Design of Mechanical Transmission Systems

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Week – 04

Lecture – 11

Lecture 11_ Automobile Gearbox: Gear Failures and Material Selection.

Good morning, so welcome to today's lecture. So far, we have been discussing automobile gearbox in this we have completed general engine operations aspects, transmission types, ray diagram which is also called as saw tooth diagram, kinematic diagram and in last lecture we have done the number of teeth in each gear as well. Now we are going to discuss about the gear failures briefly then selection of gear materials and finally find out module calculation as well. So, these three we are going to discuss in today's lecture gear failure, gear material selection and module calculation. This is the problem we have done as I said earlier, we have chosen the gear type, we have done the ray diagram, kinematic diagram. Now we are going to focus about the suitable material for main lay shafts including gears aspect and later we will do the bearing selection, lubrication, heat losses so on.

The basic failures modes of gears classified into two different aspects, one is non-lubrication related failures, the other one is the lubrication related failures right because meshing happened between the gears due to contact right. When you have a contact friction is always there okay to avoid friction you have to have a lubrication. When you say lubrication, you can have a liquid type lubrication as oil or else you can have grease both depends on the application you can choose and if you look at generally the gears are subjected to repeated load. When you say repeated load will have a fatigue failure, common fatigue failure which is a typical wear and tear. When you use any component or any system over the period will wear out okay. So due to cyclic load that is usually we say as a fatigue aspect you can see the low cycles fatigue less than 1000 cycles and high cycle fatigue these are the common thing and if you look at the overload which is beyond the design category then you can have a brittle fracture, ductile fracture plus deformation within that cold flow, hot flow, indentation, rolling all those things is there. But we will emphasize more of bending fatigue that is one thing, the other one is wear okay wear due to the contact things. Again, in the Hertzian fatigue you will have a pitting, micro pitting and surface or sub case fatigue that is case crushing that is also very critical.

In the wear aspect we have adhesion, abrasion, corrosion, fretting corrosion, electrical discharge damage of course this is very critical at the moment when you go for electrical vehicles particularly electric wheels bearings will be used of course you are right. When you have the electrical current passing through that also would damage. The other is scuffing, scuffing is nothing but when the lubricant itself is failed due to heating right when you have heating overloading heating the viscosity reduces when the viscosity reduces the film thickness reduce when the film thickness reduce you will have a direct metal to metal contact. When you have metal to metal contact you will have a severe wear it is a seizure like a seizure. So, in that category we have scoring, galling, seizing all those things there. So that means when we when scuffing happen the metal will be totally seized you cannot use any metal it is like a condemned that is the failure would experience. The relevant figures I will share later okay just to understand. Then, we talk about gear performance limits okay.

The gear performance limit is two aspects one is the pitch line velocity or circumferential velocity then the limit torque operation you can see that right. So as increase this is the x axis as increasing the speed and as increasing the load. Normally as I said at the beginning of low speed of the medium torque requirement you will have a wear aspect wear limit it is called cold scuffing which is another form of the scuffing. As you increasing your circumferential speed or pitch line velocity and the torque you would expect pitting limit. A pitting limit nothing but your contact stress aspect which is related to fatigue also okay. Pitting limit is there so if you beyond certain torque limit and the speed the pitting is limiting things. And this is the most critical aspect as you go very high speed at the torque you would end up with the scuffing limit which is the severe damage of the gear material and the tooth failure. So, we have to choose your speed as well as torque within this area where you do not have any failure. Talking about with the area with no damage. So, you try to limit your operation okay. So, when you so the question is how do we address? How do we address okay? Suppose I do not have any choice I had to operate this is the speed 10 meter per second my application with the high torque limit then you know these are going to be picture it is going to affect the performance. So, I am talking about scuffing limit or tooth failure or pitting limit. In that case I had to provide surface engineering where coating or heat treatment such a way that make the gear profile surface harder to meet the requirement. Yeah, okay that is how we need to address there are lot of things are there but our focus only design aspects so we are limiting.

