Strength of Materials
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Lecture - 06
Equilibrium Conditions

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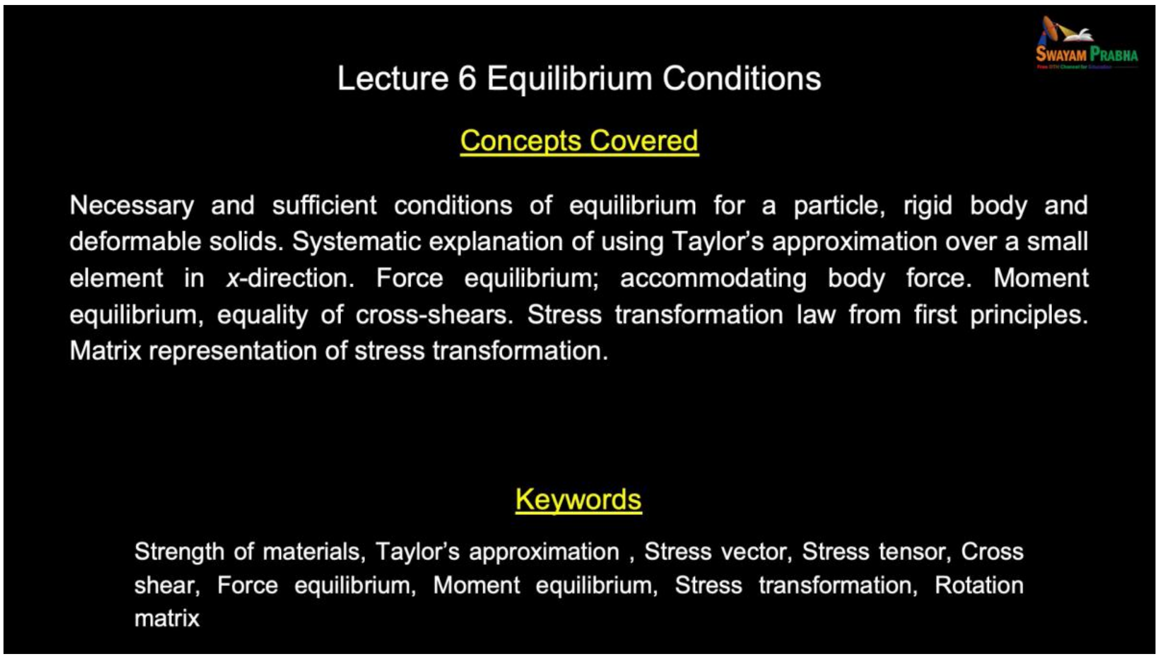

See, we have developed the concept of stress tensor. We started with P/A, then graduated to a stress vector. Then we also said, I need to find out what is the stress vector acting on all the possible infinite planes. Then we came up with the concept of stress tensor and we essentially looked at the various components σ_{xx}, σ_{yy}, σ_{zz}, τ_{xy}, so on and so forth. Now, we will also have to develop, how to check for the equilibrium once we know these quantities.

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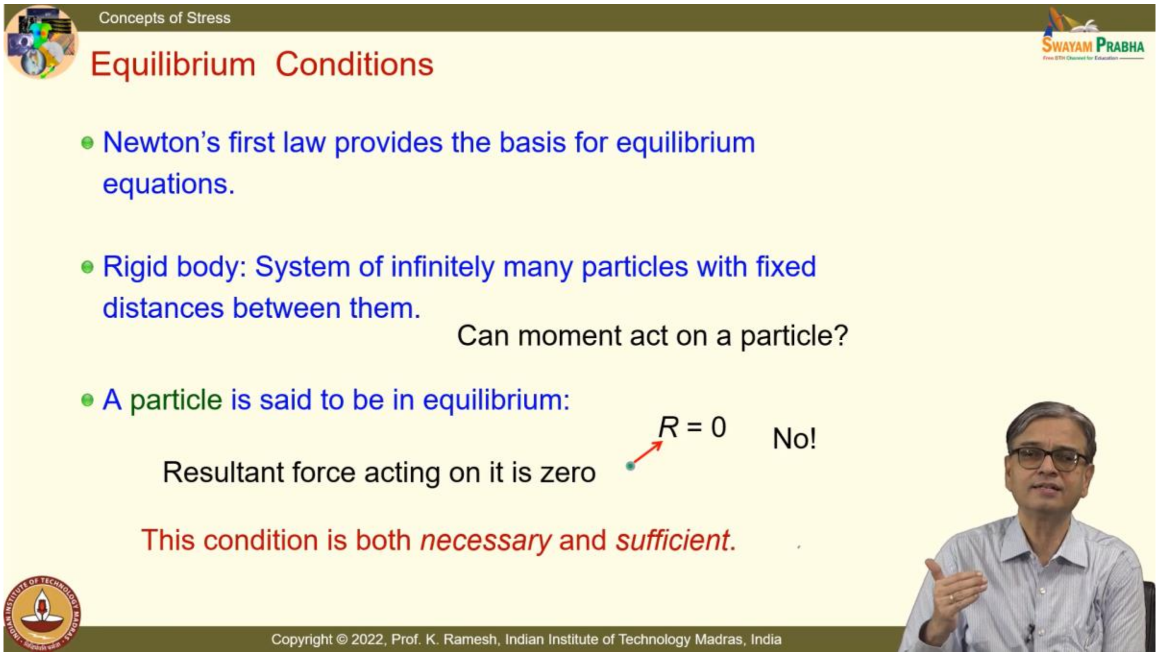

So, we move on from our basic appreciation of what we have done in rigid body mechanics though you all know that Newton's law provides the basis for equilibrium equations. And when you say rigid body, why we took in that course was, it was the simplest one to analyze when you want to start solving any problem. So, we have looked at rigid body as system of infinitely many particles with fixed distances between them. No matter what load you apply, there is no deformation.

So, we start from equilibrium of particle, it's all done in your rigid body mechanics, before we get into deformable solids, it is better to have a relook at that so that you will have a clarity. So, when you say a particle is in equilibrium? It is simple and straight forward. You say resultant force acting on it should be zero. And when you develop anything in mathematical sense, you will also have to look at whether it is necessary and sufficient.

You get the point? It is not necessary that I have a condition; the condition should be both necessary and sufficient. So, for the particle in equilibrium when you say resultant force is zero, is it necessary or sufficient or both? Tell me, you have all done that. It is both necessary and sufficient for a particle, for a single particle. I am not looking at collection of particles, fine? Can moment act on a particle? You cannot have moment acting on the particle. So, the condition is both necessary and sufficient for a particle.

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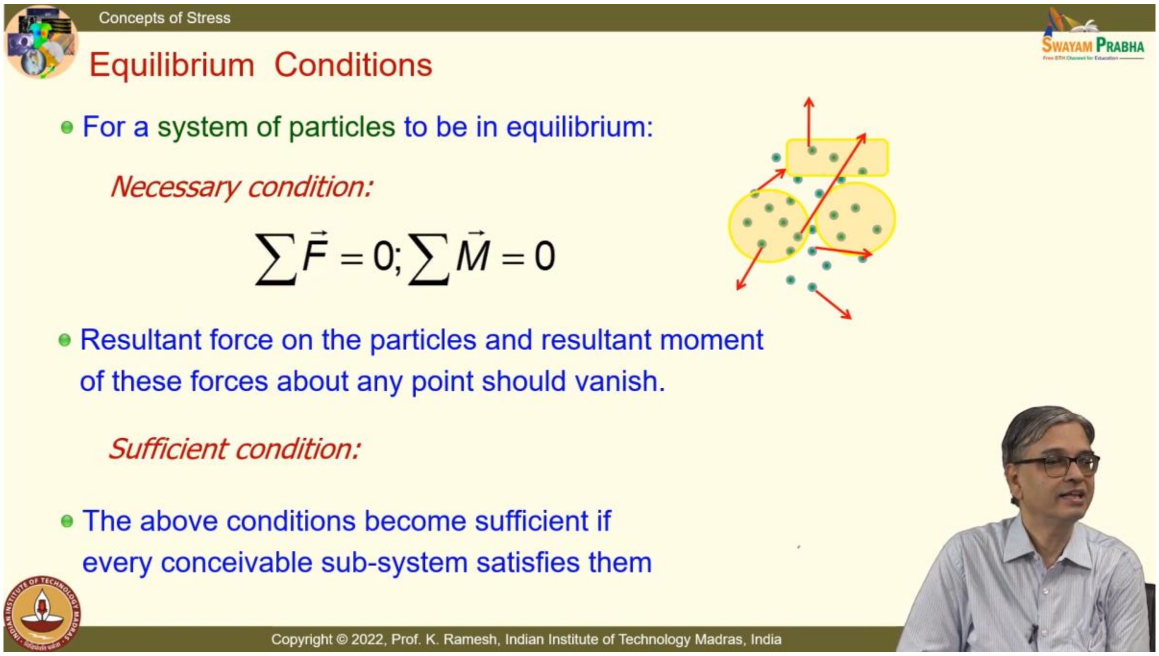

Now we graduate to system of particles. When I go to a system of particles, you have to write the necessary condition. You will also have to write the sufficient conditions. What are the necessary conditions? Here, it can also support a moment. So, I will have $\Sigma\vec{F} = 0$ and $\Sigma\vec{M} = 0$.

A system of particles can also support a moment. And what becomes sufficient? Every conceivable subsystem if I look at, if $\sum \vec{F} = 0$ and $\sum \vec{M} = 0$ is satisfied, then it is sufficient, fine? That means, I must take arbitrarily different subsystems, investigate; in practice, nobody does that. You all do it for the overall system and then say that equilibrium conditions are satisfied.

