Strength of Materials
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Lecture - 09
Principal Stresses

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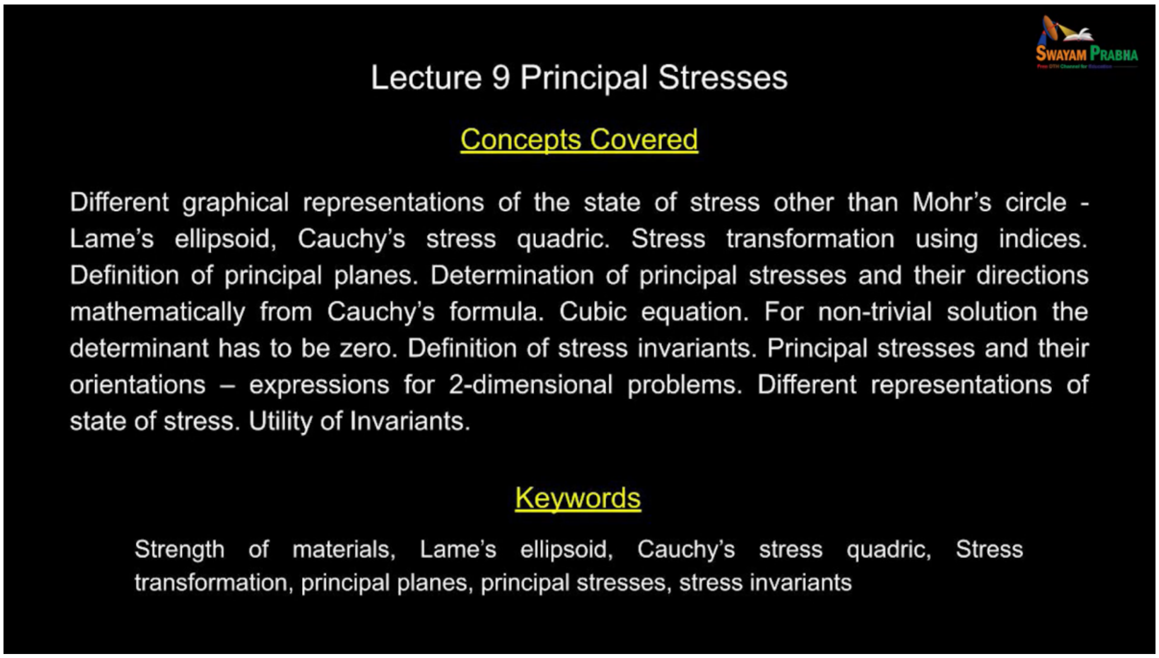

See, we have been discussing concepts related to stress. So, now you know fairly well what is the concept of state of stress at a point. And you could understand the concept still better by looking at the graphical representation of Mohr's circle. From the Mohr's circle what you learnt? You learnt, there could be planes that are very special. One such set of planes are known as principal planes in which the stress is totally normal. And we have also looked at other important planes in which the shear stress can reach a maximum.

We also learnt when shear reaches a maximum, in general, you will also have a normal stress, fine? So, you have fairly good idea of what is state of stress at a point. And we have also looked at Mohr's circle in three dimensions.

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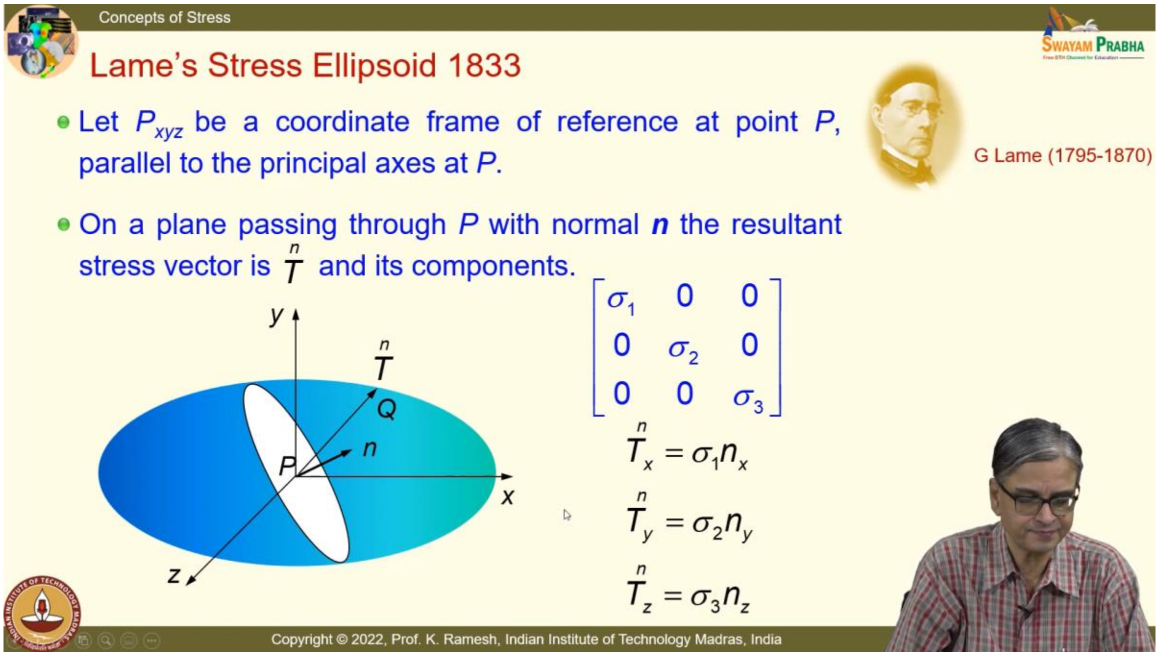

And you will appreciate still better, the utility of Mohr's circle and its simplicity when you look at other graphical representations, fine? These were developed well before Mohr's circle. And you find that Lame's stress ellipsoid was developed in 1833.

All this is possible once you understand the representation of stress tensor in the form of principal stresses that makes your life simple and your stress tensor also has many zeros. So, any mathematical manipulations that I want to do can be done quickly. And what is done in this approach is, you plot the stress vector, the magnitude of the stress vector passing through all the possible infinite planes. You have the plane and you have the expressions for the *x*-component, *y*-component and *z*-component. This graphical representation is credited to the scientist Lame.

So, you have the plane, on this plane you have the stress vector. And when you plot it for all the possible infinite planes, in general, it takes the shape of an ellipsoid. That is what you see here, which can also be mathematically proved since we have very simple expressions. We will see that subsequently.

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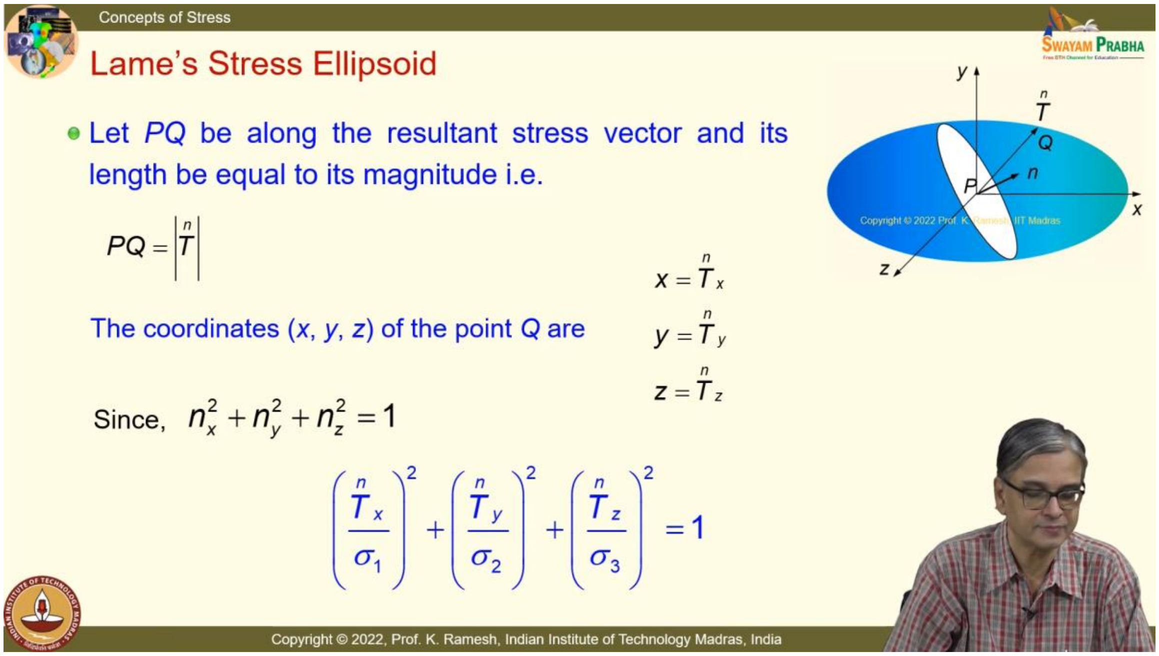

What we are plotting? We are plotting the magnitude of the stress vector, fine? And how you have these coordinates? The coordinates are nothing but $\overset{n}{T}_x$, $\overset{n}{T}_y$ and $\overset{n}{T}_z$. And you know the expression for $\overset{n}{T}_x$ in terms of the principal stress and the direction cosine. And you also have an identity that $n_x^2 + n_y^2 + n_z^2 = 1$, which is very well-known when you look at the direction cosines. So, now when I decode using this expression, I get the expression as

$$\left(\frac{\overset{n}{T}_x}{\sigma_1}\right)^2 + \left(\frac{\overset{n}{T}_y}{\sigma_2}\right)^2 + \left(\frac{\overset{n}{T}_z}{\sigma_3}\right)^2 = 1$$

So, this is nothing but an ellipsoid of revolution. But you get only the stress vector. But what we need is more than the stress vector, I need to get the normal stress and shear stress. So, there was also another graphical representation. Have you been able to take down the essentials of this line? Have you? That is the reason why I spell out the equation? When I spell out the equation, you have time to write that equation. Even if you have not written it completely, you can prove it later by understanding we have just used $n_x^2 + n_y^2 + n_z^2 = 1$. That is all the simple expression that we have used, ok?

