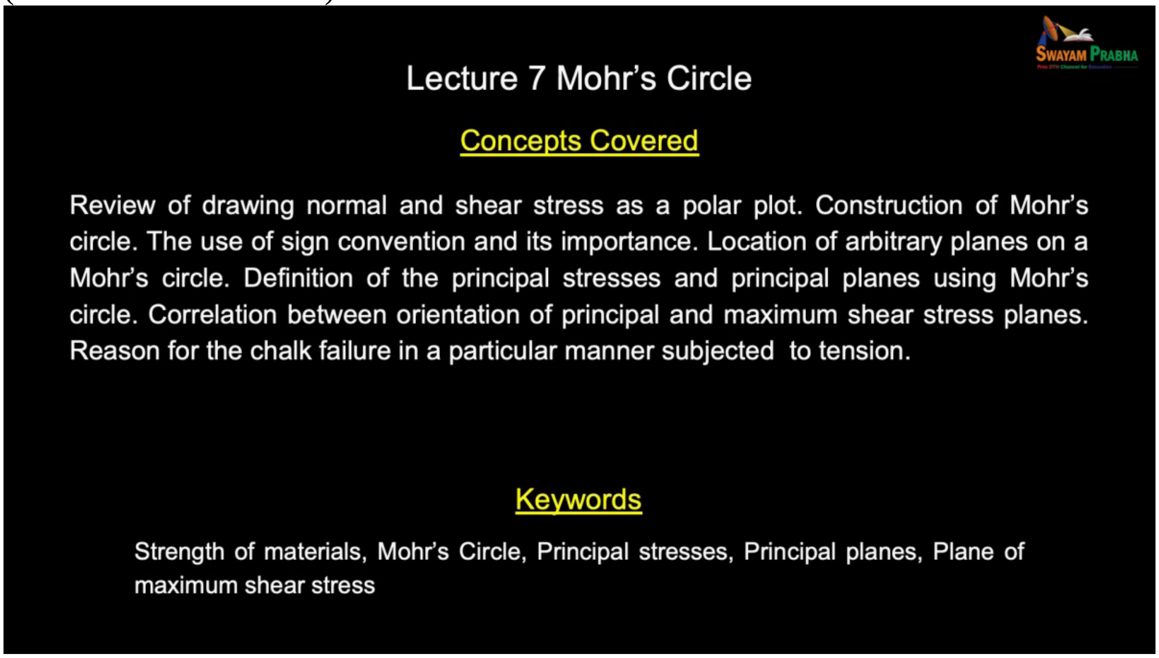
Strength of Materials
Prof. K. Ramesh
Department of Applied Mechanics
Indian Institute of Technology, Madras

Lecture - 07
Mohr's Circle

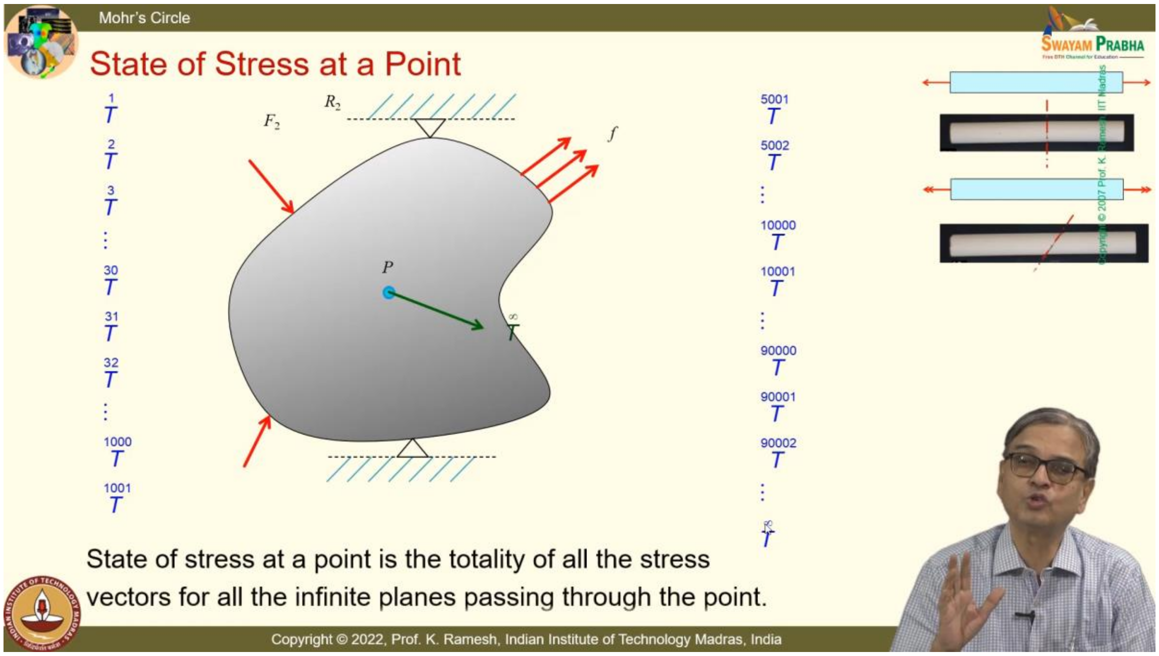
(Refer Slide Time: 00:22)



(Refer Slide Time: 00:26)

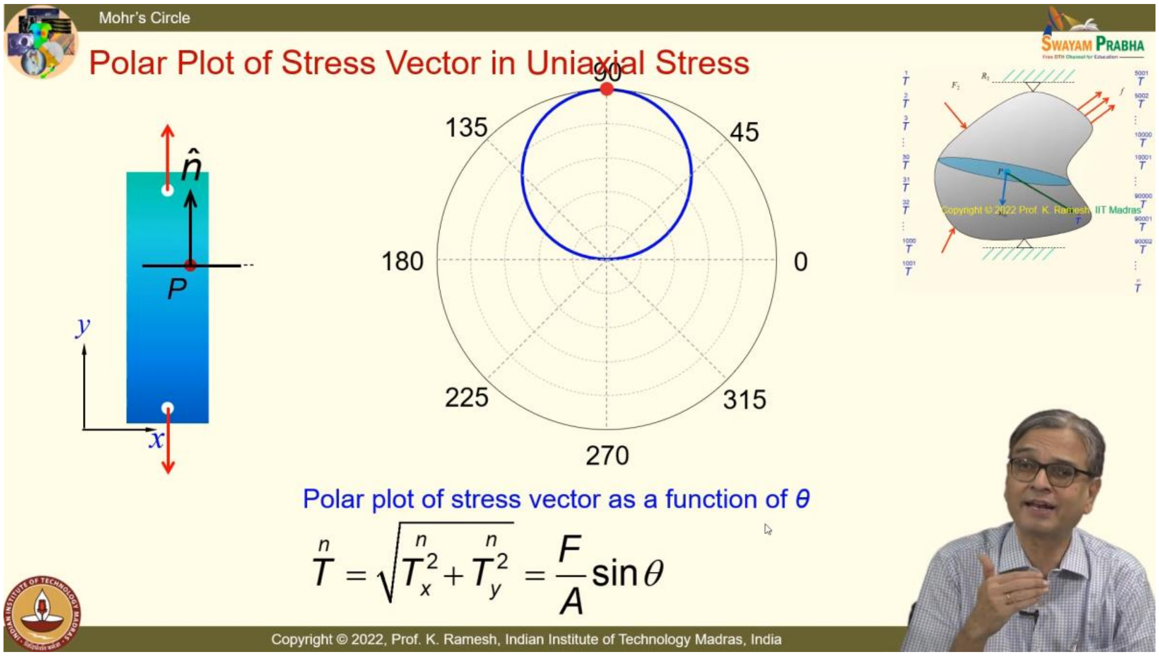
See, we have looked at stress in a very simplistic manner as P/A from then on we graduated that we need to find out what happens at a point of interest. We developed new mathematical entity like a stress vector, then we said we need to get stress vector on all the possible infinite planes passing through the point of interest. Then we developed the concept of state of stress at a point. We have looked at what is the pictorial representation, it was also showing three dimensions, then we have also written it for a planar situation. Then we have looked at what is the tensorial representation and I said what are the possible ways of graphical representation, we had seen one in the earlier class, we will continue with that.

(Refer Slide Time: 00:26)



See, the moment you say state of stress at a point, you show the stress vector like this, it is only a representation, I have not modified the size of the vector. So, do not think that every plane you have this as constant value, it is only a representation. Suppose, I put all of these points together and when I look at it, I get that as an ellipsoid when I look at from a principal stress planes, we will postpone it for the time being. So, the idea is there could be multiple graphical representations, fine. And we said that only if I have all the possible stress vectors passing through the point of interest, I get what is known as state of stress at the point. And we also said that it was looking insurmountable, then Cauchy's formula rescued you.

