect, when you say automobile vehicle aspect normally say two axle system. So, we will focus primarily for the two-axle system. So just this only further we will do the static analysis, static condition okay. So, I introduce a certain term also. So, since we are talking about the two-axle system, we have a wheel here another wheel right. This is a schematic diagram. So, this is your two-axle vehicle is a probably we will take it as a take it as a car okay. And this is you're the cg or the center of gravity of the car okay. Then from center of gravity, this is road level, the height is called h this is h okay. Then the distance between two wheels is called wheel base normally called as a L okay. h is the height from the cg to the road level and L is the wheel base and within that from the center of the front axle this is a front A okay and we will take it as a B right. So, from here to here the distance is l_1 right from the front center of the front axle to the before the center of gravity is l_1. After the center of gravity to the rear axle wheel center is l_2. So, the L again it says summation of $l_1 + l_2$ right. So, from the center of gravity, the entire load of the vehicle is acting at the down W is the vehicle weight that is clearly given. Any other information nothing right. So, at the moment this is in the static condition okay this is the static condition.

So, what do you expect the reaction forces to the front axle and rear axle. what do you understand? Normally, I will take it as a front when I say front okay maybe I will write here. Front axle, I will refer as a FA okay then rear axle, I refer as a RA. So, these are the terms you are going to use in this entire dynamic analysis yeah. So now the reaction force if you like to see the what is the reaction force when the vehicle is at the static condition okay. So, take the moment with respect to B right when you do that reaction force of W_{FA} is a front axle

$$W_{FA} * L = w * l_2$$

So, W_{FA} is nothing but is a simple supported beam kind of simple supported beam. So, $W_{FA} = Wl_2/L$, this is what you are expecting yeah. So, this is the reaction force of the front axle. Similarly, for rear axle W_{RA} what do you expect $W_{RA} = Wl_1/L$. So, this is you are expecting. So, maybe you can take it as equation a and this is equation b. So, this is static condition. Now assuming the vehicle is moving in the forward direction so when you moving forward direction what happen to the inertia of the vehicle right inertia of the vehicle. So, Wa/g. So, that is what we will take as the inertia right. So, when the brake is applied the vehicle, the deceleration will move in this direction so that is you take it as a deceleration *a* okay. So now we have done the static condition. Now we will move for the dynamic condition okay.

Dynamic condition, same way what we did same thing we will do yeah. So, we will do that when it here instead of W_{RA} I will put as a reaction force N_{FA}, okay this is the front axle with respect to rear axle equal to what do you expect? You will expect $N_{FA} * L = W * l_2$ the first right that is will be there that will be there. What other item will add, the inertia thing also will add right, the inertia thing also will add okay. That is nothing but Wa/g okay.

$$N_{FA} * L = W * l_2 + \frac{Wa}{g}h$$

This is what happened yeah. So, this is there. So, this is moving in the dynamic condition. Suppose the same vehicle stops what happened to this term, assuming that the vehicle is at rest at rest. what happened to this term. the inertia will be 0 right. So, the entire term will be goes off okay. that means $N_{FA} * L = W * l_2$. Otherwise, your reaction force at the front axle equal to $N_{FA} = W * \frac{l_2}{L}$ which is same as W_{FA}, right equation *a* yeah right. From equation *a* this is what your observation you can see that. So just to understand what exactly happening when the vehicle moves in a dynamic condition and taking rest.