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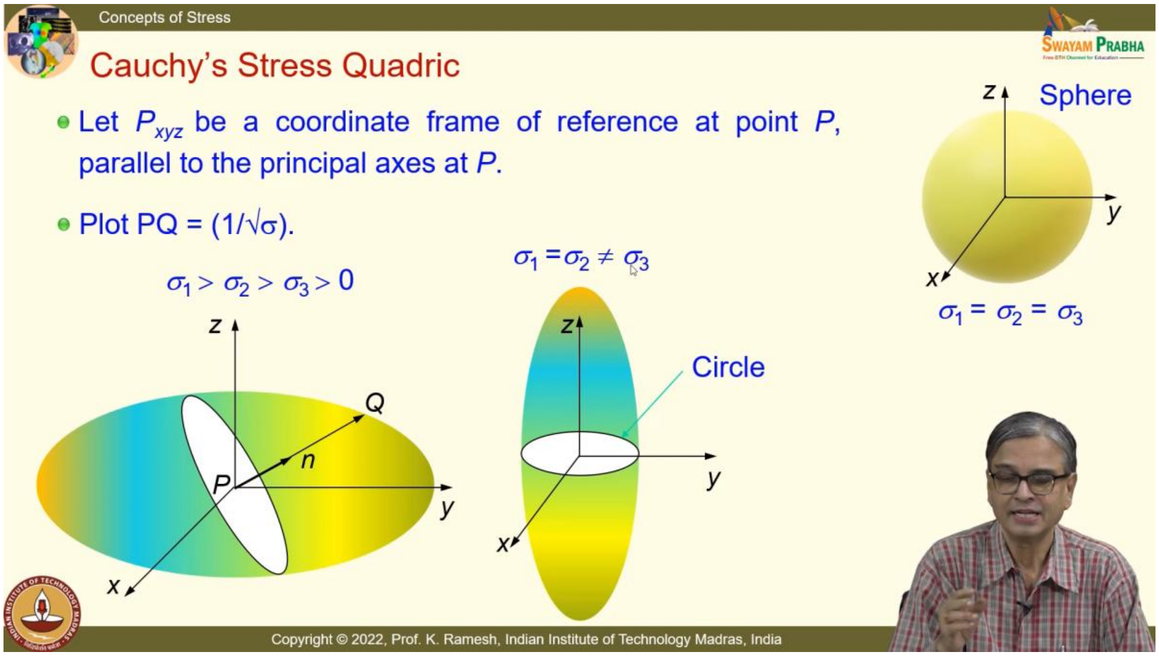

So, you have the other graphical representation called Cauchy's stress quadric. Here, he attempted to plot $1/\sqrt{\sigma}$, that is the normal stress acting on the plane of interest. In the previous case, stress vector was plotted. In this case, you have graphical representation for various representations of the principal stresses. If I have all of them non-zero, I will have this as an ellipsoid. And here, along the normal, you plot *PQ* whose magnitude is $1/\sqrt{\sigma}$. The other understanding is, suppose I plot whatever the surface I get, I find out the normal to the surface at the point. That would be parallel to the stress vector at the point of interest. I am not getting into the proof. So, you can understand these graphical representations are too complicated. And when I have a situation where $\sigma_1 = \sigma_2 \neq \sigma_3$, I get this central core as a circle. You have a special form of ellipsoid, ok? It is ellipse at all other places, but it is a circle at the center.

Suppose, I have a situation $\sigma_1 = \sigma_2 = \sigma_3$, you get a sphere. What way we got it in Mohr's circle? If I have $\sigma_1 = \sigma_2 = \sigma_3$, we just got it as a point, fine? So, it was a very simple graphical representation, very elegant. And even for other cases, even when I come down to a planar situation, you get beautiful geometrical patterns.

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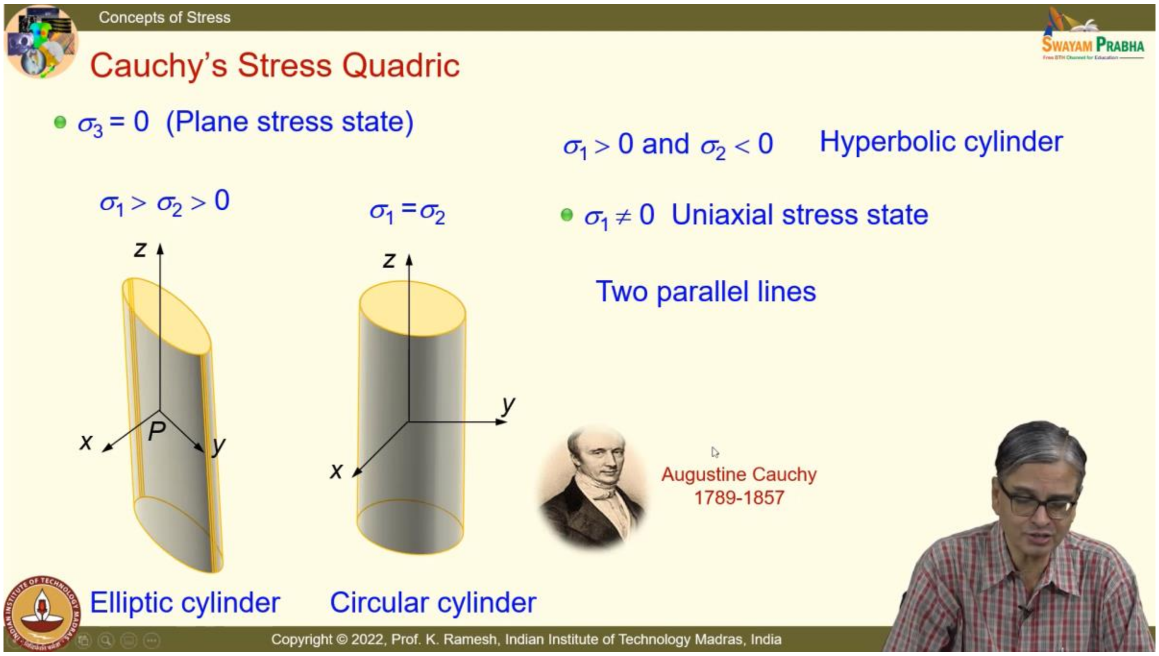

And this was credited to Cauchy and you know we have already looked at Cauchy was responsible for establishing the concept of stress tensor that happened in 1822, fine? Suppose, I take a plane stress state, if I have $\sigma_1 > \sigma_2$ and they are both positive, then I get this as an elliptic cylinder. So, likewise you know they were so happy.

See, in those days any, graphical development was considered enjoyable and people were comfortable with it. And it also adds to the happiness of the scientist that he had beautiful patterns. Suppose, I have $\sigma_1 = \sigma_2$ in a two-dimensional situation, it becomes a circular cylinder. And what was it in the case of Mohr's circle when $\sigma_1 = \sigma_2$? It was just a point. And you also have very complicated shapes when $\sigma_1 > 0$ and σ_2 is negative; I get this as hyperbolic cylinder. It was difficult for me to plot, so I left it.

And when $\sigma_1 \neq 0$, that is the uniaxial stress state, you get two parallel lines. From a graphical point of view, when people were groping in the dark, this gave some light. And there is no specific year when was this developed, but it should be much before Mohr's circle. It was credited to Cauchy, so it should be before 1857, fine? And when was Mohr's circle developed and stabilized? 1882, that is 60 years later, stress tensor was understood completely, ok? So, you find with these other examples of graphical representation, Mohr's circle is the simplest, it is just a circle for any state of stress, easy to draw, easy to get all the information that you need from stress analysis point of view.

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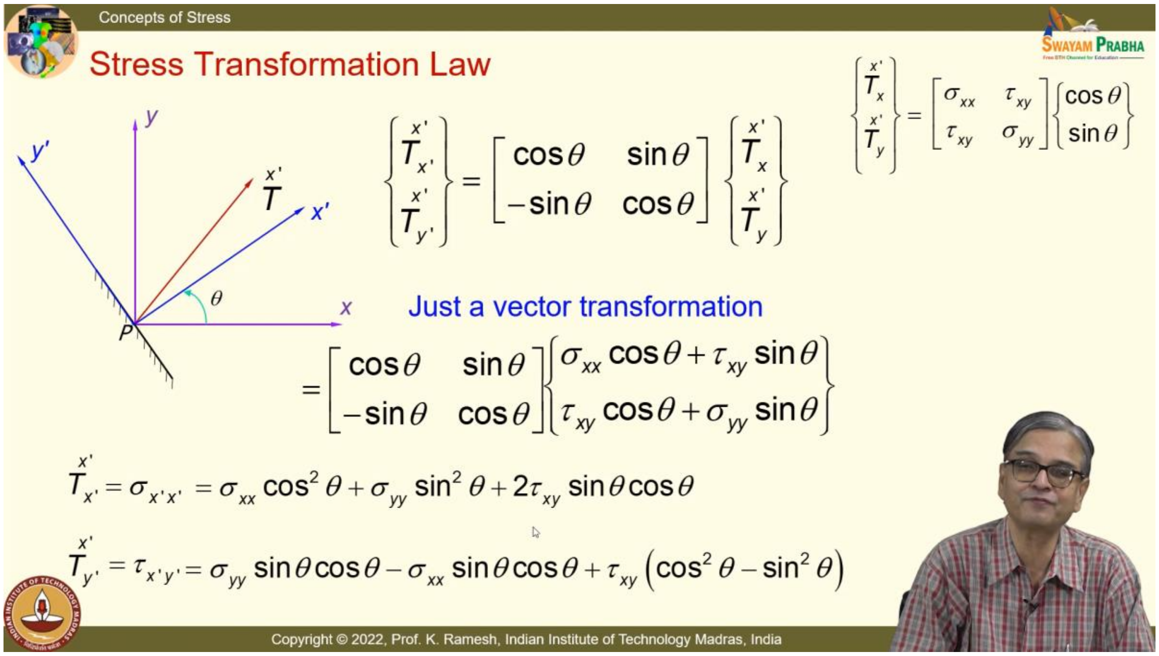

And we also know that Mohr's circle, you could draw based on the stress transformation law. And we developed stress transformation from the first principles. Again, I am repeating the same thing which we have done in the earlier class, because it is a very fundamental concept. So, when I say $\overset{x'}{T}_{x}$, I am talking about plane *x*’ and I have a stress vector. This can be achieved by the Cauchy's formula.

And if I want stress tensor with respect to *x*’ *y*’ axis, I will have to do a vectorial transformation of the stress vector, fine? If I have to get $\overset{x'}{T}_{x'}$ from $\overset{x'}{T}_{x}$, I have to go for a vectorial transformation. And this is so important until you graduate your mechanical engineering. You will be having requirement to use this again and again. So, it is worth learning how to do it from first principles. It is also worth knowing how to do it quickly.