(Refer Slide Time: 02:13)

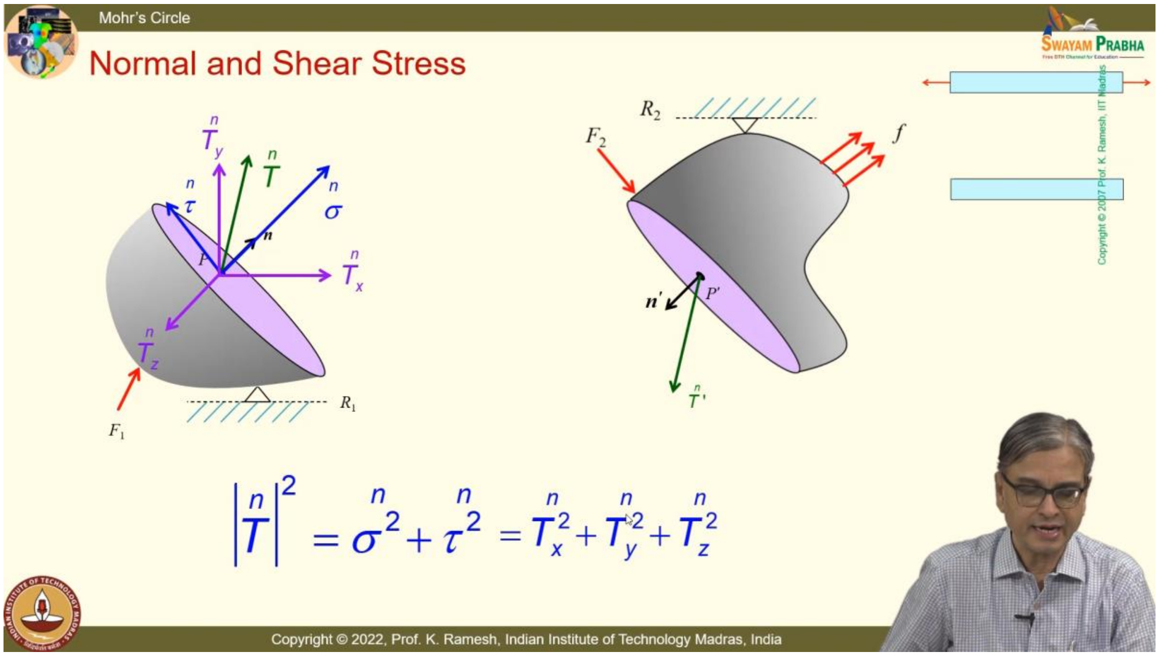


See, when you look at this, you also have the stress vector can be represented as a polar plot.

$$\overset{n}{T} = \sqrt{\overset{n}{T}_x^2 + \overset{n}{T}_y^2} = \frac{F}{A}\sin\theta$$

If a quantity is varying as a function of θ, the first step what you would have is, plot them as a function of θ, that is natural. And if you look at any of the books, they do not spend time at all in this, but this is also very important. See, if you wanted to write graphical representation, you would have started only like this, fine.

(Refer Slide Time: 02:56)

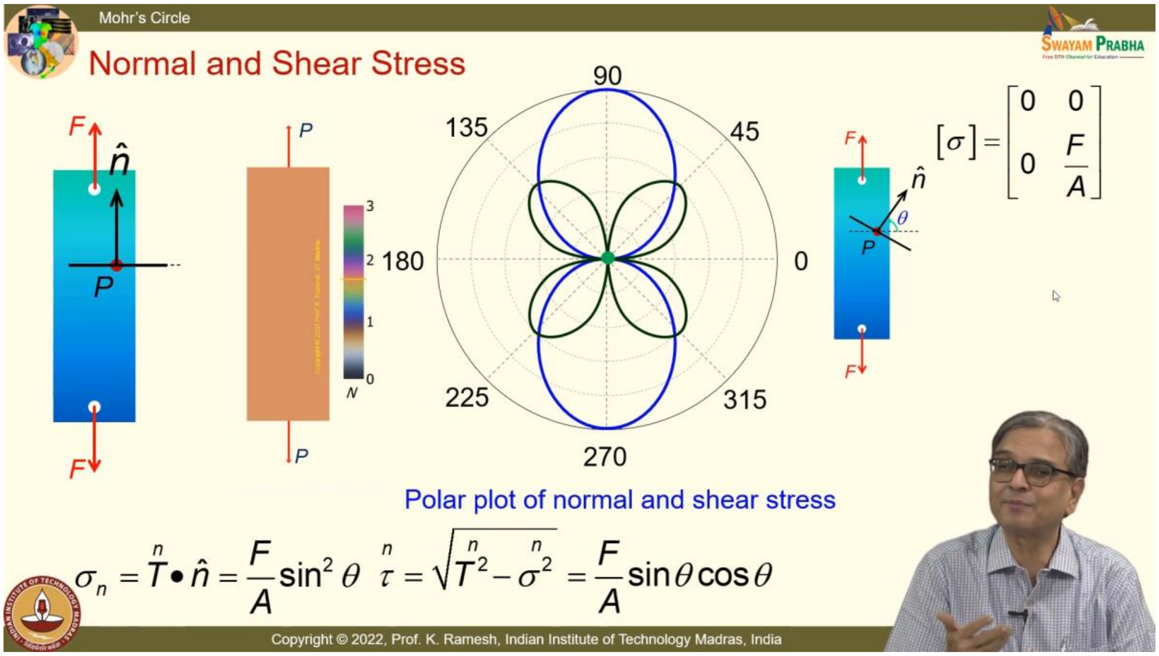


Then we said stress vector is not sufficient, we need to find out what is the normal and shear stress acting at the plane of interest, that also we have looked at. We have a stress vector that could be represented either as components along *x*, *y*, *z* directions or component along the normal and tangential to the surface. In fact, while we developed σ_{xx} τ_{xy} τ_{xz}, whatever the shear stress which is acting on the surface, we also resolved it into two additional components, fine. I suppose you understand that subtleties, because we have taken a plane passing through the *x*-plane. So, I could also write the shear stress as components τ_{xy} and τ_{xz}. Here, when I am showing this, I am having only this as on the plane what is the shear stress. So, we felt that on each and every plane passing through the point of interest, it is desirable for us to know what is the normal stress as well as shear stress, because this helps in predicting the failure theories, fine. Here, when I am showing this, I am having only this as on the plane what is the shear stress. So, we felt that on each and every plane passing through the point of interest, it is desirable for us to know what is the normal stress as well as shear stress, because this helps in predicting the failure theories, fine. So, when I have this a natural selection, we have also looked at that this can be represented in these forms and a natural selection is go for a polar plot.

$$\left|\overset{n}{T}\right|^2 = \overset{n}{\sigma}^2 + \overset{n}{\tau}^2 = \overset{n}{T}_x^2 + \overset{n}{T}_y^2 + \overset{n}{T}_z^2$$

(Refer Slide Time: 04:25)

And I said I am also plotting this as a graphical representation only, but this is as a function of load, I am plotting only one quantity. Photoelasticity also plots only one quantity, this is as a function of load, you get this at every point in the model.



This is one form of representation which is also useful, fine. At a given point, how do the normal stress and shear stress varies on each of the possible infinite planes, that is also important.

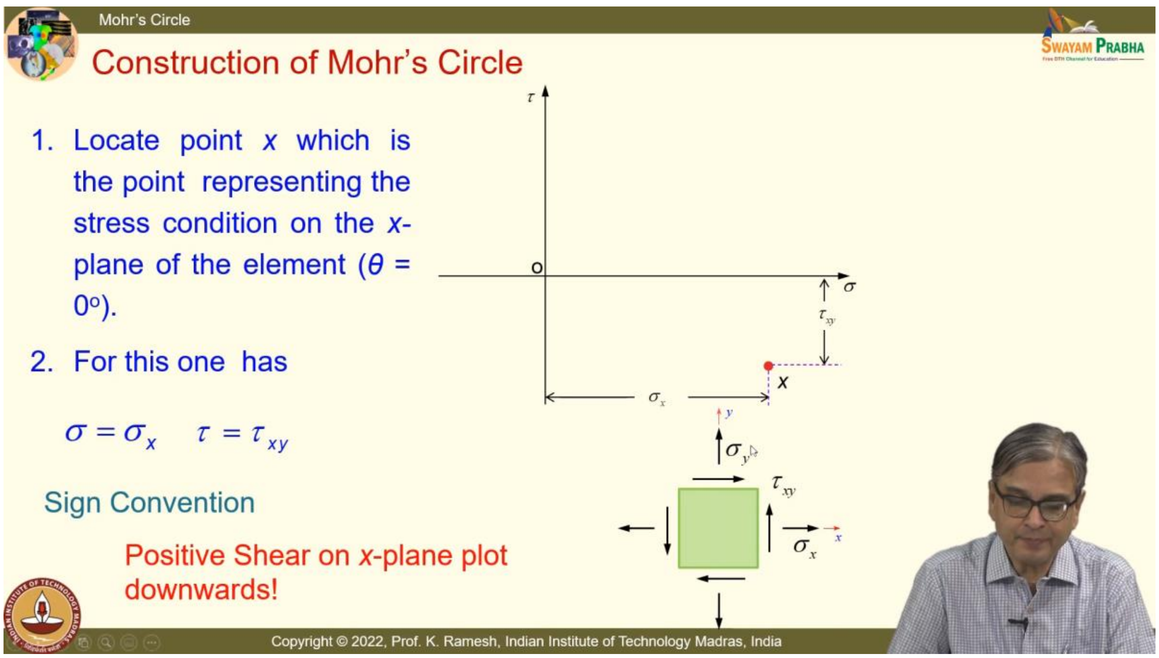
$$\sigma_n = \overset{n}{T}\bullet\hat{n} = \frac{F}{A}\sin^2\theta \quad \overset{n}{\tau} = \sqrt{\overset{n}{T}{}^2 - \overset{n}{\sigma}{}^2} = \frac{F}{A}\sin\theta\cos\theta$$

So, you have two different representations, this is obtainable from a experiment, this is also useful, because you know what happens in the neighborhood point, you have an advantage of getting a whole field information of the stress. And this is another representation what happens at a point of interest, for a given load, the load we have taken is F and then, you have this as F/A, this also we discussed, F/A appears to be a scalar. All components whether it is a vectorial components or tensorial components will appear only as numbers. I have also said you should have the practice of putting these as tensorial quantities. So, in the case of simple uniaxial tension like this, you write the stress tensor in this fashion, I have

$$[\sigma]=\begin{bmatrix}0 & 0\\ 0 & \frac{F}{A}\end{bmatrix}$$

So, that practice also you should get, this is a tensorial quantity. And you know we have also plotted the shear stress, it was looking very beautiful. And, I also made one more statement, I do not know how many of you really appreciated it, we have looked at what is the range of values of normal stress, it varies from 0 to F/A. And in the case of shear stress, it varies from 0 to $F/2A$. Fine. I have said on two planes, you have only normal stress, can you identify that from this picture, what are the two planes? Good, your hand says that, but you spell it out. So, I have the *x*-plane as well as *y*-plane, in the *x*-plane I have this as 0. Fine, In the *y*-plane I have this as *F*/*A*. And that is what is shown here in your tensorial representation, I have 0 and *F*/*A*. And it so happens that these two planes are also special planes, because on these planes I do not have a shear stress, I have only normal stress. So, they are given a special name called principal planes. And this also I made a statement that when you have a graph like this pictorially it is very nice, difficult to draw and also difficult to cull out information that you want easily. We will see another representation, we have taken simplest stress state of uniaxial tension, that itself gives you such beautiful pattern. Suppose, I have this tensorial, this one is populated only the mathematics will give you what kind of a geometric pattern that I am going to get, it may be very difficult to visualize and plot.