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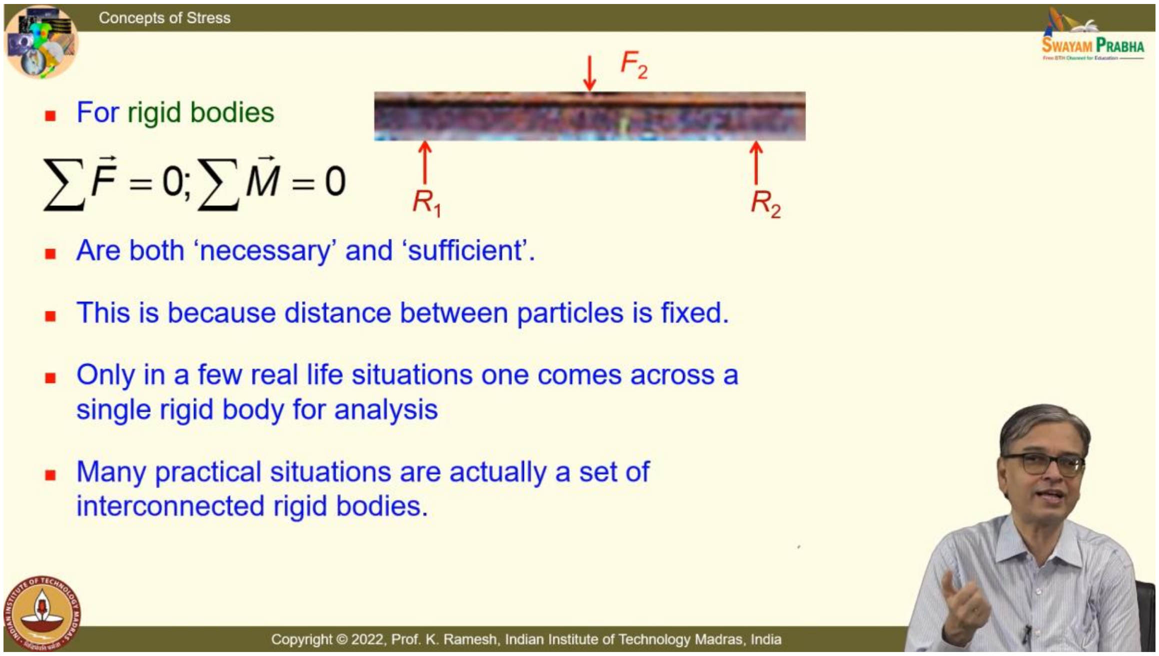

Let us now look at what happens in a rigid body. In a rigid body, if I have $\sum \vec{F} = 0$, $\sum \vec{M} = 0$, that itself is both necessary and sufficient because we have already noted the particles are fixed. The distance between them is fixed; nothing happens to them. But in practice, you know, you do not have a single rigid body, you will always have a collection of rigid bodies. You cannot have a gadget, even a simple cutting plier if you have, I have two elements connected.

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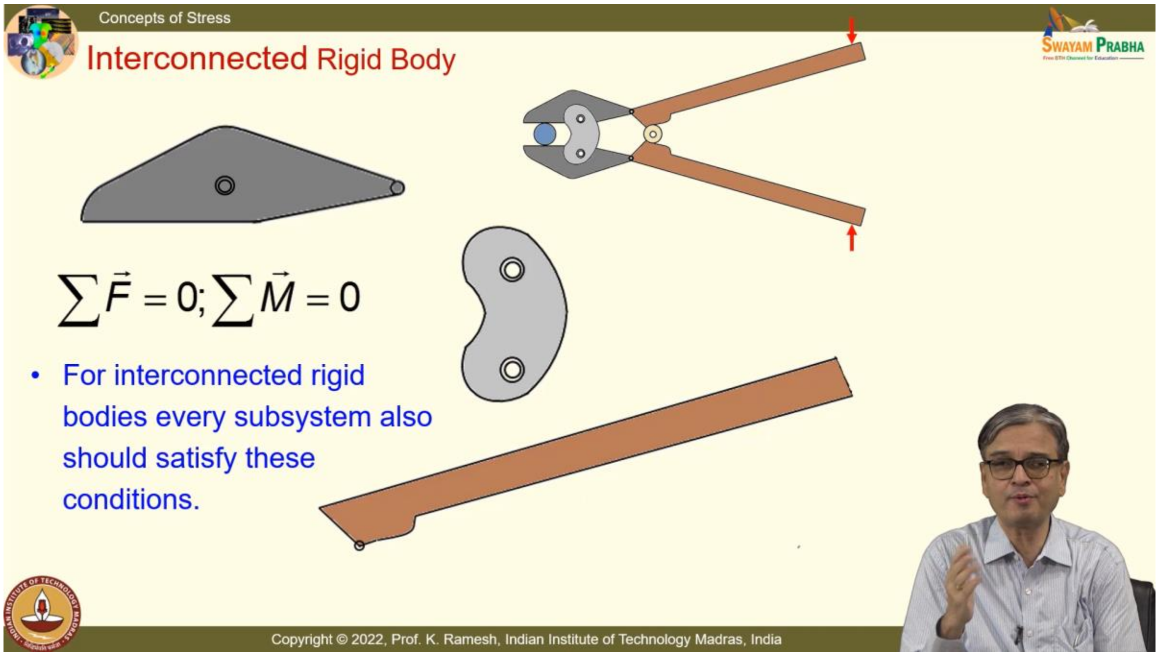

And you know, in my previous course, if you look at engineering mechanics, you have this crimping tool and I also cautioned, this acts like a double lever. Do not even playfully put your finger in the jaw. Even if you apply a small force, it is a very high magnification. And you know, in YouTube, you will find people rescue the animals in the western world. It will get caught in the wire mesh. They will have a crimping tool and cut it, metallic wire! They can easily cut that.

And this is a system of individual elements connected together. We analyze it as rigid body. So, $\sum \vec{F} = 0$ and $\sum \vec{M} = 0$ should work for the complete system as a whole. It should also satisfy for every conceivable subsystem. In this, you know, you can dismember them and you can easily have a look at it. Now, before we proceed into deformable solids, I want you to have an anticipation. We are bringing in deformation; that means, distance between the particles is changing as a function of the load.

In what form do you expect the equilibrium conditions? Are they simple algebraic equations or trigonometric equations or exponential equations or differential equations? Think about it. We will also see; we are going to develop them. Because when we develop the conditions, we want to look at whether they are necessary as well as sufficient, fine?

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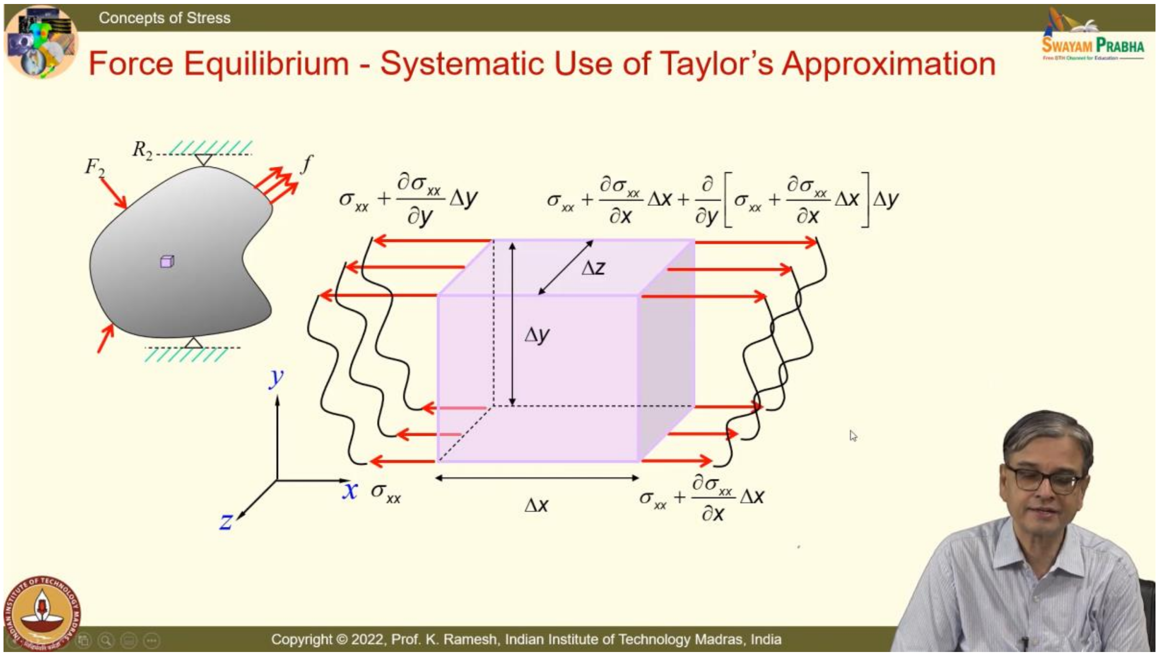

So, now we take the three-dimensional body. When we develop all these concepts, we will satisfy ourselves that the concepts are valid for a three-dimensional body. And what I do here is, I take a cube out of the three-dimensional body.

This has finite dimensions; very small; Δx, Δy and Δz. What you have to recognize is, when I go from negative x-plane to positive x-plane, the quantities vary as a function of the distance. Is the idea clear? While we developed the concept of stress vector followed by stress vector on any arbitrary plane, we took out a tetrahedron, but we shrunk that to zero. Here we are not going to shrink it to zero because we are going to investigate the equilibrium conditions. We have definitely taken an infinitesimal element of size Δx, Δy and Δz.

See, many books jump steps and then put what happens on this surface, what happens on this surface, but we will investigate it little more, so that you have clarity on applying the mathematics systematically, fine? So, what I am going to show is, let us start from a negative face. On the negative face, let me say that I have a quantity and this is varying in the x-direction. For simplicity, we will also look at, it varies in the y-direction. We will postpone the z-variation for the time being, fine? So, can I take, on this surface, what is the force that is acting. To get the force, we only know what is the stress component acting.

If I put the stress component correctly, multiplied by the area, I get the force, fine? Now, what I am saying is, on this surface, let us say that I have the stress component as σ_{xx}. That is fair enough because it is negative x-axis and negative direction is positive. So, I have put the stress direction correctly. Now, you have to tell me what happens at a distance Δx. You all have studied Taylor's approximation.

Can you write what is it that you will anticipate at a distance Δx away by Taylor's approximation? You all know! You have studied in your mathematics course and write it in Taylor's approximation and check what I have in my slide. So, I can do that as $\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x}\Delta x$, that is fair enough, no problem. Now, I move by a distance Δy. See, what I am saying is, we will consider that this remain constant over the distance Δz to illustrate the point, fine? What way would I write on this? It has moved by a distance Δy and we find that these are all functions of x, y, and z. So, I can write this as $\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial y}\Delta y$.

