Strength of Materials
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Lecture - 08
Proof of Mohr's Circle

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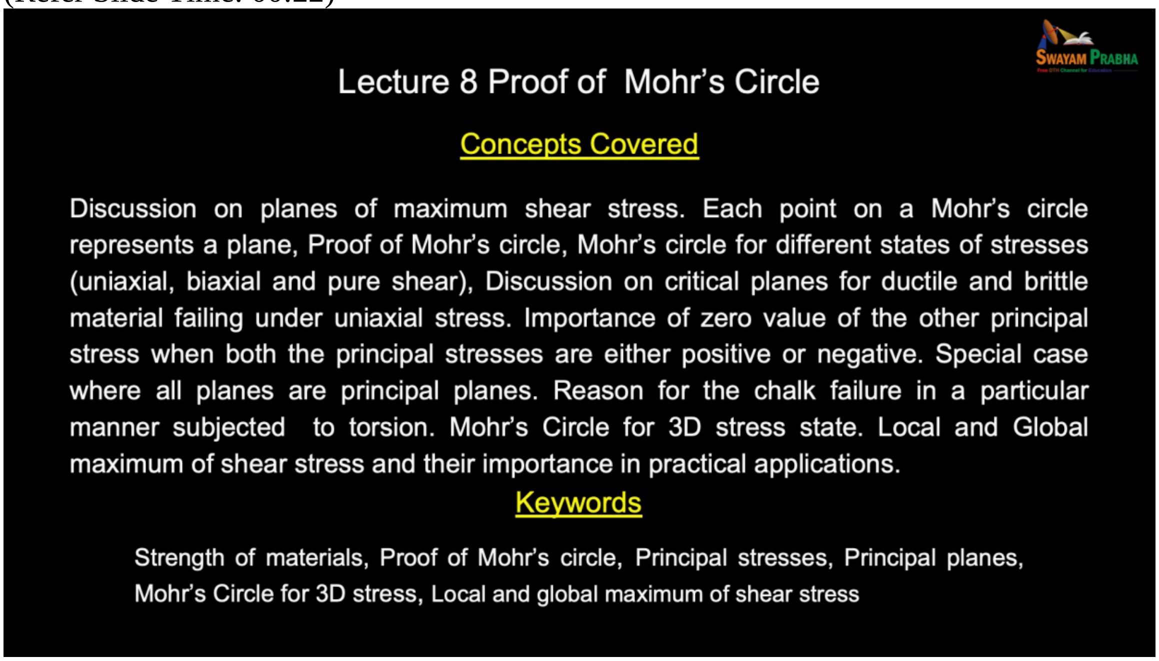

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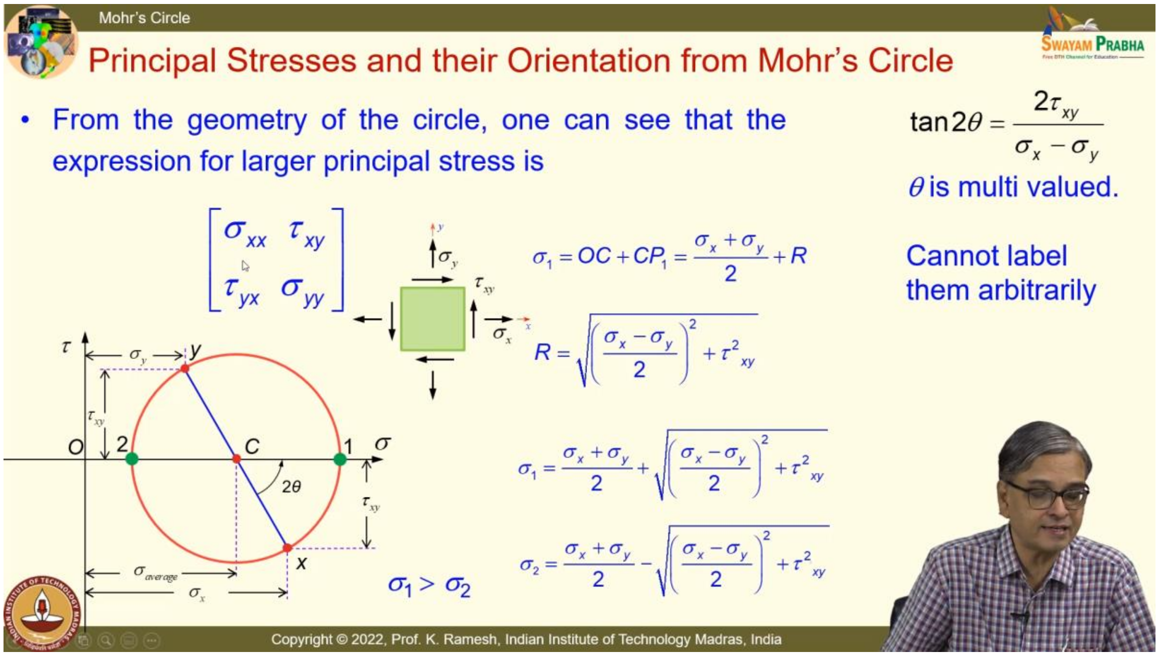

Let us continue our discussion on Mohr's circle. See, it is a graphical representation and if you look at all branches of science and engineering when they were developing in the initial stages, graphical methods played a very significant role. They did not have calculators or computers like what you have now. So, many of the key understandings they could get it from graphical representation. So, it assumes very high importance and in the last class, you know, we have developed some understanding on how to draw a Mohr's circle. And in order to make my diagram not cluttered with lines, I have taken a values of principle stresses in a manner that both are positive.

So that you know, I do not have this line coming in between. So, which will obscure some of your basic concepts. And I have also removed, we had for an arbitrary plane, all that is removed. So, I have the tensorial representation of state of stress, I have a pictorial representation of state of stress and I have a graphical representation of state of stress at a point.

And what you understand is, from the Mohr's circle, it is possible for you to get the expressions for the principle stress. Note the spelling, say it is not principle, ple, it is pal, principal stress that is how you have to do it, remember that. And it is straight forward from the diagram, I have σ_1 is given as $\frac{\sigma_x + \sigma_y}{2} + R$. And you can also find out what is the expression for R. And we have an expression for σ_1 and we have an expression for σ_1.

Ideally what you do is, depending on the numbers, you will have to find out and arrange them in the algebraically largest as σ_1, algebraically the smallest is σ_2. So, this is the convention that is being used. And I also said, you can also find out from the graph, what is the orientation of the principal stress. And that you have as $\tan 2\theta = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y}$. And what you will have to recognize is, when I evaluate θ from this expression, it is an inverse trigonometric function.

So, θ is multi-valued. Like what you have done for σ_1 and σ_2, algebraically arrange them, you cannot go and algebraically label the values of θ as θ_1, θ_2, θ_3. I have taken a planar problem. If you are taking a three-dimensional problem, you will have θ_1, θ_2, θ_3. You will have to go to Mohr's circle and then find out, what is the orientation of principal stress 1 and what is the orientation of principal stress 2 from the *x*-plane or the *y*-plane, whichever is convenient to you.

Is the idea clear? See, we follow a convention when you label principal stresses. You cannot extrapolate the same convention to θ. θ is a physical quantity. And what you have here is, from the Mohr's circle, you can find out what is the location of principal plane 1 and principal plane 2. So, from the multi-valued values of θ, you have to rearrange and then label them correctly. Labeling you have to do it correctly. And I have the tensorial representation as σ_{xx} τ_{xy} τ_{yx} σ_{yy}. From the understanding of the Mohr's circle, can I also represent the same state of stress at the point of interest in a different manner based on the principal stresses? I am free to do that. Idea of Mohr's circle is, it helps you to find out the stress vector on any of the possible infinite planes passing through the point of interest. So, if I take any two mutually perpendicular planes, I can represent the quantities forming a stress tensor, fine.

So, I can also write from the understanding of the principal stresses, a very simple matrix. See, if you have zeros, my multiplication becomes very simple. If I have to find out the stress vector, if I know the principal stresses, if I know the direction cosines, then I do less mathematics when I write it in principal stresses. And I have also said, when I am going to have failure theories, where I have multiple combination of tension, torsion, bending and so on and so forth, principal stresses is an easier approach to assemble all of them and then find out what is the net effect. Fine, I can by linear superposition, I can add the stress tensor. From the final stress tensor, what I have got, I can find out the principal stresses and I can go back to theories of failure. And number of theories of failure people have developed, it varies from material to material. And with modern manufacturing, you also have to find out, for example, when you have this rapid prototyping, where the models are grown. So, people want to find out when you grow the model, you may have some gaps in between, micro gaps. Micro gaps, how do you handle it? What is the way the material behaves? So, depending on the manufacturing method and also newer materials are being developed, failure theories keep appearing on the literature. So, you have to learn the failure theory and apply them. We will also have a detailed discussion on it later in the course.