So, that is what we are going to see in this class. From first principles, you can always derive, takes time, and some of you have very good memory, you may even memorize these expressions. But when you come to the examination, you will have a confusion whether this is $2\tau_{xy}$ or $-2\tau_{xy}$. Because when I have the expression for $\sigma_{y'y'}$, I will have a negative sign. So, now we will look at from a different perspective. You need to know and master this aspect also.

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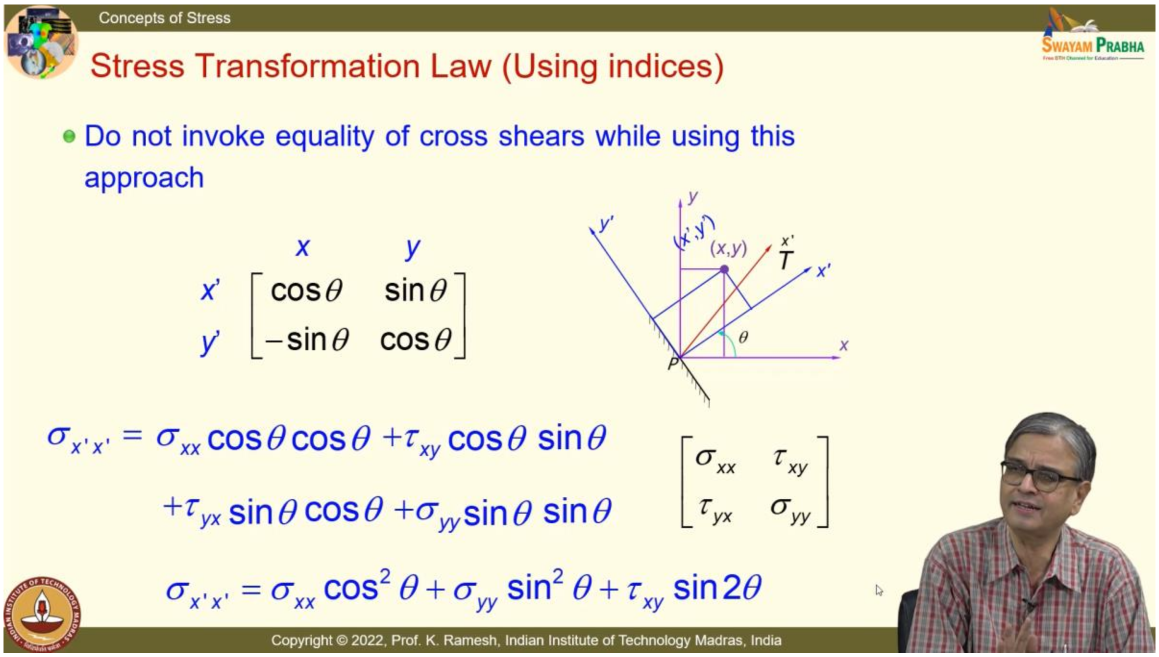

So, what I am going to do is, I will directly write the stress components in the reference axis *x*' *y*' using indices. And what is given to you? With respect to the *x y* reference, you are given the stress tensor at that point, and you also have knowledge how to express *x*' *y*' in terms of *x*, *y*. And you are given the stress tensor. And note this, the way that I have written.

What is it that I have written? I have written it in the way we originally developed. I have not used the equality of cross-shears. I have said this is τ_{xy} and this is τ_{yx}. So, when I use indices for me to calculate $\sigma_{x'x'}$, I will have four terms in the expression, and all these four terms should appear without using the advantage of equality of cross-shears. And what is important here is, I should write the vector transformation matrix properly, the rotation matrix properly. That you can easily understand. See, I have this *x*', this *x*' is definitely longer than *x*. So, it will be $x\cos\theta + y\sin\theta$, and you will have *y*' is $-x\sin\theta + y\cos\theta$. That is the mnemonic way of remembering it. If you put the *x*' *y*' and *x y* as for a generic point, you can write this.

If you write this rotation matrix correctly, you can straight away write the transformed stress component directly. What I want? I want $\sigma_{x'x'}$. This will have contribution from all the terms. So, I take σ_{xx}. And what I do is, I have *x*' *x* and what is *x*' *x* direction cosine here? $\cos\theta$. So, I will have $\cos\theta$. This is again *x*' *x*, so I have $\cos\theta$. Then I go to the shear stress component, I have *x*' *x*, $\cos\theta$, I have *x*' *y*, that is $\sin\theta$. And I go to the next term τ_{yx}, I have *x*' *y*, *x*' *y* is $\sin\theta$, then *x*' *x*, $\cos\theta$, plus σ_{yy}, I have this as $\sin\theta$ multiplied by $\sin\theta$. i.e.,

$$\sigma_{x'x'} = \sigma_{xx}\cos\theta\cos\theta + \tau_{xy}\cos\theta\sin\theta + \tau_{yx}\sin\theta\cos\theta + \sigma_{yy}\sin\theta\sin\theta$$

This is straightforward. So, I get the final expression in this form, and this is also simplified in terms of $\sin 2\theta$ or $\cos 2\theta$, whichever the case may be; this is straightforward. Is the idea clear?

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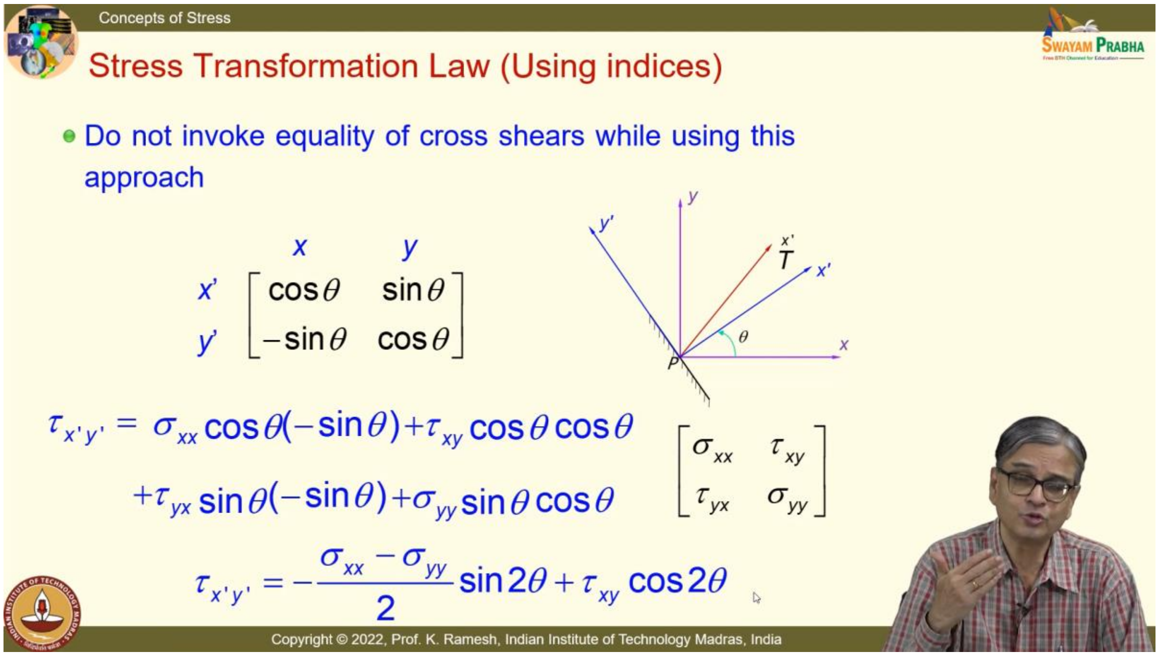

We will repeat it for tau $\tau_{x'y'}$. So, do not invoke equality of cross-shears while using this approach, that is very important. In this expression for $\tau_{x'y'}$, you will understand why I want τ_{xy} as well as τ_{yx}. Their multiplication factors will be different, fine? So, when I use these indices, you will have to do this. Can you do it? Can you do it and check it with my slide? You do not have to remember. Please do it. Take a minute to do it. I should see everybody doing that. So, if you learn how to write the transformed stress magnitudes based on indicial notation, the key point is you should write the rotation matrix. Even if you have forgotten the rotation matrix, if you go and put these axes and take a generic point, then you can construct this. If you construct this properly, rest of it is simple and straightforward; that is the key element, fine? So, I have this as $\cos\theta$ multiplied by -$\sin\theta$ and when I go to τ_{xy}, I have *x x*', $\cos\theta$, *y*' *y*, that is $\cos\theta$ again, ok? When I go to τ_{yx}, I have *x*' *y*, that is $\sin\theta$, and then *y*' *x*, - $\sin\theta$.

$$\tau_{x'y'} = \sigma_{xx}\cos\theta(-\sin\theta) + \tau_{xy}\cos\theta\cos\theta + \tau_{yx}\sin\theta(-\sin\theta) + \sigma_{yy}\sin\theta\cos\theta$$

You find these two terms are different, you get the point? So, if you use the equality of cross-shears, using the indicial notation to get the transformed stress components, you cannot apply. So, you should understand the restriction. And you similarly write for σ_{yy}, I get this as *x*' *y*, and then *y*' *y*. So, I have this and this is written in a simplified fashion here. So, I have

$$\tau_{x'y'} = -\frac{\sigma_{xx} - \sigma_{yy}}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

This is the same expression that we got from first principles also. You should know how to write it from first principles, but you should also know how to solve problems where you have to write the expression. You should not have difficulty in assigning the correct signs, ok? So, this is a recipe for you to do that.