(Refer Slide Time: 08:21)

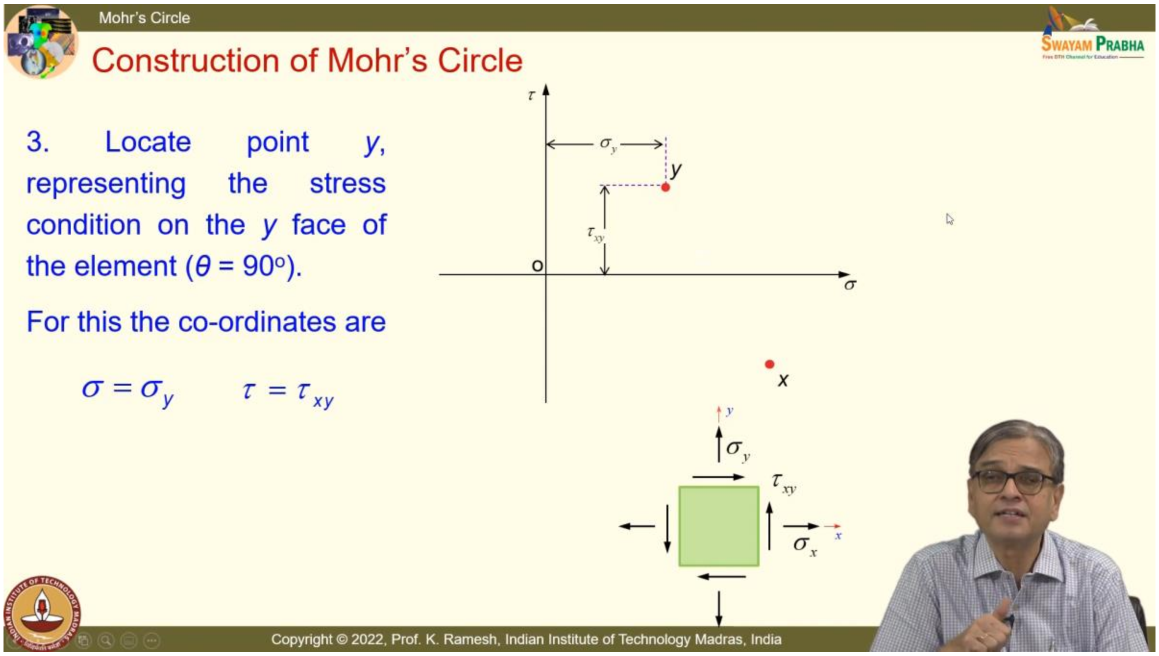


Let us see what is the way that we would put it, and this you will have to give credit to the scientist, and this has also taken sufficient time. See, we had seen in 1822 stress tensor was identified by Cauchy. This form of representation, see we want to know what is the normal stress and what is the shear stress. And I have already said in engineering and science, what you plot on *x*-axis and *y*-axis has really revolutionized the understanding of data. And when I go to this graphical representation, this also comes with an adjective that this is Mohr's circle. So even before you see what is the geometric pattern, you are going to get only a circle and circle is easy for you to draw. And what I do is, I have on the *x*-axis the normal stress and on the *y*-axis I plot the shear stress. This came about in 1882, it took about 60 years from the development of what is the stress tensor to this representation. And what we are going to focus in this lecture is the construction of it. And if you want a mathematical proof, we will have to go back to the stress transformation and we will do that in the next class, fine. Because the Mohr's circle representation is very elegant and many of the concepts that you want to understand in stress analysis can be understood very easily by looking at the Mohr's circle. And let us look at what is the way that we learn how to plot. See what we are going to plot is, we are going to plot what is the stress vector acting on plane x and what is the stress vector acting on plane y. And what you have on plane x, I have a normal stress σ_{xx} and I have a shear stress τ_{xy}. And in the representation I have also used the equality of cross shears, I am not putting it as τ_{xy} τ_{yx} and so on. We are, we have learnt already from moment equilibrium that τ_{yx} equal to τ_{xy}. In certain mathematical representations, we need to preserve this τ_{yx} and τ_{xy} that we will see later. But equality of cross shear is a useful property, it makes the stress tensor symmetric. Now, I have to plot what is happening on plane x, ok.

So, it has σ_{xx} and τ_{xy}. So, σ is on the x-axis and τ is on the y-axis. Where will you normally plot? Which quadrant it will be? We have learnt the sign convention. You can at least say which quadrant it will be? if you are given a choice. It is going to be in which quadrant? first quadrant, we will not follow that. See, I said engineering is one discipline where conventions are very very important. Why we have conventions? it helps us later. Fine, naturally when you are asked to plot, you will plot it only in the first quadrant, but I am going to break that. And I am going to have a special rule for what to do for x-plane. For y-plane, whatever you see, whatever you have said, we will do that.

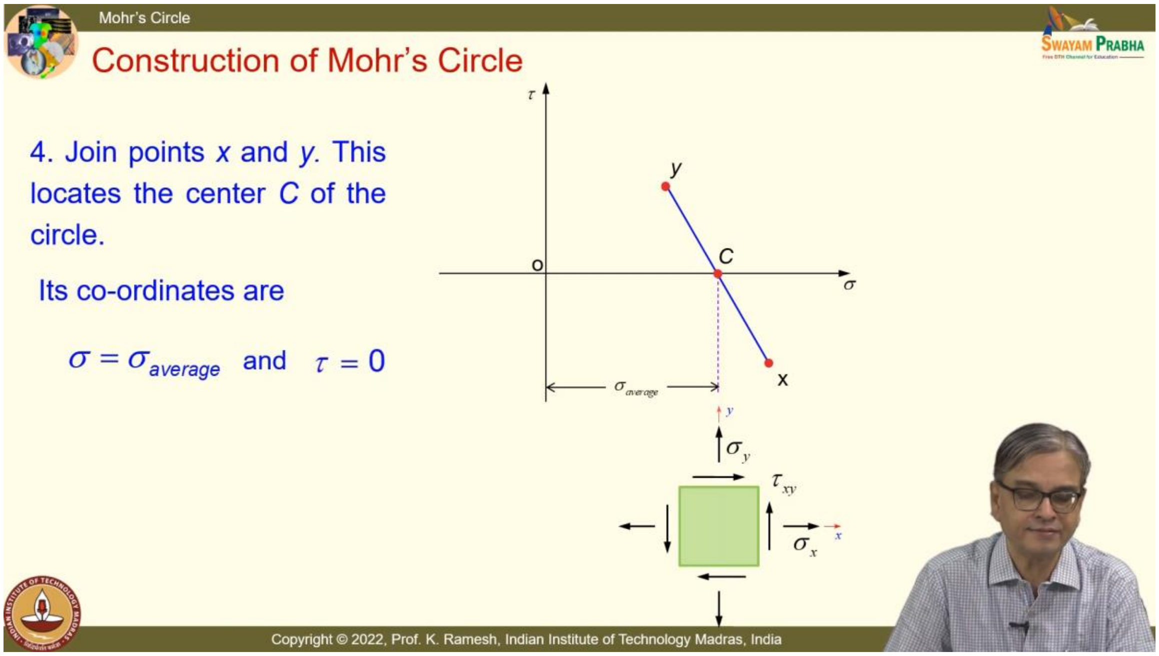
But for x-plane alone, we will plot it differently, there is a reason behind it. I have σ_x, so I have to put τ_{xy} that is easy to put, you will take some σ_x somewhere, so it is lying on this line. Now, I have to know τ_{xy}, the convention is when I have a positive value of shear on the x-plane, I would plot it downwards. I will use the same magnitude, but I will plot it downwards. So, I will have τ_{xy} marked like this, that is what the sign convention said, positive shear on x-plane, plot downwards. So, when I use that convention, I will mark τ_{xy} like this and I will have the point marked and this is the x-plane. So, on a Mohr circle, if you take a point on the circle, each point represents a plane passing through the point of interest. See, when you want to plot a circle, what all you need? You need to know what is the center, you need to know what is the radius, these are determined by the stress state, that is what you have to appreciate. From the pictorial representation, I have the information of what is σ_x and τ_{xy}, I also have what is σ_y and τ_{xy} and what is the sign for σ_y and τ_{xy} on the y-plane? both are positive.