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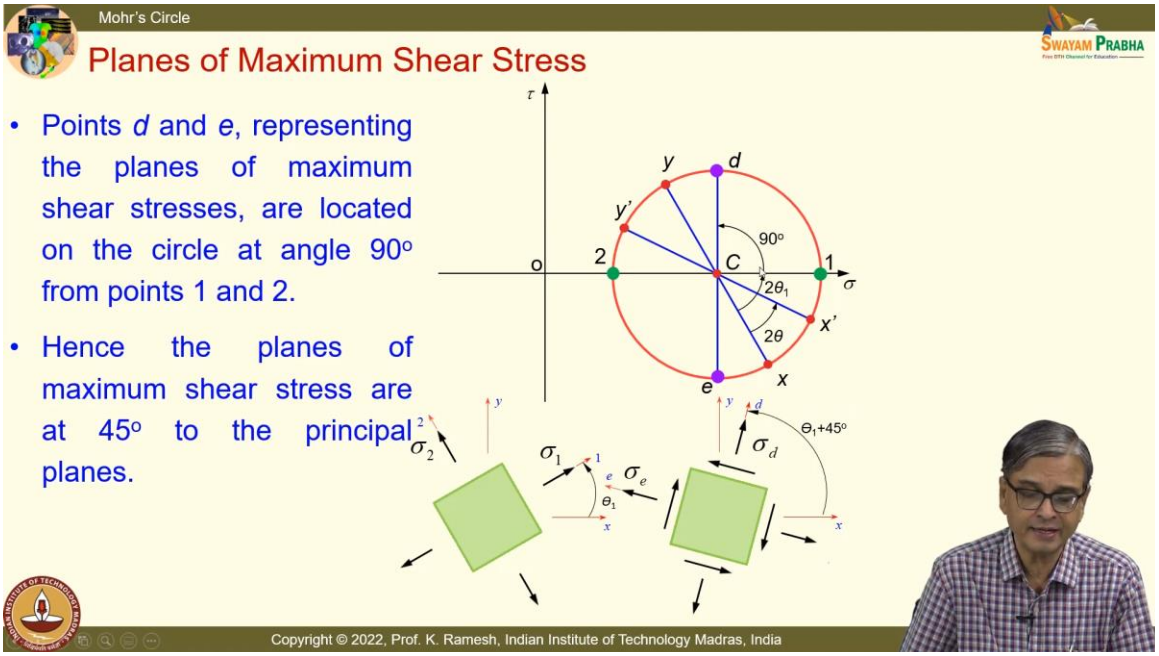

See, I said I deliberately took a diagram, which is such that the principal stresses are both positive away from this axis, so that the diagrams are much more clearer to you. And we have also made a statement that you have maximum shear stress occurring at point d and e. See, when you say mathematically something maximum, you should also look at whether it is local maximum or global maximum, fine.

For a configuration like this, this is definitely maximum, but it is not the global maximum. So, you have to wait until we develop, how do I draw the Mohr's circle for a true three-dimensional situation. The interest while developing this was, I do not want to have this line inside, so that it obscures your clarity of the basic concepts. Whatever we have discussed, it does not affect the principal stresses or understanding that each point on the Mohr's circle represents a plane. And plane is how you designate a plane? It is designated by outward normal, fine.

All those concepts you can understand, but the moment you come to maximum shear stress, you have to look at it carefully and then we will learn it towards the end of the class today, where we will look at what happens when I have a three-dimensional situation. But the idea when I have the principal planes, whether it is a local maximum or a global maximum, the way in which you look at it, the relevant principal plane and the maximum shear stress separated by the angle 45 in the physical plane and in the Mohr's circle separated by 90°, that there is no change.

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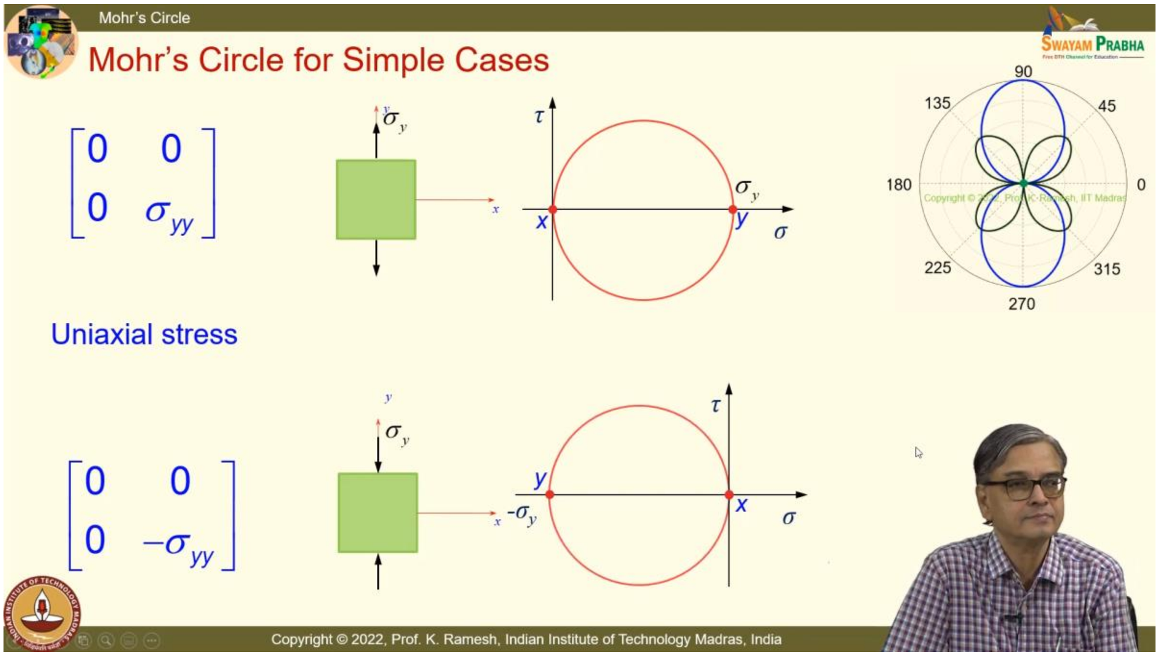

And let us draw the Mohr's circle for simple cases. You know, we have been talking about simple tension, fine. And we have looked at a very nice beautiful graphical representation. What will occur naturally to you is, you will only draw a polar plot. When it is a function of θ, one way of representing it is by a polar plot. And Mohr's circle is a real gift, fine. So, I have this as tensorial representation and it is a uniaxial state of stress, because I take the model and apply axially, fine. I have a uniaxial state of stress.

So, I have the pictorial representation. Can you draw the Mohr's circle? You know, we will be drawing Mohr's circle for a variety of cases today and you do it from first principle. Then verify with my slide whether your thought process is correct. While doing so, you have to identify what are the quantities on a given plane. So, you have to essentially identify what is a x-plane and y-plane.

And you are going to plot a graph, essentially a circle. You will have an horizontal axis as σ and vertical axis as τ. And from the given problem, what happens on the x-plane? Normal stress is zero, shear stress is also 0. So, where would I mark the x-plane? It coincides with the origin, fine. So, next step is I have to find out what is the y-plane. You are given some value of normal stress on this y-plane. So, I will go along this direction. There is no shear stress. When you do not have shear stress on the x-plane by equality of cross shears, there cannot be any stress on the y-plane. The shear stress will not exist. So, I have the y-plane mark, that is all my job is done. I have to just draw a circle using this. Now, let us have a situation. See normally when you represent a symbol, it can be both positive or negative. For the discussion purpose and for illustration, I am really showing the next case as $-\sigma_{yy}$.

Please understand the context. What is the difference? Instead of that being a normal stress acting away from the plane, it is towards the plane. Instead of tensile stress, it is a compressive stress now. So, how would I identify the *x*-plane and *y*-plane? I have the pictorial representation. Can you find out what is the *x*-plane and what is the *y*-plane? Straight forward.

What you have done in the previous case, you have to extrapolate. In the *x*-plane, everything is zero. So, it coincides with the origin. So, I have *y*-plane which is determined by the magnitude of the compressive stress and then simply draw a circle because you do not have to worry what is the nature of the curve. We will prove the nature of the curve will always be a circle.Wait for ten more minutes, fine. Right now, let us learn through simple problems what we have discussed as the procedure to plot. Then we will go to the proof and we will also learn one or two related concepts.

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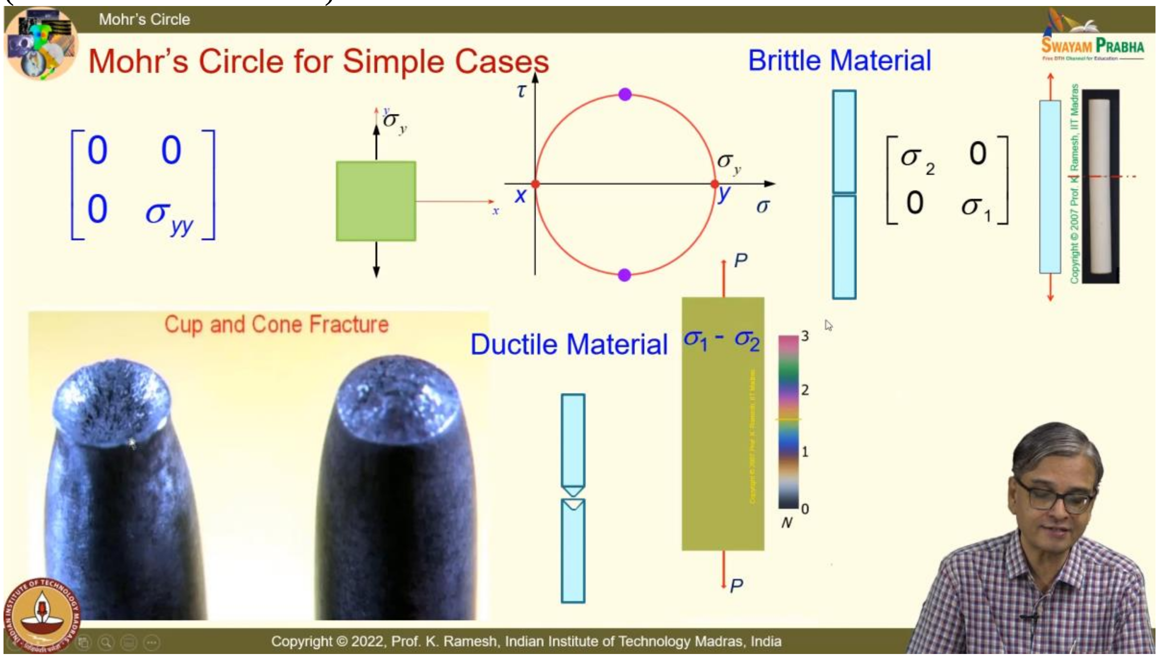

You know, you have all done an experiment on chalk and when you pulled it, it broke beautifully. You have not pre-decided where it should break. Can we investigate that from the Mohr's circle? You can draw the Mohr's circle which you have already done.