See, when we normally say the normal stress, many times we may not say σ_{xx}. Even σ_x is sufficient enough. So, it will be interchangeably used, ok? So, this is fair enough. Suppose I come to this, can you write the Taylor's approximation from first principle? See, engineers will always knock off terms that are very small. Write it like a mathematician now. Here itself, we have truncated the Taylor's series to first term. We have not written the Taylor's approximation completely. Can you write what is it that you will anticipate in this? I have this, this has varied by distance Δy. So, I can write it as

$$\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x}\Delta x + \frac{\partial}{\partial y}\left[\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x}\Delta x\right]\Delta y$$

Is the idea clear? We have already said, I have taken an infinitesimal element, Δx is small and if I have product of smaller quantities, it will be much smaller. So, we will use that advantage and simplify this expression, ok?

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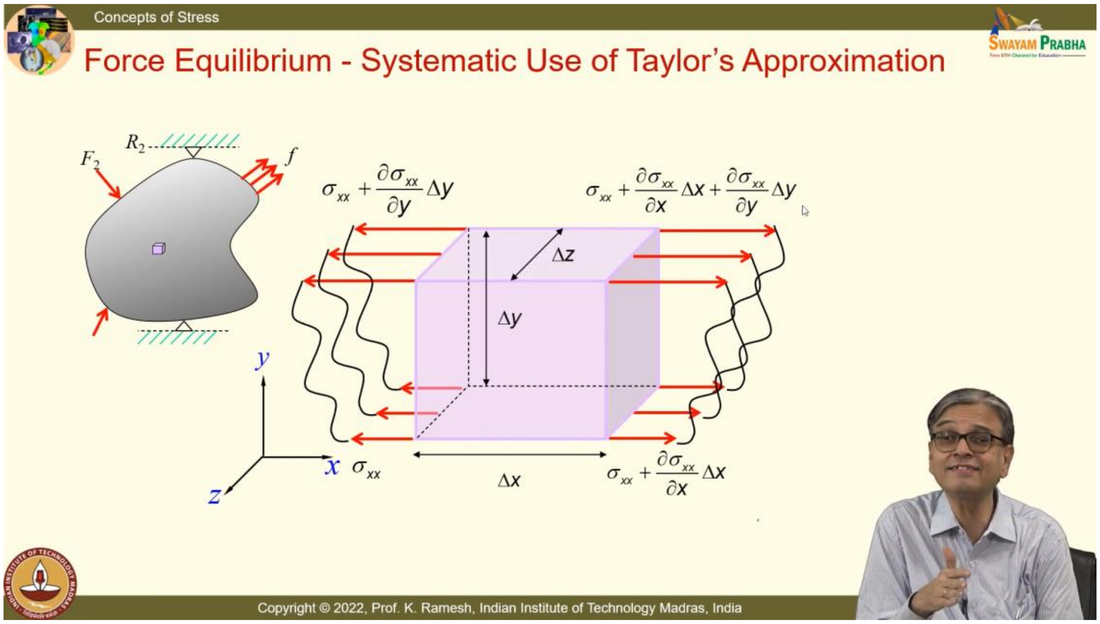

I can write this simply as

$$\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x}\Delta x + \frac{\partial \sigma_{xx}}{\partial y}\Delta y$$

In fact, you will not find this kind of a discussion in any of the books that you have access to, ok? This is normally glossed over. Now what I will do is, on this surface, I will replace it by a stress component at the center by taking the average of this. That is an approximation that we can always make. Similarly, I will do this and let us see what we get.

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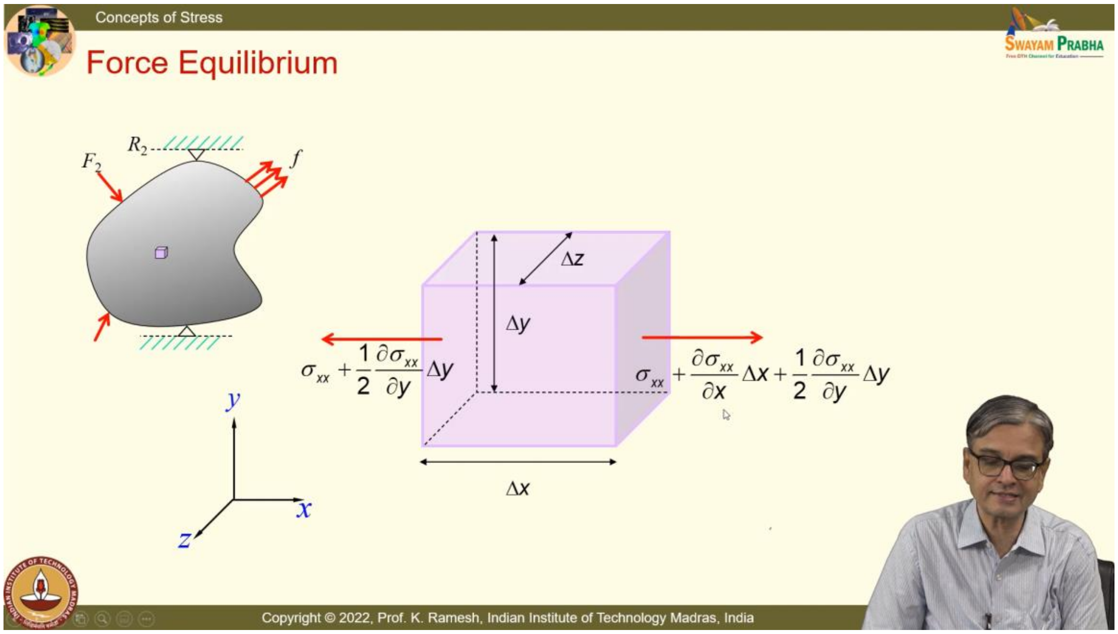

So, I have this as average of these two. So, I get this quantity $\sigma_{xx} + \frac{1}{2}\frac{\partial \sigma_{xx}}{\partial y}\Delta y$. And this reduces to $\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x}\Delta x + \frac{1}{2}\frac{\partial \sigma_{xx}}{\partial y}\Delta y$. See, this can be written in a much convenient fashion which is what is normally represented in many of the books even to start with. I can knock off this 1/2 term on both sides.

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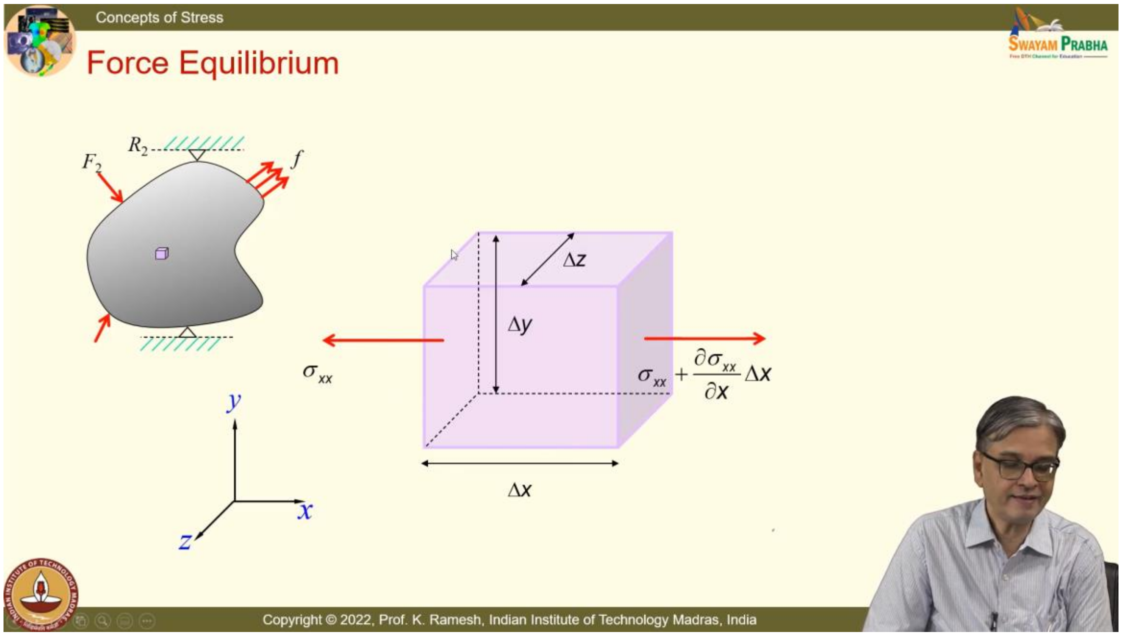

I can simply write, when the quantity is varying along the x-direction, represent this as a stress component acting at the center of the element, on this face of this element as σ_{xx}. I am looking at variation in the x-direction, put this as $\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x}\Delta x$. On this basis, you can write all the other quantities; when I am going along the x-direction what happens, going along the y-direction what happens, going along the z-direction what happens. Here, I have illustrated by taking σ_{xx} was varying in the x and y direction, fine? Even if you take x, y, z direction, ultimately you can simplify that in a form like this. So, this is the first thing that we need to know; how do you represent the components on the negative and positive faces? Because there is a change; the value does not remain constant in a generic problem. When you take a simple tension specimen, it is going to remain constant; that is a special case. We are now discussing a generic situation.