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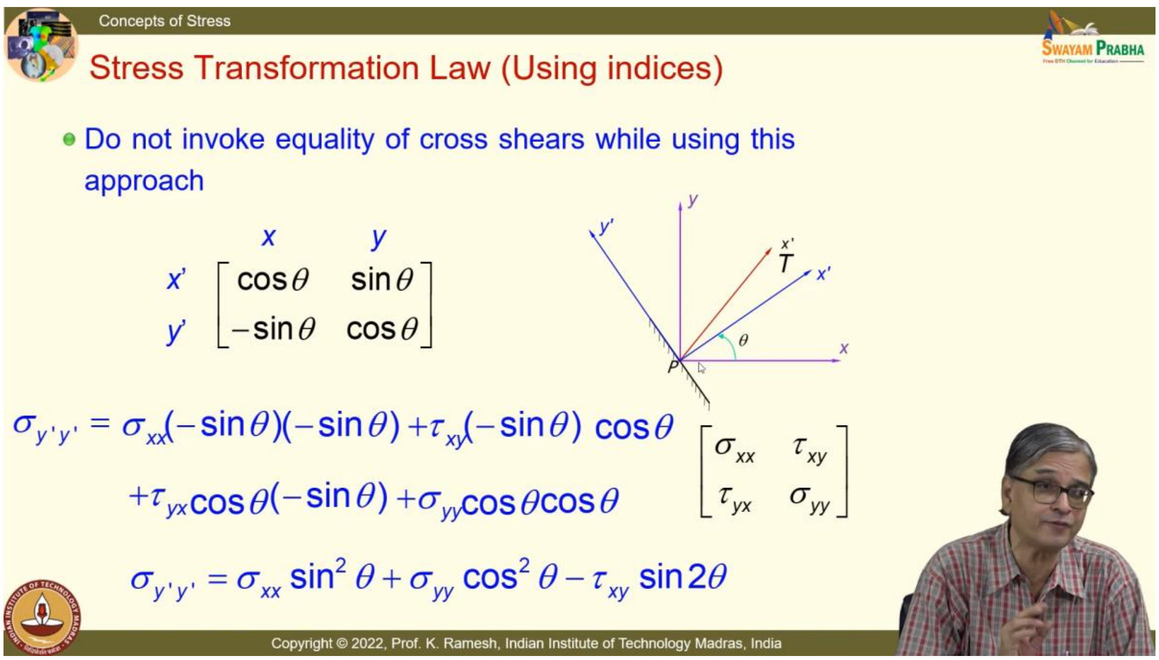

I can repeat the same thing for $\sigma_{y'y'}$. Can I expect you to do that, ok? I think I would appreciate that you do that. I am going to show the result and do that as a home exercise, ok? Because that will help you to appreciate the methodology involved so that even when you are asked to wake up from your dream, you should be able to write the stress transformation. Because this is so fundamental and important for all your future courses, ok.

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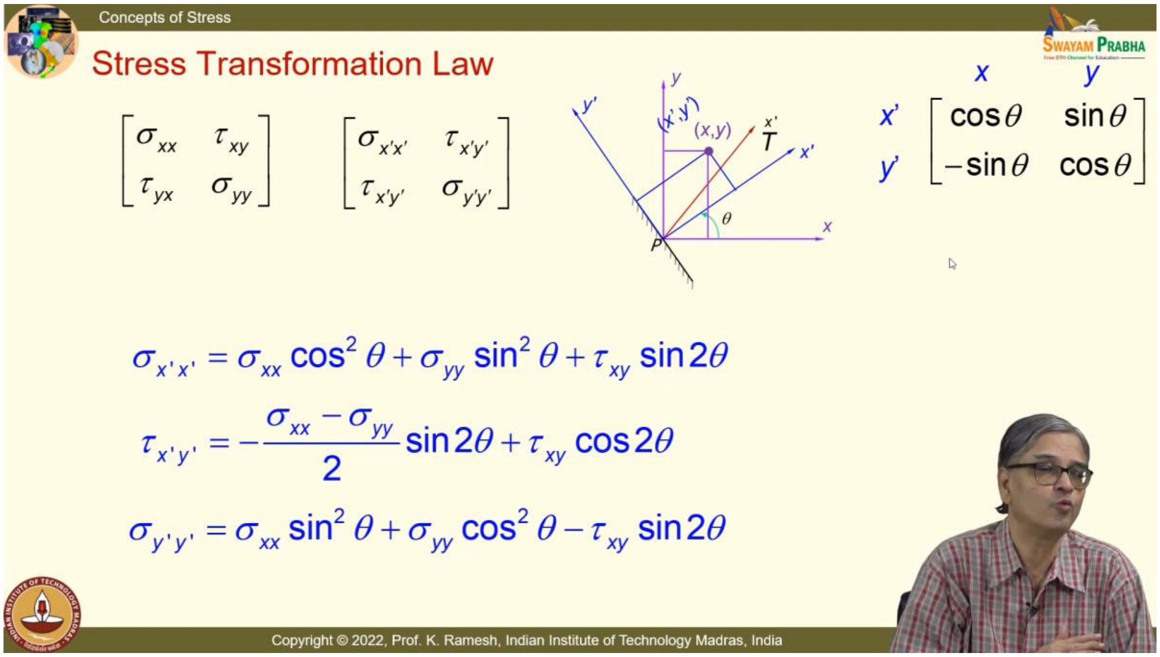

So, this is the summary you have. I have the stress tensor referred with respect to *x y* frame and then I want to find out the stress tensor referred with respect to the *x*' *y*' frame. So, I have the summary of results. You can just verify whether the expression that you have written are correct. And the message in this is that you have to write this rotation matrix and also put this *x*' *y*' here and *x y* there. If you interchange that, then again you will have a problem. So, you should know how to write the rotation matrix and what we are looking for, fine? Then, using the indicial notation, it is very comfortable for you.

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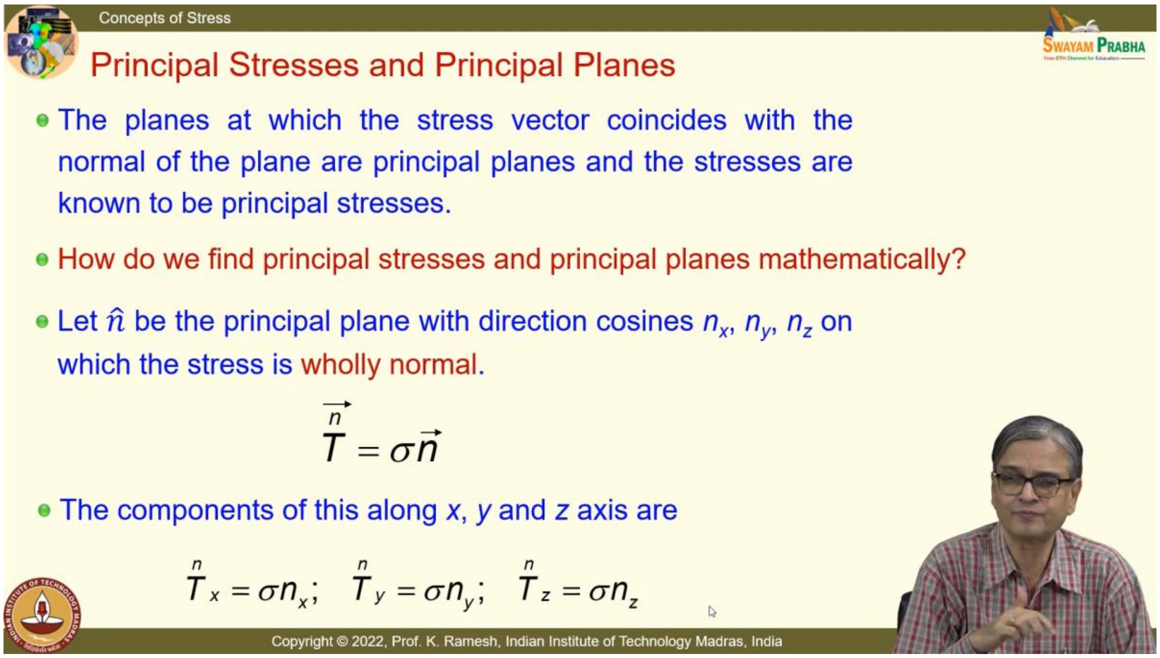

See, from Mohr's circle, you have already had an idea what are principal stresses. While we developed the Cauchy's formula to find out stress vector on any arbitrary plane, I said you can find out two classes of problems. In one class of problems, the direction cosine is given and you have to find out the stress vector. In another class of problems, specify the stress vector, find out the direction cosines. Precisely, that is what we are going to look at when we want to find out the principal stresses and principal planes, mathematically.

You already have the result from graphical representation. Is that idea clear? Mohr's circle was so useful. So, you have an idea that you can have planes where you will have only normal stress. That idea is now very well understood, ok? Now, we ask the question mathematically and we specify the plane to be $\hat{n}$ and the direction cosines are n_x, n_y and n_z.

And we also specify what is the stress vector. We have specified, on that plane, I have only normal stress component. So, that means stress vector is specified. There is no shear stress component. And I want the stress vector to be in this form which could be identified from the Cauchy's formula. I have $\overset{\vec{n}}{T} = \sigma \vec{n}$ and components of this along x, y and z axis are; we have specified this, I have specified the x-component is σn_x, y-component is σn_y and z-component as σn_z. This is what I want. What is unknown here is the direction cosines n_x, n_y and n_z. I am not sure whether you have studied eigen values and eigen vectors in your mathematical courses. So, if you have done that, then we are going to formulate this problem as eigen value and eigen vector type of development, ok?

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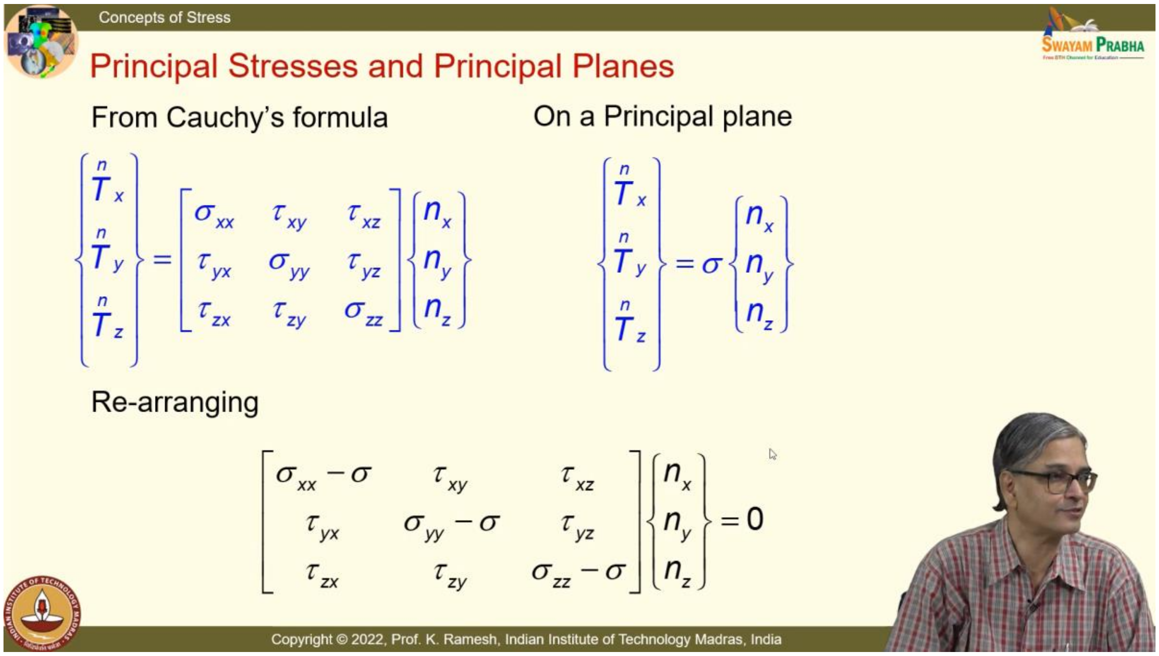

So, from Cauchy's formula, I have the generic expression. This is the given stress tensor and I have this direction cosine n_x, n_y and n_z. And in this specific problem, we have specified what should be the nature of stress vector.