And for us to find out whether the chalk will break or not, I should know what is the failure criteria for a brittle material, fine. For the current discussion, you look at that when the normal stress reaches a critical value, failure would occur, fine. And I have the Mohr's circle like this. Mohr's circle is nothing but the plane and you have *y*-plane. So, this the value of σ_y, the σ_y is increasing, what will happen? Mohr's circle also will be increasing.

The *y* will be plotted away from the origin and at some point when it becomes critical, which is the critical plane? We have said wherever you have the maximum normal stress, that is the critical plane and for the Mohr's circle, it is nothing but the *y*-plane. So, *y*-plane is this. So, you are able to explain from the Mohr's circle, what is the plane in which you could expect material separation for chalk being a brittle material straight forward. Fine. Suppose, I have a ductile material, we have also done the tension test, I have shown you the diagram and I have also shown this, a ductile material does not fail perpendicular to this.

So, the failure mechanism and the criteria are definitely different for a ductile material. Suppose, I say for the current discussion, it fails by maximum shear. It does not fail by the critical value of normal stress, but critical value of the shear stress. So, wherever the shear stress, whichever plane orientation the shear stress is maximum, you will have material separation along that plane. Is the idea clear? I have this as a cup and cone and I have also identified from our earlier knowledge, what are these points? They are the points that have the maximum shear and from the Mohr's circle, how much they are separated from the *y*-plane? From the *y*-plane, it is at 90°on the Mohr's circle, physically it is at 45°, ok.

And if I look at the cup and cone fracture and then put it like this, you will have the cup like this, you have this at 45° and you can go back and explain, this is what has caused from a simple experiment, you are able to apply Mohr's circle and identify the plane on which failure can occur. Is the idea clear? In one case, it is dictated by the plane on which the normal stress exceeds the limit. In another case, it is on a plane where shear exceeds the limit. These planes are different. See, one of the difficult concept in stress is, we are also going to have a detailed discussion on a free surface.

When you take a point, what happens on a plane is what you will have to look at. You can have infinite number of planes. If I have a plane like this, which is *y*-plane on which I will have the maximum normal stress, but suppose I have this surface, that is a free surface, you will have stress vector is zero, but there you are looking at a plane which is *x*-plane. So, you should know that will take some time. Once you keep discussing these issues, you will be in a position to appreciate how to look at the planes.

And I said even the photoelastic fringe pattern is also a graphical representation. This is also needed because I need to know what happens to the neighboring point. It is a beautiful technique where we are able to see what happens in the field as a whole. And you find that stress is constant and it is all the points in the cross section experience the same value of stress away from the loading point. I am only plotting what happens away from the loading point and we have P/A as the stress.

And now when I go back to this, from your understanding of your principle stresses, can you also write it in terms of σ_1 and σ_2? Can you write it down? Because this is a stress state where I have no shear stress and you have learnt what are principle stresses. Can you replace this as principle stresses σ_1 and σ_2? Let me see whether you write it correctly. You are learning new symbolism. You agree σ_{yy} is one of the principle stresses. Do you agree? Because its stress is acting completely normal to the *y*-plane.

What we have discussed as principle stresses on the plane of interest, the stress should be only normal. There should not be any tangential component. Now the question is, can I write it like this? Is it right? Some of you nod your head that this is right. You should never forget the convention. We have said σ_{yy} is a finite quantity and that is also a positive quantity.

So, I have to write this as zero as σ_2 and σ_{yy} as σ_1. You have to understand whenever we use the symbolism $\sigma_1, \sigma_2, \sigma_3$, algebraically the largest value you call it as σ_1. So, do not jump to write and replace it. See when I have this as a constant value, when *P* is increasing, *P/A* is going to increase, you can intuitively feel that this should be related to σ_1.

Am I right? You can say that. Is it not so? This is a graphical representation. What happens at a point of interest on all the possible infinite planes? This is again a graphical representation. Fortunately, the experiment is so colorful, you get colorful fringes and then we have seen at every point, it is having a constant color for a given load. So, you understand that the complete material participates in load sharing and the only non-zero quantity is σ_1 and you can say that this is related to σ_1, but if we go to crystal optics and develop systematically what is the way photoelasticity provides this, this basically gives contours of $\sigma_1 - \sigma_2$ and in this context what happens? σ_2 is zero. So, it is one and the same as σ_{yy}. Is the idea clear?

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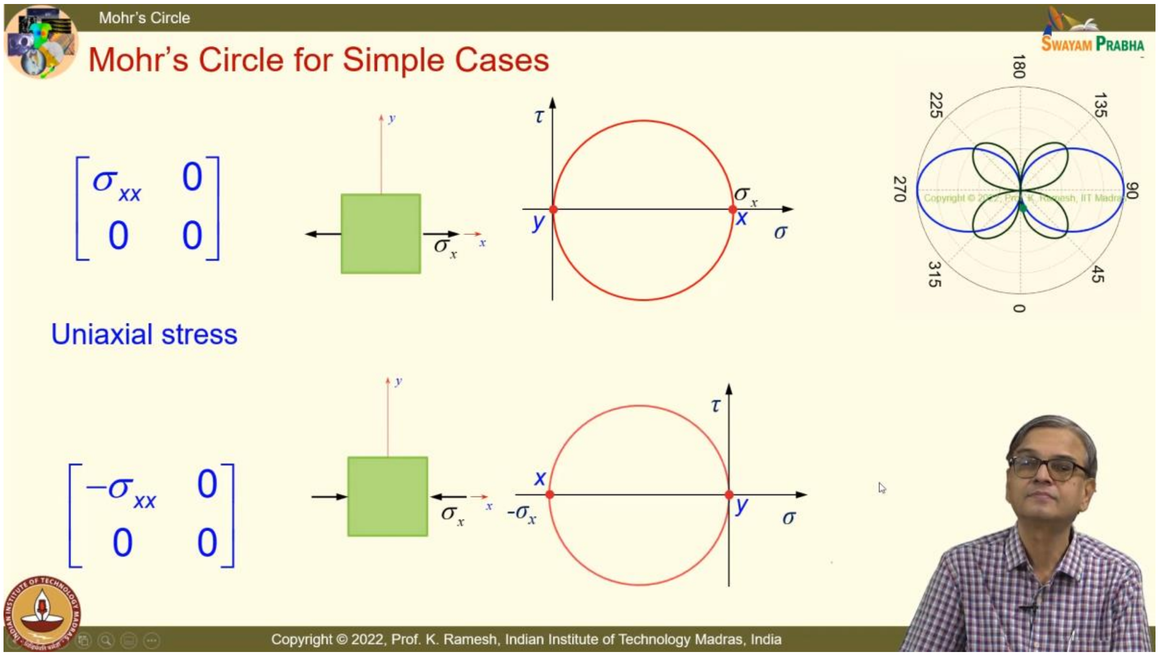

Now, we will take up other simple cases like we have the normal stress along *y*-plane, we have σ_{xx}. It is child's play for you. You should be able to draw it within a minute. Within a minute, you should be able to draw the Mohr's circle and this is again a uniaxial state of stress. So, the issue is you have to find out how to locate the *x*-plane, how to locate the *y*-plane and simply draw that as a circle.

All that you know because *x* and *y* form a diameter, fine. And similarly, when I go for $-\sigma_{xx}$ instead of a tensile stress, I have a compressive stress. This is again you can easily do and you know whatever the plot that I had done it for normal stress σ_y, I have just put it horizontal. When I do a polar plot, it will look like this. I have not changed the angle. So, it will start from 0, 90 like this you can visualize that, but the graph shape is very important. So, here again I can easily draw the Mohr's circle. I have the point the plane *x* and then plane *y* and simply draw the circle.