Now look at what are the things we look for when you select materials for gear application. It should have a high tensile strength, good endurance limit, should sustain for longer duration usually it is called million cycles where I should operate in with million cycles as well as low coefficient of friction, wear resistance then should be very smooth and without any noise and the cost should be as low as possible okay. And the gear metal selection you can see that usually we will choose cast iron material the reason is it has a low cost, good machine ability, high wear resistance and good noise abatement. Then steels within the steels they are preferred due to higher strength, wear resistance improve with the heat treatment HT means this is heat treatment okay. So, heat treatment, heat treatment and within that a Brinell hardness is come into the picture. The hardness is very critical parameter for wear aspect higher the hardness lesser the wear right. So, when the Brinell hardness less than 350 that can be used for light and medium duty drivers and Brinell hardness greater than 350 you can go for a heavy-duty application. So, two categories within the steel material. In fact, we can see the table where it gives a comprehensive the material selection.

Look at this okay we have a cast steel, a machine construction steels, through hardened steels where when you have through hardened the entire material will be hardened right that is why through hardened. Carburizing steel which is increase the hardness only on the surface level because only the gear profile is going to have a contact right and of course the fatigue failure is happening only surface and surface level subsurface level okay. So as long as you protect the teeth by providing a carburizing aspect then it will be withstood for longer duration. Similarly flame induction hardening steels, cyanide steels and grey cast iron okay and this is the important information. Sometime this is actually this is your bending stress σ_b and σ_h or σ_c both are same which talk about your contact stress right. The bending stress related bending failure your contact or a Hertzian, contact stress or pitting stress or wear stress are the same okay. And let us look at the certain material as you increase in operation you would see that the stress level is increasing right. The stress they are increasing similarly as you go for a higher treatment from maybe you can see that through hardening to carburizing flame the hardness also increasing right that you can see that. So more the better treatment you will have a harder material okay and more important I would like to emphasize suppose if you have chosen C45 as your gear material it is a conch and hardened and the Brinell hardness somewhere 185 and above then your σ_b 160 MPa whereas your contact stress or Hertzian wear stress 540 MPa okay. Now I will choose another material 35NiCr8 another variation that is hardness is very higher 400 similarly the bending stress increases to 230 whereas my contact stress right is a 1200 MPa okay. If you look at the relationship when the bending stress is 160 at least three times higher you would see the contact stress at least three times higher. So randomly we check 230 so can you see that we have 1200 so maybe we would choose another material in carburizing steel 18CrNi8 so this is 330 right and 330 and you have an 1820 can you see that. So, what is that you can infer from the table your contact stress even more critical compared to your bending stress okay. So, because the failures most of the failures are related to fatigue and the wear aspect rather than the bending things.

Now we will go for the module calculation so the first thing is to avoid undercutting and interference we need to have minimum number of teeth you can see that if gears are cutting using generating methods automatically the interference portion of the teeth are removed this is called undercutting to avoid interference the minimum number of teeth needed in spur and pinion gear if you having both the gear ratio one that means both are having same teeth then you can use this equation,

$$N_P = \frac{2k}{3\sin^2\phi}\left(1 + \sqrt{1 + 3\sin^2\phi}\right)$$

The ϕ is nothing but your pressure angle okay this ϕ is nothing but your pressure angle is a pressure angle full depth means maybe I can show the picture let us say, this is what happen this is the when that this is called full depth FD and this is the full depth and this instead of that if gear is very broad and very short right this is called stub because it is very broad it is called stub teeth it is called stub teeth right this is one thing that is variation you should know that then if the mating gear is as a more teeth than the pinion then minimum number of teeth again will change depends on your gear ratio this is the equation,

$$N_P = \frac{2k}{(1 + 2m_G)\sin^2\phi}\left(m_G + \sqrt{m_G^2 + (1 + 2m_G)\sin^2\phi}\right)$$