The stress vector should be of this nature. The question is, how to find out n_x, n_y and n_z. Is the idea clear? So, now I can combine these two. I can replace $\overset{n}{T}_x$, $\overset{n}{T}_y$, and $\overset{n}{T}_z$ in terms of this and I can do a mathematical simplification. And I can rewrite this in this form. I have this as

$$\begin{bmatrix} \sigma_{xx}-\sigma & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy}-\sigma & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz}-\sigma \end{bmatrix} \begin{Bmatrix} n_x \\ n_y \\ n_z \end{Bmatrix} = 0$$

So, what is our interest? We need to find out what are the possible values of σ and what are the possible values of the direction cosine n_x, n_y and n_z. In reality, you can have this as an eigen value and this as the eigen vector. Is the idea clear? And obviously, I want to have a non-trivial solution. If I have n_x, n_y and n_z equal to zero, everything goes to 0. We do not want that as a solution.

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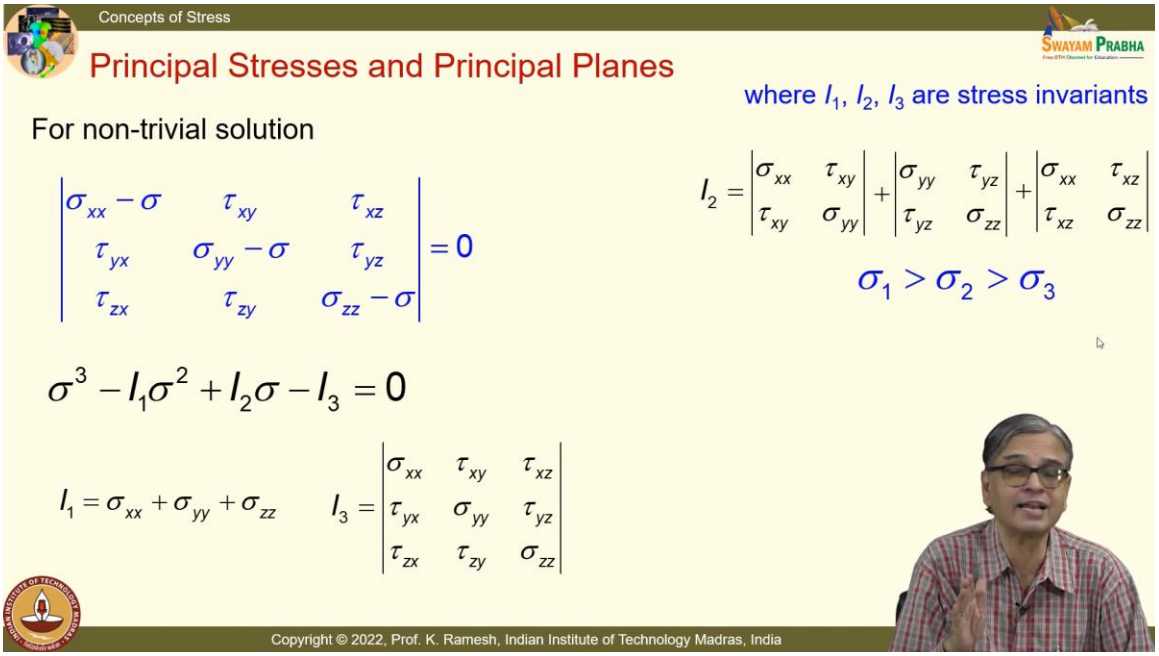

For a non-trivial solution, what I should have? I should have that the determinant goes to zero. See, you have to get the determinant, but I will try to make your life simple and faster. I will rewrite it in a convenient fashion. The final expression of this determinant can be written as

$$\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0$$

And we will soon understand why I_1, I_2 and I_3 are called stress invariants. We have just given a definition that these are stress invariants. Now, I will also have to define what is I_1, what is I_2, and what is I_3. Once I define what is I_1, I_2 and I_3, it is easier for you to write the expression for the determinant comfortably to make your life simple and also to have your involvement in development of these expressions. I_1 is given as $\sigma_x + \sigma_y + \sigma_z$. It is a very famous invariant and we should also know how to use this invariant effectively in our computations.

And you have the definition of I_2. So, look at the I_2 definition. It is very interesting. If you see the animation, you will never forget how to write I_2. I_2 has $\begin{vmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{xy} & \sigma_{yy} \end{vmatrix}$, taken as the first 2×2 portion of this 3×3 matrix plus this portion of the matrix $\begin{vmatrix} \sigma_{yy} & \tau_{yz} \\ \tau_{yz} & \sigma_{zz} \end{vmatrix}$, and all the four corner elements $\begin{vmatrix} \sigma_{xx} & \tau_{xz} \\ \tau_{xz} & \sigma_{zz} \end{vmatrix}$. So, if you know how you have sequentially written down I_2, you can easily write out from the given stress tensor.

I want you to compute this I_1, I_2, I_3. You can quickly compute I_2. It is not difficult. Please work it out and check it with my expressions, ok? What I have said is, when I say this determinant should go to zero, I should get $\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0$. I_1 is straightforward, no computation involved.

I_2, you have to write down the expressions. Similarly, I_3 also you have to write down the expression and I_3 is the determinant of the complete matrix like this. So, I_3 is the determinant of this matrix.

$$I_3 = \begin{vmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{vmatrix}$$

So, the idea is, if you find out these, you can write this expression comfortably. And let us see the expression in the long hand, ok? And finally, once you get these quantities, I will have three roots for this expression and they have to be arranged algebraically. The largest is σ_1, in between one is σ_2, and the least is σ_3. This you should never forget. There is a convention in how to label σ_1, σ_2, σ_3. It is not that arbitrarily whatever the roots that you get, you label them σ_1, σ_2, σ_3. You have to look at, arrange them in an algebraic manner, then only label as σ_1, σ_2, σ_3. Is the idea clear?

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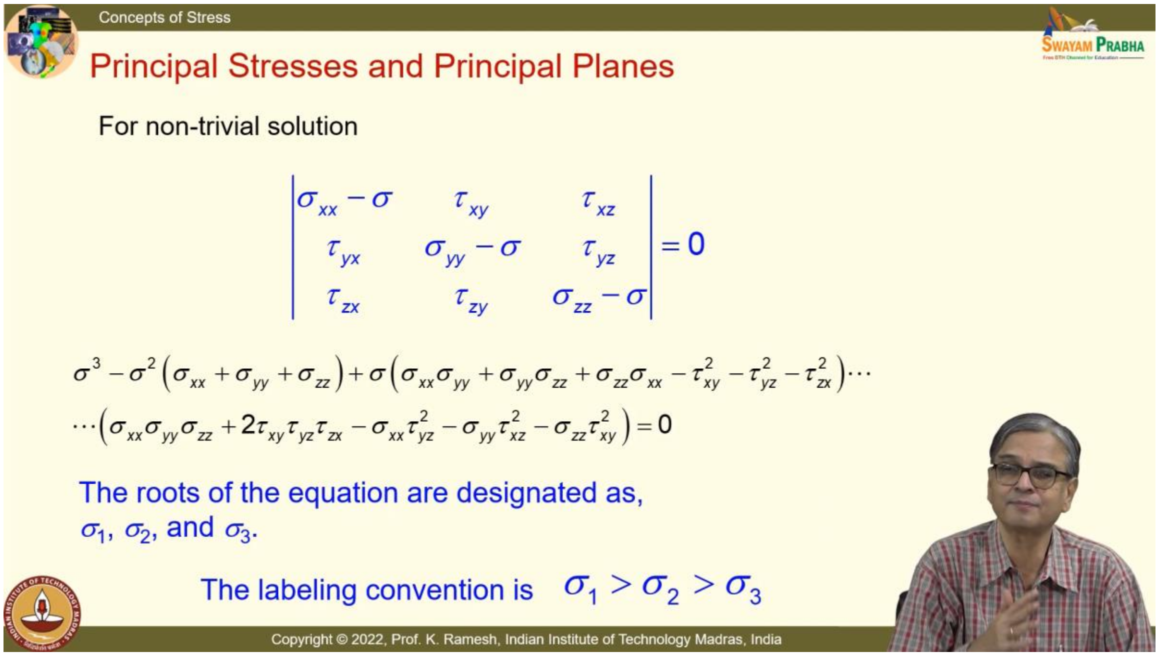

So, when I look at this expression, please verify with your computations. So, I have

$$\sigma^3 - \sigma^2(\sigma_{xx}+\sigma_{yy}+\sigma_{zz}) + \sigma(\sigma_{xx}\sigma_{yy}+\sigma_{yy}\sigma_{zz}+\sigma_{zz}\sigma_{xx}-\tau_{xy}^2-\tau_{yz}^2-\tau_{zx}^2)\cdots$$
$$\cdots(\sigma_{xx}\sigma_{yy}\sigma_{zz}+2\tau_{xy}\tau_{yz}\tau_{zx}-\sigma_{xx}\tau_{yz}^2-\sigma_{yy}\tau_{xz}^2-\sigma_{zz}\tau_{xy}^2)=0$$

See, you could have also written it from the determinant and arranged and grouped them. My interest is that you will also have to know what is I_1, I_2 and I_3 because they are very useful if you know how to use them.