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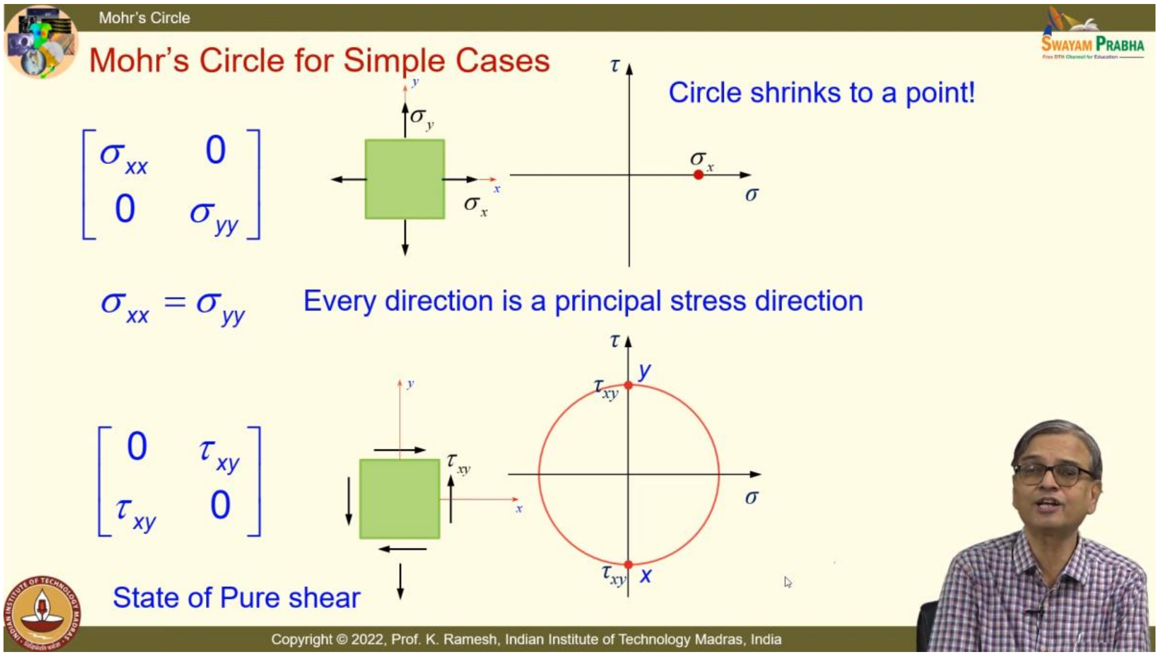

Let us now graduate to other set of problems. I have σ_{xx} and σ_{yy} such that the magnitude of σ_{xx} and σ_{yy} are identical. What way you expect the circle? Please try, try, try. Are you just try, are you getting a circle? When I have these two magnitudes are equal, I simply get that as a point. Circle shrinks to a point and this is also a very special case. In this, every direction is a principal stress direction. It is a very special case, ok. Now, let me also take another simple case where I will go for a pure shear stress state, ok.

So, I have only τ_{xy} and τ_{yx}. I have done this interchangeably. Some slides I would have τ_{xy} and τ_{yx}. Particularly, when I do indicial notation, I cannot use the equality of cross shears. I have to write them separately. Since we have recently learned equality of cross shears that we have got. Can you attempt how you draw the Mohr's circle for this case? See, the process is very simple. Look at the *x*-plane and then plot the quantities and you have a positive shear acting on the *x*-plane and there is a convention. The convention says for a positive shear, I should plot it downwards. So, I will have my Mohr's circle like this.

You have to identify the *x*-plane and *y*-plane correctly. Where will *x* come? Downwards, ok. So, I have this and then I have *y*-plane. So, now, I draw the circle. Is the idea clear? You are in a position to find out for simple cases uniaxial stress, simple biaxial stress and you have the pure shear stress.

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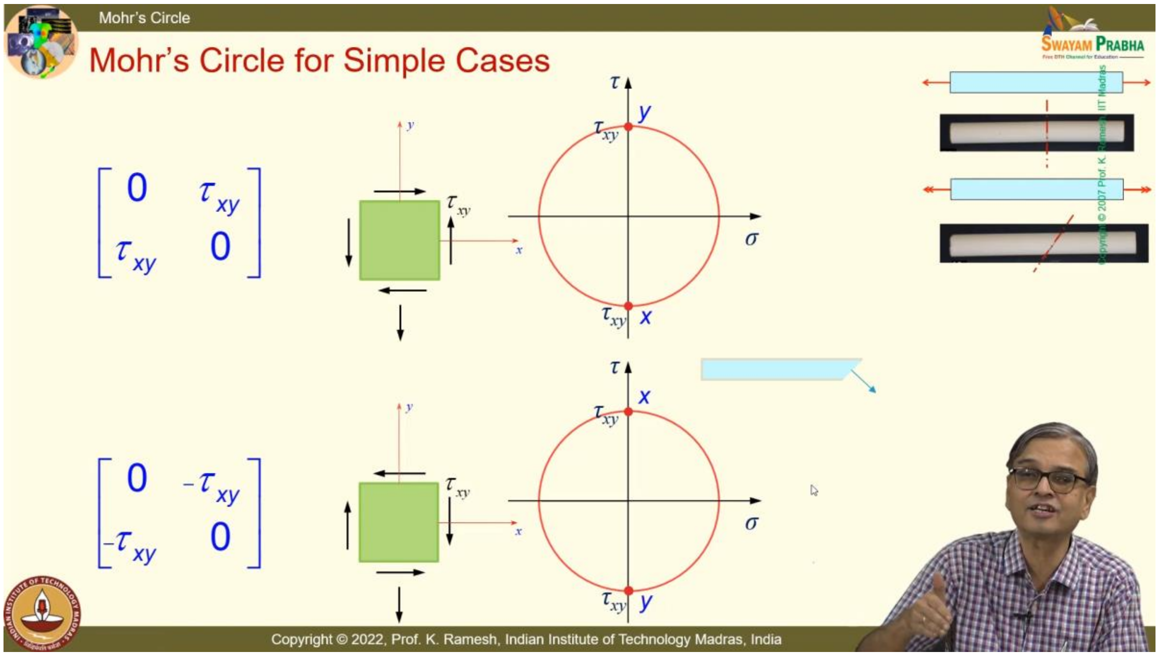

And we will also have for two different cases, I have a positive shear, I will also have a negative shear. Because you all know since chalk is a brittle material, a brittle material will fail when the normal stress reaches a critical value. You have looked at from Mohr's circle which plane the chalk will break. Now, we will graduate and find out how to reason out when I apply a torque, it fails at 45. We have seen without a proof that torsion introduces pure shear stress state which we have seen it in the very first lecture.

So, I have this, I have two cases. This is a positive shear, I have the graph, Mohr's circle is drawn. I also have the negative shear. What will be the difference between the two? x and y will be swapped. And what way you have seen in the chalk? It fails at 45° like this. It fails at 45° like this. It fails at 45°like this. So, I have the plane marked. So, let us take this portion of the chalk. I have drawn the chalk and I have also put the plane. Can you locate it on the Mohr's circle? Because we have said only a normal stress will precipitate a failure in a brittle material.

And what I have here, suppose I have this as x-axis, I have the x. First of all you should know whether I should use this graph or this graph. Now, I am taking the chalk and then applying anticlockwise. I am applying anticlockwise. So, I have an action reaction pair. So, this portion of the chalk is acting on this and I am rotating anticlockwise. That means, I am applying a shear downwards. Is the idea clear? So, I should use this to explain what happens on this chalk. And what I find is the normal stress is maximum on this plane which is separated by 45°. So, I have this. The plane is determined by the outward normal. So, I rotate from the x direction clockwise because I rotate clockwise in the Mohr's circle 90°.

In the physical plane from the *x* direction, I rotate by 45°. Now, you are convinced that we are in a position to explain the behavior of chalk even though we have not completely studied what kind of stresses developed in shear from first principle. You have taken the knowledge that it introduced pure shear stress state. From the Mohr's circle, you are able to investigate this, fine.

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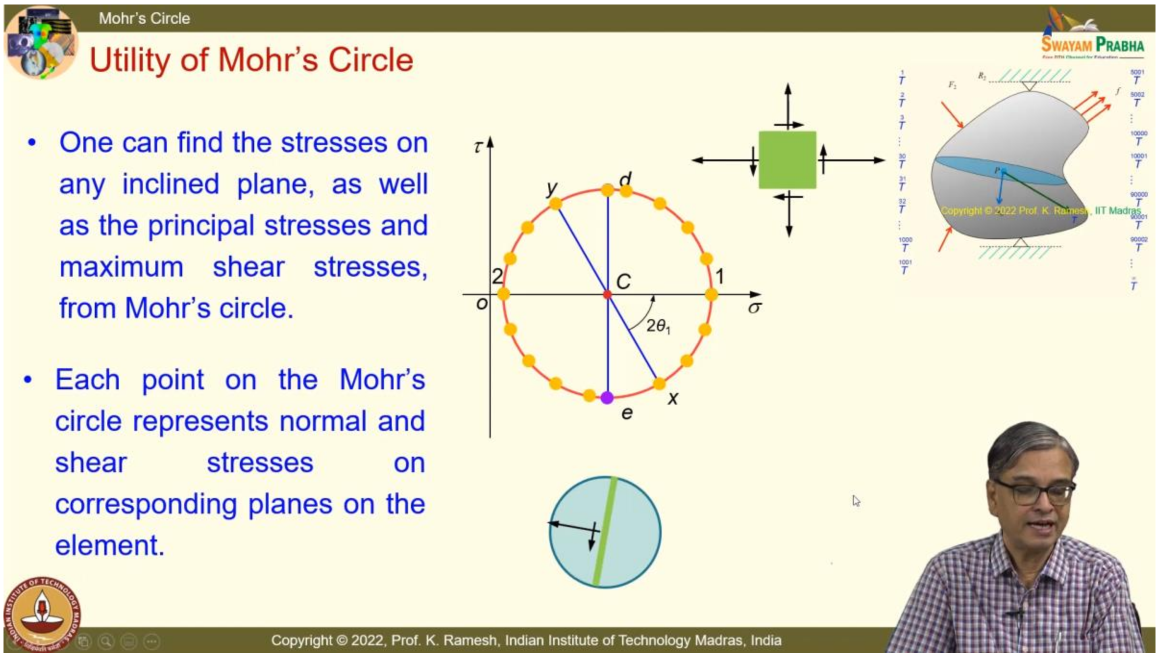

See, the utility of Mohr's circle is, you know, you have to settle in your mind that each point on the Mohr's circle is representing a plane. And in the state of stress at a point, we have demanded that I should get stress vector on all the possible infinite planes passing to the point of interest and which is achieved in a Mohr's circle for a planar problem we are discussing, ok.