Now, we will proceed assuming the vehicle is moving on the road, when you moving on your thing is,

$$N_{FA} * L = W * l_2 + \frac{Wa}{g}h$$

That is clear right. So, this is Wl_2/L, even more simplify that okay. Take it as a N_{FA}, yeah.

$$N_{FA} = \frac{Wl_2}{L} + \frac{Wah}{gL}$$

That is fine. Previously, we found Wl_2/L equals to what? W_{FA} right, that is what we know that So, W_{FA} if you transfer that. Then,

$$N_{FA} = W_{FA} + \frac{Wah}{gL}$$

So, this is the reaction force you would experience when the vehicle moves in dynamic condition okay. So, what did you understand, I know this is already there right this is already there your reaction force already there. So, what is happening now. What is the thing additionally adding into your front axle. This much weight transfer added right. Can you see that yeah. So this much when you applying the things this much weight is transferred, it is added from where is added somewhere else right somewhere else is added. the vehicle weight is anyway W, it cannot that cannot be changed vehicle weight is W so somewhere else it taken and add into the front axle and what happened to the somewhere else. maybe I am sure you know N_{FA} when you see that the vertical forces balancing, $N_{FA} + N_{RA} = W$ right yeah must be equivalent to W okay. So, if I want to find out what will be the N_{RA} right. Then,

$$N_{RA} = W - N_{FA}$$

That is the thing yeah. So, I substitute this what is the N_{FA}, already I know that

$$N_{RA} = W - \left[W_{FA} + \frac{Wah}{gL}\right]$$

I think yeah this is correct only. Shall I move on to the next slide yeah okay. So, if you rewrite again,

$$N_{RA} = W - \frac{Wl_2}{L} - \frac{Wah}{gL}$$

Now, it is obviously you know the answer, okay.

$$N_{RA} = \frac{WL - Wl_2}{L} - \frac{Wah}{gL}$$

So,

$$N_{RA} = \frac{W(l_1 + l_2) - Wl_2}{L} - \frac{Wah}{gL}$$

$$N_{RA} = \frac{Wl_1 + Wl_2 - Wl_2}{L} - \frac{Wah}{gL}$$

This term will cancel each other. Finally, your N_{RA} right the reaction force at the rear axle is,

$$N_{RA} = \frac{Wl_1}{L} - \frac{Wah}{gL}$$

So, what is this term called now the same as the weight added at the rear axle is subtracted right. This weight is removed. So, that is what you see when the brake is applied okay yeah. So, this is weight, this much weight is removed right. So, whatever we did its ideal condition what are we know now we found out the reaction forces of the front axle and rear axle in ideal condition. In realistic condition, what happened to the braking effort. What will be the braking effort, how the vehicle stops. The braking force alone is enough or it has to overcome other forces. See one thing you should very clear, the force to stop the vehicle that is a braking force okay that is one thing is there. Other than that, the other forces also the brake force should overcome what are they. Remember when we talked about the machine tool gearbox and automobile gearbox, we said certain forces, tractive effort. So not only rolling resistance not only a rolling resistance then you have a drag coefficient which is the aerodynamic resistance, air resistance that two thing and third it's a very important your gradient, when the vehicle is moving uphill so that's also will add up right. So, though those things should do overcome not only that you have to overcome those three resistances plus the braking things. so that's we are going to do in realistic condition now. So, what I'm going to do.