When you do the computation, this is what I said, when you make the computation as engineers, speed is not important, accuracy is important, ok? So, you should have inbuilt checks in verifying your results so the roots of the equation that designated as σ_1, σ_2, σ_3. Again, the labeling convention, see, the idea of repetition is that you should recognize that this is important. While learning a subject, healthy repetition is very important, ok?

See, now we will live in a two-dimensional domain, ok? Let me live in a planar situation. Even before I find out what are the expressions for the principal stresses in two dimensions, you have them from your Mohr's circle. So, whatever I do from a mathematical approach, it should satisfy identically. Only then my graphical approach is correct and my mathematical approach is also correct.

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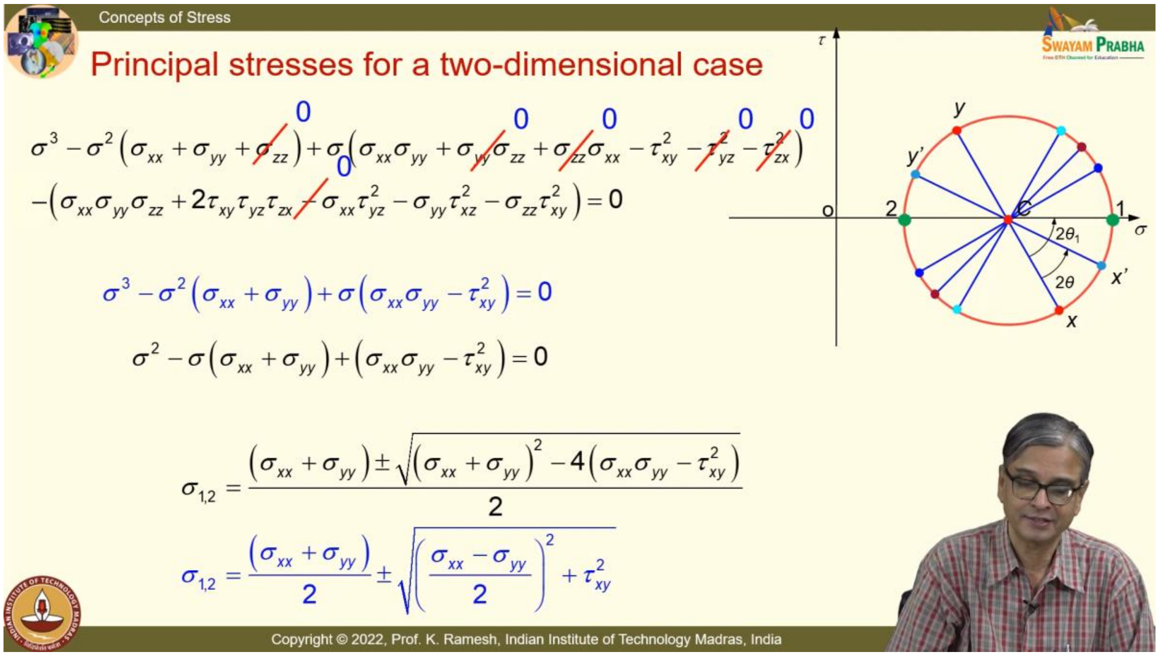

So, you have to give credit to the pioneers that they have verified the development from multiple different ways, ok? So, when I say two-dimensions, I have only σ_{xx}, σ_{yy} and τ_{xy} are existing; all other stress components with z are zero, ok? So, when I do that, I will have σ_{zz} is zero and this term goes to zero and this term goes to zero; these two terms also go to zero. And if you look at this expression, every term has a quantity which has a subscript z, so all these quantities also go to zero. So now, this expression can be simplified and rewritten as

$$\sigma^3-\sigma^2\left(\sigma_{xx}+\sigma_{yy}\right)+\sigma\left(\sigma_{xx}\sigma_{yy}-\tau_{xy}^2\right)=0$$

And I can take out σ and I get a quadratic expression. Once you have a quadratic expression, you know how to find out the roots of the quadratic expression. But when you write the roots, it is directly not similar to what you have got in Mohr's circle. You have to do one more step of simplification because you have this $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ is what you have generally learnt as roots of a quadratic equation. So, when you do that directly, I will get

$$\sigma_{1,2}=\frac{\left(\sigma_{xx}+\sigma_{yy}\right)\pm\sqrt{\left(\sigma_{xx}+\sigma_{yy}\right)^2-4\left(\sigma_{xx}\sigma_{yy}-\tau_{xy}^2\right)}}{2}$$

When I simplify this, this reduces to the same expression like what you have got based on the Mohr's circle that we have seen earlier.

There is absolutely no difference. So, these are very famous expressions. I have this as

$$\sigma_{1,2} = \frac{\left(\sigma_{xx} + \sigma_{yy}\right)}{2} \pm \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

And from Mohr's circle point of view, what is this expression? This was related to the radius of the Mohr's circle, ok? See, there, by looking at the graphical representation, we were able to find out the principal stresses as well as its direction. Now we will also have to find out the direction based on the mathematical approach, ok? And before we go into that, let me also say, since you are very comfortable with Mohr's circle, the stress state can be represented in terms of the principal stresses. It can also be represented in terms of any point forming the diameter of this Mohr's circle.

Is the idea clear? I can have multiple representation of the stress tensor. And why I call this I_1, I_2 and I_3 as invariants? Suppose I start with my stress tensor referred to *x*' *y*', that must also give me the same principal stresses σ_1 and σ_2. Is the idea clear? It cannot give any other value of this, because it all represents what happens in a particular point of interest on all the possible infinite planes. The stress tensor per say can be represented with respect to *x y* reference, *x*' *y*' reference, 1 2 reference or any other diameter. This is a very subtle concept, you know this can be easily illustrated if you know Mohr's circle, because we have spent sufficient time on understanding the Mohr's circle. Now you are equipped. So, you can understand the roots of the equation, no matter what is the starting point of the stress tensor, should remain same.

So, the coefficient should remain identical, the individual numbers for you to multiply and get the final value, final value should remain same, but individual value can be different; that is what you see as different stress tensors. And this is also an inbuilt check when I want to find out stress transformation, ok? I can use that as an invariant.

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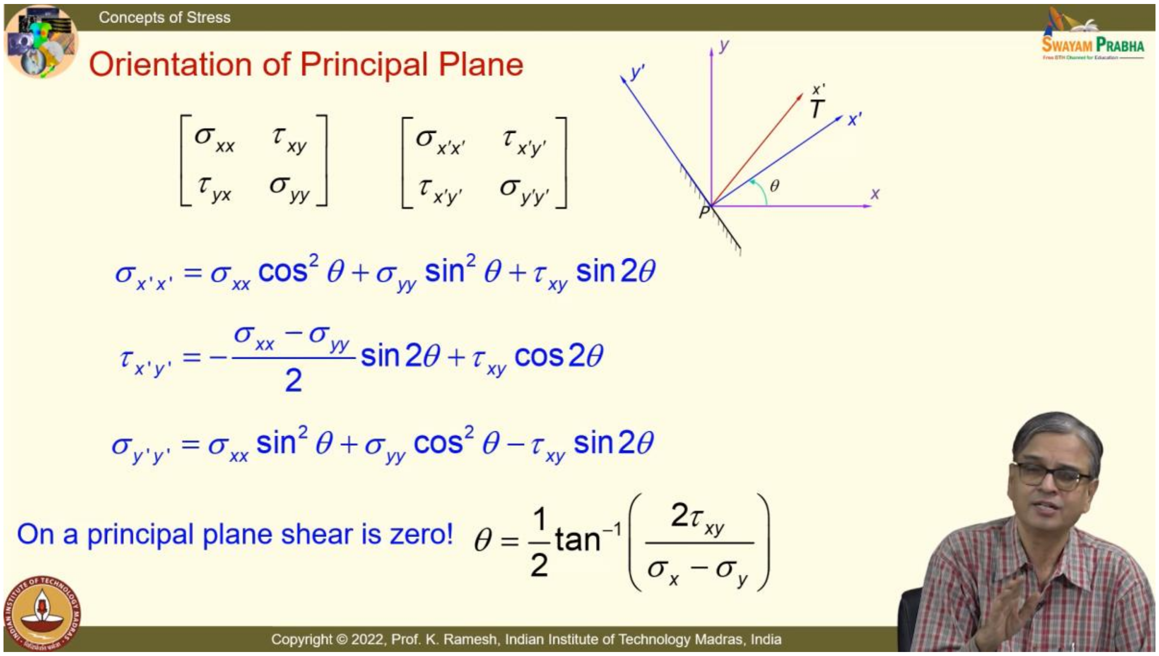

Now I said I have to find out the orientation of the principal stress plane, mathematically. That means, I should apply the condition mathematically. What is the definition of a principal stress plane? On the principal stress plane, you cannot have shear stress.

So, I go and say, in this expression, shear stress is zero, because this is generic expression. You are finding out the stress transformation on a generic *x*' *y*'. This *x*' *y*' can coincide with your principal stress planes. So, this gives me again the same expression that we have got from Mohr's circle,

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)$$

So, you have graphical representation, you have a mathematical development, both are identical. Is the idea clear? See, that is how scientific development has been.

