Course Name: Theory of Fire Propagation (Fire Dynamics)

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Lecture – 02

Module 4 – Burning of Liquid Fuels

Theoretical analysis – governing equations:

Mass conservation in terms of mixture density and x-direction velocity is written as,

$$\frac{d}{dx}(\rho u) = 0 \Rightarrow \rho u = \dot{m}'' = \text{ constant}$$

Fuel conservation is written in terms of mass fraction of the fuel (Y_F) and the net volumetric rate (kg/m^{3}s) at which the fuel is consumed ($\dot{\omega}_F'''$).

$$\dot{m}'' \frac{dY_F}{dx} = \rho D \frac{d^2 Y_F}{dx^2} - \dot{\omega}_F'''$$

Conservation equation of the oxidizer is written as,

$$\dot{m}'' \frac{dY_O}{dx} = \rho D \frac{d^2 Y_O}{dx^2} - \dot{\omega}_O'''$$

Theoretical analysis – conserved scalars:

Energy conservation with chemical energy source term ($\dot{\omega}_F''' \Delta h_c$), which is the heat release rate is written as,

$$\dot{m}'' c_p \frac{dT}{dx} = k \frac{d^2T}{dx^2} + \dot{\omega}_F''' \Delta h_c$$

Consider a global reaction that occurs at infinitely fast rate:

1 kg fuel + ν kg oxidizer → (1 + ν) kg products.

If a variable, $b_{FO} = (Y_F - Y_O/\nu)$, is defined, then it can be noted that $\boldsymbol{\dot{\omega}_F'''}$ - $\boldsymbol{\dot{\omega}_O'''}$ / ν = 0. This implies the governing equation involving b_{FO} does not have the non-linear source term. Such a variable is called a **conserved scalar**. Similarly, other conserved scalars can be as:

$b_{OT} = c_pT + \Delta h_c \times Y_O/\nu$ and $b_{FT} = c_pT + \Delta h_c \times Y_F$

Theoretical analysis – formulation:

To solve the transport-controlled combustion problem, a conserved scalar, b, is considered as one of the variables defined earlier: $b = b_{FO} = (Y_F - Y_O/\nu)$ or $b = b_{OT} = c_pT + \Delta h_c \times Y_O/\nu$ or $b = b_{FT} = c_pT + \Delta h_c \times Y_F$

Properties such as thermal conductivity (k), specific heat (c_p), density (ρ) and mass diffusivity (D), are considered as constants and are evaluated at a given average temperature. A constant Z is defined as 1/(ρD) or c_p/k; Lewis number

= 1. Governing equation for the conserved scalar is written as,

$$\dot{m}''Z\frac{db}{dx} = \frac{d^2b}{dx^2}$$

Under steady conditions, $\dot{m}''$ and Z are constants. Integrating,

$$\dot{m}''Zb + c_1 = \frac{db}{dx}$$

Theoretical analysis – inner region solution:

Separating the variables and integrating again:

$$\boldsymbol{\dot{m}''Zb + c_1 = \frac{db}{dx}}$$

$$\boldsymbol{dx = \frac{db}{\dot{m}''Zb + c_1}}$$

$$\boldsymbol{\dot{m}''Z(x + c_2) = ln(\dot{m}''Zb + c_1)}$$

$$\boldsymbol{b = \frac{e^{\dot{m}''Z(x+c_2)} - c_1}{\dot{m}''Z}}$$

For the inner region between fuel surface and flame, boundary conditions are: At x = 0, b = b_s and at x = x_f, b = b_f. Invoking these, the constants are evaluated. Solution for the inner region is:

$$\boldsymbol{b(x) = \frac{e^{\dot{m}''Z(x-x_f)}(b_f - b_s) - b_f e^{-\dot{m}''Zx_f} + b_s}{(1 - e^{-\dot{m}''Zx_f})}}$$

Theoretical analysis – outer region solution:

For the outer region, boundary conditions are: At $\mathbf{x = x_f, b = b_f}$. At $\mathbf{x = \delta, b = b_\infty}$.

$$b = \frac{e^{\dot{m}''Z(x+c_2)} - c_1}{\dot{m}''Z}$$

Invoking the boundary conditions, the constants are evaluated. Solution for the outer region is:

$$\boldsymbol{b(x) = (b_\infty - b_f)\left(\frac{e^{\dot{m}''Zx} - e^{\dot{m}''Zx_f}}{e^{\dot{m}''Z\delta} - e^{\dot{m}''Zx_f}}\right) + b_f}$$

Three ‘b’ variables involving Y_F, Y_O and T are used as conserved scalars. Solution of b variables in the inner and the outer regions are presented above. From this, Y_F, Y_O and T can be calculated.

