Strength of Materials
Prof. K. Ramesh
Department of Applied Mechanics
Indian Institute of Technology, Madras

Lecture - 04
Stress Vector

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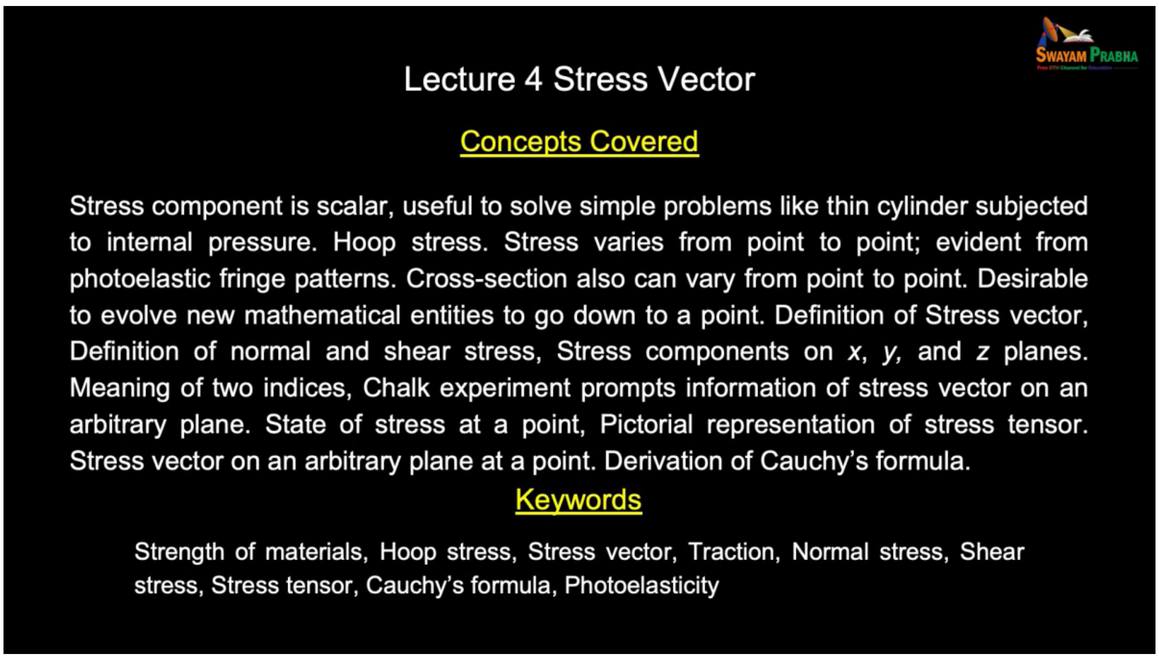

Let us develop concepts related to stress. These will be used in this course as well as in your future courses and also in advanced studies. Even though in this course, we will confine our analysis to slender members, in the development of concepts of stress, we will start with a three-dimensional body and it is also not restricted to isotropic materials. In fact, we will not bring in the material aspect at all while developing the concept. Only one aspect what we will say is, we consider this as an elastic continuum. That is the only thing that we require for a mathematical development.

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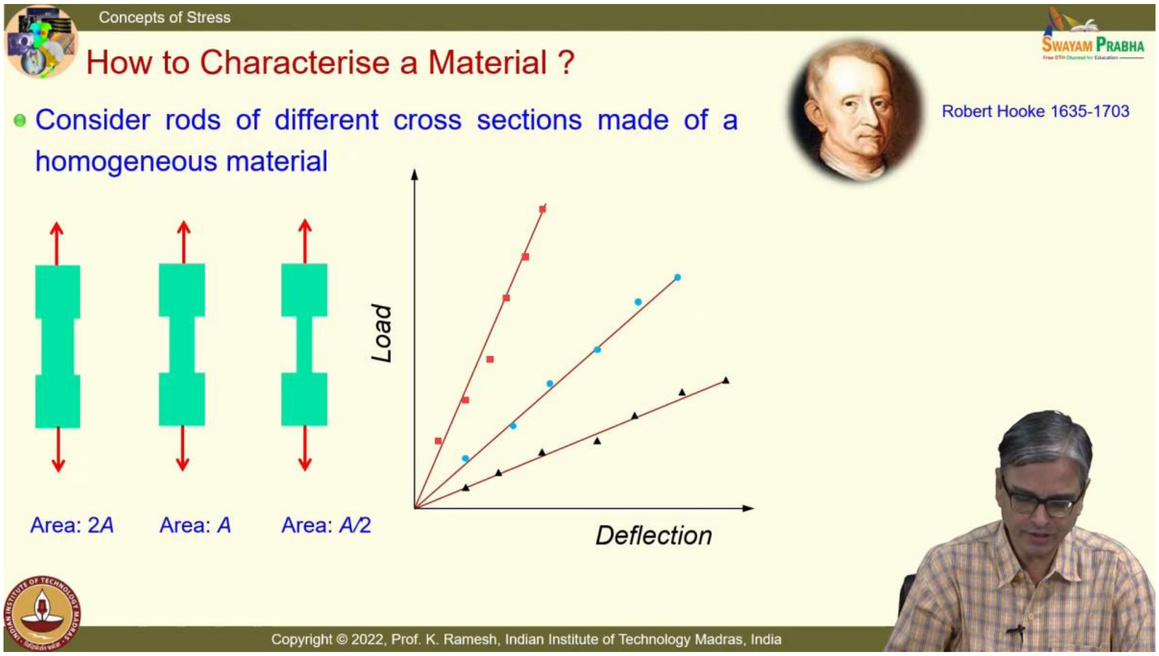

And what you will have to appreciate is, when they were graduating from rigid body mechanics, even documenting there is deformation and this deformation is linear in the restricted range is a knowledge by itself. That was credited to Hooke.

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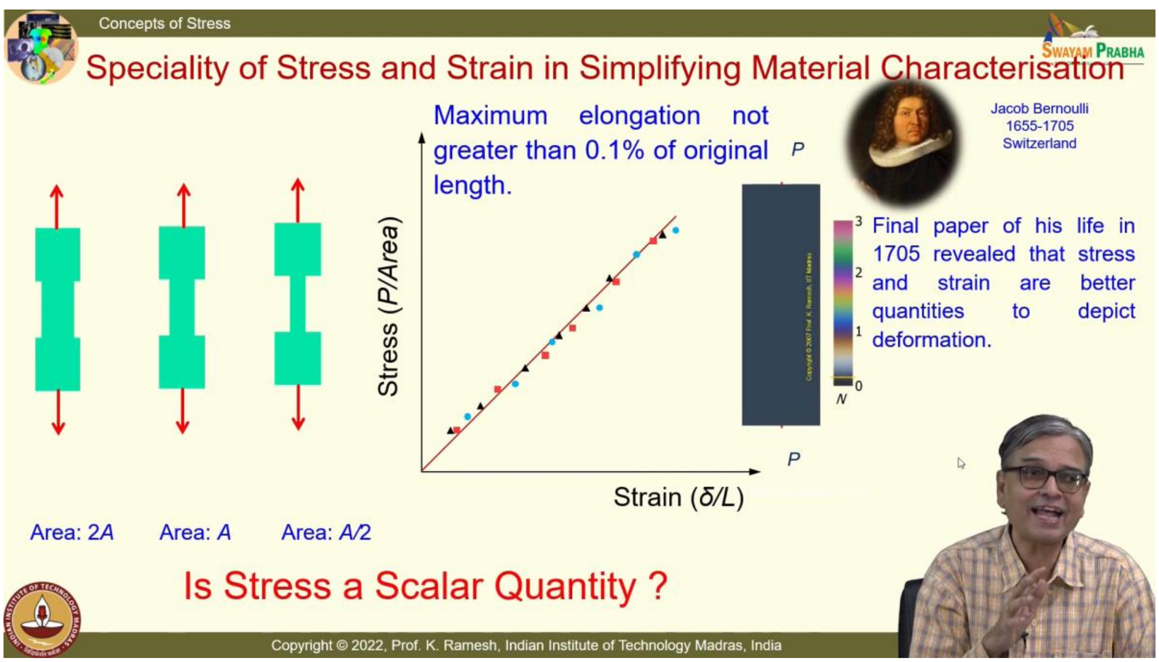

While compiling these graphs, people found, rather than simply plotting force versus displacement, if you re-plot change in length divided by original length versus force divided by area, you have an advantage that I get only a single straight line for three different specimen cross-sectional areas. This gives an impression that we are talking about stress as a number, fine? When you say a number, you will say that it is a scalar quantity, but it is not so.

It is much deeper than that. And we have also seen a simple understanding that P/A, which simplified the graph and also gives the hope that I can do a simple test and understand the material was not a casual development. This was developed by Bernoulli as his last paper.

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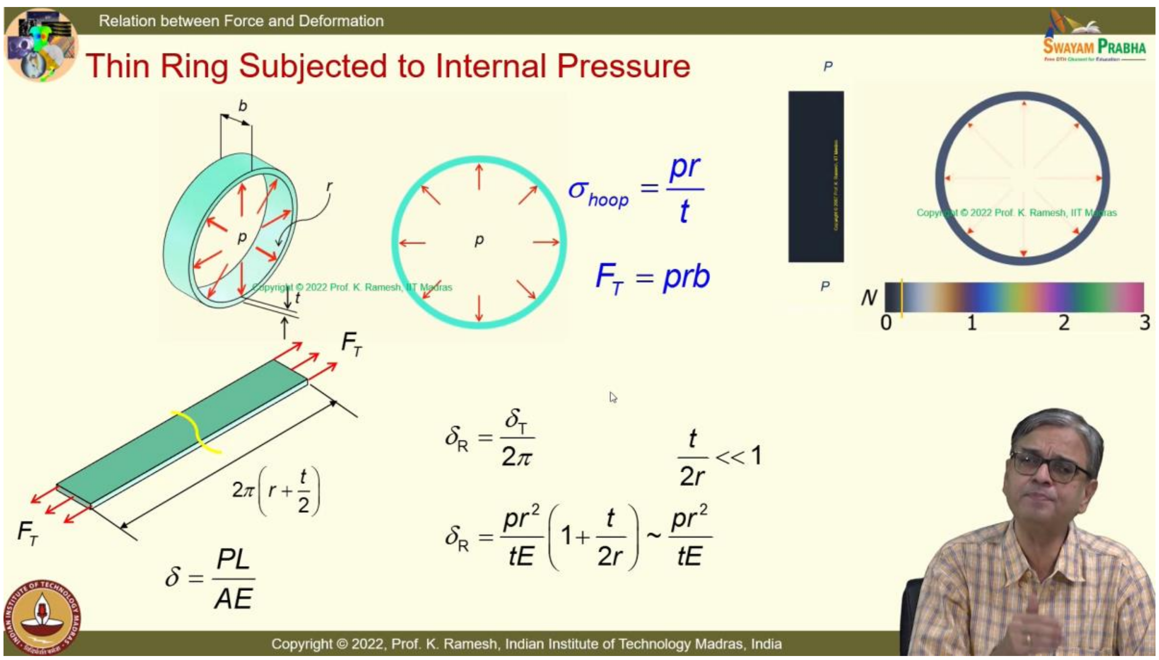

You know, we utilize this information of P/A in a very meaningful manner to solve a very simple useful problem. That was for a thin ring subjected to internal pressure.