(Refer Slide Time: 13:50)



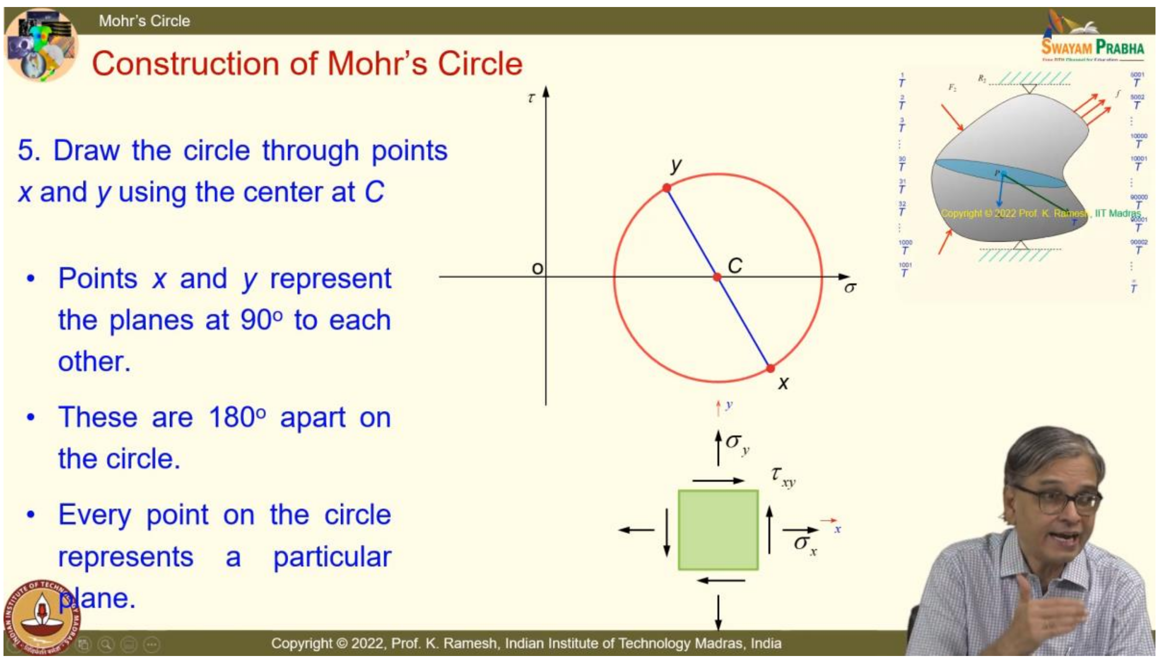
Ok. So, the next job is, I have to locate point *y* and for point *y*, we use the normal sign convention to plot. I have the *x*-axis, I have the *y*-axis, I simply mark the point *y*, I have σ equal to σ_y. So, I have σ_y and this is the line on which you can identify point *y*. Now, I have τ_{xy}, so this horizontal line meets and I get the point *y*. And what you will have to know is, the points *x* and *y* are on a diameter, that you do not know right now, fine.

(Refer Slide Time: 14:37)



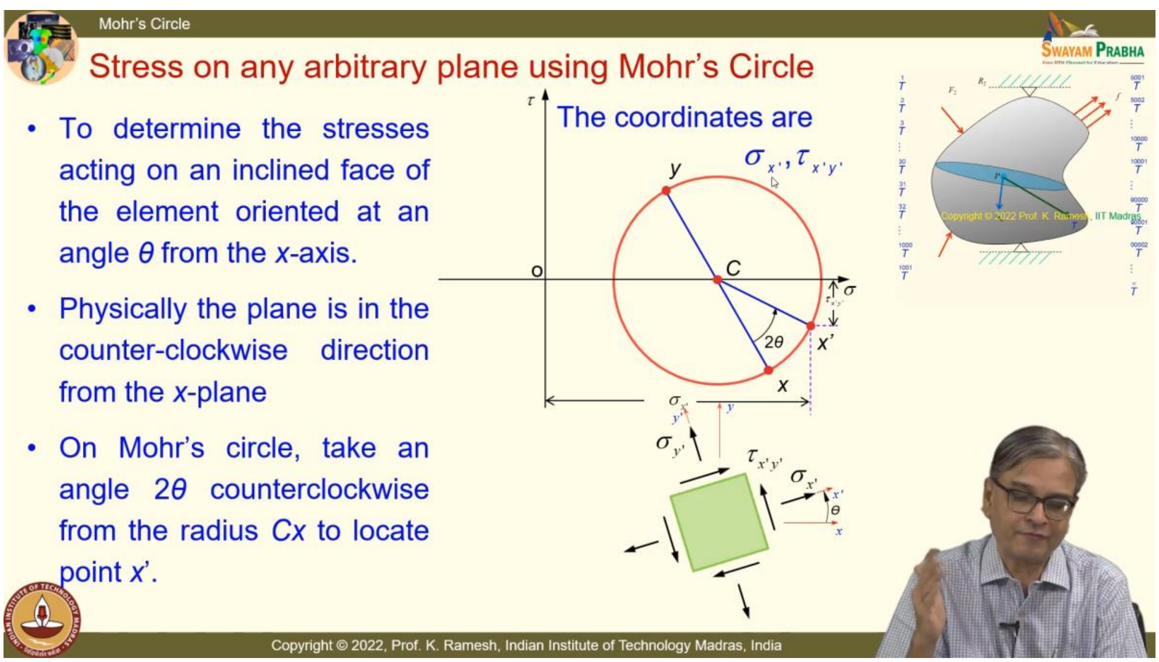
So, I join these two points. I join these two points, to locate the center. See, in those days you should appreciate, they were not even having a calculator. Now, you have a very powerful computer and even the calculator, some of them do some symbolic computations, all that has come. In the early 19th century also, they had only slide rule and most of the development, they were very comfortable in graphic representations. So, the graphical representation has really helped early development of science, in all disciplines people had graphical representations. So, you should give the due credit for those graphical representations. So, now I have the center C and I have already said that this forms a diameter, what strikes you directly? See, in a physical plane I have *x* and *y* are separated by what angle? but in the Mohr's circle how they are represented? I have this angle doubled, so that is one observation, keep that observation in your mind. Now, I draw the circle with C as the center and you can also say what is the coordinate? One thing you can say, because it is lying on this axis, the shear is 0. And what is the value of this? Suppose, I have this as σ_x and σ_y, I can call this as average of that, we will see what is the average later, keep it as $\sigma_{average}$ and τ as 0. So, we know the coordinates of C. Now, I can draw a circle passing through the points *x* and *y*, I know the radius, so I can draw the circle.

(Refer Slide Time: 16:30)



See, I have taken up a generic problem, do not confuse that we have discussed the polar plot for a uniaxial stress, whether this circle belongs to uniaxial stress? No, it is a generic problem, because I have taken a generic state of stress, I have some σ_x, some τ_{xy}, some σ_y. So, I have both shear and normal stress existing. So, you learn for a generic case. Later on as an exercise, we can do it for a variety of cases, we have looked at uniaxial tension, we have looked at pure shear, likewise we can learn how these circles appear. So, what you have here? The graphical representation of the state of stress that is what happens in infinite planes passing through the point of interest is simply a circle, no matter what is the state of stress represented. The circle size may be different, the origin may be shifted and the planes *x* and *y* may be relocated, but the geometric shape remains a circle, do not you feel it is an advantage? It is a real breakthrough, we will prove that it is a circle from a mathematical perspective that we will see, fine. At this stage, we learn how to do the construction, because in the construction we follow a convention, why do we follow a convention? That will become apparent when we see what happens in an arbitrary plane. So, this is what is summarized here, what we discussed in the previous slide. So, what you find here is the planes *x* and *y* appear along the diameter separated by 180° on the Mohr's circle. So, whatever the angle that you see in Mohr's circle is twice the physical angle and I go from *x* to *y* in anticlockwise direction, I can go from *x* to *y* in the anticlock direction in the Mohr's circle provided I followed the convention, that will become apparent when we seek a arbitrary plane.

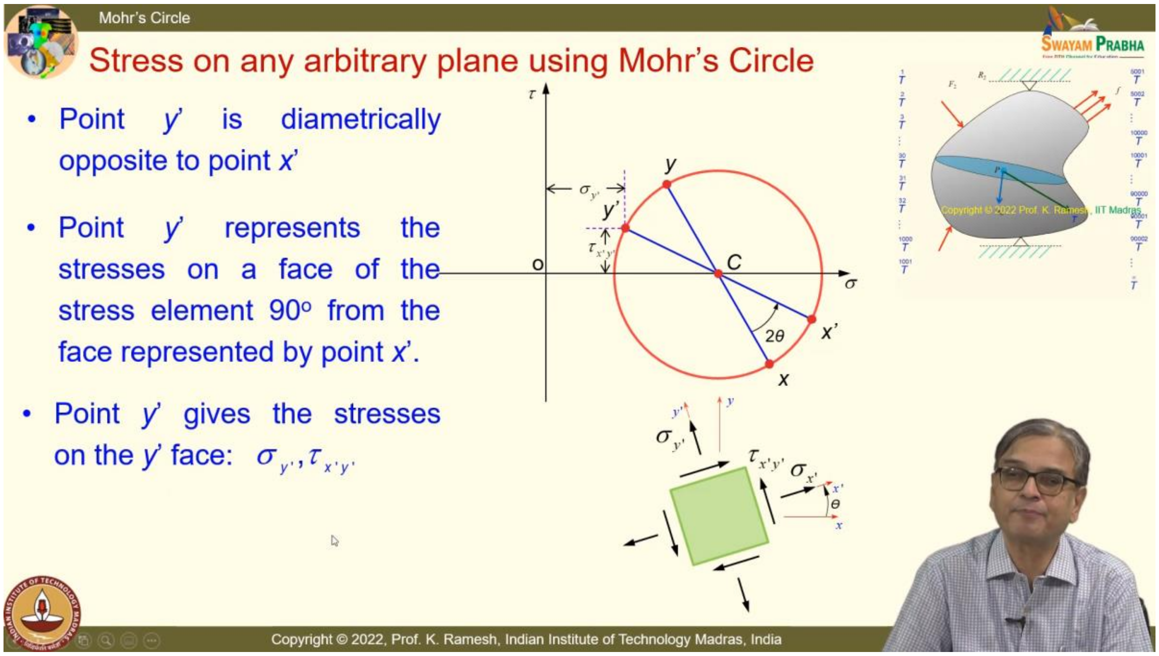
(Refer Slide Time: 18:50)



See we have done a stress transformation law from vectorial transformation to tensorial transformation, you have those expressions. Now, what we are going to do is from the Mohr's circle, we will find out if I have a plane oriented at angle θ from the *x*-axis, how will I get it? Fine, I have to locate the point, the issue is I have to locate the point on this Mohr's circle appropriately, because once I have located the point, the magnitude of normal and shear stress are immediately known, they are nothing but the coordinates of that point.