Here, the same k is your full depth or sub depth full depth k=1, sub depth k=0.8, where m_G is nothing but your i_G, m_G is nothing but your i_G because the gear ratio is also play a vital role and the smallest pinion that will operate with the rack and pinion this is for rack and pinion and N_P, P indicate for the pinion please understand this is suffix P is talk about pinion because pinion is the one which transfer the load transmit the things so we always concerned focus based on the pinion rather than the mating gear yeah tooth stresses you could see and this is a very fantastic figure taken from TJ Dolan okay 14th eastern photo elasticity conference proceeding which was published in 1941 still is valid look at this so what happens so this is the gear okay you try to move when you move it the force will acting on the teeth in this way okay this is in this thing when you do that will go for the bending and also make a rotation that is what happening right in fact I will give you in a minute now so look at this so what is what you think this wherever you see the patterns or stress concentration, stress concentration so okay when you give the like this which is the weakest portion the weakest portion here you can see this these are the root you can say this is the root portion right root radius as the root portion this is the weakest portion another weakest portion where you apply the load you can see that right so when the force is contacting try to rotate the mating gear the stresses will happen so when we design we need to focus this stress concentration because those are the very critical for the failure aspect in the gears yeah so photo elasticity module force per gear tooth is given now we will go for the gear equation of course this is covered in your undergraduate I just go through quickly so because from there we are going to derive module equation.

So, usually the assumption are a full load is applied to the tip of the single tooth it is neglecting radial load okay the load is uniform across the width neglect a frictional losses and the stress concentration is negligible they omit all the critical aspect look at this the load never be applied only the single tip of the tooth okay that's and the radial load when the gear is there it is subject to the I will show maybe here again right this a gear assuming that so when this happening load is like this happen we have to split the load into two different aspect okay this is your resultant load this is the apply load so one is F_T is called tangential load other one is F_R is the radial load, okay the tangential one which makes the rotation right which makes the rotation and also transfer the torque whereas F_R will try to make a contact between the two gears will minimize the separation okay so you could see that the when load is applied here they have taken the only the F_T they are not considering the F_R that's another information no friction and the stress concentration they are not even considering the stress concentration in this aspect right so the bending strength for the gear tooth equation I have taken from the Shigley you can see that as I mentioned earlier the load is transferred here okay then we can split into two aspect F_R and F_T, the t is the tooth thickness tooth thickness okay and this is called height tooth height h there one more parameter the means this b is the face width of the gear right so assuming that you have a so this is the face width of the gear right so assuming that you have a gear just for representation right this is your gear that to protect in the three dimensional like this yeah so if you look at this this is your face width of the gear right so this face width b right and this is your thickness t yeah I am sorry this should be your height h this is the height and this is your thickness and this is your face width so load will act here only at the tip it is like a cantilever beam the load will act here only this portion so this is what happening if you look at this you can see this diagram where you can see that the height is given when you cross section b is given and the F_T, and this is a fixed portion that is why I just hide it like this yeah so that is your tooth thickness okay so what happened the Lewis is the one who derived the equation he assumed parabolic curve so that along the parabolic the strength will be uniform right with that we derive the equation right so by considering the strength will be uniform otherwise the strength if you take as such the gear shape it is a invalid right the strength will be variable which we do not want to have it because we have to start the equation in simplistic formation so using standard sigma the bending equation finally derived as,

$$\sigma_B = \frac{My}{I} = \frac{6F_T}{bt^2}$$

So, substitute for a high value moment of area moment of inertia I is the length of the length when it bends to and M is the moment and substituting you will finally get it.

So, now we will go for the agma standard that is a standard method for finding other things so the since we omitted all the important parameters it is our job to get the exact solution the question is if you find the stresses without taking the all those critical parameter you end up with the bigger size right engineering is always for the optimization you want to have a as good as possible as small as possible so then you need to do the correction methods when you do the correction methods you have to think about the pitch line velocity I need to think about the manufacturing accuracy which is going to be responsible and the contact ratio stress concentration degree of shock loading the way you operate the load that is also a matter then accuracy and rigidity of the mounting because the mounting is not good enough or structure I am talking about structure is not good enough then you cannot place the gears so that is also you have to consider right so in fact we will discuss more of structural aspect later stage the moment of initial gears rotating machinery we talk about gears rotating machinery we are talking about is not considering the other part such as oh your flywheel right flywheel anything else clutch those are the rotating parts right that has to be considered right how about break also will come into the picture but at the end not before that these two things are will come before so now by taking consideration we will get the equation sigma bending equation,

$$\sigma_B = F_T K_O K_V K_S \left(\frac{1}{bm_t}\right)\left(\frac{K_H K_B}{Y_J}\right)$$