And what we said was, you rip open the hoop for the complete length. It is not just the semicircular one; that was one of the question a student raised. It is a complete length of the hoop. You have this as $2\pi\left(r+\frac{t}{2}\right)$. And we have found that force transmitted by this is prb. And there is also a very famous expression for extension in simple tension; $\delta = PL/AE$. So if we use that, you are in a position to find out the elongation of this strip. And you can also find out what is the radial elongation as $\delta_T/2\pi$. We have got this expression. And what is more important from stress analysis point of view is, you find out what is the stress developed. You also say that this as a hoop and you have this value as *pr*/*t*, you simply call this as a hoop stress. So, we were able to solve a simple problem.

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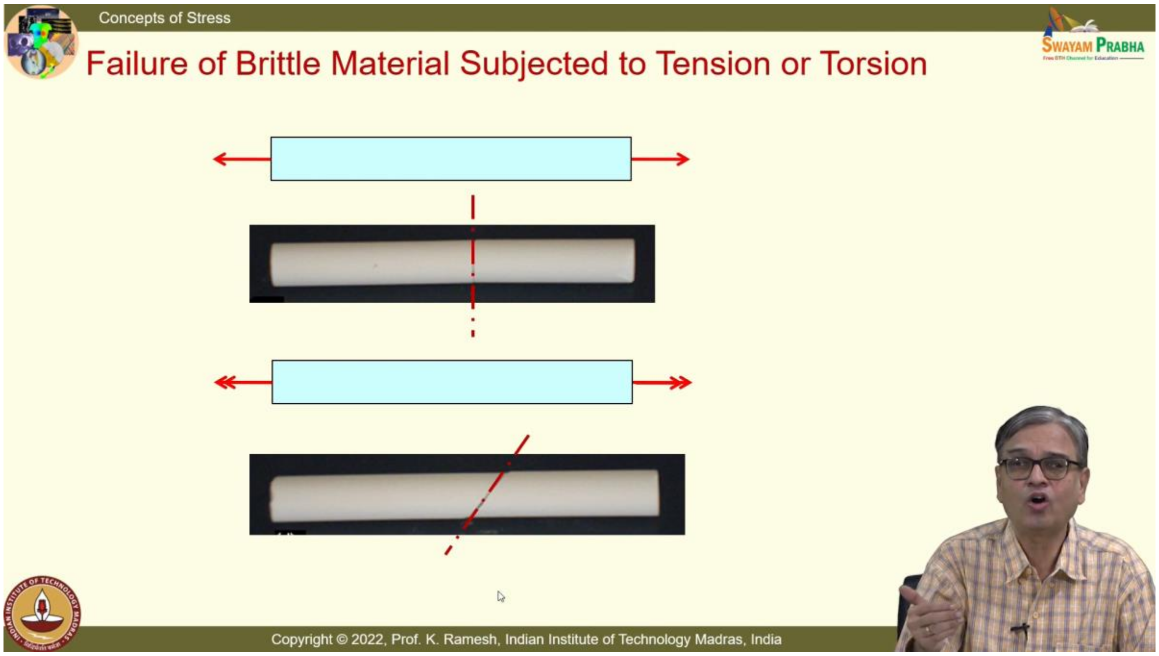

But you know, when you look at the experiment on chalk, we found that when you put an axial tension, I get a failure like this, which is very natural to expect. It is also coinciding with the cross-section. On the other hand, when you apply torsion, what you find is, you have this happening at 45^o.

So, you cannot live on your concept of *P*/*A* saying that you have a constant cross-section. You have to go deeper and look at what happens in all the possible infinite planes. This is one aspect of it.

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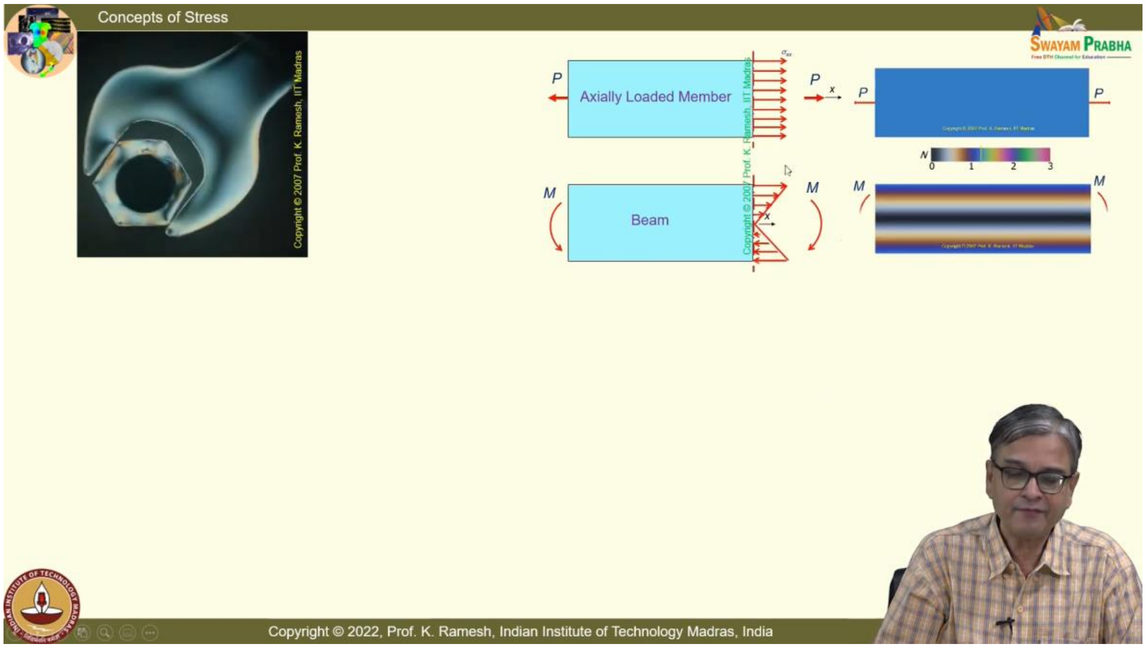

Other one is, see, we have also looked at simple tension and bending. We have utilized whatever information that was available from photoelasticity. And we have seen that in the case of beam, whatever we call it as a resistance varies from point to point. Even though the cross-section remained constant for the length of the beam, the resistance was varying from point to point. And you have the spanner in which the stresses vary from point to point and also the cross-section is also varying. It is not constant.

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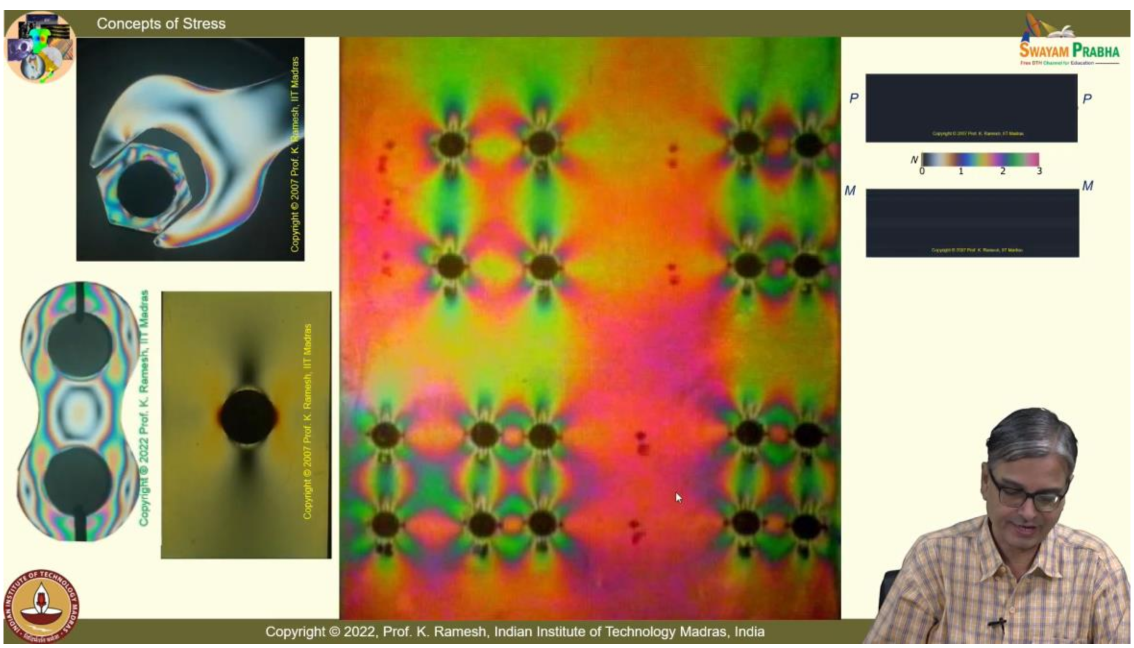

And let me also show one more example. Can you say what is this? Have you seen this kind of an element in your day-to-day life? See, engineering, you learn by observation. You look at your cycle; you have a cycle chain. If you look at one of the elements there, you have it like this. And when you apply the axial pull, it develops beautiful patterns. Even though you will not be able to solve it completely in this course or even the next level course, photoelasticity being an optical method, thankfully it solves the differential equations optically and presents you with beautiful patterns.

Here again you find, I have to find out something point by point. It varies from point to point. And you also have a plate with a hole where the fringes are developing like this. And when you travel home, you should watch the bridges built. You will find a number of riveted joints.

So, you have a panel with several riveted joints. That means, you have a pin inserted there and this figure, the courtesy goes to my student Jeby Philip from VSSC. So, there is a need to graduate from what happens in a cross-section to what happens at a point. Is the idea clear? Ok.

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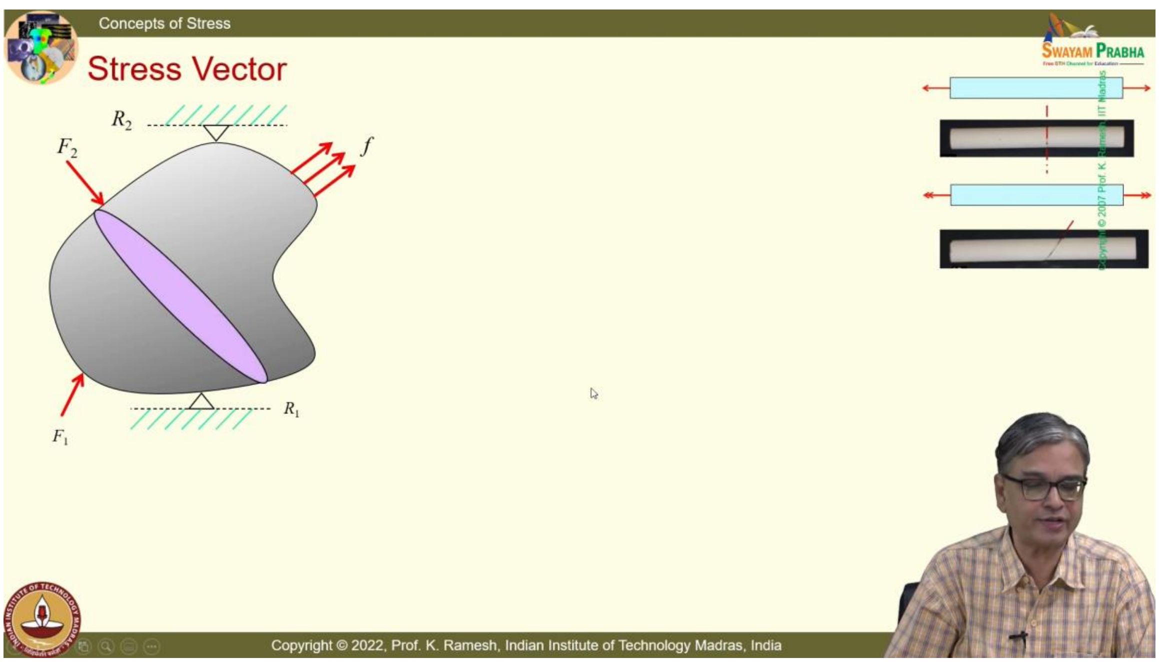

And I said that, you know, we can take up a three-dimensional body. The body is in a state of equilibrium. This is subjected to both body and surface forces. Body force acts on each volume and element of the body. Please make a neat sketch. Please make a neat sketch. And what I am going to do is, I am going to take a generic point and cut an imaginary plane passing through it. That is what I am going to do.