So, I have the stress tensor given and then I want to find out what happens at a plane θ and we have labeled that axis as x' and y'. In the earlier case also we had labeled it as $x'\ y'$, what we determined was we determined $T_{x'x'}$, you have to interpret that as σ_{xx}. So, I was cautioning you, it is not like a vectorial transformation, it is slightly more than that because you have two subscripts, ok. Now, the x'-plane is separated by angle θ, how do I locate the plane in the Mohr's circle? From whatever the discussion we have had, can you say how do I, because I know where is the location of *x*-plane on the Mohr's circle. Now, I have to locate x'-plane, how do I do it? 2θ, so that is fair enough, is not it? So, I have to move by angle 2θ from the *x*-plane and I would get the x'-plane on the Mohr's circle. So, I go by 2θ, so I locate this as x', is the idea clear? Because I want to preserve how do I move in the physical plane to Mohr's circle, we have a special convention on how to plot quantities on a *x*-plane. If I plot positive shear downwards on *x*-plane, it is easy for me to locate the planes from the physical plane to the Mohr's circle, in the same manner. The only difference is if it is θ, I should look at that twice the angle 2θ. So, once I know this x', I can find out what is $\sigma_{x'x'}$ as well as $\tau_{x'y'}$ that is nothing but reading it from the graph. So, your stress transformation becomes simple and straight forward. So, that is a greatest advantage. What you have to realize is every point on the circle represents a particular physical plane. See here we have taken a two-dimensional representation. Suppose, if I want to go to three dimensions, there again Mohr's circle is useful which we will also see in the next class tomorrow. So, what you have here is I have to locate the point *x*, locate the point *y*, then find out what is the center and what is the radius my job is done. And if I want to find out for any arbitrary point, it is easy for me to do that. Let us also look at some of the special points. So, the coordinates are $\sigma_{x'}$ and $\tau_{x'y'}$. I have already said we will interchangeably say $\sigma_{x'}$ or $\sigma_{x'x'}$ because it is understood when you put the symbol as σ, it is normal stress. So, it is automatically understood in one fashion. So, we take the liberty to call it like that.

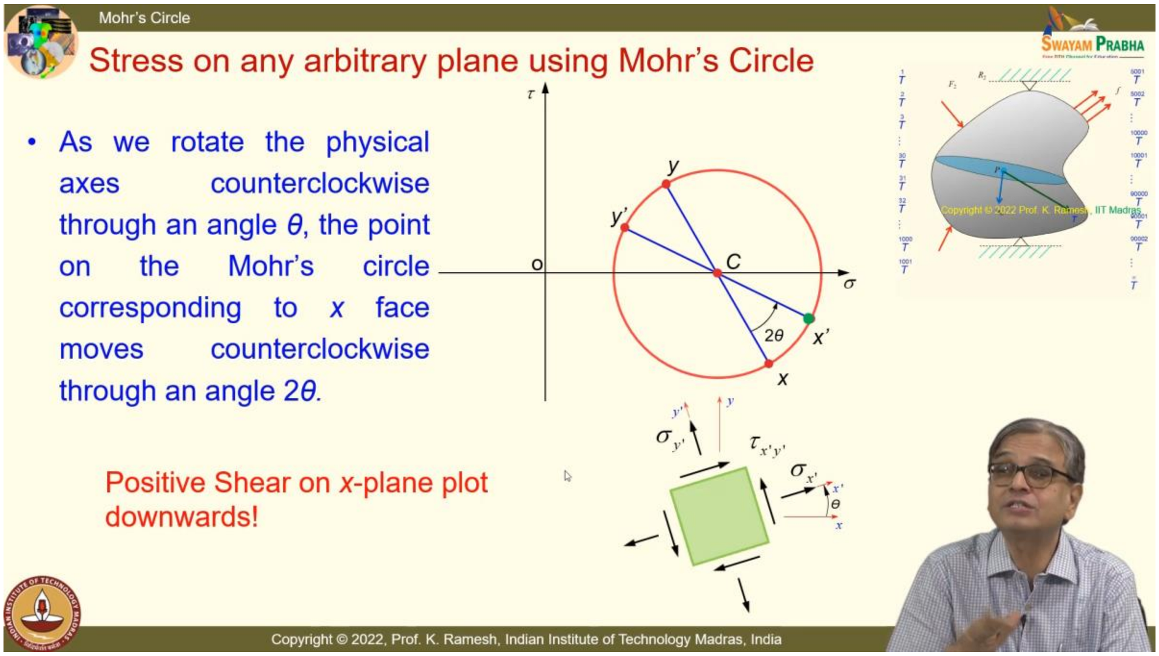
(Refer Slide Time: 23:05)



And how do I locate the y'-plane? We have already seen that x and y are actually separated by 180°. On similar fashion x' and y' are separated by 180°. I simply extend the radius to the diameter. So, I locate the point y'. So, I can read what is $\sigma_{y'}$ and $\tau_{x'y'}$ from this diagram.

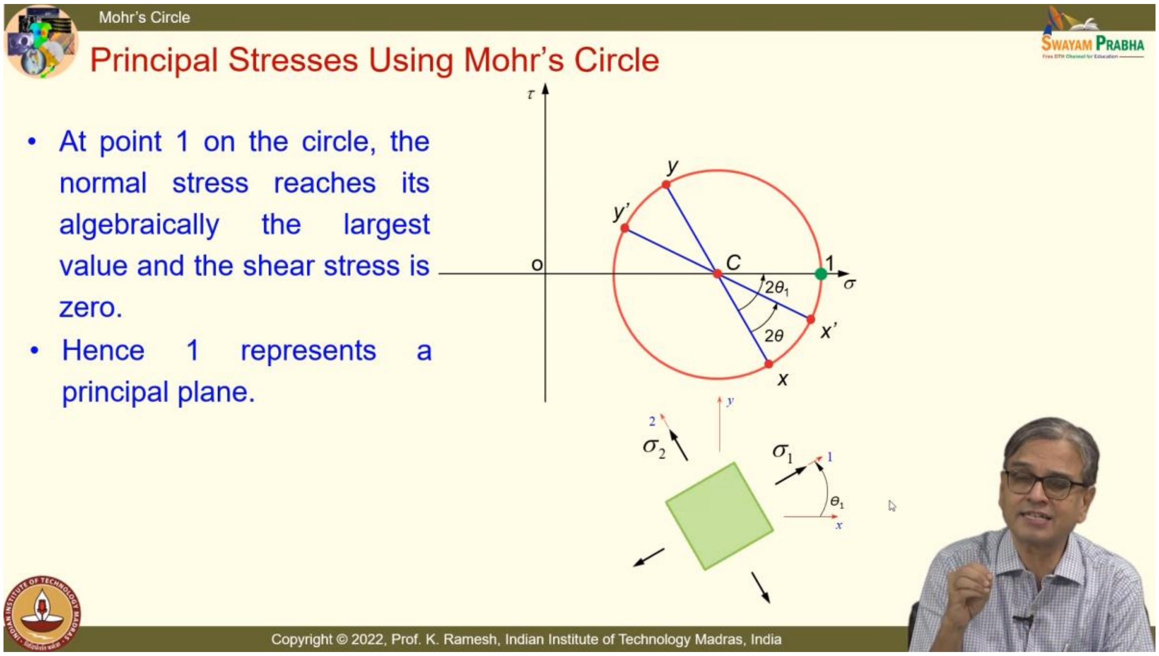
So, it is as simple as that. See many times we may not make the calculation from the Mohr's circle, but we may want to find out the sense from the Mohr's circle. We will also see what is that, how it can be used, sense of rotation from the Mohr's circle. So, point y' gives the stresses on the y face. They are nothing but $\sigma_{y'y'}$ and $\tau_{x'y'}$. So, stress transformation is simple and straight forward.

(Refer Slide Time: 24:09)



There is no difficulty at all, ok. And this is emphasized that it is moving by the angle 2θ. And by looking at this circle, can you identify certain interesting planes? Which was difficult to find out when I had the polar plot, only when you look at it very intently you will be in a position to identify. I can easily see the circle cuts the *x*-axis which is nothing but σ-axis at two points. What is the implication of that? On those planes I have only normal stress and we have already said something very special. So, whenever I look at any stress state, when I draw a circle with σ and τ, it will always cut that horizontal axis. So, I am always going to have planes on which only normal stress exists. They are very special, they are called as principal planes. So, that is one thing. There is also another information you can gather. I have this as the maximum value of shear stress. When I have maximum value of shear stress, in general you will always have a normal stress. That is what we had seen in the polar plot also. That is very clear here. Is the idea you are able to appreciate? Ok. So I have this direction θ, what I have in the physical plane is represented as 2θ is reemphasized in this slide. So, I move to this. So, I have this as 2θ. That is what you have. On a similar vein, we will also go and find out what is the principal stress, ok. And for me to do this anticlockwise rotation θ and anticlockwise rotation 2θ is possible only when I follow the sign convention. If I do not follow the sign convention, I lose that advantage. So, if I am going to have an advantage, better to follow a convention. See, we are in a society where you know people if they break the convention, they think that they are more close to people. It is not so. In science and engineering, you will have to follow the convention because it has its definite role to play. Is it not? Now, you find the celebrities they break the convention and the fans are very happy. That does not work in engineering. You have to follow the convention.