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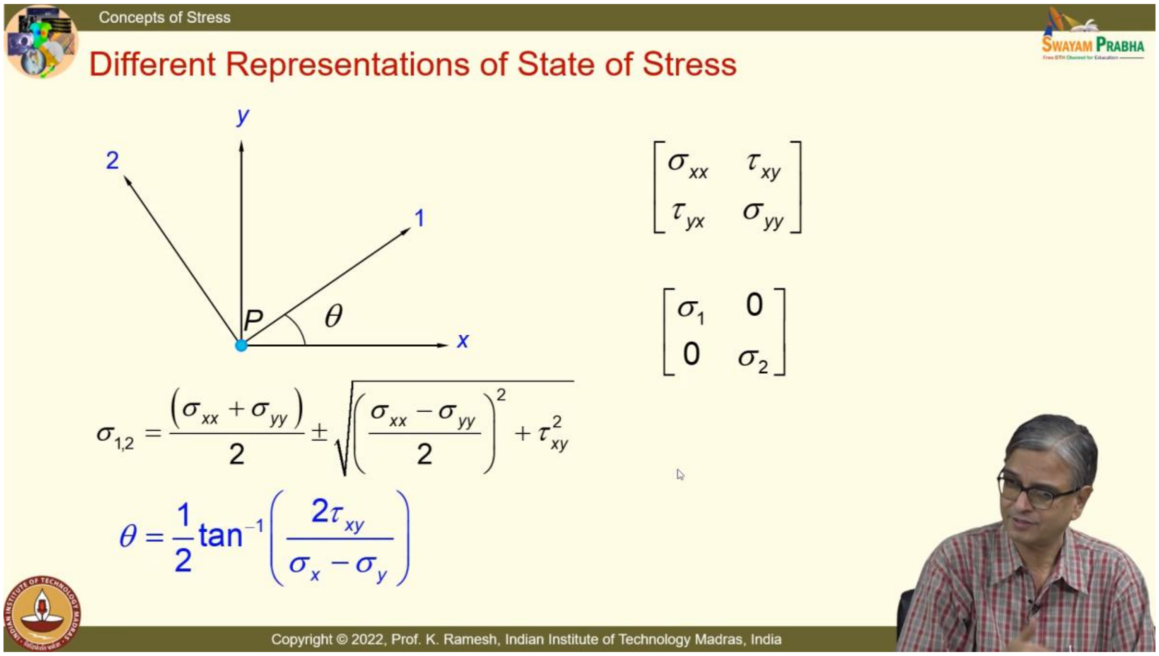

Now, we will also appreciate and put it in black and white whatever I have said. I can represent the stress tensor as $\begin{bmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & \sigma_{yy} \end{bmatrix}$ or I can also express it in a simplistic manner as $\begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}$. Is the idea clear? My reference axes become different.

This you should understand. See, I have a populated matrix here. Here I have only the diagonal terms available; non-diagonal terms are zero. It brings in lot of comfort in mathematical development. We will take advantage of this, ok? We will take advantage of this.

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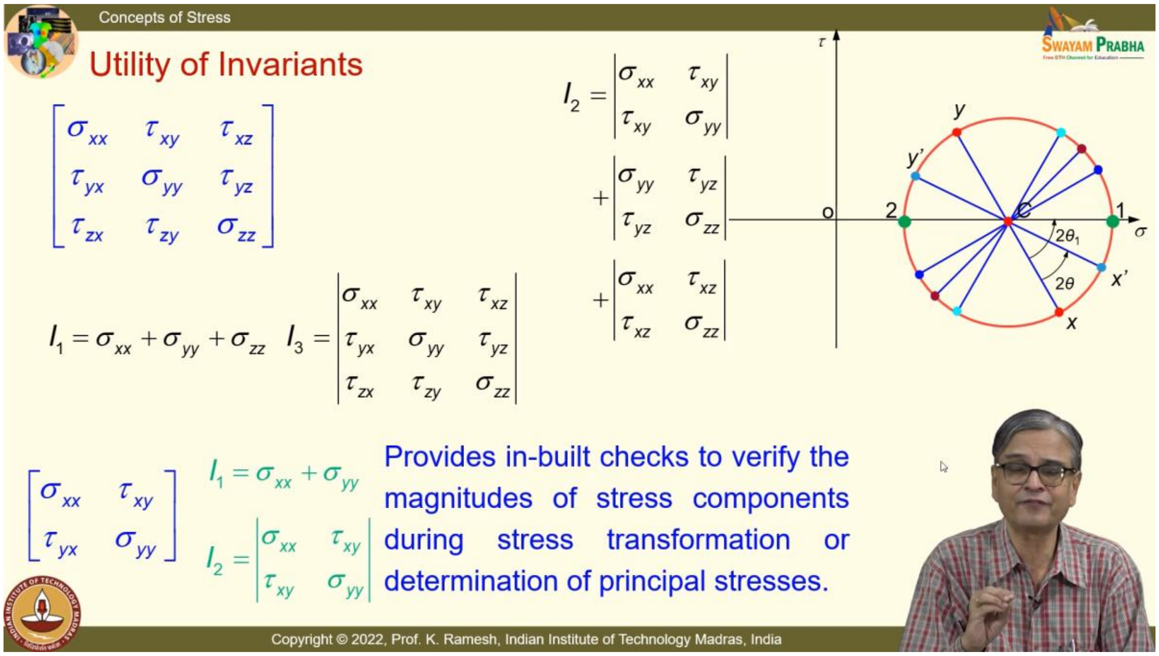

As I said we have to utilize the invariants. This is again repetition of the same slide where we have looked at what are I_1, I_2 and I_3. So, look at this and understand how you get I_1, I_2 and I_3. I have the four corner elements forming the third determinant for finding out this I_2 and I_3 is the determinant of the complete matrix. And we will confine ourselves to a two-dimensional representation. So, I have I_1 reduces to $\sigma_{xx} + \sigma_{yy}$ and I_2 is $\begin{vmatrix}\sigma_{xx} & \tau_{xy}\\ \tau_{xy} & \sigma_{yy}\end{vmatrix}$, ok? So, as I mentioned earlier, when you do stress transformation or when you do determination of principal stresses, you have an opportunity to verify your values. What way you will verify? Simplest one, at least you check whether I_1 remains same when I go from $\sigma_x + \sigma_y$ to $\sigma_1 + \sigma_2$ or $\sigma_{x'} + \sigma_{y'}$. Is the idea clear? And I would expect you to do this verification in all your calculations.

Speed is also important. When I say speed is not the only thing, that is the idea of saying that speed is not important, accuracy is more important. It is not you said speed is not important, you give me two hours to solve a half another quiz. Do not do that, ok? There is also a reasonable time in which you have to do it. This is not the only utility of invariants. From the Mohr's circle, you can again reconfirm that you have the advantage that the stress tensor at a point of interest can be represented in multiple ways; infinite ways in fact, ok? And some of them will be very simple like when you refer it with respect to principal stress planes, you have zeros, mathematics is simple.

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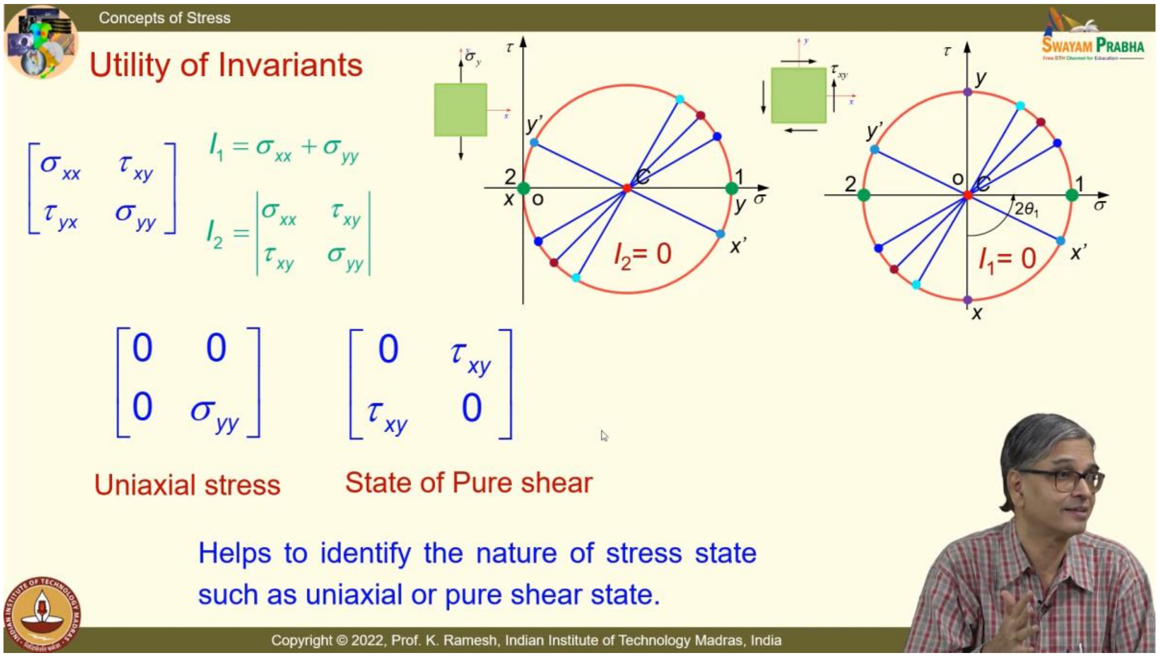

And you can also use the utility of invariants in many other ways, ok? I have a uniaxial state of stress, can you plot the Mohr's circle for this? I have it; you can also draw it. And what we have learnt? We have learnt the uniaxial stress state can also be represented in multiple different ways. When I represent it in *x*' *y*' or any of the other directions, the complete matrix will be populated, fine? Now tell me by utilizing the invariant, suppose I give a matrix which is populated, how will you verify whether it is representing a uniaxial state of stress? You have a clue right on your screen, I say that you have to utilize the invariants, please think about it, ok? Think about it. And we will also see another important state of stress; what is this state of stress? It is a pure shear state.

And you can also draw the Mohr circle, the graphical representation is very simple. The only thing is the origin is shifted. I shift the origin and I re-plot. And again, when I have a pure shear stress state, when I represent the matrix in one reference frame, I have beautiful zeros and the non-zero number. Pure shear stress state also has zeros and non-zero number. But when I go to the Mohr's circle, I have infinite possibilities in which I can represent the same stress state.

Is the idea clear? Now with the utility of invariants, can you tell me how will I use the invariants to identify if a given matrix is corresponding to uniaxial stress state or shear stress state? Anybody? Very good, very good, very good. Because you know how to compute the invariants for this. And it is also a direct check that invariants are really invariants. So, when I have I_2 is zero, I will have this as uniaxial stress state.

When I_1=0, I will have that as a shear stress state. So, you should know, appreciate what are invariants, also assimilate what are invariants and use them in your computation. See, that is the purpose why you are learning it, fine?