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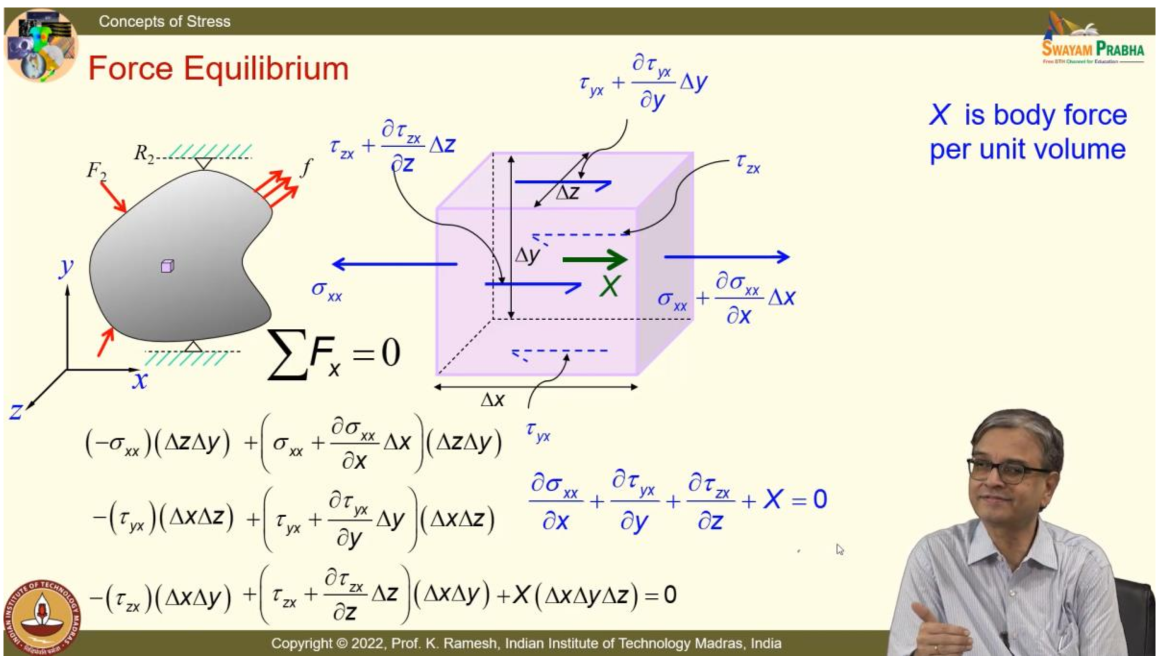

So, following this, we would write and then evaluate what is sigma $\sum F_x = 0$. And while I do this, I will write only components that will contribute to this equation on each of the faces. And you all know, if any of the shear stress component which has the second subscript as x, that is what going to cause, ok? So, now what I am going to do is, I have written down the force and while writing this, I have already cautioned you, we follow the convention to write initially.

But when I sum it up, I look at the coordinate system, if it is pointing towards the negative direction, I will have -σ_{xx} multiplied by the area, I have $\Delta z \Delta y$, that gives me the force. And on this face, I have this as $\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x}\Delta x$ multiplied by the area $\Delta z \Delta y$. It is very simple, what you have done in rigid body mechanics, same thing you are going to do. But when you want to find out the forces acting on these different faces, you need to start from the stress component multiplied by the appropriate area. Now, we look at the component that will cause force equilibrium in the *x*-direction.

So, I have on the negative face τ_{yx}, which is written using the convention. This varies in the *y*-direction. So, when it varies in the *y*-direction, if I use Taylor's approximation systematically, this can be simply reduced to $\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y}\Delta y$. If you understand this, rest of this is straight forward, no difficulty at all. Now, I can also find out the quantities related to force.

So, I have $-(\tau_{yx})\Delta x \Delta z + \left(\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y}\Delta y\right)(\Delta x \Delta z)$. So, we need to have what is the component acting on the other face, which will contribute to equilibrium along the *x*-direction. So, I have τ_{zx} on the negative face, on the positive face; *Z* is varying, so, I put this as $\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z}\Delta z$. Is the idea clear? Is there any difficulty at this stage? Once you have written this, you can always write the force.

So, we will write the force; $-(\tau_{zx})\Delta x \Delta y$, $\Delta x \Delta y$ is the area. And I have $\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z}\Delta z$; the area is $\Delta x \Delta y$. Have we accounted for all the forces acting on the system? Yes, or no? Body force, we have not written it down. Because I said initially, in order to make the diagram simple, we will not confuse with more of this.

So, we have to take care of the body force. Let the body force be written as *X*; this acts on volume element. So, I will have to multiply it by the volume, that is $\Delta x \Delta y \Delta z$ equal to zero. Can you simplify this? Can you simplify? Please simplify and verify what I have in my screen. I get a nice equation, in which form? Because I raised, before we start this, what way do you anticipate the equilibrium condition, because it should be both necessary and sufficient. So, I get this as a nice differential equation, fine? Because I have infinitely many systems possible when I have collection of particles.

When I am looking at a deformable solid, the differential equations take care of that. So, I get the equilibrium condition along the *x*-direction as,

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + X = 0$$

Now, I can repeat the same thing for force equilibrium in the *y*-direction, force equilibrium in the *z*-direction. You can even write it cyclically, fine? But we will go step by step, at least at a faster pace, fine?

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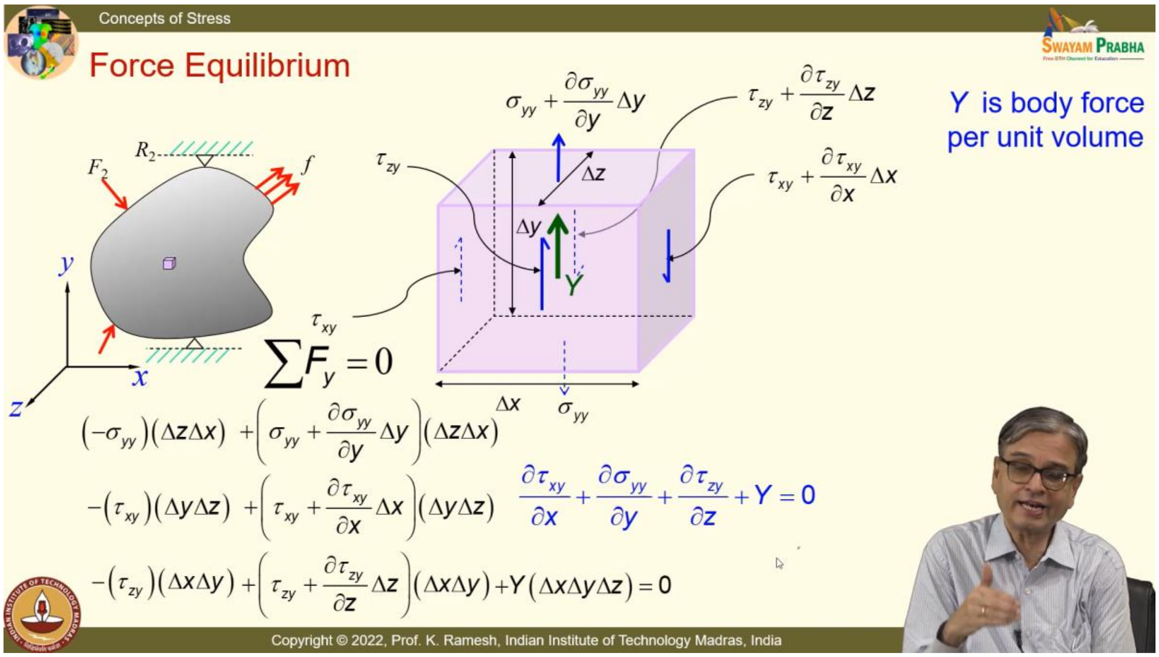

So, I want to do $\sum F_y = 0$. So, I put what happens on the *y*-plane. So, I have σ_{yy} and $\sigma_{yy} + \frac{\partial \sigma_{yy}}{\partial y}\Delta y$. You should understand how we have got this. We have to look at, we are varying along which direction and write the Taylor's approximation comfortably. So, I can write the relevant forces for summation that is given in the screen and we will do it on the other faces. So, I have τ_{xy} and $\tau_{xy} + \frac{\partial \tau_{xy}}{\partial x}\Delta x$ and here the area becomes $\Delta y \Delta z$.

So, I can write the force equilibrium in the *y*-direction. In fact, you can do it yourself, keep doing it yourself. And we will also have to take the body force. And when I have the body force, I get the final expression in a simplistic form like this. I have this as

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + Y = 0$$

Even before writing this, if you look at the cyclical form of writing, you can write for the *z*-direction. But nevertheless, we will have a look at the forces individually.

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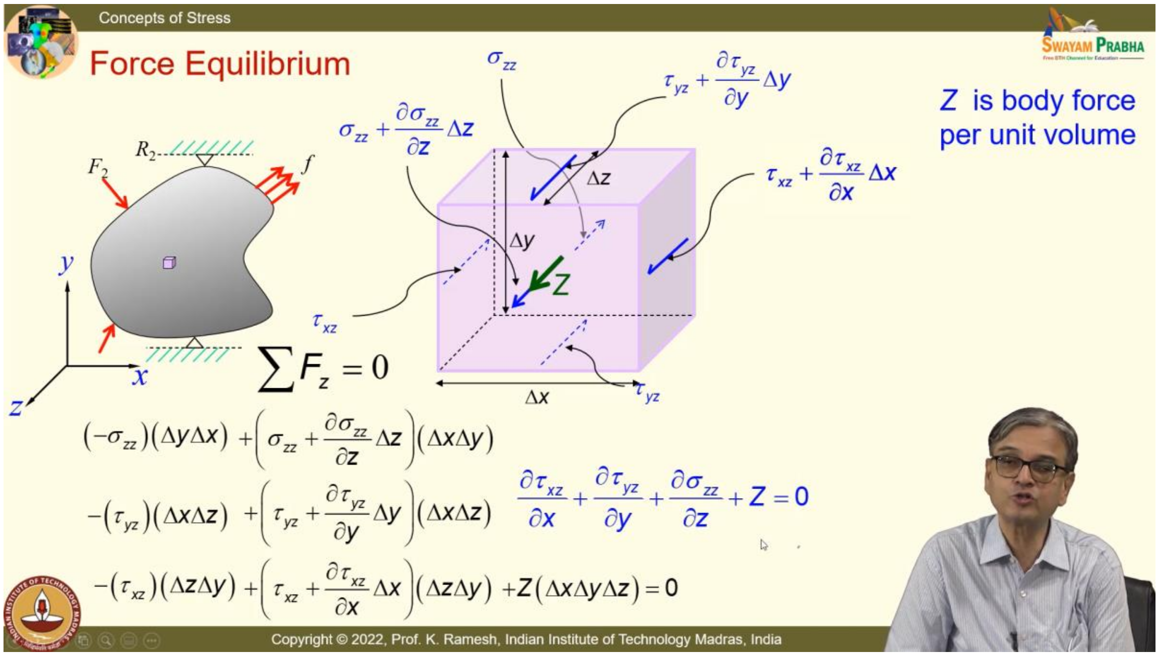

We will develop what is $\sum F_z = 0$. We start from the *z*-direction and we have σ_{zz} on the negative *z*-plane and I have this as $\sigma_{zz} + \frac{\partial \sigma_{zz}}{\partial z}\Delta z$. I can find out the forces multiplied by the area $\Delta y\Delta x$.