Boundary conditions for primitive variables:

Boundary conditions for Y_F, Y_O and T

x	**Y_F**	**Y_O**	**T**
0	**$Y_F = Y_{F,s}$**	**0**	**$T = T_s$**

	$\dot{m}'' = \dot{m}'' Y_{F,s} - \rho D \left.\frac{dY_F}{dx}\right\|_{x=0}$		$k \left.\frac{dT}{dx}\right\|_{x=0} = \dot{m}'' h_{fg}$
x_f	**0**	**0**	$T = T_f$ $k \left.\frac{dT}{dx}\right\|_{in} = k \left.\frac{dT}{dx}\right\|_{out} + \dot{m}'' \Delta h_c$
x_δ	**0**	$Y_O = Y_{O,\infty}$ $\left.\frac{dY_O}{dx}\right\|_{x=\delta} = 0$	$T = T_\infty$ $\left.\frac{dT}{dx}\right\|_{x=\delta} = 0$

‘b’ variables and associated boundary conditions

X	b_{FO}	b_{FT}	b_{OT}
Definition	$\frac{Y_F - Y_O/\upsilon}{Y_{F,s} - 1}$	$\frac{c_p T + \Delta h_c Y_F}{h_{fg} + \Delta h_c (Y_{F,s} - 1)}$	$\frac{c_p T + \Delta h_c Y_O/\upsilon}{h_{fg}}$
$x = 0$ $\frac{db}{dx} = \dot{m}'' Z$	$\frac{Y_{F,s}}{Y_{F,s} - 1}$	$\frac{c_p T_s + \Delta h_c Y_{F,s}}{h_{fg} + \Delta h_c (Y_{F,s} - 1)}$	$\frac{c_p T_s}{h_{fg}}$
$x = x_f$	0	$\frac{c_p T_f}{h_{fg} + \Delta h_c (Y_{F,s} - 1)}$	$\frac{c_p T_f}{h_{fg}}$
$x = \delta$ $\frac{db}{dx} = 0$	$\frac{-Y_O/\upsilon}{Y_{F,s} - 1}$	$\frac{c_p T_\infty}{h_{fg} + \Delta h_c (Y_{F,s} - 1)}$	$\frac{c_p T_\infty + \Delta h_c Y_{O,\infty}/\upsilon}{h_{fg}}$

These boundary conditions are used to calculate $\boldsymbol{T_f}$, $\boldsymbol{\dot{m}''}$ and $\mathbf{x_f}$. Here, T_s and $Y_{F,s}$ are unknowns and are related using thermodynamic equilibrium at the interface.

Steady mass burning rate & flame temperature:

From this approach, for steady burning of a liquid pool/film, the expression for mass loss rate is given as,

$$\dot{m}'' = \frac{1}{Z\delta} ln(1 + B)$$

The Spalding's transfer number is expressed as,

$$B = \frac{C_P(T_\infty - T_s) + Y_{O,\infty} \times \Delta h_c/\nu}{h_{fg}}$$

Flame temperature and its location are given as:

$$T_f = T_s + \frac{\left(\nu B - Y_{O,\infty}\right)}{\left(\nu + Y_{O,\infty}\right)} \frac{h_{fg}}{C_P}$$

$$\frac{x_f}{\delta} = 1 - \frac{ln\left(1 + Y_{O,\infty}/\nu\right)}{ln(1 + B)}$$

Mass fraction of the fuel at the interface is:

$$Y_{F,s} = \frac{B - Y_{O,\infty}/\nu}{(B + 1)}$$