(Refer Slide Time: 27:06)



Now, we will find out the principal stresses. So, by definition principal planes are those planes where the stress state is such on that plane you have only a normal stress. Apriori you do not know at what angle with respect to the *x*-axis do you have the principal plane. So, this is one of the quantities that you have to find out in a physical problem.

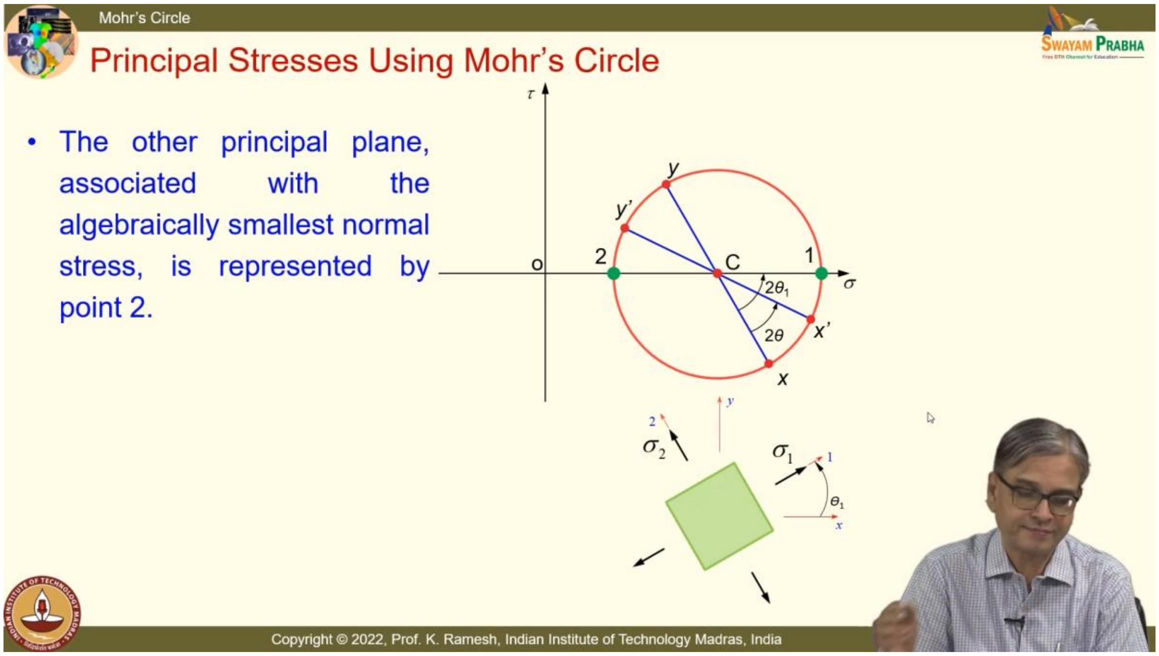
Second aspect is what is the magnitude of the stress? I need to know what is the magnitude and what is the orientation of the principal plane. All that can be easily gathered from Mohr's circle. Instead of telling this as an arbitrary plane, now we have fixed. Find out a plane on which you have only normal stress. The same question I raised when we had the Cauchy's formula where I had

$$\overset{n}{T} = [\sigma]\{n\}$$

$$n = \cos\theta\hat{i} + \sin\theta\hat{j}$$

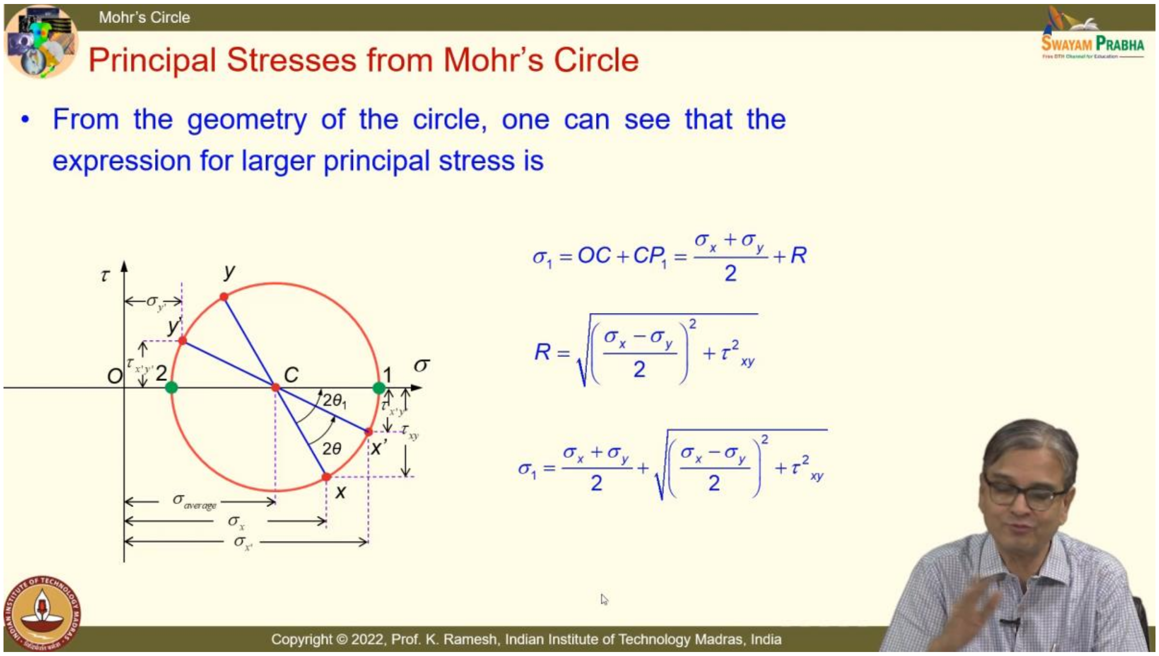
I said two class of problems can be formed. In one class of problems you are given the direction cosines, find out what is the stress vector. In another class of problems find out the direction cosines if you dictate what should be the nature of the stress vector. Mathematically that requires little more calculations, but graphically it is a child's play, because in a point of interest, among the infinite planes passing to the point of interest, the principal planes are always important. You will find when I said we live in linear elasticity, you have a greatest advantage. You will see the advantage of the concept of principal stresses, because when I have multiple loads acting, you have torsion, bending and axial forces acting on a member, I can do the principle of superposition very easily, ok. Failure theories are written with the help of principal stresses. So, now I have this as $2\theta_1$. So, you can find out what is the orientation of the principal stress direction in the physical plane, it is θ_1. Fine. So, when I go to θ_1, make a neat sketch, you will have to recognize on the principal planes I have only normal stress, they are very special. And I have labeled this as 1 and I can also find out when I have this, the other principal plane is perpendicular to that in the physical plane. On the Mohr circle, it is separated by angle 180. So, I will have this 2 as marked here. And you know there is also a convention, how do you label the principal stresses? Suppose in a problem I solve it mathematically, I would always label the algebraically larger value as σ_1, this is again a convention. In the diagram I have shown this, compared to this, this has algebraically larger value, fine. When I say σ_1 at a given point of interest, if I have multiple normal stresses, I can have in a three-dimensional problem three principal planes. Of these, the algebraically the largest I will call it as σ_1, the next algebraically largest as σ_2, the least value as σ_3. So, this is the convention that is also used.

(Refer Slide Time: 30:51)



So, the other principal plane you can represent it by 0.2 and this is separated by angle 180 from the principal plane 1 and I have this as σ_1 and σ_2, I have this angle as 2θ. See from the diagram, I can also write mathematical expressions of what are the magnitudes of these stresses and what is the orientation, I can do that, we will also do that.

(Refer Slide Time: 31:29)



So, you have a summary that is I have a Mohr's circle, I have the arbitrary plane is represented and also the principal planes are represented and I want to find out what is the magnitude of this from the Mohr's circle. Suppose, I want to find out what is the value of σ_1, how will you write σ_1? So, I have to know what is the distance OC and what is the distance C1.

Can you find out what is the distance OC? We have written that as $\sigma_{average}$ and that is nothing but $(\sigma_x + \sigma_y)/2$ and your C1 is your radius R. See from the Mohr's circle, we are also in a position to write the expressions for you to find out these quantities mathematically. And what is the value of R? Because we know only the stress tensor at the point given with respect to the reference axis x and y, we call that as σ_x τ_{xy} and σ_y. Can I find out what is the expression for radius R? You can easily work it out because we know what is the, what are the coordinates for x and y.