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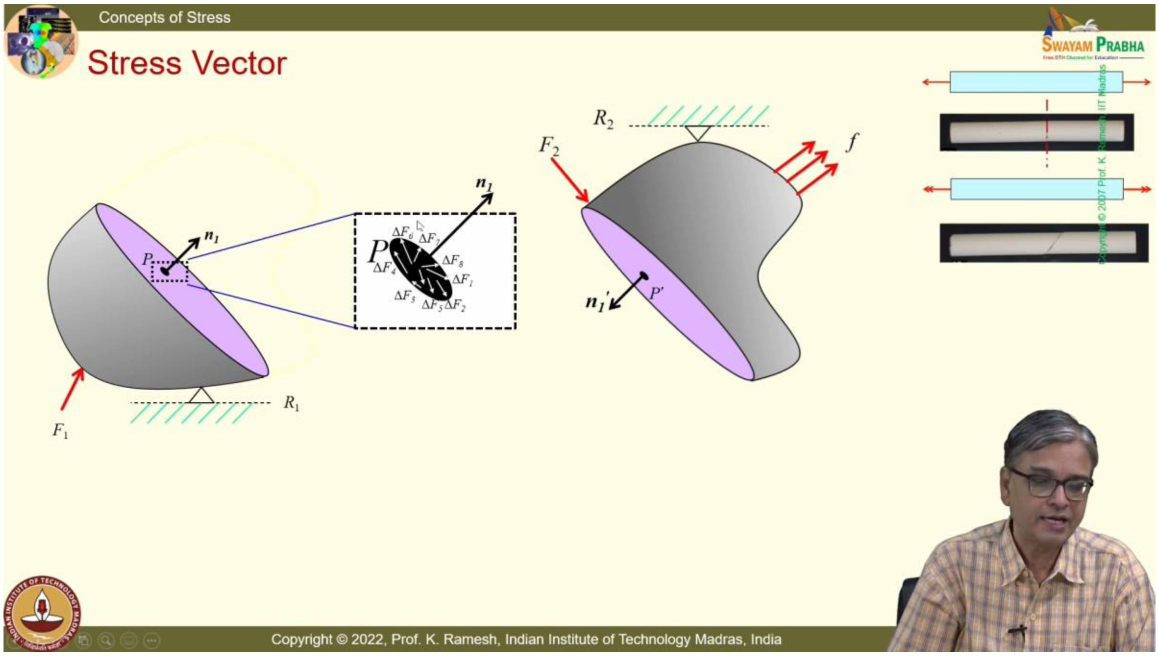

And what I am going to look at is, I am going to look at what happens at a point. We label that point as P. You should not confuse when I separate these two, I am trying to find out what is the reaction developed in this complete surface. No! We are not talking about that. We will confine our attention only to the point. So, we will take a small area surrounding that point.

And when I take this point as P, I can have a reflection of it in this and I have this. And what you have this as n_1 and n_1' are depicting the planes. Plane is denoted by the outward normal, fine? So, I have taken a generic plane passing through the point of interest. And I am also going to develop new mathematical entities for me to develop the concepts further. So, I will take a small area surrounding that point. I will have some kind of a force distribution. I can find out its resultant.

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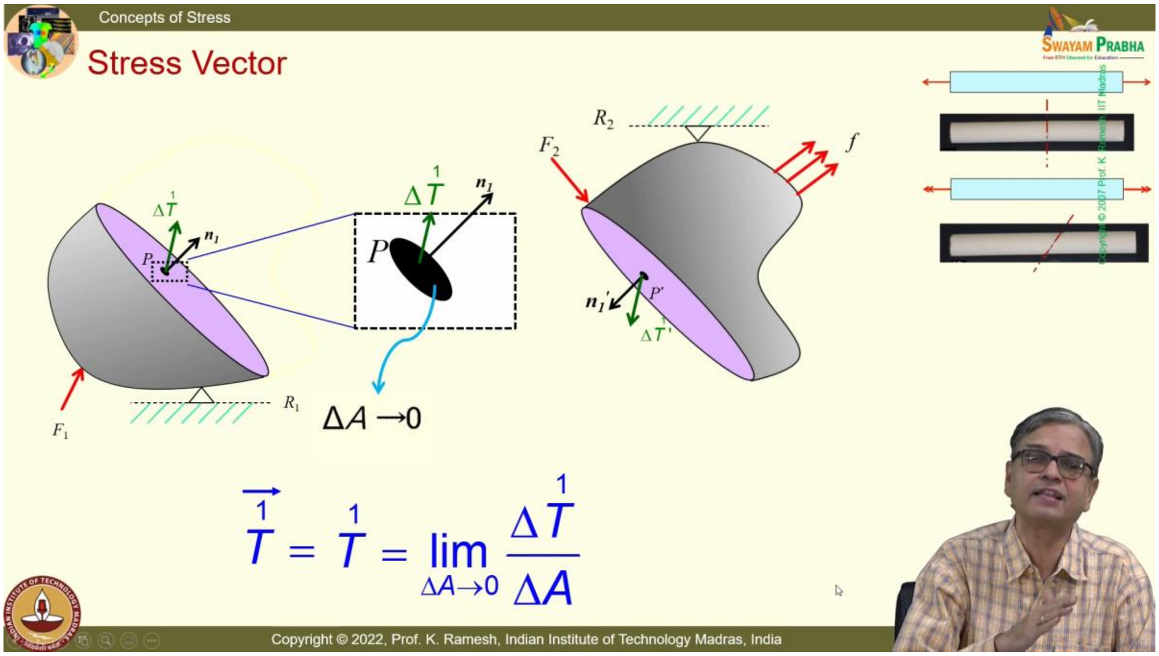

Let the resultant be labeled as $\Delta \overset{1}{T}$. There is a new symbolism. We put, because I am looking at the plane 1, I have also put this 1 on top of it. We are developing a new symbolism. The moment I come and say a force, force has magnitude as well as direction. It is a vector, fine? And in the limit, we want this area to go to zero. So, I have the quantity $\vec{\overset{1}{T}}$, where I show distinctly the vectorial symbol. But for writing convenience, we will simply have $\overset{1}{T}$.

Whenever I have $\overset{1}{T}$, I will say that this is denoting a vectorial quantity. And we say $\lim_{\Delta A \to 0} \frac{\Delta \overset{1}{T}}{\Delta A}$ is a finite quantity. Is the idea clear? We are defining a new quantity. And we say that this is a vector and this vector is known as stress vector. And we also understand the new symbolism.

I will simply have this as $\overset{1}{T}$. You understand immediately that this is a vectorial quantity. It is for writing convenience. When I put the components with a suffix, you can find out that these are scalar quantities. Always the component is only a number, fine?

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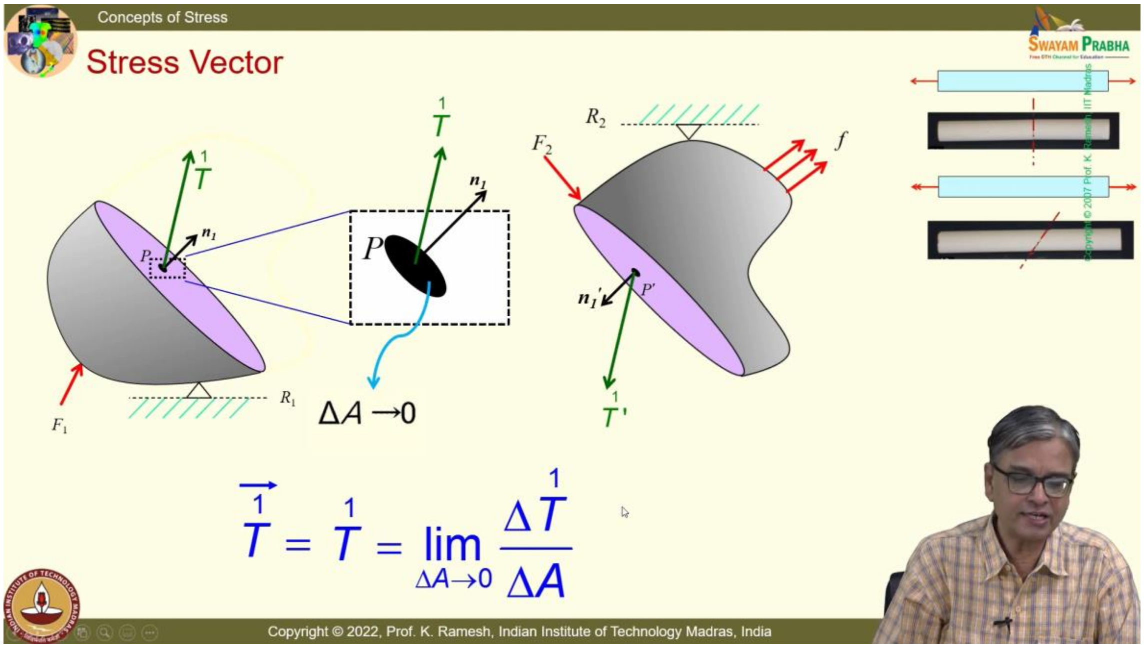

I can also depict this on the free body diagram that we have written. So, on a plane 1, I will have a normal depicting the plane, and also I have the stress vector $\overset{1}{T}$. In general, the orientation of the stress vector need not lie along the normal. In specific cases, it can coincide with the normal. Now I have taken plane 1 and I have defined this quantity. I call this $\overset{1}{T}$ as stress vector.