So, you have identified that there is a principle stress, principle stress 2 and maximum shear stress. I said whether it is local maximum or global maximum. Let us postpone the discussion and wait for some time. And I have the orientation of the principle stress denoted as $2\theta_1$. And what you need to understand is, I take the plane *x*, this is the stress tensor that is given to you.

Stress tensor is given with respect to *x y* direction, they are mutually perpendicular. So, if I take up *x*-plane here, I have a normal stress and the shear stress which can be culled out from the Mohr's circle. Suppose, I go to the next point, I can also find out what is the shear stress. So, as I go closer to the σ-axis, my shear will keep decreasing. That is what is depicted here to visualize. And you know, it is a schematic. I have not measured it correctly and then put it this. You will get the sense that normal stress is increasing, it reaches a peak.

It reaches a peak when it reaches the point 1. So, I have only normal stress acting on this plane. Then you know, once I go here shear changes its direction, shear keeps increasing and shear reaches a maximum.

And the knowledge what you have is, when shear reaches a maximum, you will invariably have normal stress also in a generic situation. In one situation, you will have normal stress is zero. What is that situation? We have just now seen. When I have only pure shear, when I have pure shear in the maximum shear stress plane also, you will have only shear stress. So, then the values keep changing and when I come to plane 2, it again has only a normal stress and then shear starts appearing and you reaches the maximum shear and the story goes. So, you have to recognize on a circle, I can have infinite points. So, it accommodates all the infinite planes. So, that is the beauty of the Mohr's circle.

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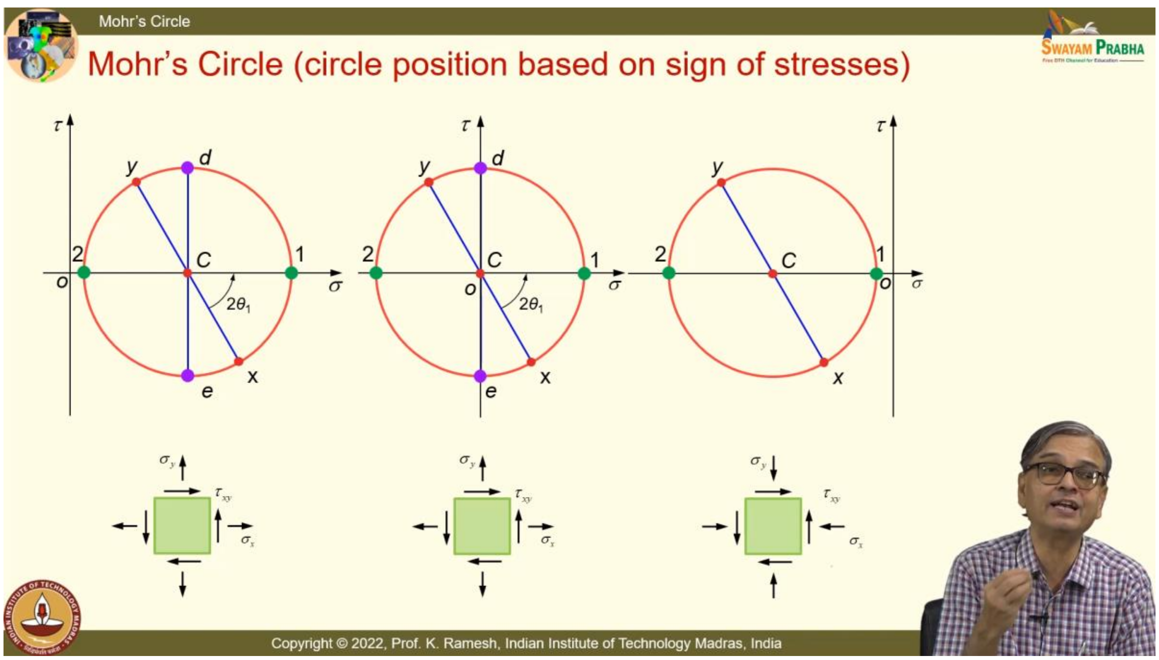

And you know, depending on the stress tensor I take it, the location of the Mohr's circle will keep changing. See the shape is always a circle geometrically, but its location and also the location of the *x* and *y* planes will differ from problem to problem because when you have a state of stress, it is different for different situations. So, when I have this, that is what I said. When I represent it, I am representing this as positive quantities. So, this is what we have done. This is σ_1 and σ_2 are both positive in this case. So, that is one possibility.

I can have another possibility where I have σ_y is compressive. When I have σ_y is compressive, I would have a Mohr's circle which occupies something like this, fine. I have a Mohr's circle which occupies like this. And I can also have another situation where

Mohr's circle is completely shifted to the left. I have σ_y is compressive as well as σ_x is compressive. I can also have a Mohr circle like this. So, what you will have to appreciate is the graphical representation remains a circle. Its location can get shifted. In a special case, it can become a point. And you will have to carefully identify what is the *x*-plane and *y*-plane on the given Mohr's circle. Or for a given stress state, you first locate the *x* and *y*-plane, then plot them. So, that is the message in this slide.

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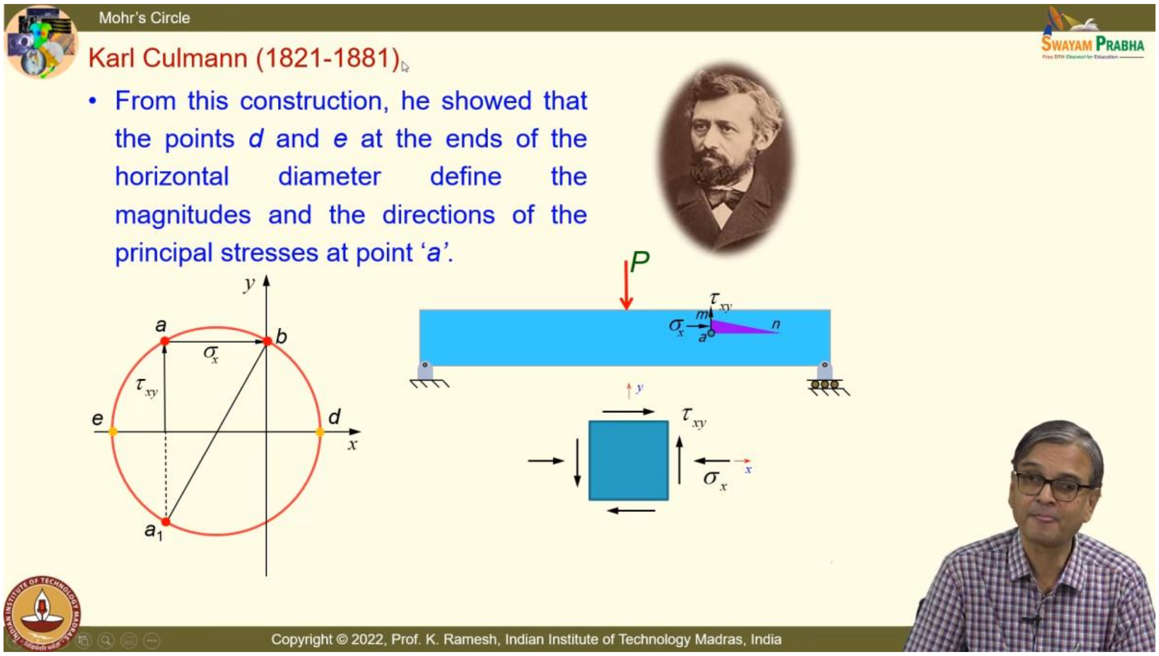

And I said, you know, people were very good at graphical methods earlier and if you look at the history, it all got developed by Karl Kalman. And he plotted for the first time, he has not followed the procedure what we have done. Only for the problem of a beam, when it has both normal and shear stress, he followed a procedure like this. I am going to follow what he has done. So, he has plotted on the *x*-axis and *y*-axis. So, he plotted shear stress and labeled that as point *a*. Whatever the shear stress at point *a*, he had put the shear stress. And he had put the normal stress and put it like this. This is actually a compressive stress here. And then he identified the point *b* and located the *y*-axis. Then he mirrored this point *a* to a_1, joined these two and got a circle. No other explanation was given. So, the first attempt to get a circle to represent state of stress is credited to Karl Culmann.

And you know, I said Mohr's circle was developed in 1882. So, he is passed away even before that. And he also identified that there could be two planes where the stresses are completely normal. So, he also had an idea of what is the principal stress.