And similarly, account for the shear stresses acting on the relevant faces, fine? So, I will have τ_{yz} is now taken up; τ_{yz} varies as $\tau_{yz} + \frac{\partial \tau_{yz}}{\partial y}\Delta y$. This, you should be able to write from understanding. That is why I am going slow; I am not going in a hurry. But for force summation, you all know. Once you write this, force summation is straight forward. There is no difficulty at all.

And here, the area is $\Delta x\Delta z$, that is what you have here. So, similarly, I write τ_{xz} on the negative *x*-plane and $\tau_{xz} + \frac{\partial \tau_{xz}}{\partial x}\Delta x$ on the positive *x*-plane. I can write the relevant forces. And as before, I will also have to account for the body force, ok? When I put the body force as *Z* and multiplied by the volume of the element $\Delta x\Delta y\Delta z$, when I simplify, I get this expression as

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + Z = 0$$

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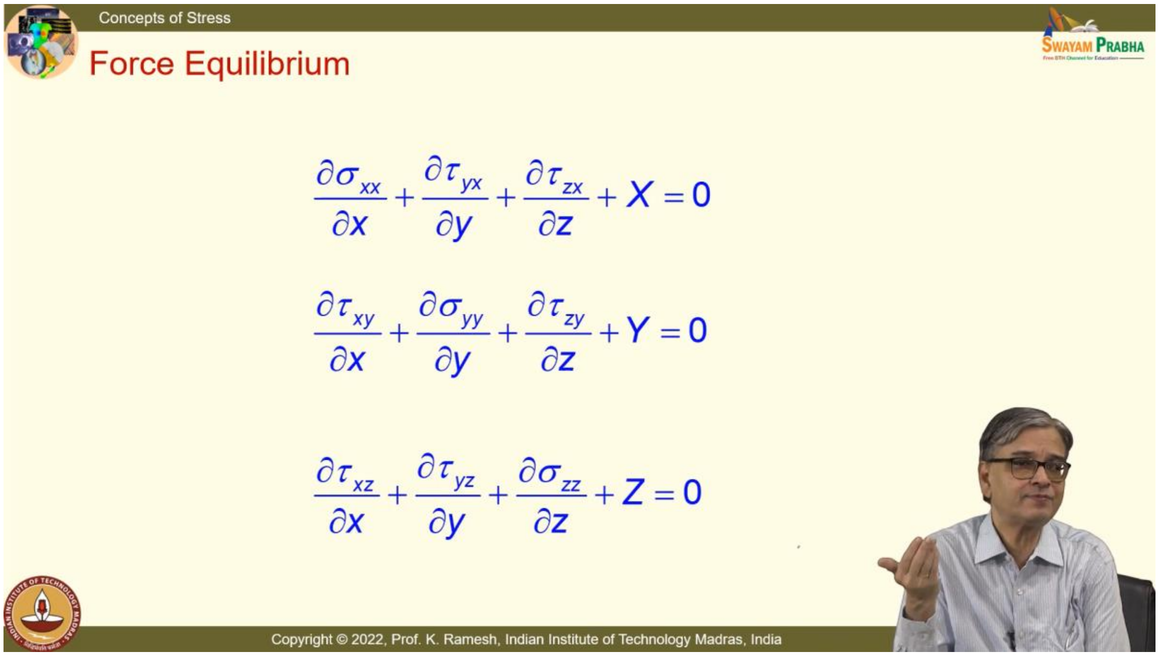

So, I can summarize this in a single slide. So, I get essentially differential equations governing the force equilibrium. From the discussion that every conceivable sub-system has to be verified; it is implicitly accommodated in a differential equation. So, if I satisfy these differential equations, the force equilibrium is both necessary and sufficient. There is nothing more that you need to do. If you solve a problem, if it satisfies the equilibrium conditions, you are happy with it. See in rigid body mechanics, we have looked at force equilibrium, we have also looked at moment equilibrium. Let us look at what way the moment equilibrium equations are.

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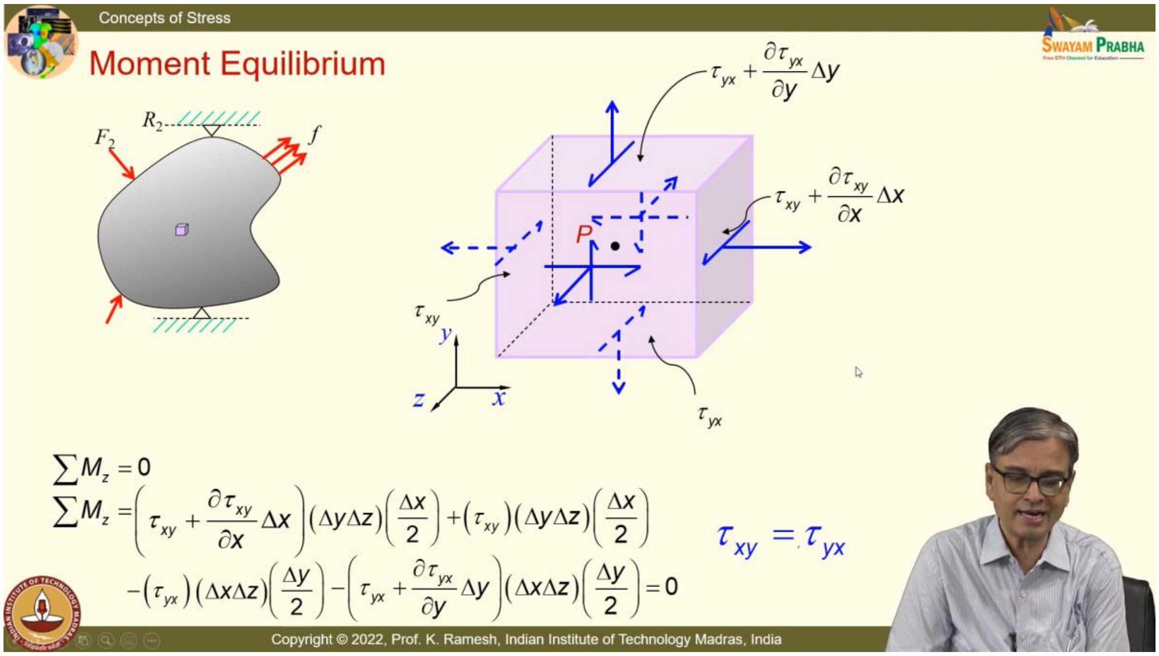

And here, all the components are put and what we are going to do is, you have to take moment about any convenient point. We will take a point which is at the center of the element, we will take a point *P*. So now, what we are going to look at is, we will first find out what is the moment about the *z*-direction.

I have to make $\sum M_z = 0$ and from your understanding of engineering mechanics, you can find out which of the stress components will contribute to moment in the *z*-direction. If there is any component which is passing through the point, the axis is passing through the point, it is not going to cause a moment, fine? That understanding you have. So, we will only flicker those components that will contribute a moment in the *z*-direction. You agree with me what components are flashed? I have τ_{xy}, τ_{yx} flashed on the positive and all the axis, ok? And now you know how to write them on the negative plane and the positive plane.

So, you have to write the quantities correctly. So, I have τ_{xy} and because it has a finite dimension Δx, you will have $\tau_{xy} + \frac{\partial \tau_{xy}}{\partial x}\Delta x$. This, you should never forget. Because I am taking a small element with sizes Δx, Δy and Δz. So, when I look at the negative plane and the positive plane, the quantities will vary as a function of distance. So, on similar lines, I can also write what is τ_{yx} and this is $\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y}\Delta y$.

So, you should know where to put $\frac{\partial}{\partial y}$, where to put $\frac{\partial}{\partial x}$, ok? So, that comes from the appreciation of what we have discussed earlier. So, I can write the forces, I can find out the moment. What is the moment arm? This is the force. I have the stress component multiplied by the area. What is the moment arm for τ_{xy}? It is simply $\Delta x/2$, fine? And I have the other one as $(\tau_{xy})(\Delta y \Delta z)\left(\frac{\Delta x}{2}\right)$.

So, similarly, I write it for the y-x components, $-(\tau_{yx})(\Delta x \Delta z)\left(\frac{\Delta y}{2}\right)$, and I have this as $-\left(\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y}\Delta y\right)$ multiplied by the area $\Delta x \Delta z$, and the moment arm is $\frac{\Delta y}{2}$, that goes to 0. Can you simplify this expression? When you simplify this expression, I get a very very famous result. What we find is, whatever the shear stress acting on this surface, if we call it as τ_{xy} is identically equal to τ_{yx}; we call this as equality of cross-shears. In fact, we used this result while we develop the stress vector on an arbitrary plane. When you wanted to write the Cauchy's formula, we have written it in a matrix notation where this was helpful.

The moment you see, if you find out the moment about the z-direction, gives me τ_{xy} equal to τ_{yx}. You can extrapolate. If I do it for x-direction, y-direction, I will get all the other shear components; the equality of cross-shears will exist. When I reverse the subscripts, they remain identical. So, we will go one by one, but at a little faster pace.