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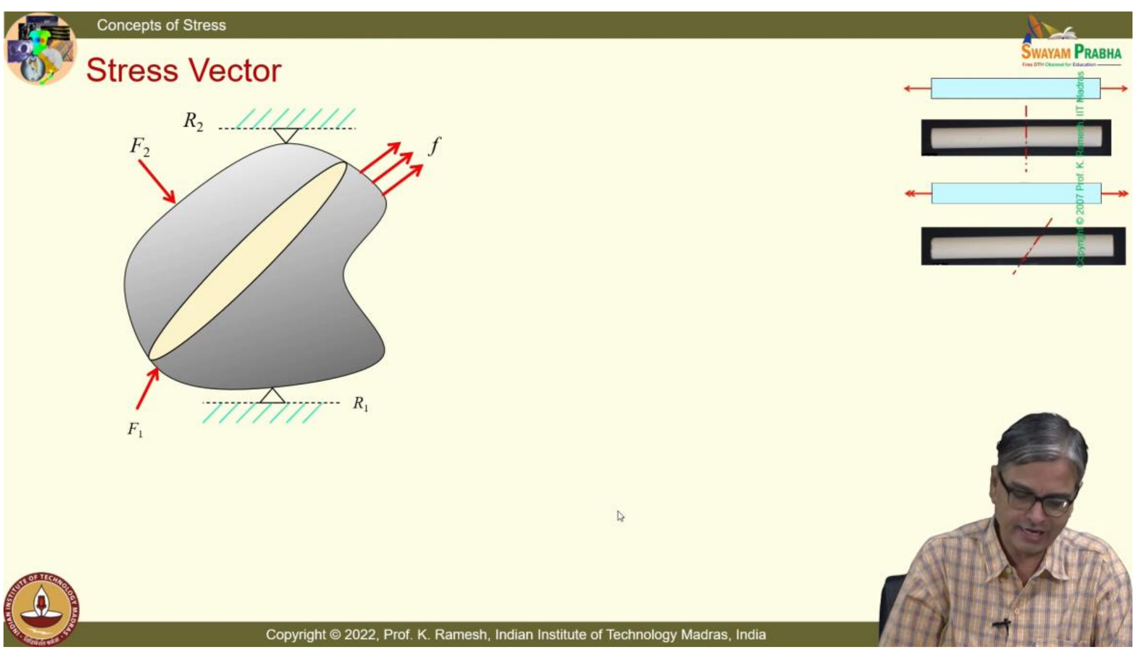

Now what I do is, I go to another plane passing through the same point, which is totally different.

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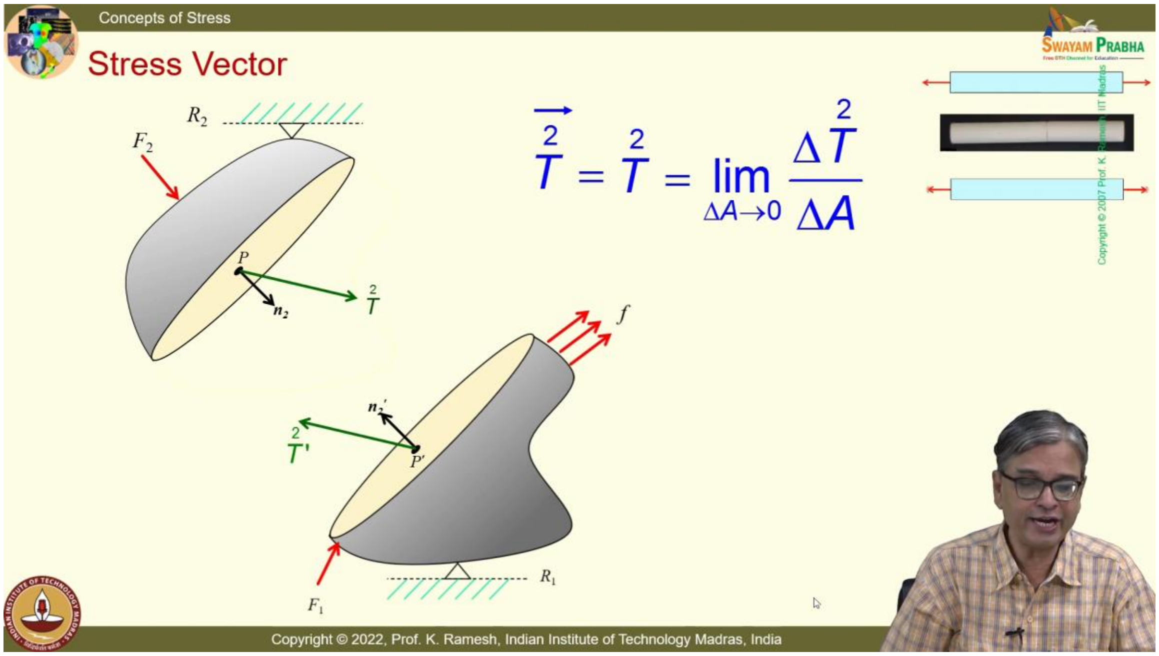

Here again, I can confine my attention to the point of interest. Take a small area and shrink that area to zero. For us to illustrate, let us call this as plane 2. So, I can define the quantity $\vec{\overset{2}{T}}$, fine? $\vec{\overset{2}{T}}$ is a vector. We simply write $\vec{\overset{2}{T}}$ as like this for our convenience.

We understand that it is $\lim_{\Delta A \to 0} \frac{\Delta \overset{2}{T}}{\Delta A}$. So, I have this, by Newton's third law, they are equal and opposite. And the body is in a state of equilibrium under the action of both body and surface forces. You know to avoid clumsiness in the drawing, the body forces are not explicitly shown, fine? Now we have to say what is the concept of state of stress at a point. See, our goal is, after you understand all this, you should be able to explain why the chalk when it is twisted fails at 45°, fine?

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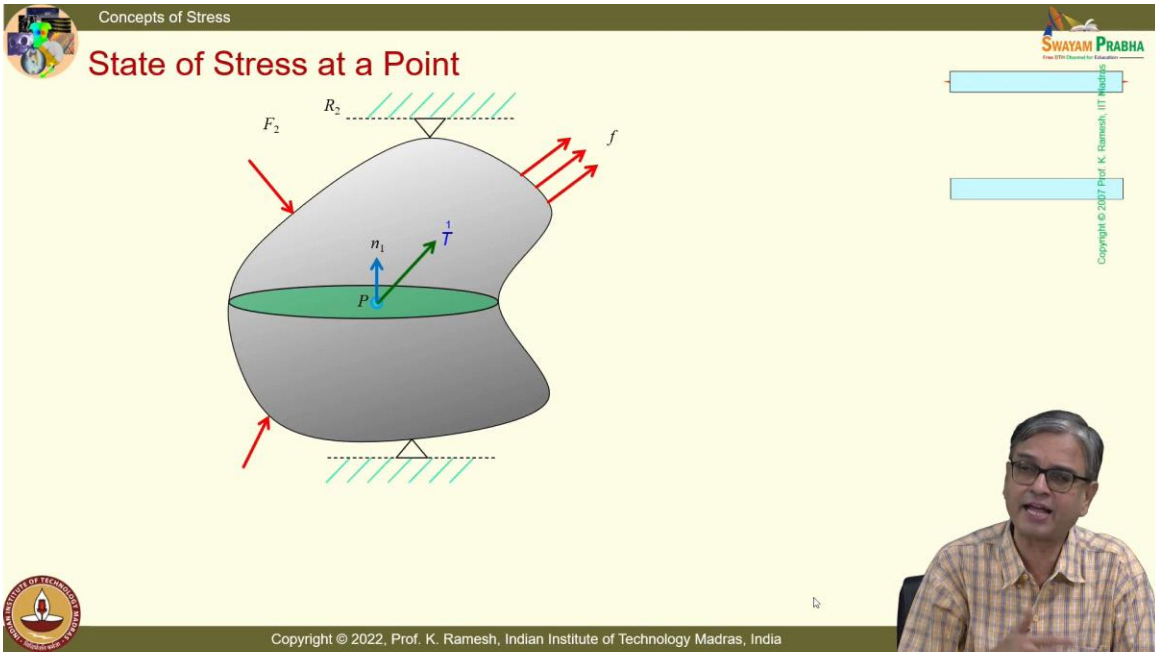

So, we want to find out stress vector on all the planes passing through the point of interest. When you say a point, how many planes can pass through? Infinite number of planes. So, I can have a point P. I can have a plane 1. Please make a sketch. I can have an outward normal. Plane is denoted by an outward normal. I can have a stress vector $\overset{1}{T}$.

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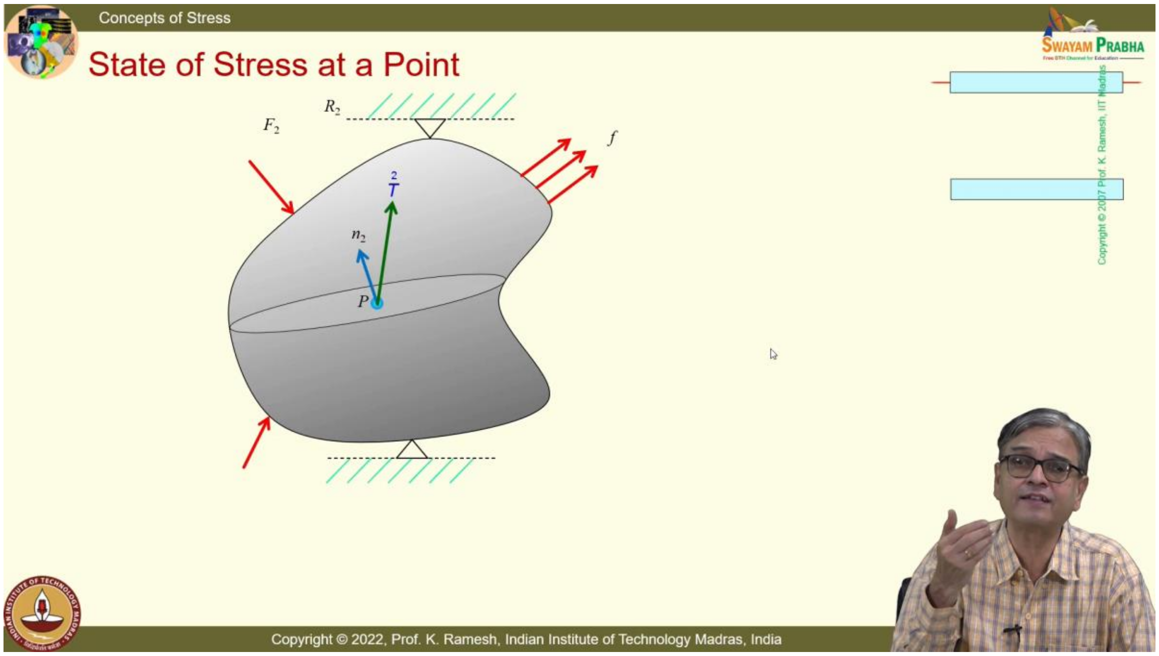

I can have another plane arbitrarily. And in all these cases, we shrink the area surrounding that point to zero and in the limit by borrowing the idea force divided by area, we define the resultant force divided by the area; in the limit, the area tends to zero. So, what we actually want is a mind-boggling quantity. I want stress vectors passing through infinite number of planes through the point of interest.