From these, you can easily find out an expression for the radius. Please work it out and then check it from my slide. It is not very difficult, it is very simple, simple property of circle is what you will have to use. So, I can get the expression for R as simply

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2_{xy}}$$

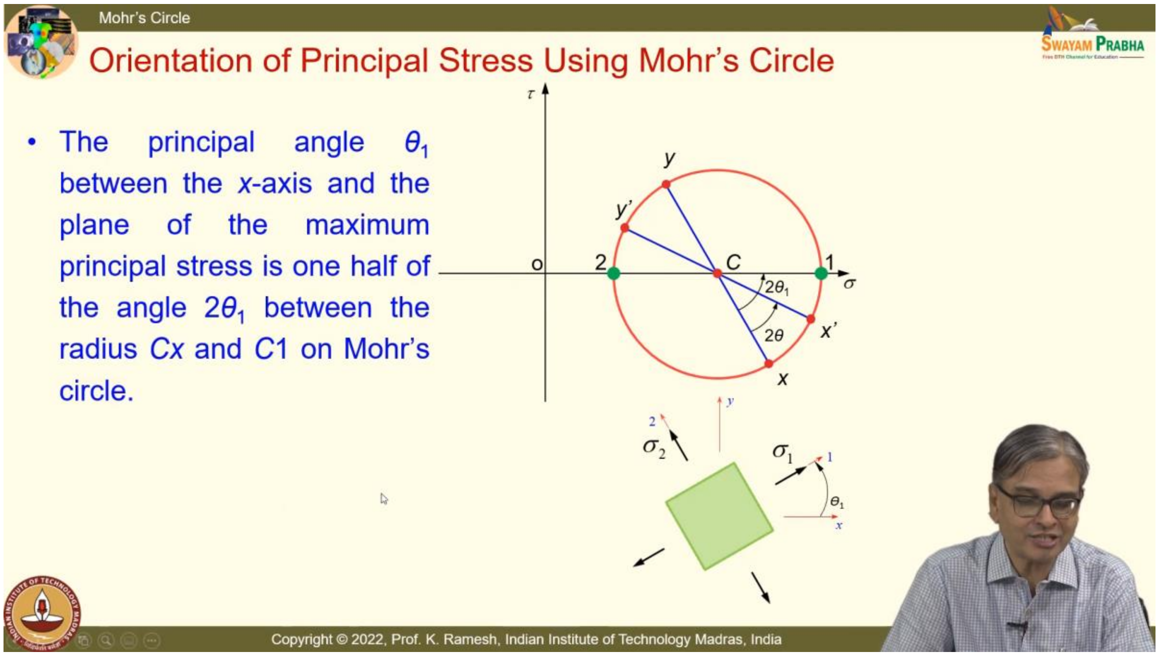
And whenever I have a square root, when I want to find out σ_1 or σ_2, I will call this as plus or minus. I have σ_1 is represented this way

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2_{xy}}$$

and σ_2 will be, I will have a minus sign here.

So, I can find out what is σ_1 as well as σ_2 from the Mohr's circle. Now let me ask one more question. Can I find out the angle from this? I do not have the expressions here. Can you work it out? Can you work it out and tell me? It is again a very famous expression, which we will again determine from mathematics later. From Mohr's circle itself, you can get the expression. The clue is you have written R, from there you have to proceed. Anybody has the expression? What is the expression for $2\theta_1$. or in a simplistic term find out what is $\tan 2\theta_1$? What is $\tan 2\theta_1$? It is very simple I say, $\frac{2\tau_{xy}}{\sigma_x - \sigma_y}$, fine. Ok, let me ask one thing. See many times when you learn engineering, you will have to bring in all aspects of your learning from mathematics. I have only $\tan 2\theta_1 = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$. If I want to find out θ_1, what happens to inverse trigonometric functions? Is it single valued function or multi valued function? It is a multi valued function. So, you will always have a tie, whether I have to take θ_1 as the given value that I get from a calculator or do I have to add additional 90°? You can resolve that easily if I have a circle diagram like this, I do not have to draw it with geometric perfection. If I want to find out this, I have this $2\theta_1$ is an acute angle, if I want to find out $2\theta_2$, it is an obtuse angle. So, when you get the mathematical expression, find out the θ, if you want to associate that correctly to σ_1 direction, because σ_1, σ_2, σ_3 we have said it is dictated by the algebraic value. When I have the expression $\tan 2\theta_1 = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$, it is not θ_1 or θ_2 or θ_3. You cannot say algebraically arrange them and then call it as like we said σ_1, σ_2, σ_3, θ_1, θ_2, θ_3 also we will do it algebraically, you cannot do that. You have to identify the associated direction. So, to find out the associated direction, because these are multi valued functions, you can use the Mohr's circle very effectively. You do not have to do a geometric sketch, your Mohr's circle can directly give you whether it is an acute angle or obtuse angle. So, you have an advantage.

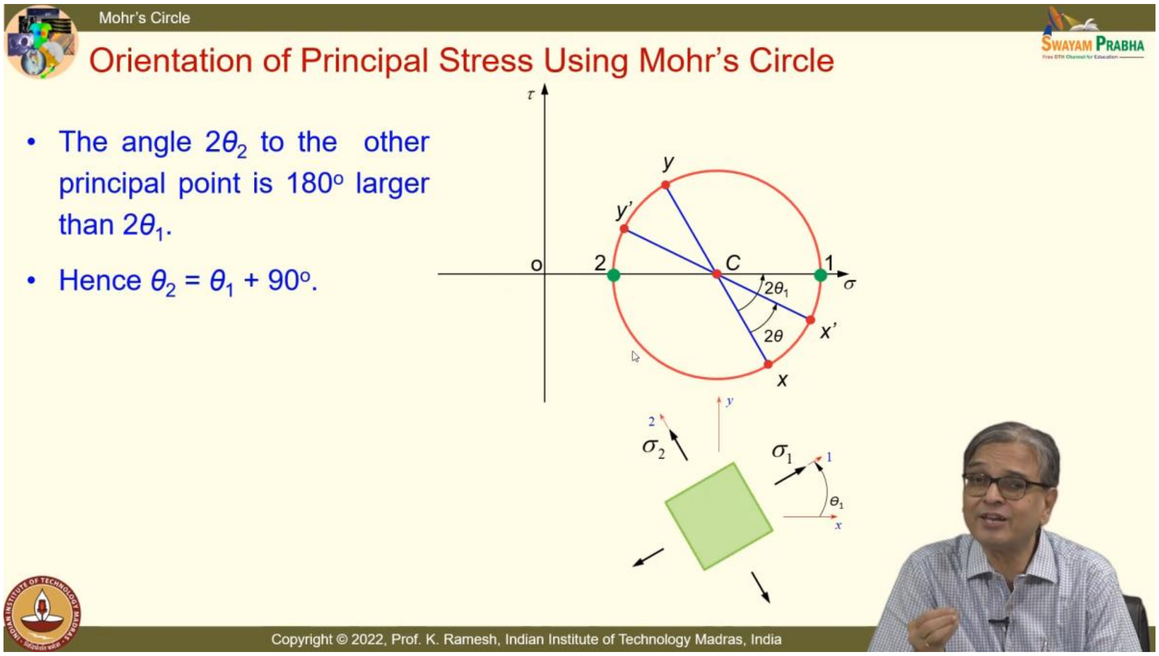
(Refer Slide Time: 36:56)



So, this is what I had discussed and what I have discussed, I have not put the expressions here and I can use this as the basis for me to find out how to get the principal stress direction comfortably from your Mohr's circle.

Even when you solve it mathematically, you will have to use the multi valued function. Many times you know when you take the square root, you do not take plus or minus, you always achieve it as plus. You agree with me? When you have solved many problems, because many times you get the correct result with plus. So, you have not even gone and checked. Like I said, when you are having a multiple rigid bodies, inter-connector rigid bodies, if you want to apply the equilibrium condition, you should ideally apply to the system as a whole as far as all the subsystems, which I have emphasized when I developed the engineering mechanics course. But many times people simply assume, if you satisfy $F= 0$, $M = 0$ for the entire system once, solution is correct. And I had a counter example that your mathematics should help. Idealizations are very important in engineering. If you do not do the correct idealization of the support, if you do not check $F= 0$, $M= 0$ for the subsystem, it will show up. So, you have to go back and improve the idealization. So, do not violate. See, once you start practicing engineering, it is not the quickness with which you solve the problem is going to retain you as an engineer. Correctness with which you solve the problem is very important. So, you should have checks and balances, which we will also see when we want to do stress transformation, there are certain things are invariants. We will use that to verify whether the result obtained is correct. That inbuilt checking approach is needed for engineering. Otherwise, when you bridge, when you build a bridge, it will collapse because I was told they had built a bridge in Andhra Pradesh without putting steel. What to do? You know the contractors who fund those projects, they want to maximize their profits and if you build a bridge without reinforcement, how will it withstand? It cannot withstand at all. That came up for retrofitting in our civil engineering department, those professors were communicating and if not IIT, which institution can salvage this kind of a situation. So, they have to put extra supports and make sure that the build that bridge at least last for the minimum service life. So, you have to have inbuilt checks in all these aspects of calculation, it is not speed.