So, these are the correction factors you can see this for bending where K_O is the overload factor, K_V is the called dynamic factor, K_S is the size factor, P_D is the diametral pitch okay but we use a different value we will go for SI units and this is you think about this is the these are the important thing and these three parameters others are correction factors and this is the important by keeping them right m_t is the transverse metric module and again b is the face width in millimeter and K_H is the low distribution factor, K_B is the rim thickness factor okay and Y_J is the geometric factor right for bending strength okay which is nothing but a Lewis form factor Y is the Lewis form factor yeah what we are going to do for simplicity and we will make it all the correction factors one and similarly this is for the pitting stress okay the pitting stress or wear resistance okay wear equation or contact stress all the same.

$$\sigma_C = Z_E \sqrt{F_T K_O K_V K_S \left(\frac{K_H}{db}\right)\left(\frac{Z_R}{Z_I}\right)}$$

So, this is σ_C right, you have all the parameters except K_B would not have any K_B here but we have other new parameters Z_R and Z_I okay so where when we talk about Z_E is the elastic coefficient I will discuss what is that mean elastic coefficient, Z_R is the surface condition factor and b is the face width which is already known and pitch diameter d is always represent pitch diameter suffix p which indicate about as I said earlier for the pinion diameter okay now Z_I is a geometric factor for pitting resistance yeah. Now we have to find out the module calculation that is what our aim okay and do you understand difference between stress and strength, stress is coming from you apply the load within the system or within the what you call it the component that is called stress strength is coming from the material both are different right both are different okay so what we try to find out sigma allowable σ_{all} is the bending strength is coming from the material right,

$$\sigma_{B,all} = \frac{S_t}{S_F}\left(\frac{Y_N}{Y_\theta Y_Z}\right)$$

this is from the sigma allowable from material where, S_F is the factor of safety I will just tell you what is that mean S_F is the factor of safety so I will discuss what is that mean yeah okay. So, S_t is the allowable bending stress that is coming from the material from the material we do the correction factors is the Y_N is the stress cycle factor for bending stress which nothing but the endurance curve okay and Y_θ is the temperature factor or what is the temperature and we are going to operate, Y_Z is the reliability factor. The reason is the gears are subjected to fatigue failure when you say fatigue failure it is a question matter of reliability whether you want 90 percent assurance 95 percent assurance or 99.999 so that is the reliability factors based on that again your allowable stress will change and S_F is the factor of safety which is stress ratio and this is the sigma all okay if I take out this alone separately right you can see that so S_F is coming in this nicely.

Similarly for the pitting should not exceed allowable stress,

$$\sigma_{C,all} = \frac{S_C}{S_H}\left(\frac{Z_N Z_W}{Y_\theta Y_Z}\right)$$

Here, we have other S_C is the allowable contact stress taken from the material then these are the correction factors. S_N is the stress cycle factor for pitting stress, Z_W is the hardness ratio for the pitting stress, Y_θ, Y_Z are the temperature and reliability factor respectively and finally $S_H$is the AGMA factor of safety which is the stress ratio for pitting aspect okay for the pitting aspect yeah.

Now we will do the calculation okay for this we do for the bending stress. So, factor of safety S_F is the ratio between your maximum allowable stress and σ_B from the calculation yeah. So, we know from the equation σ_B given as,

$$\sigma_B = F_T K_O K_V K_S \left(\frac{1}{bm_t}\right)\left(\frac{K_H K_B}{Y_J}\right)$$

This is the equation we have. So, see these are the correction factors nothing to do with the equation for simplicity what we are going to do we will take the all these five correction factors as a K_{Total} okay. So,

$$K_{Total} = K_O K_V K_S K_H K_B$$

We will take it as it is. By doing that I simplify the equation,

$$\sigma_B = \frac{F_T K_{Total}}{bm_t Y_J}$$

This is the equation okay so this is the equation one, yeah. Now we will go for the sigma allowable stress from the material, σ_B equal to your S_T strength of the material, Y_N is the correction factor, Y_θ, Y_Z is equation number two. As far as the design consideration your material strength should be greater than your design strength right material strength should be greater than the design strength that means,