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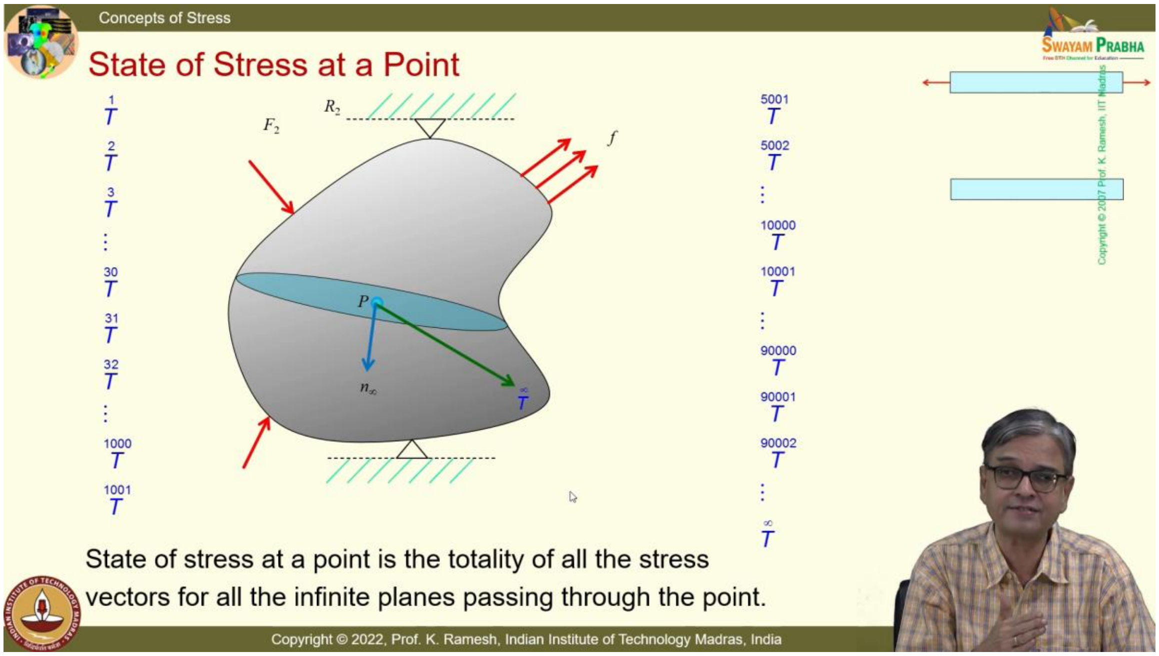

So, I can have n = 10000; I can have n maybe it is 50000 and 90000. I can have 100000 and then I have simply infinity. So will your notebook be sufficient to represent the quantity? It is not possible. So, it is not convenient at this stage for us to understand what is state of stress at a point unless there is a simplification possible. So, what we call as state of stress at a point is the totality of all the stress vectors for all the infinite planes passing through the point.

So, we want to have that. Only then, we can graduate and then analyze what happens in an arbitrary plane in a material; how the failure is precipitated, so on and so forth. So what you will have to appreciate is, from a cross-section, we have graduated to what happens at a point. At a point, we have a challenge that we have to look at all the infinite planes.

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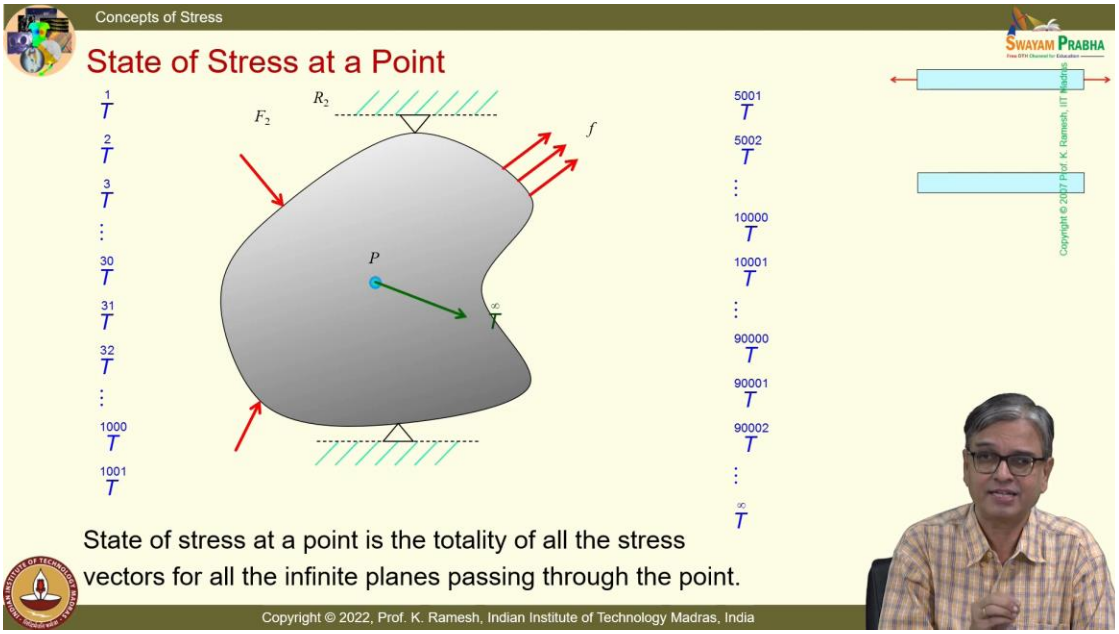

So, this can also be depicted in a different sense where I have the stress vectors $\overset{1}{T}$, $\overset{2}{T}$, $\overset{3}{T}$,… depicted like this and I have all these infinite quantities. So, the idea is to appreciate that we are looking at a problem which looks unsurmountable at this stage. But engineers do not leave it like that. You know you have a via media that is why we have developed the concept of state of stress at a point. Now what we will do is, we will select orthogonal planes that are three mutually orthogonal planes and look at what are the quantities.

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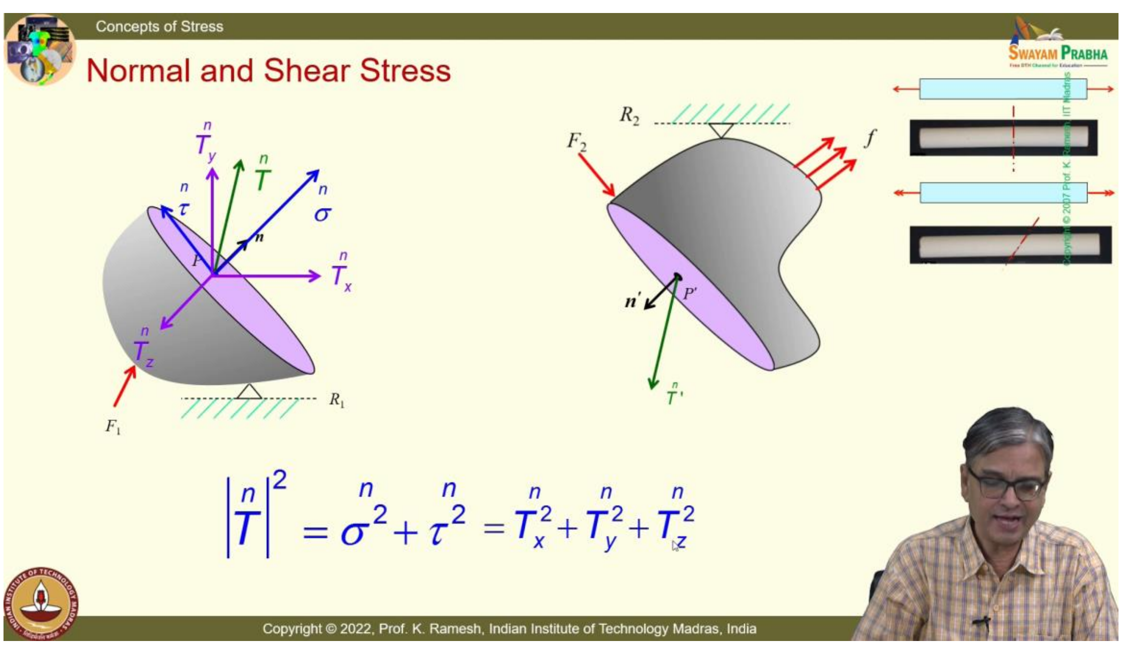

Even before that, we will also understand when I have an arbitrary plane, I can also look at what way the stress vector can be resolved. Now we know, whenever I take a plane, if I say plane is *n*, denoting it is an arbitrary plane and when I have a stress vector acting on that plane as $\overset{n}{T}$, I can resolve it along the normal and tangential to surface. This is one way of resolution. If I do that, I have a special symbol, I call this as $\overset{n}{\sigma}$. For the normal stress, we use the symbol σ and then for a tangential stress, we use the symbol τ. And when you look at these quantities, the components are always numbers. You understand that point? Components are always numbers; they are scalar quantities only. You know what is the direction along which you are writing it. So, direction is inherently embedded in a different manner.

So, I have these as quantities usually denoted by N/mm^2; you call it as MPa and so on. I can also resolve the vector $\overset{n}{T}$ in space into components along the *x*-axis, *y*-axis and *z*-axis. So, what you need to understand is the moment I put the suffix as *x*, *y* or *z*, understand we are not talking about a vector, we are talking about the component of the vector. If you understand the symbolism, writing is easier for you. So, when I have the magnitude of the stress vector, I can either understand this as consisting of the normal stress as well as the shear stress or the components along the *x*, *y* and *z* directions of the stress vector, fine? Now, what we will do is, we will try to find out what happens on three mutually perpendicular planes.

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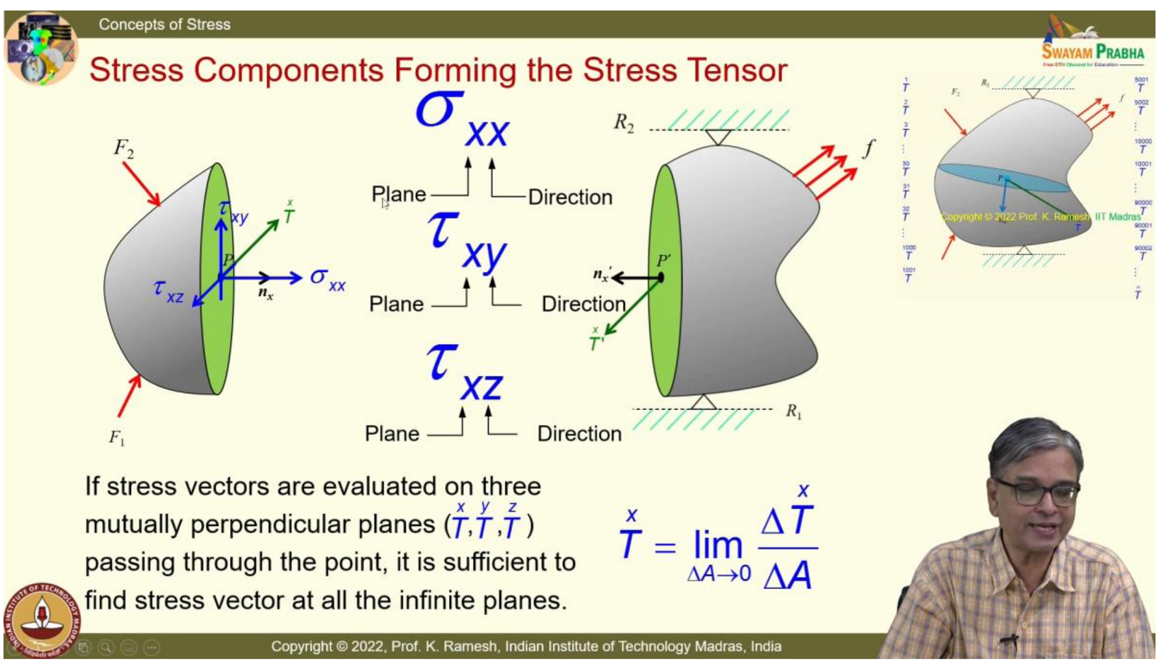

We have a choice and we will have the simple Cartesian coordinate system and then it is easier for you to have the first plane as along the *x*-direction, I call this as *x*-plane, fine? I have a point and then I denote it by an outward normal. And we have already developed on this plane, you can have a stress vector $\overset{x}{T}$ which is a vector. I can resolve this in a manner along the normal and tangential to the surface. You all know how $\overset{x}{T}$ is defined, $\overset{x}{T}$ is defined $\lim_{\Delta A \to 0} \frac{\Delta \overset{x}{T}}{\Delta A}$, borrowing the same idea, fine? There is nothing new here. So, I get this as a component along the *x*-direction, a component along the *y*-direction and a component along the *z*-direction.