So, I will take again the slope in realistic condition. This is the slope; the vehicle is moving upward direction assuming this is the θ for the gradient aspect right. This is for gradient aspect. So, this is one wheel, this is another wheel okay. The vehicle is moving like this yeah. That's there. So, now we have a center of gravity CG right, we have center of gravity center of gravity, it's moving up so you have inertia this way right inertia Wa/g inertia is there. Then, you have a deceleration rate that's a okay. This is supposed to be the weight and will act angularly because it's moving in slope right. So, W will be there. That's vehicle weight. Of course, you will have a another, when you divide this way when you split the weight, you will have two components. One component in horizontal direction, another component in vertical component okay. So, this is fine right. So, from here, if you draw like this right. This would be your wheelbase L. Again, from center of gravity to rear axle is the l_2. From center of gravity front of rear front axle is the l_1 okay. This is fine okay. What else we have right, your rolling resistance will come into this direction right. Rolling resistance will come and what are opposing thing air assistance, rolling resistance and the gradient all those things will be coming okay. Now, I will take it as this is the aerodynamic resistance, R_a this is air resistance or aerodynamic resistance okay or else aerodynamic resistance okay. That's fine. So, still it's very not clear, we have to see that the balancing of the forces right that we need to find out, yeah. tell me, so this will be a reaction force right. This will be your reaction forces. N_{FA} front axle reaction force and this is your rear axle reaction force that's fine. What else you expect now apart from you will have a rolling resistance also right rolling resistance will move the oppose the vehicle movement right. So, rolling resistance I will say RR rolling resistance it's a rolling resistance okay. So, you can have $(RR)_{FA}$, okay. That's a front axle. Again, you will have a $(RR)_{RA}$ also okay. So, now we have rolling resistance okay. Now, when you apply a brake, what do you expect. Where will be the braking force to stop. Vehicle is moving in this direction. Please understand that the vehicle is moving in this direction. The braking force you would expect in this direction right yeah. Another braking force will come here. so $(P_b)_{RA}$ and this is $(P_b)_{FA}$. P_b is a braking force okay. Anything else or did we miss now. I think we covered right. Generally, your braking force right the braking force P_b supposed to be

$$P_b = \frac{T_b - \Sigma I\alpha}{R_W}$$

This is a braking force the braking force mainly related with your braking torque and this is actually nothing but rolling masses. This is for rolling masses, whereas R_W is the tire radius the tire radius. I is the moment of inertia, α is the angular acceleration that is given now. That is generic but in in realistic condition, the retardation that overall retardation effect is combination of your braking effort, rolling resistance, aerodynamic resistance, gradient and transmission resistance also there, okay. So, in fact I will write down the total retard resistance is

$$P_{Res} = P_b + R_a \pm W \sin\theta$$

I have given the plus or minus. I will tell you what is the mean plus or minus that will give. So, normally you have a two component, one component will have a gradient and the other component will interact with the ground. When you interact with the ground which is related to your rolling resistance right $\mu_R W \cos\theta$, okay. Plus, R_t is your rolling resistance okay.

$$P_{Res} = P_b + R_a \pm W \sin\theta + \mu_R W \cos\theta + R_t$$

Now, I will again explain that this is your gradient $W \sin\theta$ is a gradient okay, right. When do you use the plus, when do you use the minus. Plus, when you move up that is add up your resistance right. Plus moving up. Minus moving down okay, that is what you use it yeah. That is fine. So, R_t is the transmission resistance it is a small quantity, okay. Now, so it is the small quantity, we can neglect this R_t okay. How about this component $\mu_R W \cos\theta$. Already, I have mentioned that μ_R is the coefficient of friction in rolling right, rolling coefficient of friction, okay. What is the typical value μ_R equals to 0.02 to 0.03. So, W is the weight of the vehicle $\cos\theta$ anyway will give you even if it assumes as 0, you will have maximum load. But, with that if you multiply 0.02, do you think that makes sense. Does not right. So, we will omit that term also okay. We will omit that term. Finally, we will have two things right. So, we will have the gradient aspect, we will have the braking aspect and rolling aspect. With that, we will try to find out how these forces are acting when the vehicle is braking with respect to front axle as well as rear axle.

So, from this I will write the rewrite the equation. So, during braking, right. so,

$$N_{FA} * L = W * l_2 + \frac{Wa}{g} h - Wh\sin\theta - R_a h$$

So, I rewrite the equation now right,

$$N_{FA} L = W l_2 + h\left[\frac{Wa}{g} - R_a\right] - Wh\sin\theta$$

Then, within that I can take another thing,

$$N_{FA} L = W l_2 + h\left[\frac{Wa}{g} - R_a - W\sin\theta\right]$$

And, if we want alone the front axle the reaction force is,

$$N_{FA} = \frac{W l_2 + h\left[\frac{Wa}{g} - R_a - W\sin\theta\right]}{L}$$

So, this is we can take it as equation 1, okay. So, this is for front axle. Clearly, you know that front axle this much is a added weighted right, it is added one. So, similarly if I want for a rear axle what happens to my reaction force,

$$N_{FA} = \frac{W l_1 - h\left[\frac{Wa}{g} - R_a - W\sin\theta\right]}{L}$$

So, this is the equation number 2. This is what happened okay. Now, we got the reaction forces both front axle and rear axle okay. Now, we will move on to the next one yeah.