$$\sigma_B \leq (\sigma_{all})$$

which is talk about your allowable right from the things yeah okay. I think this is the equation number three. So, what we are going to do we will substitute one and two in equation three to get the simplify equation maybe I can do it here. So, I know what is the σ_B here. So, you can substitute equation one into three I would expand this but F_T okay $F_T.\frac{K_{Total}}{bmY} \leq (\sigma_B)$. For me I want to find out the module right that is the aim of our thing so module greater than equal to if you keep the module alone the other comes will comes as like this $m \geq \frac{F_T K_{Total}}{(\sigma_B)Yb}$. This is what happens so I will rewrite again, $m \geq \frac{F_T K_{Total}}{(\sigma_B)Yb}$ right and σ_B allowable things okay this is the equation. So, I want everything in terms of module aspect right. What is the equation for torque equation guess torque equal to torque equal to your tangential force into half of the pitch diameter right. Can you say that yeah $T = F_T * \frac{d}{2}$ that is you can say that okay. So, this is okay so can I substitute that one so that means I can replace F_T with respect to *T* okay. So, $F_T = 2T/d$ that is I can do that $\geq \frac{2TK_{Total}}{(\sigma_B)bdY}$, K_{Total} will be remain there okay. what happened module equal to what is definition of module equal to pitch diameter divided by number of teeth $m = d/Z$ because I have d term here so I can replace I can replace if I do that right. So if I do that $m \geq \frac{2TK_{Total}}{(\sigma_B)mZbY}$ and y it take this m to here, $m^2 \geq \frac{2TK_{Total}}{(\sigma_B)ZbY}$ yeah so *Z*, I will have *b* will be there and *Y* is there this is the equation I have okay. So, this is the equation. Now I introduce one important parameter is called structural support okay structural support right. The structural support nothing but ψ is the structural support $\psi = b/m$. While doing the problem I will explain about the ψ aspect okay remember we are not talking about the only the design in the gear right we are talking about designing of the gears for gearbox aspect okay. So, one of the main functions of gearbox what is it you should provide the structural support also right that is one of the important information we have discussed earlier. So, it is not only to provide the transmission at the same time you should give structural support also right. So that is the very critical parameter for operation of gearbox in the design of gearbox. So that is why when you use one new term b/m structural support okay. So instead of *b,* I can put $m\psi$ that is the value. So, if you take it that *m* here right, $m^2 \geq \frac{2TK_{Total}}{(\sigma_B)Zm\psi Y}$. I will rewrite again here, $m^3 \geq \frac{2TK_{Total}}{(\sigma_B)Z\psi Y}$. So far, we have not taken the factor of safety right we have not taken the factor of safety. If I taken the consideration of factor of safety, then,

$$m^3 \geq \frac{2TK_{Total}S_F}{(\sigma_B)Z\psi Y}$$

So, this is the equation we should use it for finding out the module okay this is called cubic module equation. This is again for bending stress remember σ_B is given this is for bending stress and similar way we need to find out for the contact stress also right we have to find out the contact stress for bending based on the bending stress.

Now we will do the for-contact stress contact stress where right or else what you can say there okay contact stress, wear or pitting all are same all are same right. So,

$$\sigma_C = Z_E\sqrt{F_T K_O K_V K_S \left(\frac{K_H}{db}\right)\left(\frac{Z_R}{Z_I}\right)}$$

So, as we did, we will also have all the k correction factors as a $K_{(Total),C} = K_O K_V K_S K_H$. This is different, there was a K_B factor was there right for bending but there is no K_B factor here so that is why we need to have another correction factor $K_{(Total),C}$ we will assume that and then and when you do that, we will do the even take out the square item right. What happen when you do square both sides,

$$\sigma_C^2 = \frac{Z_E^2 \left(F_T K_{Total,C}\right) Z_R}{dbZ_I}$$

So, as we did earlier for bending stress similarly, we try to do that. We would expect, I am straight away going to the equation rather than expanding everything because of that same procedure we have to follow. So,

$$m^3 \geq \frac{2TK_{Total,C} S_H^2 Z_R Z_E^2}{(\sigma_C)^2 Z^2 \psi Z_I}$$

So, this is the cubic equation for based on the pitting stress. So, now we know that we derive the equation module equation cubic equation for both a bending as well as pitting stress aspect okay. I think I will stop now we will do the problem right, based on this equation in next class. Thank you.