And we have a way of naming that also. We have two subscripts, how these two subscripts are named? I have σ_{xx}; the first subscript denotes the plane on which the component is acting. So, it is acting on the *x*-plane and along the direction *x*, so I call this as σ_{xx}. So, that is how you have to interpret the symbol. We are talking about a component, the component will always be a number and usual units is MPa. And we have the component τ_{xy}. We denote this as τ indicating that this is acting tangential to the surface.

So, when I have the first subscript, it denotes the plane *x* and second subscript denotes direction *y*. See, I am deliberately going slow, because later, you do not need to remember, but you should be able to write on a particular plane, what are the components. And we have also utilized a sign convention. On a positive surface, positive direction is positive.

That is how these quantities are written. Now I have τ_{xz}; the first subscript denotes the plane and the second subscript denotes the direction, fine? Now the saving grace is, if stress vectors are evaluated on any of the three mutually perpendicular planes, you can find out stress vector on any arbitrary plane. You have to wait for the proof, fine? So that means when we saw that we have to get infinite number of quantities of stress vector, the problem was unsurmountable. But now, by this statement, what I need to find out is, what is $\overset{x}{T}$, $\overset{y}{T}$ and $\overset{z}{T}$. For the plane *x*, we have seen these components, fine?

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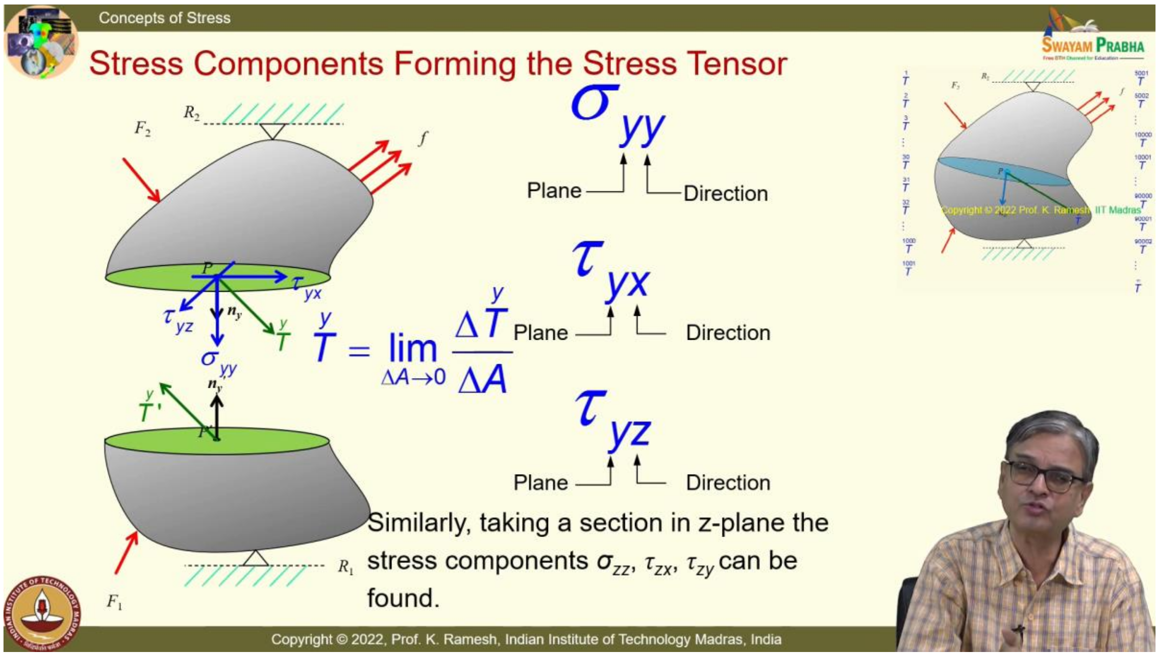

We will write it for one more plane and for the third plane, you try to write it yourself. So now, I take the *y*-plane. Again I take the same point; the point remains the same. The only thing is the plane which I am passing through is different. These are mutually perpendicular. Here again, because it is *y*-plane, I put this as $\overset{y}{T}$. And once I have this $\overset{y}{T}$, I can resolve this into components normal to the plane and tangential to the plane.

So these are depicted as the stress magnitudes. I call this as σ_{yy}. Now you can understand why I call this as σ_{yy}, why I call this as τ_{yx}, why I call this as τ_{yz}. So if you look at, I have σ_{yy}, it shows the plane and the direction. Similarly, I can write it for τ_{yx}. First subscript denotes the plane; second subscript denotes the direction.

Similarly, I have τ_{yz}. First subscript denotes the plane and second subscript denotes the direction. So on similar lines, if you take a plane passing through the *z*-plane, you can write the quantities what happens on the *z*-plane, fine?

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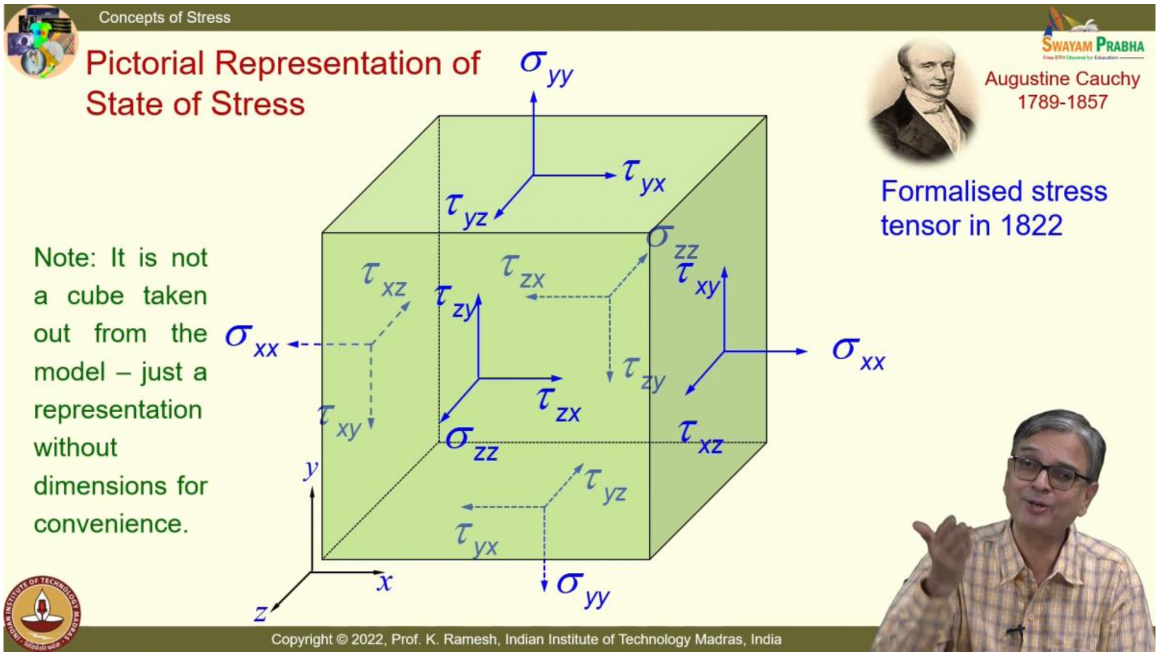

So, now what I am going to do is, I have taken a cube of size zero. This is for representation purpose, because when I look at a point, I can have a plane, positive plane along the x-direction, negative plane on the x-direction. Similarly, I can have a positive plane on the y-direction, negative plane on the y-direction. Similarly, for the z-direction. Now please write down, what are all the stress quantities on the positive plane of x. Please write down. Please write down on the positive plane, then check it from my slide. You should get it by looking at the reference axis, you should write it on your own, write it, make an attempt.

If you make a mistake, do not worry. Only by making mistakes, you will remember the concepts better. Because there is no charm in just copying down what I am popping up in the slide. Your mind should be involved in the development of the concept, fine? So what I have on the x-plane positive is, I have σ_{xx}, τ_{xy} and τ_{xz}. Now, we move on to the negative plane, write down the quantities. You should write down the quantities in the positive direction, that is the training for you.

And we have said on the negative plane, negative direction is positive. You should absorb that idea and then write down the quantities. So when I do that, σ_{xx} is pointing towards the negative direction, τ_{xy} is pointing towards the negative direction, τ_{xz} is pointing towards the negative direction. Is the idea clear? And this you should be able to do at any point in time without any difficulty. You should not remember this, but you should understand the symbolism. How do we denote the subscripts? First subscript denotes the plane on which it is acting, and second subscript denotes the direction, ok? And similarly, you can write it for the y-plane and you can also do it for the z-plane.

This, you should be able to do it without any difficulty. And you should keep in mind, it is not a cube taken from the model; only for a representation purpose, this is put. The length of each side is zero. It summarizes what happens at a point of interest. And the credit goes to Cauchy. And it is also very interesting; he developed this in 1822.

So we are celebrating the bicentennial; this is 2022. So what happened 200 years back, we are learning it today, fine? Ok.