Now, we will take the entire the breaking force, we are now considering the breaking force. look at the figure again, we will take the breaking force. When you take the breaking force,

$$P_b + \mu_R W = \frac{Wa}{g} - R_a - W\sin\theta$$

So, we are talking everything with respect to equilibrium condition at the moment. So, but we know this term right $\frac{Wa}{g} - R_a - W\sin\theta$ already is known to us okay. So, when you substitute in equation 1, okay we may take it as an equation number 3. Substitute in equation number 1 what do you expect, we will have

$$N_{FA} = Wl_2 + h(P_b + \mu_R W)$$

I am just replacing the entire thing with this term and substitute in equation number 1 by L.

$$N_{FA} = \frac{Wl_2 + h(P_b + \mu_R W)}{L}$$

You can take it as equation number 4. Similarly, for N_{RA}

$$N_{RA} = \frac{Wl_1 - h(P_b + \mu_R W)}{L}$$

This is equation number 5 okay. So, the μ_R is the rolling resistance okay. It is the rolling friction between the tire and road okay. But actually, I will introduce one more thing is called μ_S. It is the road adhesion coefficient okay, right. So, road adhesion coefficient is nothing but indicating about in sliding direction, in sliding aspect. Let me ask question which resistance is more it is a sliding resistance or rolling resistance. Sliding right. Usually, the sliding resistance will be more. It is obvious that any given time your μ_S is much greater than your μ_R that is clear right. So, in that case when do you expect the maximum braking effort. When do you expect the maximum braking effort, right. I am talking about $(P_b)_{Max}$, when do you expect maximum. When $(P_b)_{Max}$ should be your road coefficient with the load right. This is the condition we expect the maximum effort right. This is where getting maximum braking effort right. So, that is as a whole, if you respect to the individual front axle, rear axle. I can rewrite again,

$$(P_b)_{Max} = \mu_S N_{FA}$$

This is equation 6. Similarly, my $(P_b)_{Max}$ depends on for rear axle right.

$$(P_b)_{RA} = \mu_S N_{RA}$$

But you know that what is the values of already you know N_{FA}, N_{RA} also. if substitute them, I would expect the $(P_b)_{Max}$ of my front axle is,

$$\left(P_{b_{Max}}\right)_{FA} = \mu_S \left[\frac{Wl_2 + h(P_b + \mu_R W)}{L}\right]$$

And, you can rewrite again in in simplistic way, we take the load alone and keep it outside,

$$\left(P_{b_{Max}}\right)_{FA} = \mu_S W \left[\frac{l_2 + h\left(\frac{P_b}{W} + \mu_R\right)}{L}\right]$$

So, this is the thing right. So, but we know $P_b, W,$ what is that. Your maximum braking effort depends on what. Just now I told you in the previous slide. It depends on what. Your surface friction, the braking effort is depends on the your surface friction, road adhesion friction right. That's what happens. So, we can rewrite again,

$$\left(P_{b_{Max}}\right)_{FA} = \mu_S W \left[\frac{l_2 + h\left(\frac{P_b}{W} + \mu_R\right)}{L}\right]$$

But, $\frac{P_b}{W} = \mu_S$, if I substitute, I expect,

$$\left(P_{b_{Max}}\right)_{FA} = \mu_S W \left[\frac{l_2 + h(\mu_S + \mu_R)}{L}\right]$$