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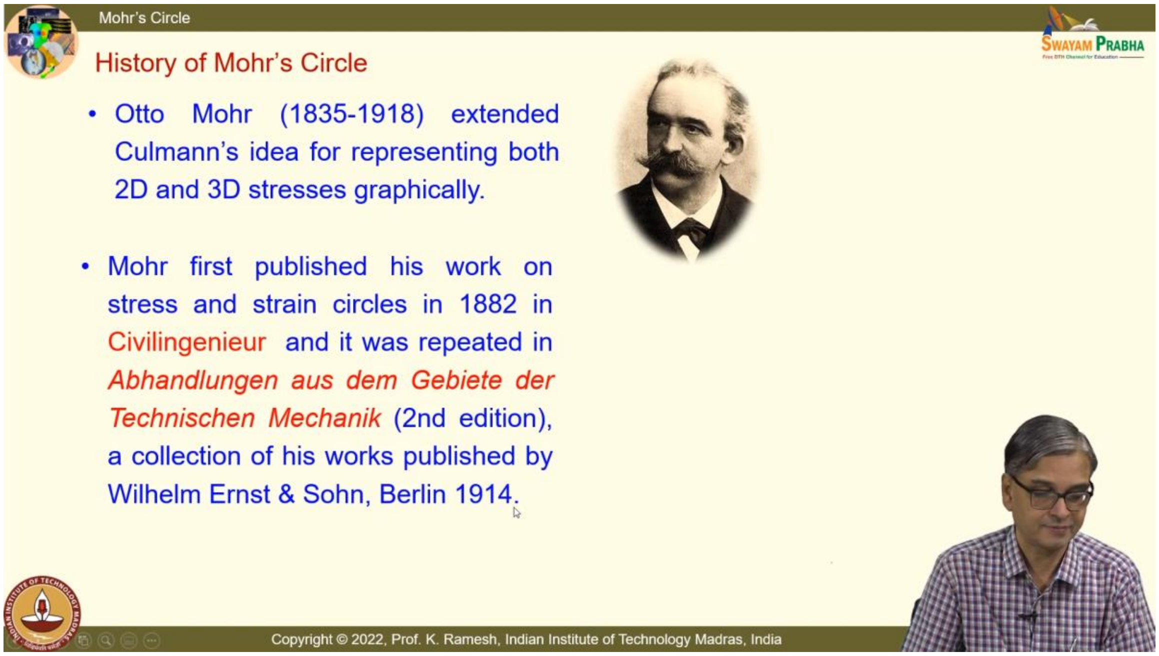

And this was later developed by Otto Mohr, not only for the two-dimensional situation, but also for three-dimensional situation. It was done by Otto Mohr. He published his work in 1882 in *Civil Engineer*. And there is, I think it is a German title of the book. It is a collection of works published in 1914.

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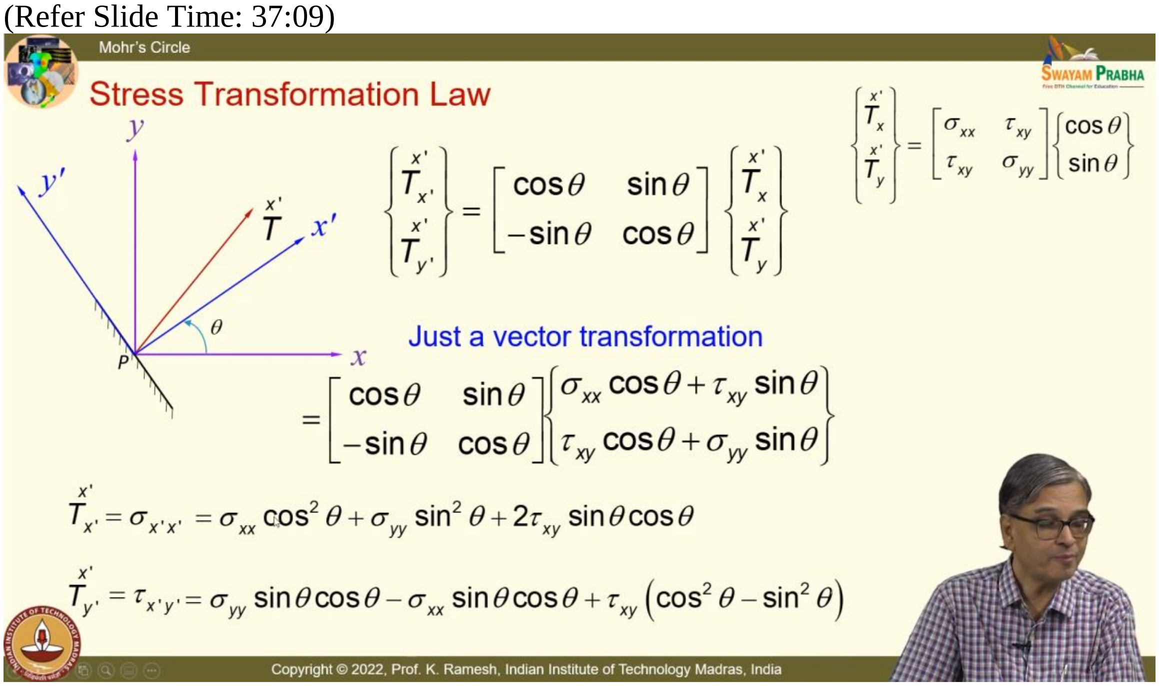

And now we go back to proving why the Mohr's circle is a circle. That stems from your stress transformation law. See, stress transformation law, until you graduate, you will have to be with it. You will be repeatedly doing it in your higher courses also. So, there is no harm on repeating it again and again. And we have seen it from first principles. I have to get the stress vector $\overset{x'}{T}$ and stress vector you get it by Cauchy's formula. From the Cauchy's formula, you transform $\overset{x'}{T}$ to $\overset{x'}{T}_{x'}$ by a vectorial transformation. This is a vectorial transformation. So, this takes care of one index, this takes care of the, you have two indices. The first index is related to the plane, second index is related to the direction, ok. So, you cannot simply say σ_x and $\sigma_{x'}$ as $\sigma_x \cos\theta$ as $\sigma_{x'x'}$.

No, $\sigma_{x'x'}$ means, what is the normal stress acting on plane x' in the direction x'. So, that has to be done in two stages. In which we have done in the earlier class and I have also asked you to simplify the expression. I do not know how many of you have simplified it.

Have you simplified the expression? All this we have seen in the earlier class. It is nothing new to you. I am also going to reproduce the same result, I am not going to simplify it here. Is there a possibility for simplification? The second expression I can simplify. I can put this as $\cos 2\theta$, I can put this as $\sin 2\theta$, fine. Then I will go and manipulate this, this is what I am going to do, ok.

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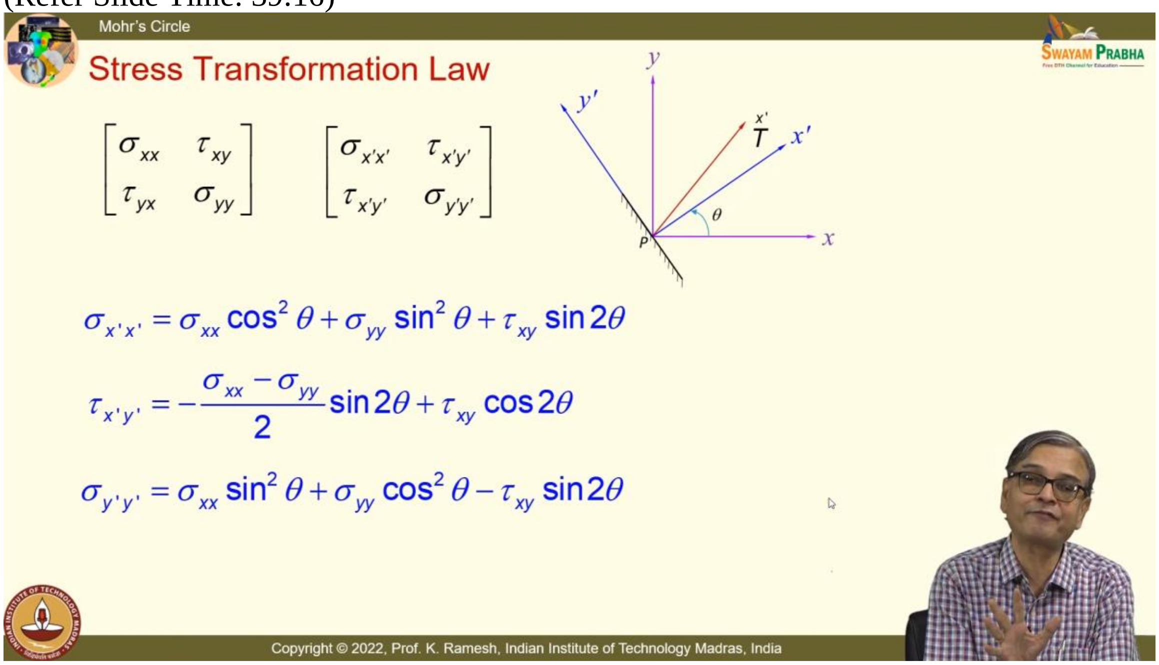

So, I have stress tensor σ_x as,

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & \sigma_{yy} \end{bmatrix}$$

which you will have to get it as in the new reference axis $x'\,y'$. It would appear like this.

$$\begin{bmatrix} \sigma_{x'x'} & \tau_{x'y'} \\ \tau_{x'y'} & \sigma_{y'y'} \end{bmatrix}$$

This is basically from your transformation law. Now the question is, I am going to use the first two expressions, modify it in a manner, I will write them in a parametric form. The moment I write it in a parametric form, I will get an idea what the curve it represents. So, from the stress transformation, you do get a proof that why you get a circle. So, this is a very intelligent way of recasting the same equation.