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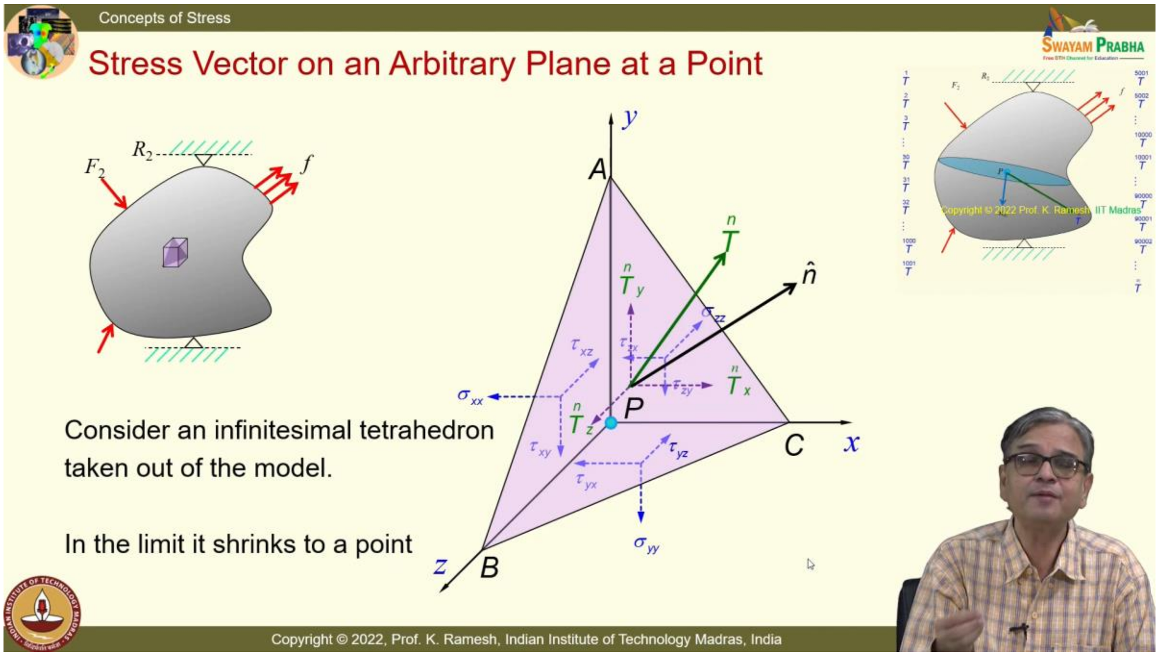

Now, what we have to do is, our goal is to find out if I know the stress vectors on three mutually perpendicular planes, how do I get the stress vector on an arbitrary plane? That is the problem posed before us. So what I do is, you consider an infinitesimal tetrahedron taken out of the model.

I have just drawn a cube. I will show the tetrahedral face. Now, I take it out from the model. When I take it out, from your understanding of free body, whatever the surface interaction, I should replace it by the force interaction there. Now you have learnt, when I have a plane, how to write the stress components. These stress components become forces when you multiply by the area.

Is the idea clear? So now we take out this tetrahedron and drawn very big. And mind you in the limit, this shrinks to a point P. That is why we want to say that is an elastic continuum. When I shrink it to zero, I have the material point still present, you have a finite quantity so on and so forth. So, now what you will have to do is, when I have the tetrahedron, I have all these are negative sides, negative x-plane, negative y-plane and negative z-plane when

I have the reference axis denoted like this. Can you write the stress components in those faces? Please write it down and then check it with my diagram.

You know the three-dimensional object is in a state of equilibrium under the action of both body and surface forces. When I write the final expression, I will bring in the body force also. To have the diagram simple and clear, we are not indicating the body force explicitly even at this stage. If you have written down what happens on the negative *x*-plane, first denote that this plane *ABC* is denoted by an outward normal $\hat{n}$ and we write the stress vector acting on this, $\overset{n}{T}$.

Our goal is to find out what is $\overset{n}{T}$, that is the goal. What we know from our earlier definition is, we will have normal stress components σ_{xx}, σ_{yy}, and σ_{zz} acting in these fashion and the respective shear components will act in these directions. You should be able to write it on your own. You do not have to see my picture and write it. Write it on your notebook and verify it with my picture.

So, what happens on *y*-plane? I have τ_{yz} like this, τ_{yx} like this. You should recognize that you are writing it on a negative plane. You should see whether it is a positive plane or a negative plane. And I have this for the *z*-plane. So, this is what is happening on the tetrahedron. In addition, you also have a body force. Now what we will do is, we will try to find out what are the components $\overset{n}{T}_x$, $\overset{n}{T}_y$ and $\overset{n}{T}_z$.

If I find out the component based on the equilibrium of this, whatever concepts that you learnt in rigid body mechanics, same thing you are going to do. Now you know, you have to find out the area and then multiply it appropriately to get the force. That is what we are going to do.

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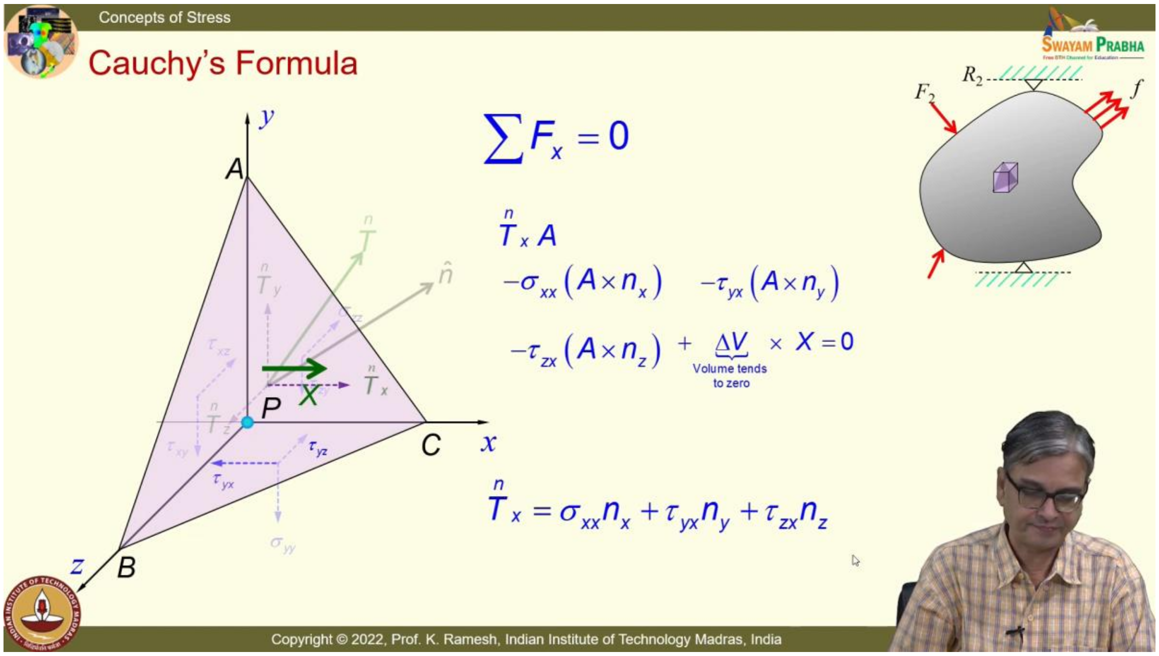

So, I have to do $\sum F_x = 0$ and let me represent the area of the triangle ABC as simply A, ok? And then, in order to denote that I am looking at that area, I also color it differently. So, I am looking at this area and if I have to find out what is the force, I have the component.

I have to find out the component $\overset{n}{T}_x$; it is not a vector. I multiply by the area of the triangle ABC. So, when I multiply the stress with the area, I get the force. I get the force in the x-direction. And what are the other quantities that you will have to find out? You have to find out what is the force acting on this surface, ok? And can you write this area as $A \times n_x$? Please note that this is a simple multiplication sign, it is not a cross product. A is used as a scalar, $A \times n_x$ will give you; n_x is the number; the component of the direction cosines. These are all numbers.

So, I have the force. When I write the equilibrium equation, I must recognize that this is pointing in the negative direction. The same thing you have learnt in the earlier course also. Engineering is one subject where conventions are very important and we have followed a sign convention. While writing the quantities on the negative face, negative direction is taken as positive and you write. But when you sum it up as $\sum F_x = 0$, you must look at the global picture and put the sign appropriately; do not make a mistake.

Once you have written for one quantity, rest of the quantities are very simple because we know, whichever is ending with the symbol x is what we are going to write. So, I have τ_{yx} and this is related to the area of this plane, that is nothing but projection of the plane ABC, which can be easily written as $A \times n_y$ and you have this as $A \times n_z$. And you also have, now I bring in the body force, because you have to understand that we have written down the equilibrium equation under the action of both body and surface forces, fine? So, I have the volume ΔV that tends to zero. And this is again a multiplication sign and I put the body force as X; it is force per unit volume. So, force per unit volume multiplied by volume will give you the force. You can divide by A and when you do a simplification, I get a very nice expression $\overset{n}{T}_x$, the component of the stress vector in the x-direction is

$$\overset{n}{T}_x = \sigma_{xx} n_x + \tau_{yx} n_y + \tau_{zx} n_z$$

It is as simple and as straightforward from this. The moment I write for x-direction, I can follow the same thing for directions y and z, I can do that?

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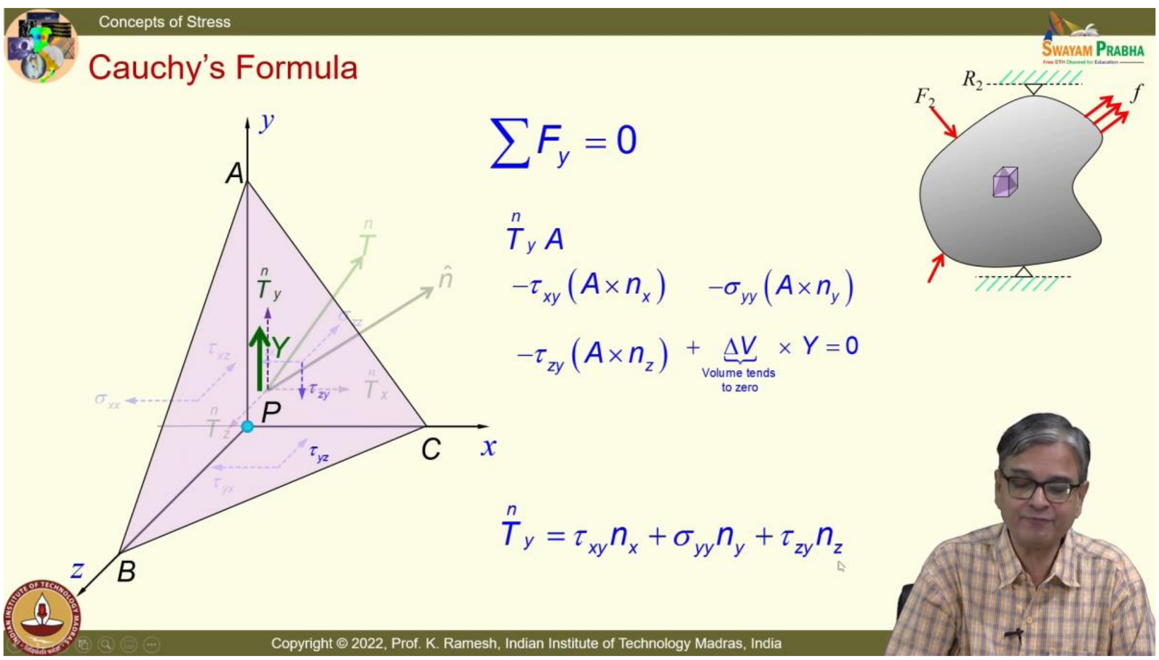

So, I go to $\sum F_y = 0$, I do the same thing. Now I have to find out what is the force in the y-direction. I have this as the component $\overset{n}{T}_y$ multiplied by A, then whatever the quantities that have the second subscript as y.

So, I have looked at the negative x-plane, then negative y-plane, then negative z-plane. So, if I accommodate all these quantities, in addition, I also write the body force that is depicted as Y and I get this equal to zero. Is the idea clear? So, your training on how to write the components on the faces should come naturally to you. We have understood and explained the symbolism. If you have understood that, then it is easy to write.