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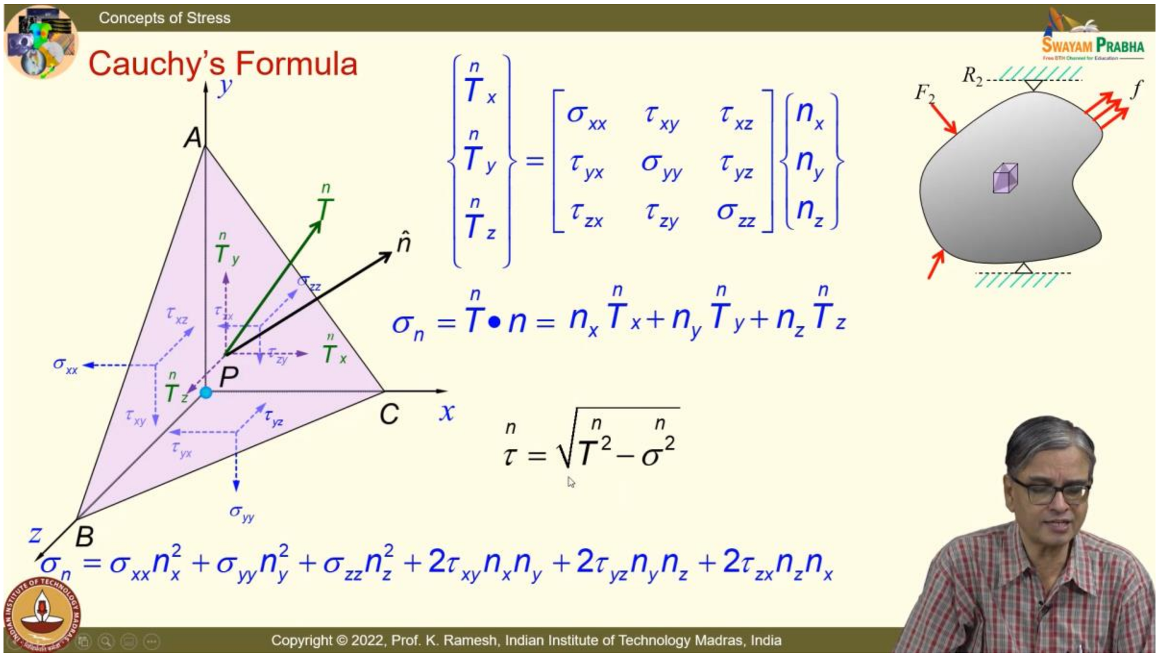

And there are also advantages when I represent it in principal planes. See, we have earlier seen this Cauchy's formula. And we have also said that I should know how to write normal stress and how to write the shear stress in long expression which we have already seen.

I have shear stress is $\sqrt{\overset{n}{T}^2-\overset{n}{\sigma}^2}$. I can find out this and this looks very long and difficult to compute. Suppose I express the same stress state in terms of the principal stresses, all these expressions simplify. And not only that, you know researchers also wanted to develop new entities. Even for development of new entities, it was convenient for them to visualize when it is expressed in principal stress planes.

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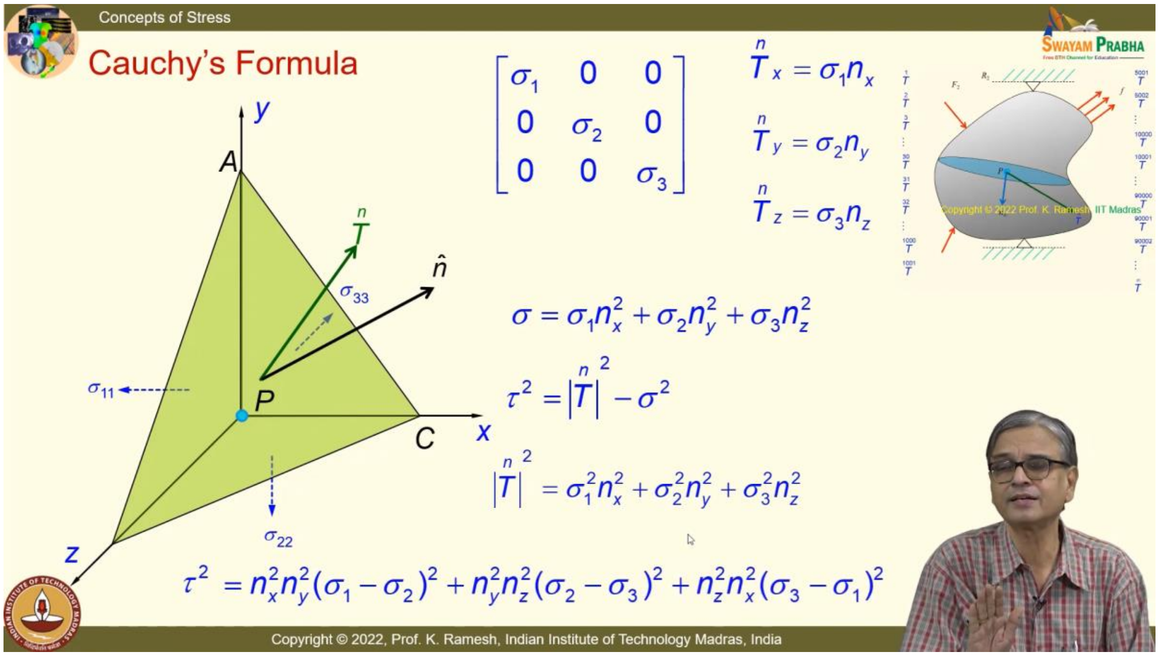

Suppose I rewrite the same expression, just to show that I am handling a principal stress plane, I have shown this tetrahedron in a different color. And mathematically you know when you want to have a reference axis *x y z*, it is arbitrary, I can still put *x y z* in a convenient fashion and then say that they correspond to principal stress direction 1 2 3.

Is the idea clear? In order to distinguish from a generic representation, I have changed the color of the tetrahedron. And I have also put this as only normal stress existing. And if you want to rewrite, you can also write it as σ_{11}, σ_{22} and σ_{33}. Can you write the expression for normal stress and shear stress? Go from Cauchy's formula, find out what is $\overset{n}{T}$, find out $\overset{n}{T} \bullet n$.

Write the expression for normal stress, write the expression for shear stress. Please do that, please do that and then verify it from my expressions. I have very simple expressions for $\overset{n}{T}_x$, $\overset{n}{T}_y$ and $\overset{n}{T}_z$. They are simply $\sigma_1 n_x$, $\sigma_2 n_y$, and $\sigma_3 n_z$. And I have

$$\sigma = \sigma_1 n_x^2 + \sigma_2 n_y^2 + \sigma_3 n_z^2$$

Very elegant and nice looking. Compared to what we had seen for a generic representation of stress tensor, this is very elegant. And I also have the shear stress as

$$\tau^2 = \left|\overset{n}{T}\right|^2 - \sigma^2$$

So, I have

$$\left|\overset{n}{T}\right|^2 = \sigma_1^2 n_x^2 + \sigma_2^2 n_y^2 + \sigma_3^2 n_z^2$$

And I can substitute it in this. I am showing the expression for you, ok? But I expect you to do that as a homework; please do that as a homework. I want to give one more homework today, fine?

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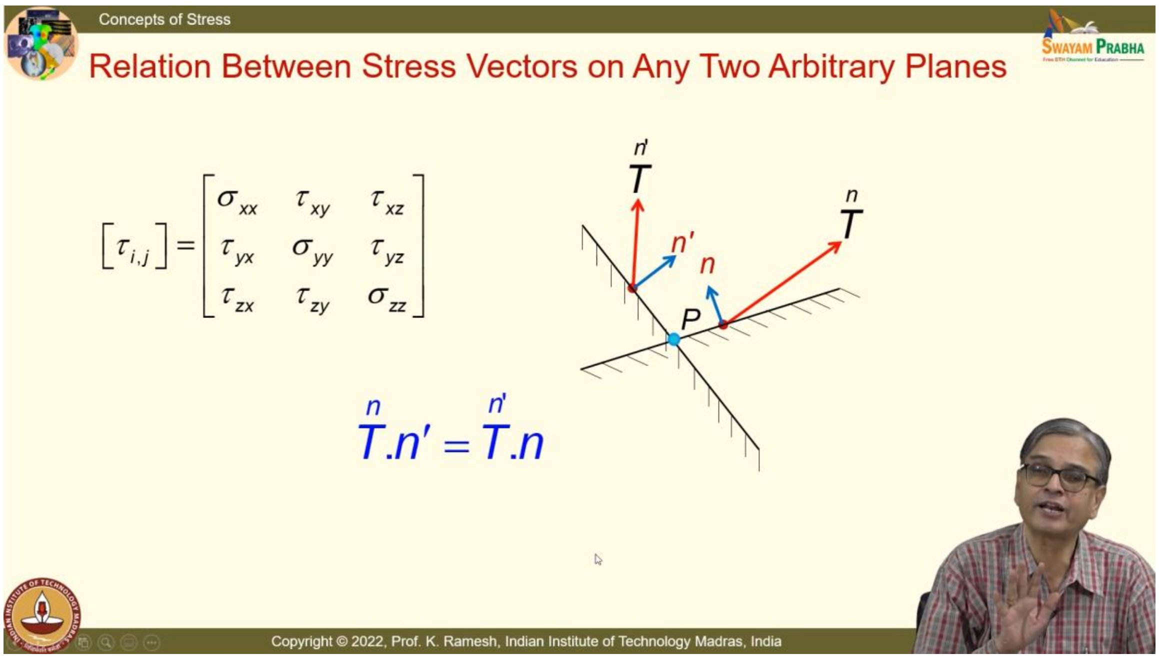

The next homework is what is the relation between stress vectors on any two arbitrary planes? See, this is a very useful result; I take a point P, draw this sketch, I have a plane, the plane is represented by an outward normal and it has a stress vector $\overset{n}{T}$.

I can also pass another plane n', I have a n' plane. I have stress vector as $\overset{n'}{T}$. And I want you to prove $\overset{n}{T} \bullet n' = \overset{n'}{T} \bullet n$. See, these are all vectorial quantities and this is a dot product. Please work it out in long hand and come with this identity, because we will use this identity to develop certain other interesting concepts related to state of stress at a point.

So, in this class, we have looked at two more graphical representations of state of stress at a point. They showed that Mohr's circle is, by far, the best graphical representation for state of stress. Then we mathematically developed, what are the principal stress magnitudes as well as the directions. And in the process, we identified new set of quantities called invariants I_1, I_2 and I_3. And I said that when you do the stress transformation or principal stress determination, you have an inbuilt check; if you check the invariants, your computation is correct. And I expect you to do that whenever you solve a given problem.

And we have also looked at how to identify a given stress state is uniaxial or a pure shear stress state. See, in the early developments, people were struggling. When they were understanding material behavior, they found uniaxial was easy to handle, biaxial - some more understanding is required. So, even if you know the given stress state at a given point is uniaxial, the problem was lot simpler. And looking at the stress tensor represented in terms of the principal stress magnitudes really help the mathematicians to come out with newer entities, ok? Thank you.