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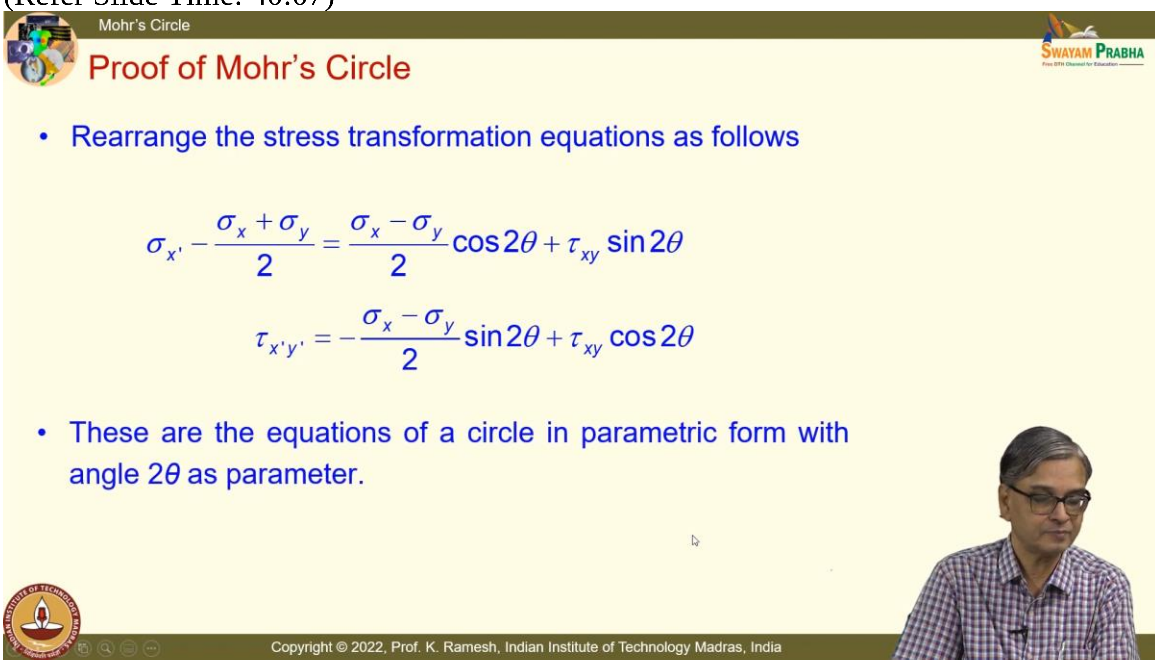

You have to go back to your trigonometric identities and then play with this. This is same as the expression for $\sigma_{x'x'}$ what we have written, but written in a convenient manner like this. And I have

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

So, now what I do is, I square and add and eliminate the parameter.

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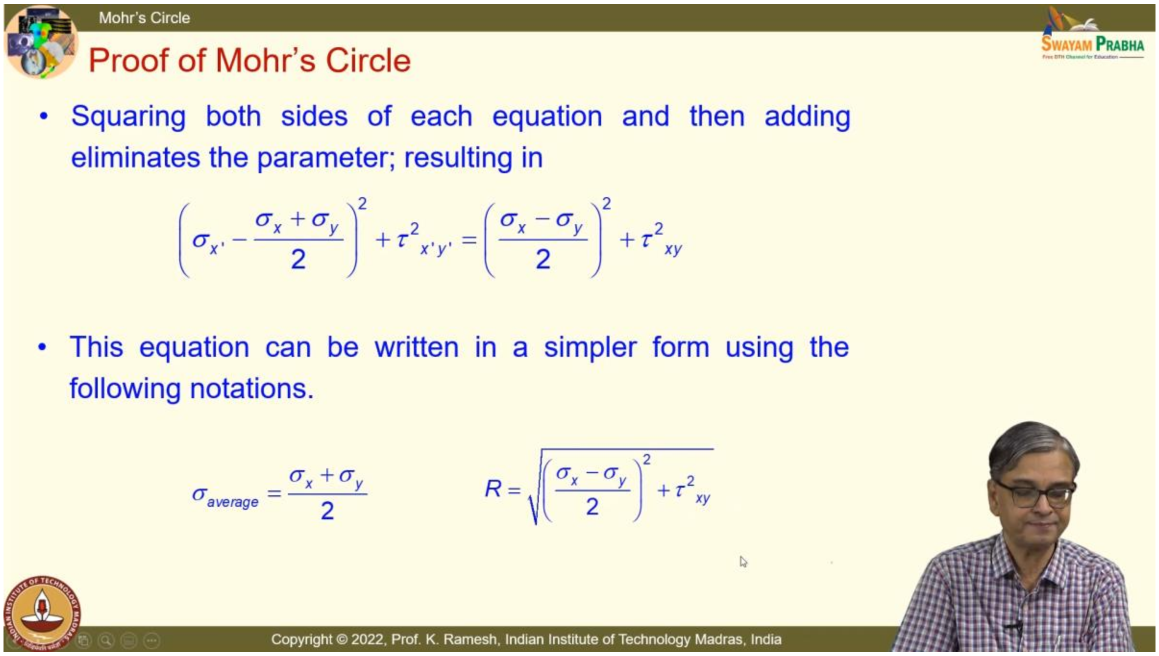

Let me look at what is the equation that I get. Squaring both sides of each equation and then adding eliminates the parameter that is resulting in

$$\left(\sigma_{x'} - \frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau^2_{x'y'} = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2_{xy}$$

which I can also replace it in terms of the radius and also the origin of the circle. Origin of the circle we have said that as σ_{average}, τ_{xy} is zero, is not it. So, I can rewrite the same expression in terms of bringing the new notation

$$\sigma_{average} = \frac{\sigma_x + \sigma_y}{2} \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2_{xy}}$$

So, if I do this my equation results in the form,

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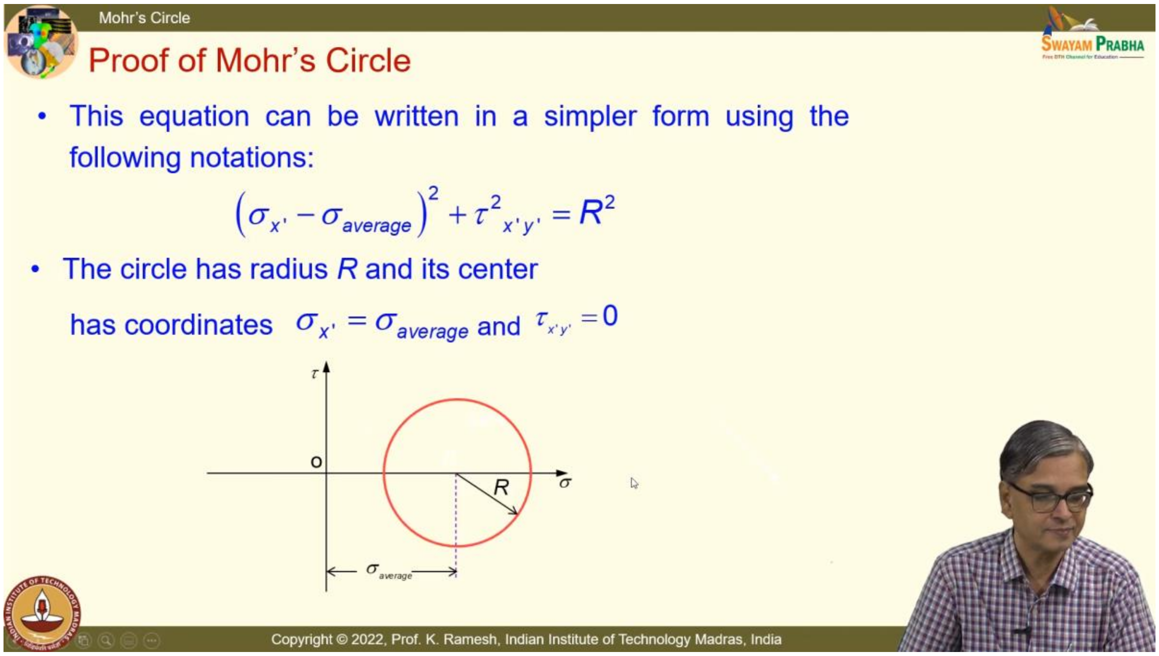

I have

$$(\sigma_{x'} - \sigma_{average})^2 + \tau^2_{x'y'} = R^2$$

which is nothing but a circle, fine. We have learnt the procedure, now we have proved it will always be a circle, it will not be like a nice butterfly, only for a simple tension we got the shear stress appears like a butterfly. If I go for any other populated stress tensor, we may have different curves which we have not looked at all, but the beauty of Mohr's circle is it is easy to draw and you have to draw the circle with center as $\sigma_{average}$ and $\tau_{x'y'} = 0$. And we have seen from the examples that we have looked at it can be a circle of various radii depending on the problem, it can be on the positive axis, it can be on the positive as well as negative axis, it can be completely on the negative axis, it can be equally distributed between positive and negative and it can also be just a point, it can also be just a point. I have just made the point very big for you to see, do not think it is a filled circle. So, I can have point on the positive axis or the negative axis all that is possible.