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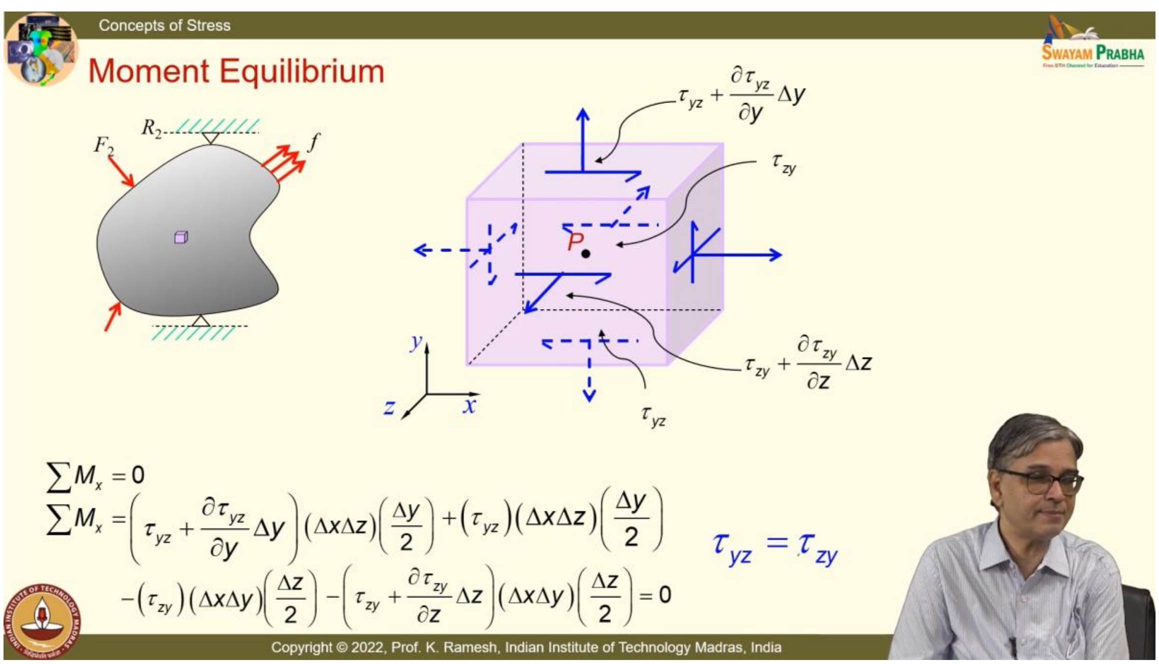

So, I want to write what is $\sum M_x = 0$ and I have taken the point at the center. What are the components of the stress that will induce moment in the *x*-direction? We will flash that first, ok? So, I have this. Can you tell me what is this? τ_{yz}, see that appreciation is important. You do not have to remember, but you should understand and interpret the subscripts appropriately. So, on the negative plane, it is τ_{yz}, on the positive plane? On the positive plane, it will $\tau_{zy} + \frac{\partial \tau_{zy}}{\partial z}\Delta z$. When I write it for τ_{yz}, I go from τ_{yz} to $\tau_{yz} + \frac{\partial \tau_{yz}}{\partial y}\Delta y$. If you have clarity, writing this is child's play. But if you memorize, you will mess it up.

No memory is required, clear understanding is what is required. Then we again do the evaluation of force, then the moment arm, and our job is done. The moment arm here is $\Delta y/2$ and I have this as $\Delta y/2$. For τ_{zy}, it reduces to $\Delta z/2$. So, I get this and I also simplify this expression. This gives me τ_{yz} equal to τ_{zy}.

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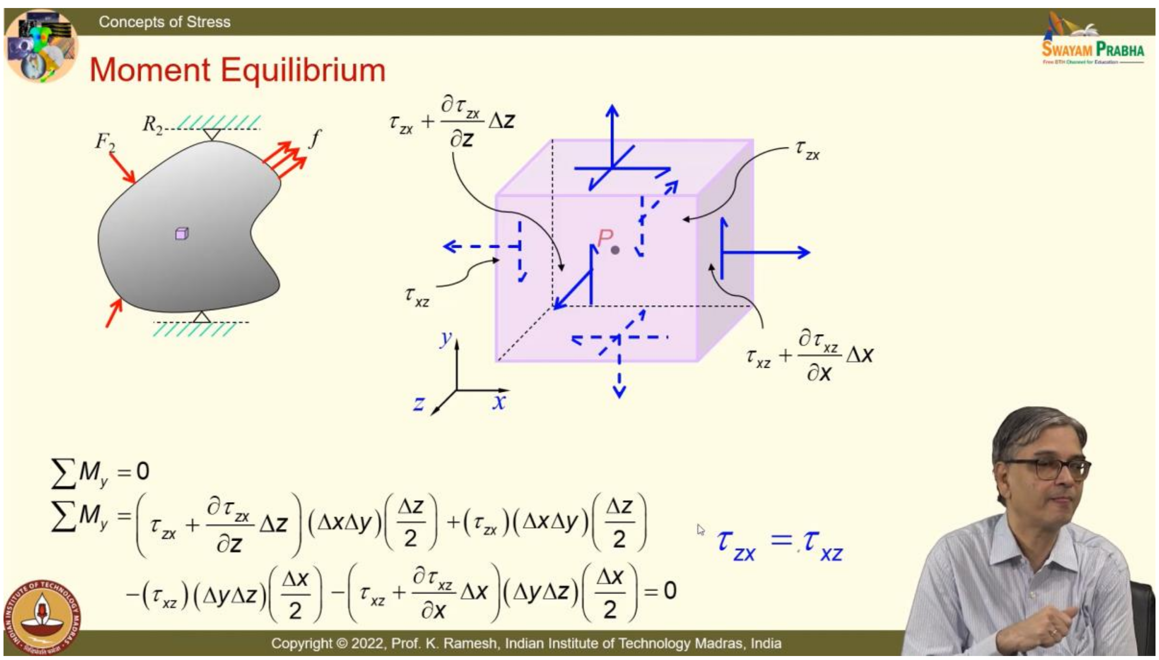

So, naturally, when I am going to look at what is going to happen in *y*-direction, $\sum M_y = 0$ if I want to sum up, I will have the identity τ_{xz} equal to τ_{zx}. So, I have $\sum M_y = 0$. So, look at the components that are going to cause a moment in the *y*-direction, we are looking at the *y*-direction. And you have this as τ_{xz}, I move along the *x*.

So, it will be $\tau_{xz} + \frac{\partial \tau_{xz}}{\partial x}\Delta x$. To drive home this understanding, I am going slow and also repeating the expressions. If you understand the concept, you do not have to memorize. So, similarly, I write what happens on the negative *z*-plane, τ_{zx} and on the positive *z*-plane $\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z}\Delta z$. And we can quickly go through this summation. And when I do this, we are going to get the result $\tau_{xz} = \tau_{zx}$ or $\tau_{zx} = \tau_{xz}$, whichever way you write it, fine?

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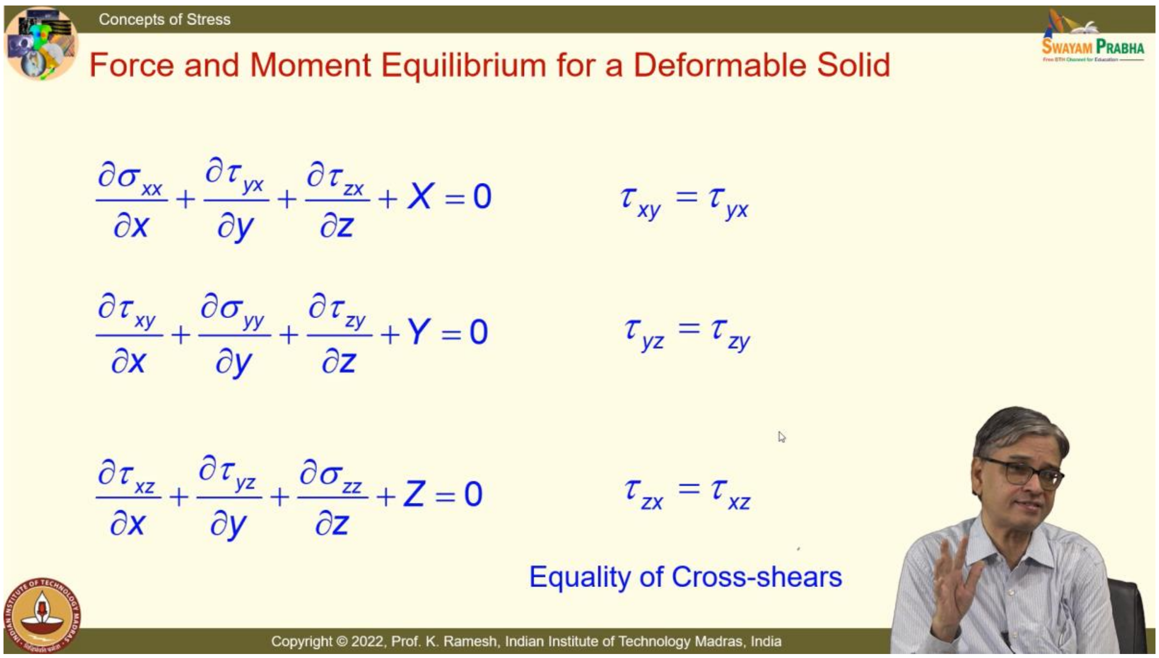

And now, we will summarize what are the force and moment equilibrium equations for a deformable solid. So, I have the force equilibrium like this; they are differential equations. And the moment equilibrium throws me that equality of cross-shears exists and this is a very famous result. Because of this equality of cross-shears, my stress tensor becomes symmetric, ok? In a large variety of problems in the engineering domain, stress tensor is symmetric. Only in a very small cases, when I go to plasticity or when I have the solid in a very strong electric or magnetic fields, you will find the equality of cross-shear does not exist. You also have a couple stress vector, like stress vector we have looked at, we will also have a couple stress vector. You rarely come across even in your advanced studies unless you take up special research in those areas, where stress tensor is not symmetric, you can comfortably live in the domain of stress tensor being symmetric, ok?