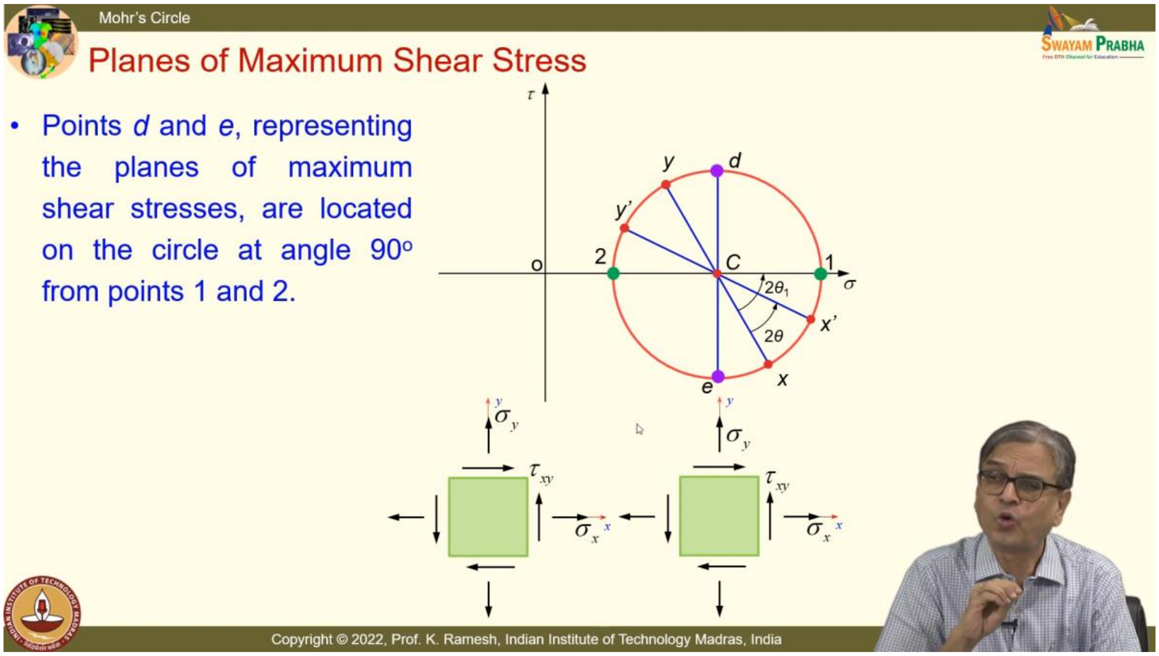
(Refer Slide Time: 40:03)



So, you forget about your JEE training, where you were all told speed is very important, speed is not important in practicing engineers, correctness is important. So, I get θ_1 from this and θ_2 is $\theta_1 + 90^\circ$, which is very clear from Mohr's circle. I have taken a problem in such a manner that I have this as acute angle and obtuse angle, it could be of any category. But when I mathematically solve, you should recognize that I am handling a multivalued function, it is not a single valued function, inverse trigonometric quantities are multivalued and you should apply all the basics. But many times people do not apply the basics and many times people have not attached what is the angle that they have got to the relevant principal stress, that is also very important, that you can easily do when you have the Mohr's circle. And we have said that you also have a maximum shear stress and can you tell me what is the orientation of the maximum shear stress in the physical plane? Can you tell me what is the orientation of the maximum shear stress in the physical plane? 45° to principal stress plane, fine.

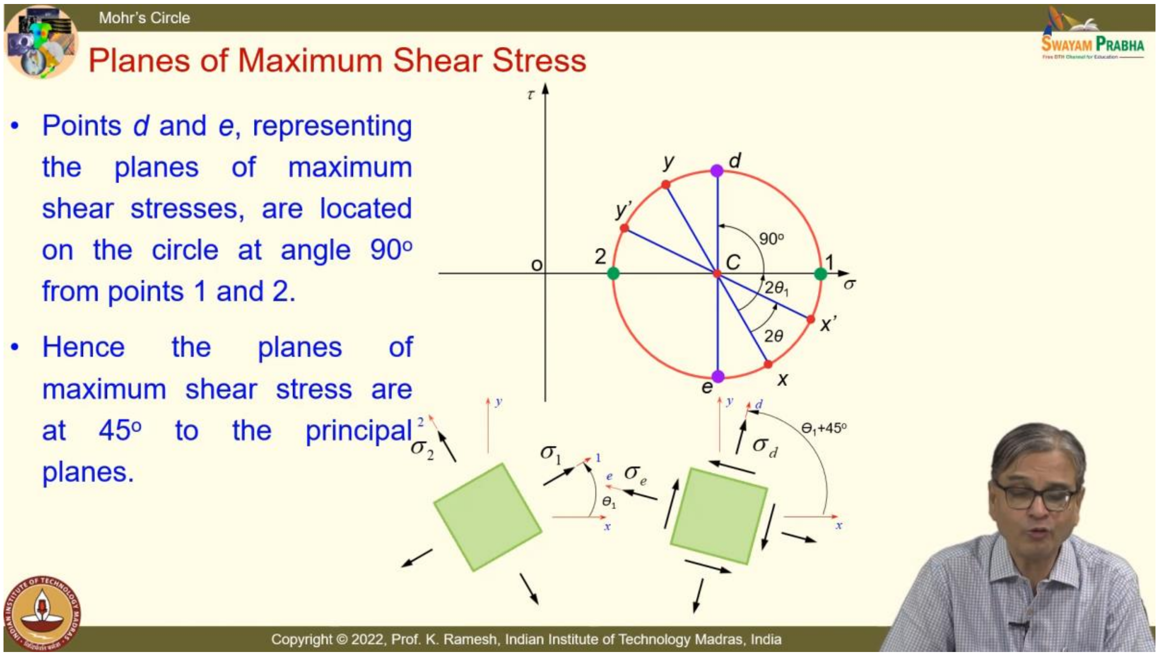
So, you understand. Now, can you go back and think something happened to chalk, we were telling 45°, is not it? So, 45° was not arbitrary, there is some meaning attached to it. So, you should know what kind of stress state is introduced when I apply torsion. When I apply torsion, what happens? You should know and you should also know what causes a chalk to failure. Suppose I say chalk fails by normal stress exceeding a limit, fine.Then you can explain what happened when you pulled, what happened when you twisted.

(Refer Slide Time: 42:18)



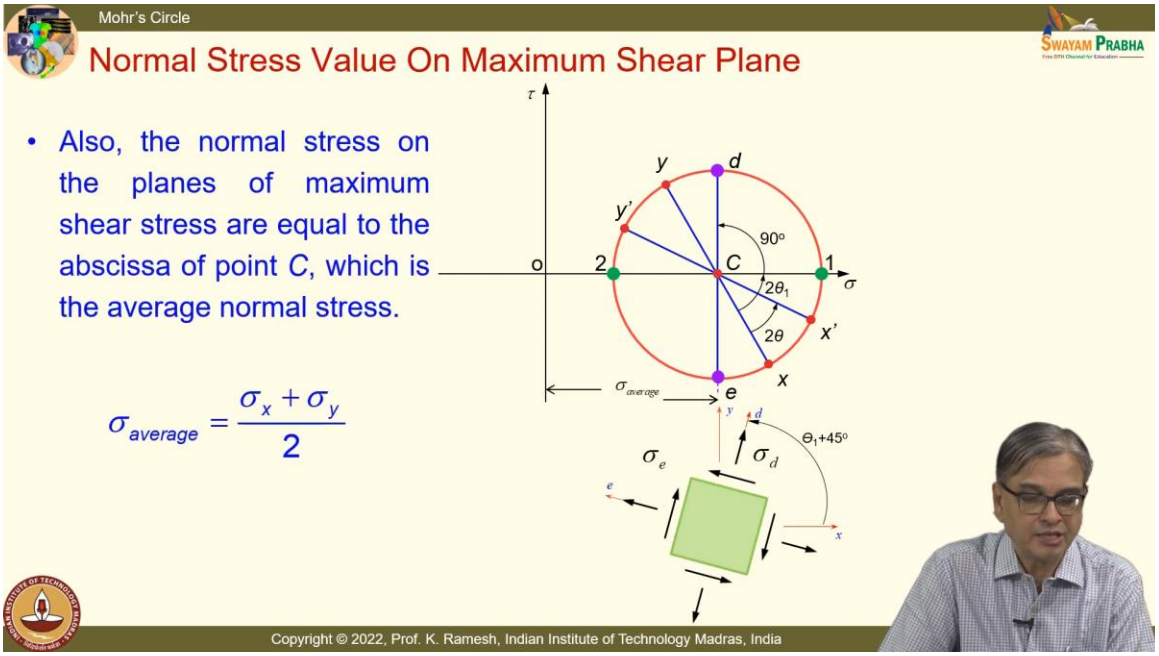
So, here it is at 45°, you have correctly identified in the physical plane, in the Mohr's circle it is at 90°. So, you have the points *d* and *e* representing the planes of maximum shear stress. And you will have to recognize maximum shear stress plane always has some amount of normal stress in general. I can also have in special situation when the center is shifted to the origin, I will have on the maximum shear stress plane only shear stress, I will not have normal stress, ok. But in general only when you have normal stress, you can identify either two planes in two dimensions or three planes in three dimensions where the stress is wholly normal.

(Refer Slide Time: 43:17)



So, they are very important, ok. So, I have this at 90°. So, on the physical plane you can identify, from the principal stress plane, they are oriented at 45°. So, now you can appreciate instead of looking at what happens as state of stress at the point of interest, if you look at from principal planes as the reference, your matrix also becomes very simple to write and you also find out what is shear. Even before we go into theories of failure, experiments have revealed brittle materials fail by maximum normal stress and ductile materials fail by shear stress. So, that is the reason why we attach more importance to finding out at a given plane of interest what is the normal stress as well as the shear stress. That is the reason why we play attention. So, this gives you a very important aspect that at a given point of interest, when I want to draw the Mohr's circle, I should identify the plane *x* and plane *y*, then draw the circle. Once I draw the circle, the two important aspects are what are the principal planes and what are the maximum shear stress planes. Whether you want to find out on an arbitrary plane is secondary. At a given point, you always want to know what are the principal stresses and what are the maximum shear stress, how they are located, the magnitude of the maximum shear stress as well as its location.

(Refer Slide Time: 45:04)



So, I have this normal stress is nothing but $\sigma_{average}$, ok and this is what you have. So, I have shear stress as well as the normal stress acting on those planes. So, in this lecture we have looked at a very powerful graphical representation that goes with the name of Mohr's circle. The scientist who determined this is Otto Mohr and I said that this was determined in 1882, 60 years later than stress tensor was determined. So, some of these quantities take such a long time to develop, we study them in 15 minutes in a class, one class to the next class we travel by 100 years, fine. So, that is the way the learning goes. Thank you very much.