First symbol denotes the plane and second symbol denotes the direction. And when I simplify, I get

$$\overset{n}{T}_y = \tau_{xy} n_x + \sigma_{yy} n_y + \tau_{zy} n_z$$

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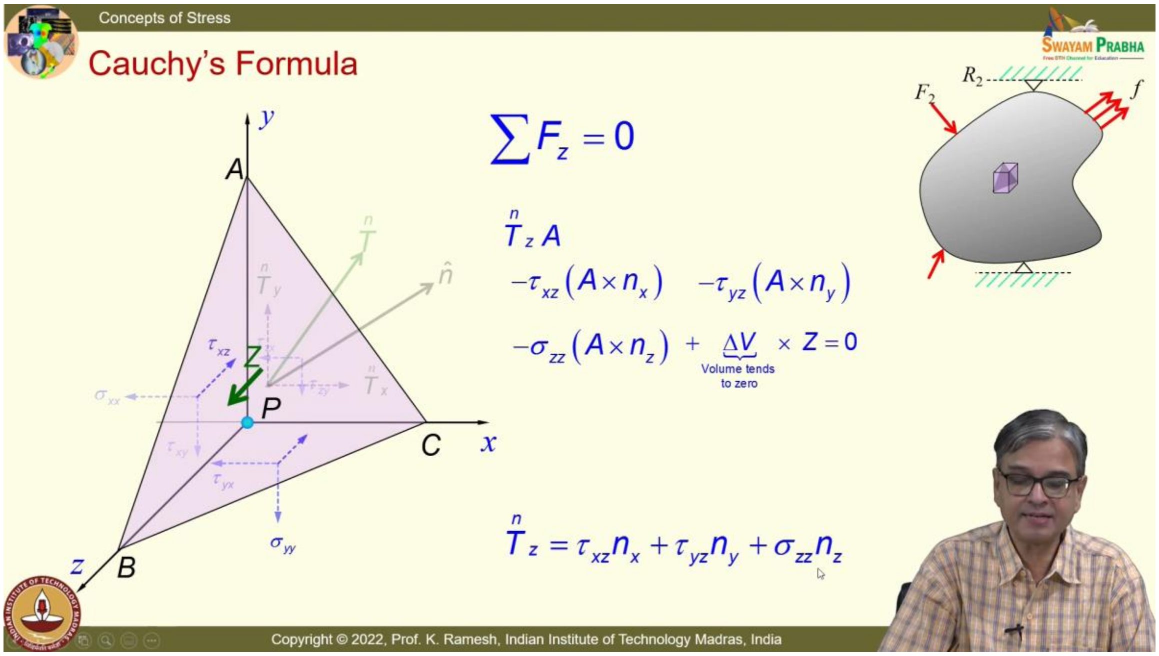

Now, I repeat the same thing for the *z*-plane. You can also make an attempt and then verify what I have written on the screen, fine? Write it down. I should see people writing it, only then, I will get convinced that you are making an attempt. So, it is a similar process, I get $\overset{n}{T}_z$ and then multiplied by the area *A*.

And I have this, whatever the quantities which have the direction *z*. Look at the negative *x*-plane; look at the negative *y*-plane; write the force; look at the negative *z*-plane; write down the force. And also bring in the body force that is put as *Z*. And when I simplify this, in the limit, the tetrahedron reduces to zero. I get an expression for $\overset{n}{T}_z$;

$$\overset{n}{T}_z = \tau_{xz} n_x + \tau_{yz} n_y + \sigma_{zz} n_z$$

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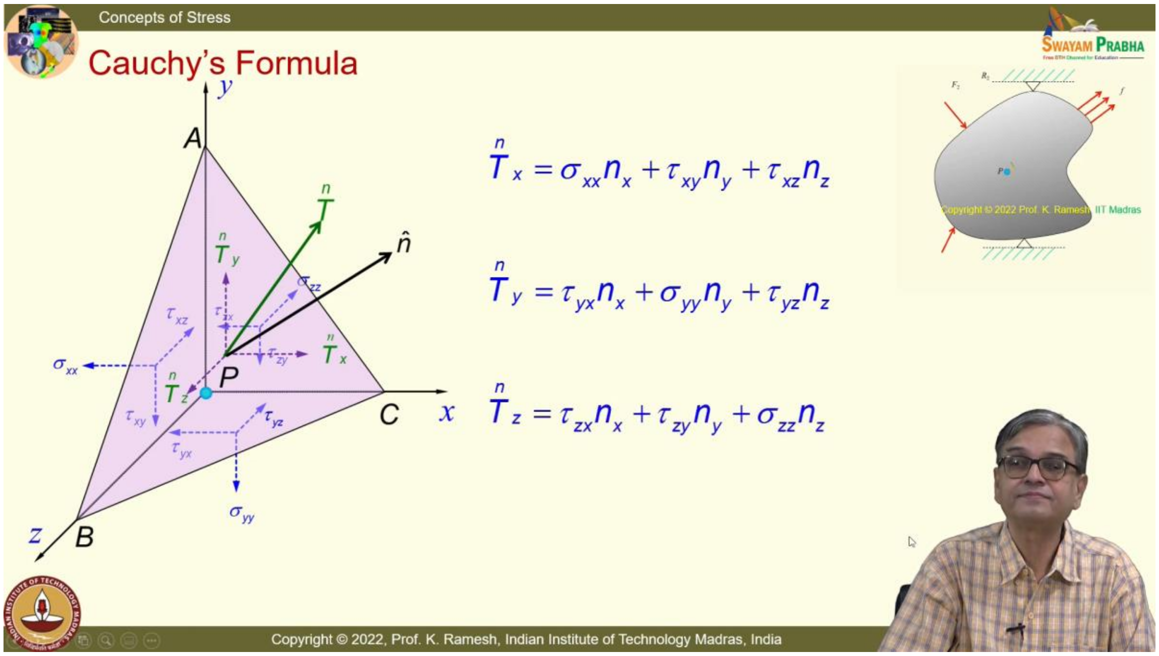

So, what is it that we have achieved? By looking at three mutually perpendicular planes, we have been able to get what is $\overset{n}{T}_x$, we have been able to get what is $\overset{n}{T}_y$, and what is $\overset{n}{T}_z$. Is the idea clear? We said we should graduate from a cross-section to a point. We borrowed the idea while looking at the experiments involving three different cross-sections. People coin a new terminology '*stress*' which is nothing but force divided by the cross-sectional area. From there, we graduated to how to handle a point. When we wanted to handle a point, we felt that there are infinite planes that can pass through it. And if I want to say what is state of stress at a point, totality of all these stress vectors on all the infinite planes is what I need. Because my idea is which way the specimen is going to fail; *a priori* we do not know.

We have to investigate. We have to investigate depending on the material, there will be a criteria and we will have to find out whether those quantities exceed in a particular plane. So, I need to have all the infinite planes. The problem was looking unsurmountable. Then we also made a statement, if I know the stress vectors acting on any three mutually perpendicular planes passing through the point of interest, I can find out the stress vector on any arbitrary plane. That is what we have achieved now.

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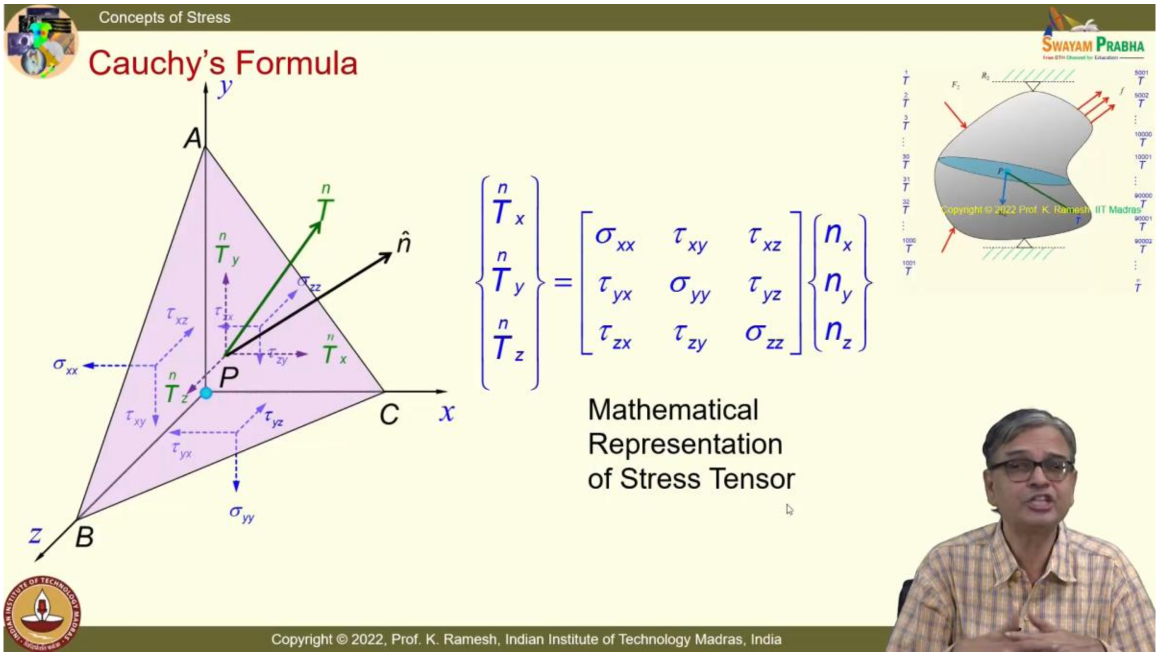

I can also rewrite it in a different fashion. I can put this as a notation like this; I have to get the vector $\overset{n}{T}_x$, $\overset{n}{T}_y$ and $\overset{n}{T}_z$. And you have a quantity which we have determined in all the three mutually perpendicular planes. They are put as a matrix. I have σ_{xx}, τ_{xy}, τ_{xz}, τ_{yx}, σ_{yy}, τ_{yz}, τ_{zx}, τ_{zy}, σ_{zz}.

This quantity is a mathematical representation and it is a tensor of rank 2, mathematically. It is much more than a simple vector. So, initially we saw P/A was looking like a scalar. I said it is no longer a scalar when we develop the concepts. We initially thought that as a vector; we called this as a stress vector.

But we asked, what happens on all the possible infinite planes. For that, we found a via media; if I know the stress vector on any three mutually perpendicular planes, I can find out the stress vector on any arbitrary plane mainly because I have the direction cosines. So, if I specify the plane of my interest, and if I know what is this for a given point, I can find out stress vector on any of the arbitrary planes. One to infinity, no problem. Is the idea clear?

So, in this lecture, we have developed the concept of state of stress at a point. We started with a simple understanding of P/A. Then we said, by looking at the experience on breaking a chalk, we have to find out what happens on arbitrary planes. Then we also found in a generic situation, stress varies from point to point. You cannot hold on to the cross-section only as a basis to do it. We have to graduate to what happens at a point of interest. Then we developed a new mathematical entity, where we defined in a region very close to the point of interest $\Delta T/\Delta A$, in the limit ΔA tends to zero remains finite.

You are able to do that because it is an elastic continuum and we defined that quantity. This is also known as *traction* in advanced studies, fine? And from the concept of stress vector, we graduated to how to find out stress vector on any of the arbitrary planes, if you know stress vector on any three mutually perpendicular planes. That quantity resulted in a matrix, we said that is nothing but a tensor of rank 2. Thank you.