This is what happened. So, this is the maximum braking effect you would expect in front axle. you can take it as equation number 8. Then similarly,

$$\left(P_{b_{Max}}\right)_{RA} = \mu_S W \left[\frac{l_1 - h(\mu_S + \mu_R)}{L}\right]$$

So, this is the equation number 9, okay. So, my question is what is that you see now. So, we have found your braking effort right for the front axle and rear axle individually yeah. Remember, earlier when we talked about activation force $P = \frac{M_n - M_f}{C}$. What is this and what is that. This is the maximum braking effort it's happening between the road surface and the tire. I am talking about this term all right. This is what happening. Whereas, this is happening from your effort braking effort, which is coming from your pedal force right. From the pedal force to the where the brake drum aspect right. that's what happening. So, my question is, as a brake design engineer, what are the things are you looking for? What are the things are looking for? Look at the equation right. So, your μ_S is variable right. You can have a skiddy road or a pitman road or else muddy road also. The μ_S is variable and μ_R also variable okay. Then, which one we have to consider during the brake design. Your weight (W), h and L. Now, can you see that how these dimensions are crucial for designing your brake. Can you observe that. So, these are the very important thing okay. But they are fixed or variable. They're fixed, they're fixed one. So, when you talk about designing you have to take care of your weight, take care of the the height and the the wheelbase that is there, but again the performance will change. Based on your road adhesion factor, μ_S as well as your rolling distance μ_R. Can you see that okay. So, these are the things now. Why we are discussing is, even though when you apply a brake you have already observed that certain amount of weight is transferred from the rear axle to the front axle right. So, when you do that when this is happening, we have to make sure that the braking effect also should be divided right, should be optimized right. In fact, the distribution of braking force as primarily depends upon your hydraulic pressure and wheel cylinder. When you apply a brake pedal right, so from the brake pedal fulcrum to will go to vacuum booster, vacuum booster to master cylinder, from the master cylinder to it goes to the finally wheel cylinder right, from the wheel cylinder then it will expand to activate things that's what happening. So, in between how to optimize it right. So, the distribution of making force between the front and rear axis exactly. So, whatever you got $(P_b)_{Max}$ front axle, rear axle must be same during your the pedaling effect also.

So, in fact I will introduce a new term called $(K_b)_{FA}$ is a brake distribution factor front axle. $(K_b)_{RA}$ distribution again for rear axle. If, I do that then,

$$\frac{(K_b)_{FA}}{(K_b)_{RA}} = \frac{\left(P_{b_{Max}}\right)_{FA}}{\left(P_{b_{Max}}\right)_{RA}} = \frac{l_2 + h(\mu_S + \mu_R)}{l_1 - h(\mu_S + \mu_R)}$$

Can you see that now? The distribution is purely depends upon the your the vehicle dimension, wheel base, height as well as μ_R and μ_S, okay. So, let me ask one question, when brake when you apply a brake, you would expect the braking forces should be developed same time on the both front axle and rear axles. Then, on the vehicle stop. But, in realistic condition this is valid only for one particular μ_S value. The optimum can be maintained only one particular μ_S okay. But road condition keeps changing. So, because of that either the front wheel lock first or rear wheel lock first that will be there okay. So, in fact this is called brake distribution factor K_b. So, normally the K_b

$$K_b = (K_b)_{FA} + (K_b)_{RA} = 1$$

That has to be maintained. obviously right. So, if you make a hundred percent effort certain portion will go to the front axle, remaining go to the rear axle. So, any given time your K_b front axle plus K_b rear axle must be equivalent to one. That cannot be changed. But your load condition keeps changing okay. So, that a thing we will discuss in tomorrow lecture. So, in fact we are going to discuss in detail, what happened when the front axle lock what are the problem you would expect? And, the rear axle lock what are the problem you are going to expect, okay. So then how to address the problem like which is the most safety in that condition, how do I know which axis lock first that’s also we will discuss. okay so thank you. I'll stop now we'll discuss the remaining thing tomorrow lectures thank you.