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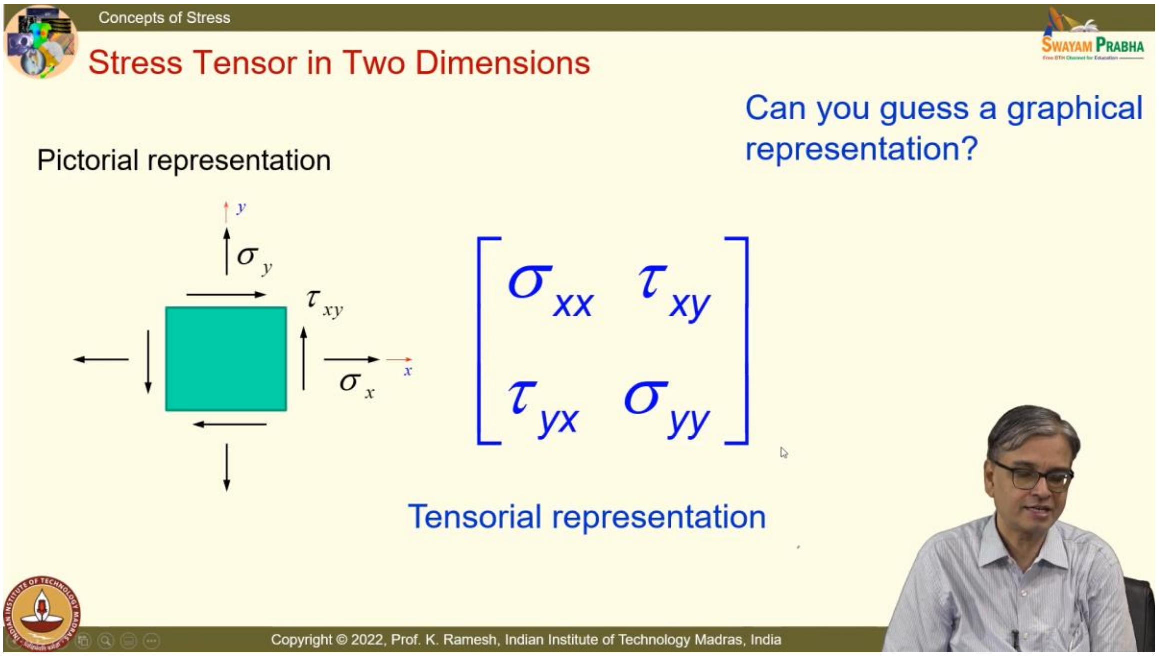

Now we will look at some of the application of what we have learned to simple problems, ok? We have looked at three-dimensional one and again I say that you have a pictorial representation of stress. And when I take this, you have to appreciate, this is not a finite square, it is representing what happens at a point of interest, its dimensions go to zero, fine? It is only a representation; it is a pictorial representation and the tensorial representation is $\begin{bmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & \sigma_{yy} \end{bmatrix}$.

Many times, you know, you will have a uniaxial analysis, that is what we have looked at when you have a simple member in tension, we have done a uniaxial stress, fine? You know, this is a tensorial representation and I again ask a question, can you guess graphical representation? We will postpone the discussion to the next class, fine?

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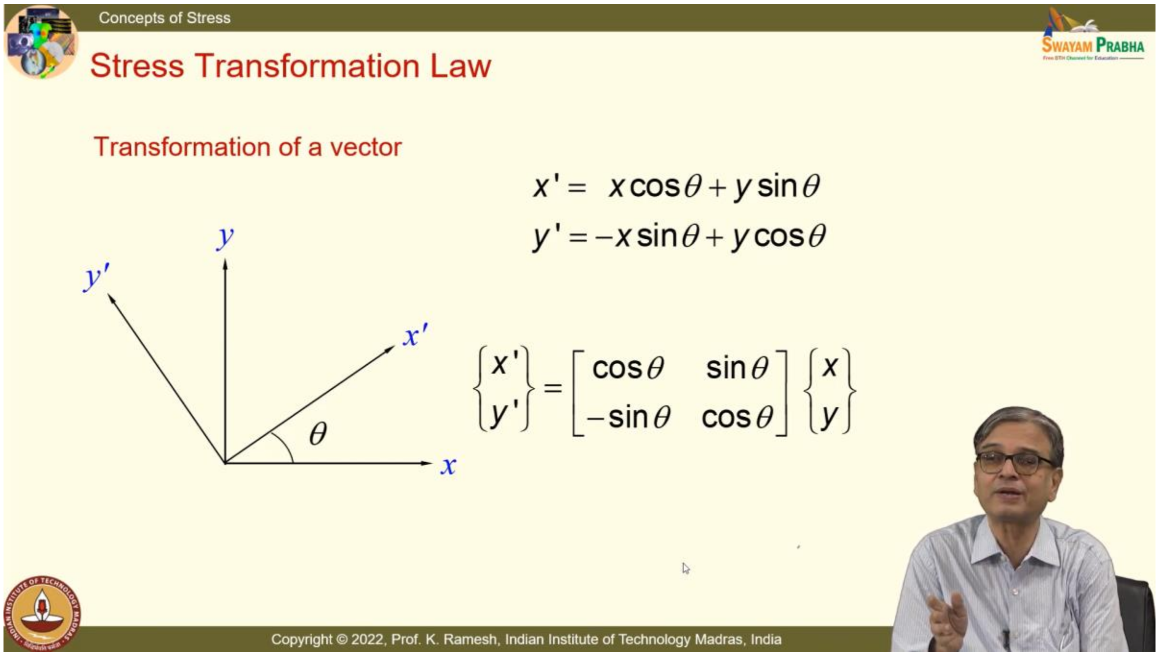

And what we want to know is, we also need to know how to transform from one reference axis to another reference axis. You all know how does a vector transform, no difficulty at all, ok? So, I have a coordinate system *x y*. I have another coordinate system *x*' *y*' which is at an angle θ. Suppose I take an arbitrary point on this, I can relate how *x*' is related to *x* and *y* and how *y*' is related to *x* and *y*.

Can you write that? This you all know; you have done it umpteen times. So, you can write these expressions; *x*' is nothing but $x\cos\theta+y\sin\theta$ and *y*' is nothing but $-x\sin\theta+y\cos\theta$. I can also write this expression as a matrix notation. I can put this $\begin{Bmatrix} x' \\ y' \end{Bmatrix}$ as a vector and I can put these quantities appropriately as a matrix. So, I want to go from $\begin{Bmatrix} x \\ y \end{Bmatrix}$ to $\begin{Bmatrix} x' \\ y' \end{Bmatrix}$ and what I need this here is a matrix. This matrix is known as a rotation matrix: $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$.

And you should know how to write this, not by memory but by making a sketch like this and quickly finding out where you will put minus. The confusion comes whether it is $-\sin\theta$ or $\sin\theta$. So, if you know how to write it, you can easily transform a vector.

But transformation of stress components is little involved. I have two subscripts. I have to account for how these subscripts have to be handled. So, we will develop this in stages. We will first find out the stress vector; from the stress vector in one reference axis, we will transform the stress vector to another reference axis. And in the process, we would have accommodated both the subscripts, fine? Ok.

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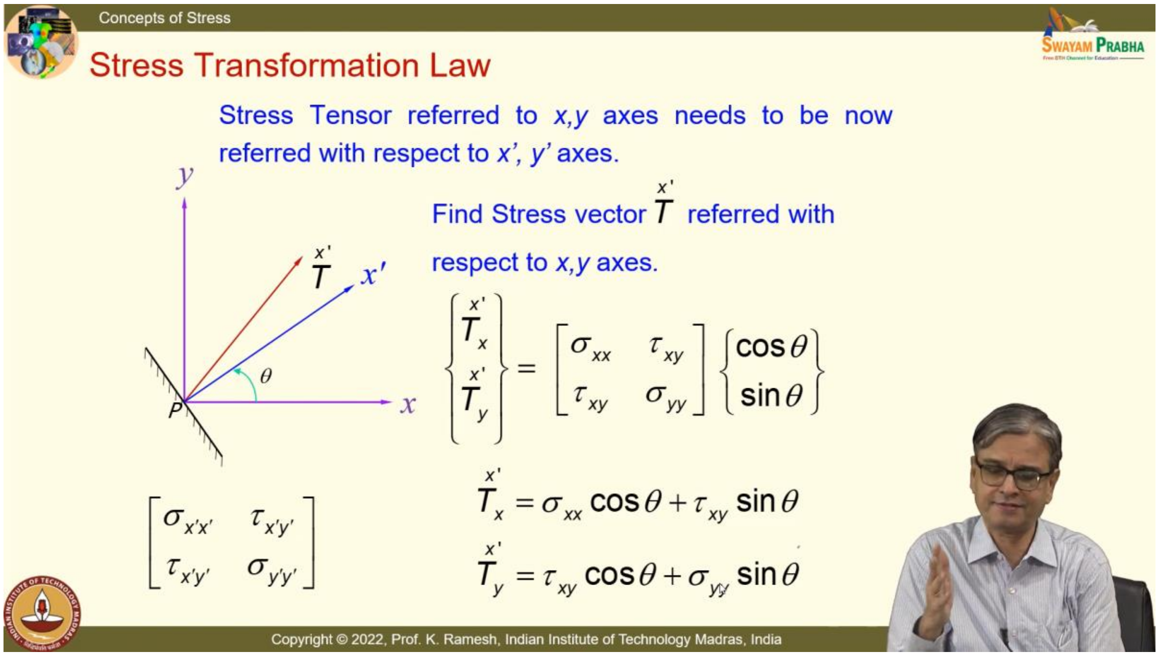

So, the idea is, we have the reference axis *x y* and I have stress tensor referred to *x y* axis. We will label that as $\begin{bmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{xy} & \sigma_{yy} \end{bmatrix}$. Look at here, I have brought in the equality of cross-shear. Whichever way I write, it does not matter in this kind of a development. We will also develop transformation law from multiple perspectives. You can do it by many different ways. And what I want is, my demand is, I want to get the stress tensor referred to axis *x*' and *y*'.

What is the meaning of it? I actually want $\begin{bmatrix} \sigma_{x'x'} & \tau_{x'y'} \\ \tau_{x'y'} & \sigma_{y'y'} \end{bmatrix}$; it is not like a vectorial transformation, fine? So, I have to look at what is the *x*' plane, on the *x*' plane, the normal stress component is sigma $\sigma_{x'x'}$, ok? Let us see how we go about doing it. So, I have to find out the stress vector first. I have the plane denoted by the outward normal. And we have developed the Cauchy's formula. If I know the stress vector, if I know the direction cosines of an arbitrary plane, can I find out the stress vector? I can find out the stress vector on plane *x*', so I will have the cap as *x*', ok? So, I want to get the stress vector.