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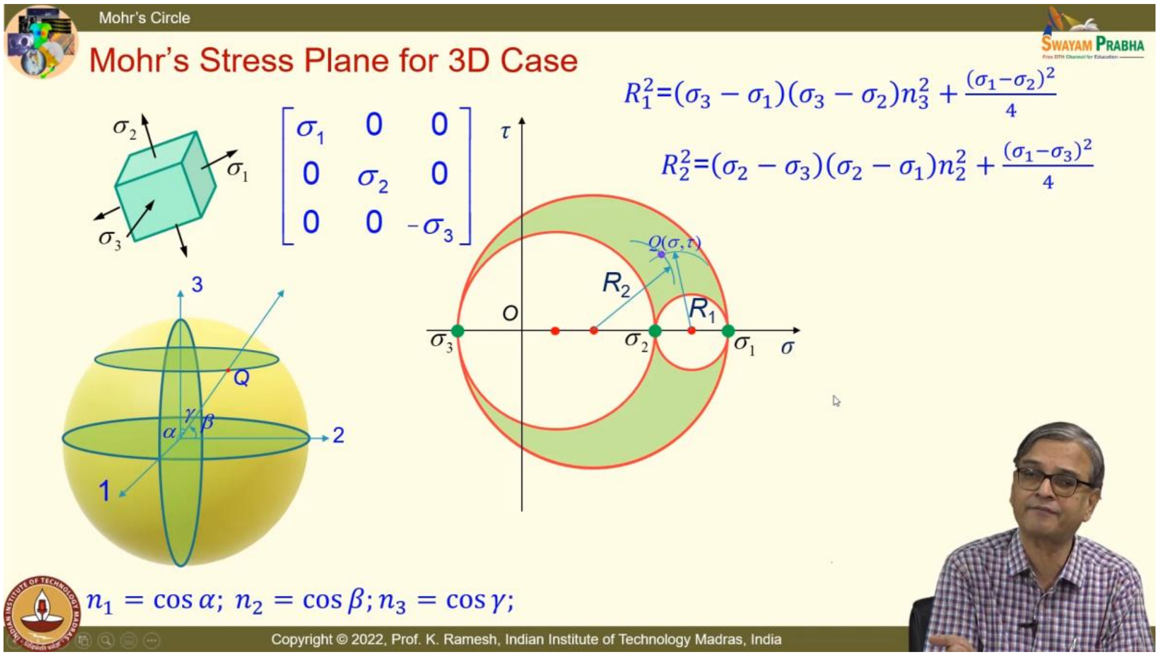

Let us go for a three-dimensional case. Now, you are all accustomed to, I can always raise I can always write a stress tensor in terms of the principle stresses. Now, I have taken σ_1 σ_2 -σ_3, this is what I have put it here. So, I have this as the pictorial representation, because the principle planes can be oriented at an arbitrary direction that is why it is tilted, ok.

Now, what Mohr has said is like you have drawn a circle between σ_1 and σ_2, you repeat the same thing for σ_2 σ_3 and σ_1 σ_3. So, instead of one circle for a three-dimensional case, you will get three circles that is the, I am going to do this without a proof, we will just use the result, we will not attempt to make the proof that you can see in some of the text books readily available. So, I have the circle, this is what we have seen and you draw the circle and for points lying on the circle, you know what is the value of σ and τ and you also know how to locate the plane that is straight forward. And what Moh has said is the in between zone is the zone in which all the points in all the possible infinite planes in a three-dimensional body exist, all the points in this zone, what is zone as green bounded by the red circles, this is the area if you take any point you can locate the point in the three-dimensional space. So, that means, I should know the direction cosines how to find out, I will take an arbitrary point and I will tell you how to mark and locate the point on the Mohr's plane, because when I locate this I will be able to find out what is the normal stress and what is the shear stress, ultimately that is what we want to get.

So, I take a because once I go to three-dimensional, I should go into a three-dimensional representation of the direction cosines, can not restrict to one single plane. And the reference axis are the principal planes 1, 2 and 3 and I have a point *Q*, when I join it with origin, I will be able to find out from mathematics what is the direction cosine α, β, γ. So, in the when you want to apply the Cauchy's formula also you need to know the direction cosines, I get the direction cosines and once I get the direction cosines

$n_1 = \cos\alpha;\ n_2 = \cos\beta;\ n_3 = \cos\gamma;$

you have an expression, ok. I have radius R_1 is given, because I know already what is the value of σ_1, σ_2, σ_3. From the point of interest I know what is the direction cosine, so whatever the quantities that you have on the right side is completely known. So, I can estimate what is R_1, and R_1 is a arc of the circle from the first circle, I have σ_1 and this is labeled as R_1. And I also have

$R_2^2 = (\sigma_2 - \sigma_3)(\sigma_2 - \sigma_1)n_2^2 + \frac{(\sigma_1 - \sigma_3)^2}{4}$

So, I do it from the second one, so I can locate the point Q on the Mohr's three-dimensional representation of the stress state. So, this gives you the point Q and you get the value of σ and τ is the idea clear? For all the points on the boundary of the circle, whatever we have done for Mohr's circle you will have. So, here you will have in the plane 1 and 2, I will have a local maximum shear stress, in the plane 2 and 3 I will have local maximum shear stress, but the global maximum is between σ_1 and σ_3, so this is the global maximum. That is very important, see you have a failure criteria called Tresca criteria, when they apply people forget this, that is what we are going to look at now, whether it is a local maximum or global maximum.

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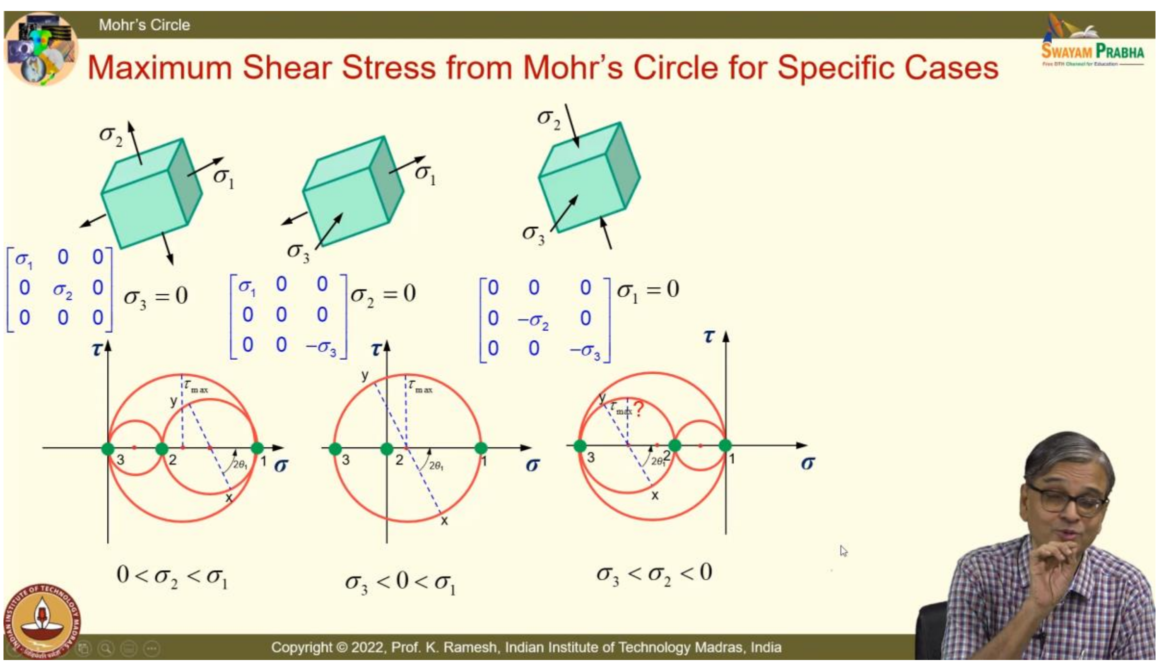

I have taken this, I have σ_1 and σ_2 existing and your σ_3 is zero. You know we have plotted for all our discussion, we have taken this because we have a diagram where this axis is away, so the diagrams are very clear. And we have also said this is what $\tau_{maximum}$, that statement is correct, but in the plane12, normally what you will do 0 is ignored, 0 plays a nuisance value, but once you go and want to earn a salary after your course, you want to have many zeros after 1. So, 0 plays a very important role, so the nuisance value of 0 plays, so I have to recognize 0 and recognize this as σ_3 and draw the circle for the sake of completeness, I can also draw the circle between 2 and 3. And what happens to the maximum shear stress, maximum shear stress is not this globally, globally this is the maximum shear stress, is the idea clear.

This is one of the points where people always make a mistake as students, when both the principal stresses are positive or both the principal stresses are negative, you have to be alert in finding out what is the maximum shear stress. Go back and think what is the plane in which it is acting, even the plane is different, see we always want to have live in two dimensions, fine. Everything is not two-dimension always, even a two-dimensional problem, you may have to look at a plane which is in the three-dimensions for you to worry about the shear stress. Suppose, I have σ_1 and σ_3 which are like one is positive and negative, whether I worry about the sigma σ_2 as zero, it is immaterial, it is not going to affect me as long as I am finding out what is the maximum shear stress, because invariably I will find out what is the global maximum. On the other hand, when I have both these stresses are negative, then I have this situation. I will again recognize the zero value and σ_1 is zero here. So, you are also indirectly trained, when you have multiple quantities, even zero can be a maximum principal stress, because algebraically that is the largest. So, you have to look at what is algebraically the largest, label that as σ_1.

So, in this class, we have looked at various aspects of Mohr's circle. See, we have also proved that when you plot these quantities, you will get only a circle, fine, that you have looked at it. And we have also looked at how a Mohr's circle will look like in a true three-dimensional situation. Instead of one circle, I will have three circles and the in between area is what I have to worry about. For any of the data points, I should know how to locate that, based on the direction cosine and the values of σ_1, σ_2, σ_3, I can locate them, then I can find out normal as well as shear stress. Thank you.