I have shown the plane for your understanding, I have to get the stress vector $\overset{x'}{T}$, which is also denoted here. What is the Cauchy's formula? I have to multiply the stress tensor by the direction cosine. The direction cosine is very clear here; $\cos\theta$ and $\sin\theta$. So, I can find out the stress vector components, that is $\overset{x'}{T}_x$, that is x component of it and $\overset{x'}{T}_y$ simply as stress tensor multiplied by the direction cosines.

The direction cosines for this axis are $\begin{Bmatrix}\cos\theta\\ \sin\theta\end{Bmatrix}$. So, I have determined the stress vector on the plane x'. Is the idea clear? Once I have a vector, I can go back to my vectorial transformation and transform it to the new coordinate system, fine? That is, we want to get $\overset{x'}{T}_{x'}$; that is what we want to get. So, we want to get this. So, when I write it in expanded form, I get

$$\overset{x'}{T}_x = \sigma_{xx}\cos\theta + \tau_{xy}\sin\theta$$
$$\overset{x'}{T}_y = \tau_{xy}\cos\theta + \sigma_{yy}\sin\theta$$

So, I have determined the stress vector on the plane x'; there is no difficulty. We have just applied the Cauchy's formula which we developed in the previous class.

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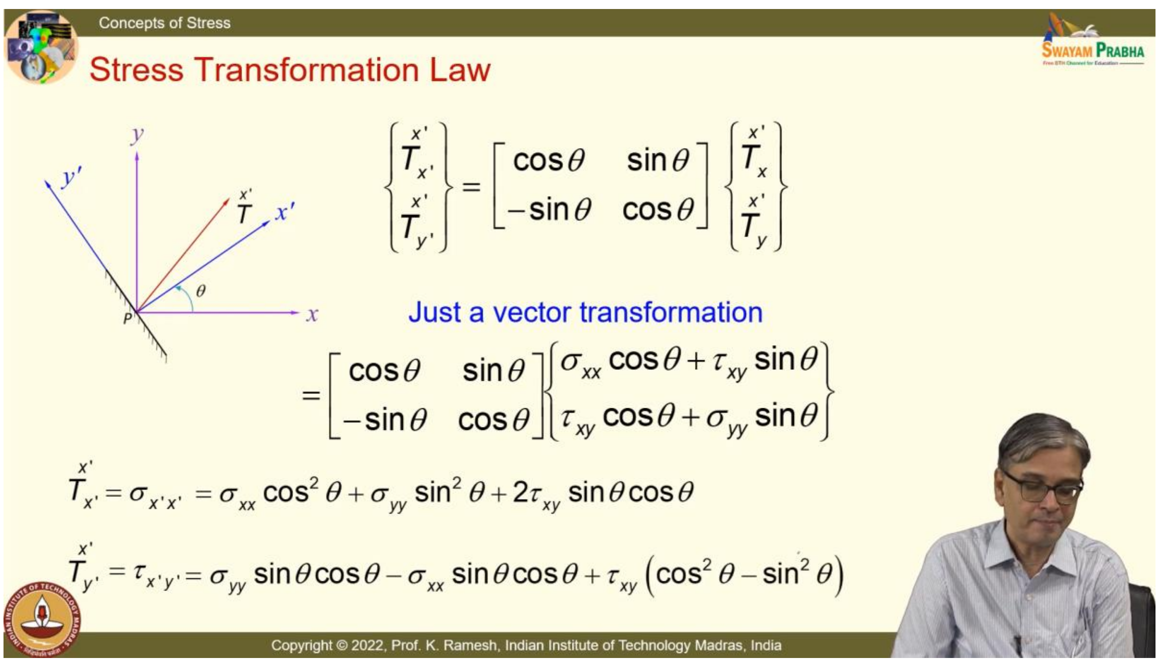

Now I have to transform this to the reference axis x' y'. So, you should understand what is $\overset{x'}{T}_x$ and what is $\overset{x'}{T}_{x'}$. Is the idea clear? You should understand how we evolve. Now, I have a vector given in the reference axis x y. I want to transform this vector to x' y' axis; it is a simple vectorial transformation. How is it achieved? You use a rotation matrix. Is that idea clear? And you already know what is the rotation matrix; we have seen it.

So, I want to get $\overset{x'}{T}_{x'}$ from $\overset{x'}{T}_{x}$. So, that I have to put a rotation matrix here. I have

$$\begin{Bmatrix} \overset{x'}{T}_{x'} \\ \overset{x'}{T}_{y'} \end{Bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} \overset{x'}{T}_{x} \\ \overset{x'}{T}_{y} \end{Bmatrix}$$

So, when I do the actual multiplication, let us see what happens. This can be expanded and we have the $\begin{Bmatrix} \overset{x'}{T}_{x} \\ \overset{x'}{T}_{y} \end{Bmatrix}$; we have seen it in the last slide, ok? And I have this, when I multiply it, I get $\overset{x'}{T}_{x'}$ is nothing but $\sigma_{x'x'}$; that is what you have to interpret. So, $\sigma_{x'x'}$, I get this as

$$\overset{x'}{T}_{x'} = \sigma_{x'x'} = \sigma_{xx}\cos^2\theta + \sigma_{yy}\sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

So, the tensorial transformation is little involved. When I have this, you cannot simply multiply only $\cos\theta$ and $\sin\theta$ and be done with it. You have to appreciate, you have to graduate to $\overset{x'}{T}_{x}$, from $\overset{x'}{T}_{x}$, you have to find out $\overset{x'}{T}_{x'}$. That symbolism you should appreciate; only $\overset{x'}{T}_{x'}$ is $\sigma_{x'x'}$, fine? So, that is the subtlety you have to keep in mind. So, I can also get $\overset{x'}{T}_{y'}$, that is $\tau_{x'y'}$. I get this as

$$\overset{x'}{T}_{y'} = \tau_{x'y'} = \sigma_{yy}\sin\theta\cos\theta - \sigma_{xx}\sin\theta\cos\theta + \tau_{xy}\left(\cos^2\theta - \sin^2\theta\right)$$

This can be further simplified. So, is the idea clear? You are learnt from a vectorial transformation to tensorial transformation. And all tensors will have the same transformation law. We are also going to develop strain tensor, where we will not spend time on the transformation law. We borrow whatever we have learnt for stress directly to strain, ok?

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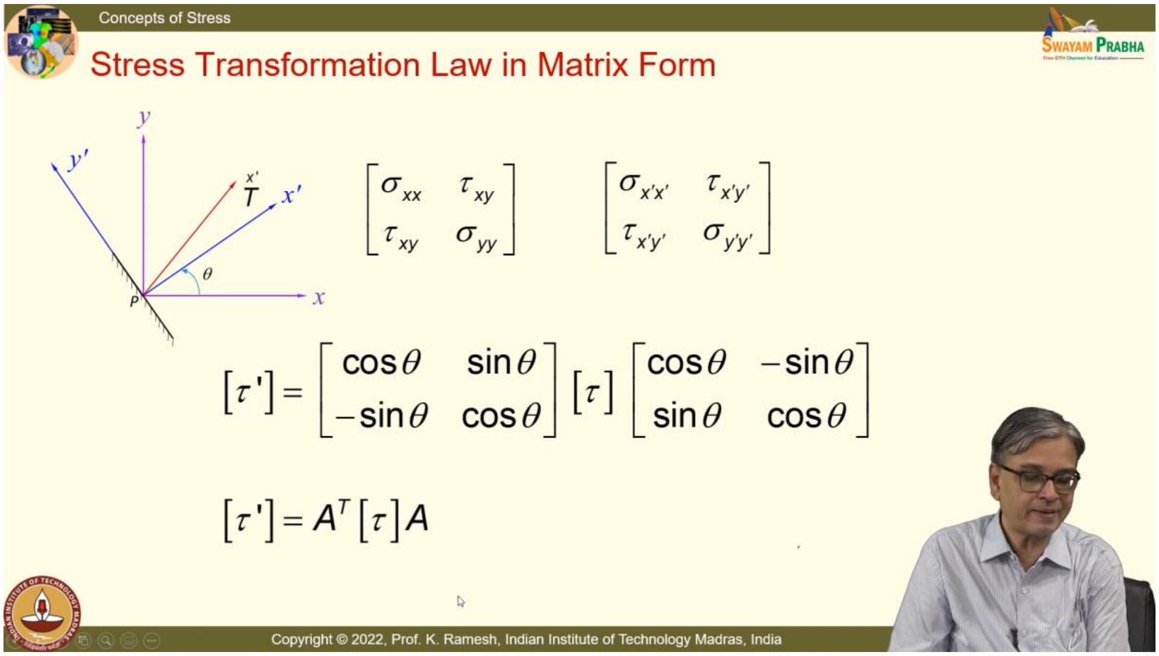

This can also be written down in a different form; that is how in advanced studies people write it. I want to get from $\begin{bmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{xy} & \sigma_{yy} \end{bmatrix}$ to $\begin{bmatrix} \sigma_{x'x'} & \tau_{x'y'} \\ \tau_{x'y'} & \sigma_{y'y'} \end{bmatrix}$. And it is also usually represented, the complete matrix is represented as simply as [τ'], ok? So, it is

$$[\tau'] = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} [\tau] \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

So, I can write this [τ'] as

$$[\tau'] = A^T [\tau] A$$

This is the form you will see repeatedly in higher studies in various forms. And you know, books do not discuss transformation law in this perspective, ok? But this is the most convenient way of doing it. And there are also other simpler ways that we will take it up in the classes to come.

So, in this lecture, we have looked at, from our understanding of equilibrium conditions for a particle, collection of particles, rigid body and articulated rigid bodies to deformable solids. And in the deformable solids, we got the force equilibrium as differential equations. And the moment equilibrium gave a very important result, it said $\tau_{xy} = \tau_{yx}$. We also said that it is known as equality of cross-shears, which we use it repeatedly in our simplifications. This makes the stress tensor symmetric.

Then we also learned how to transform a vector from *x y* coordinates to *x*' *y*' coordinate system. From that, we also developed how to transform a tensor referred to *x y* coordinate system to *x*' *y*' coordinate system. That was arbitrary. So, that means, once you know the transformation law for any coordinate system, the same transformation law can be used. Thank you.