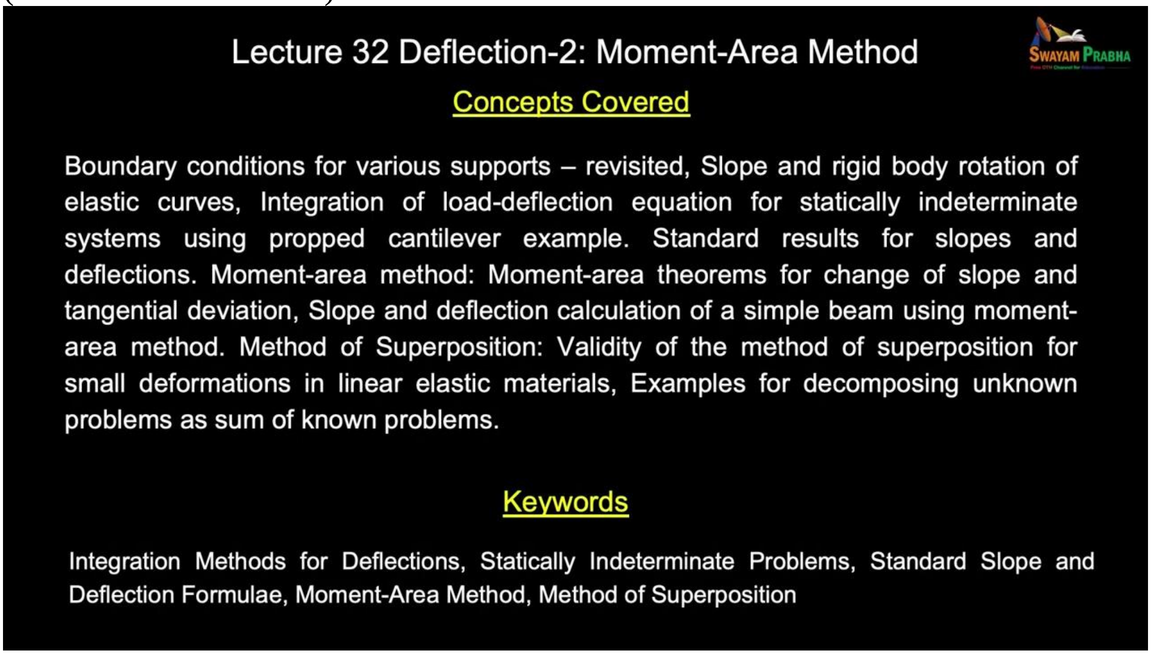
Strength of Materials
Prof. K. Ramesh
Department of Applied Mechanics
Indian Institute of Technology, Madras

Lecture - 32
Deflection 2 Moment-Area Method

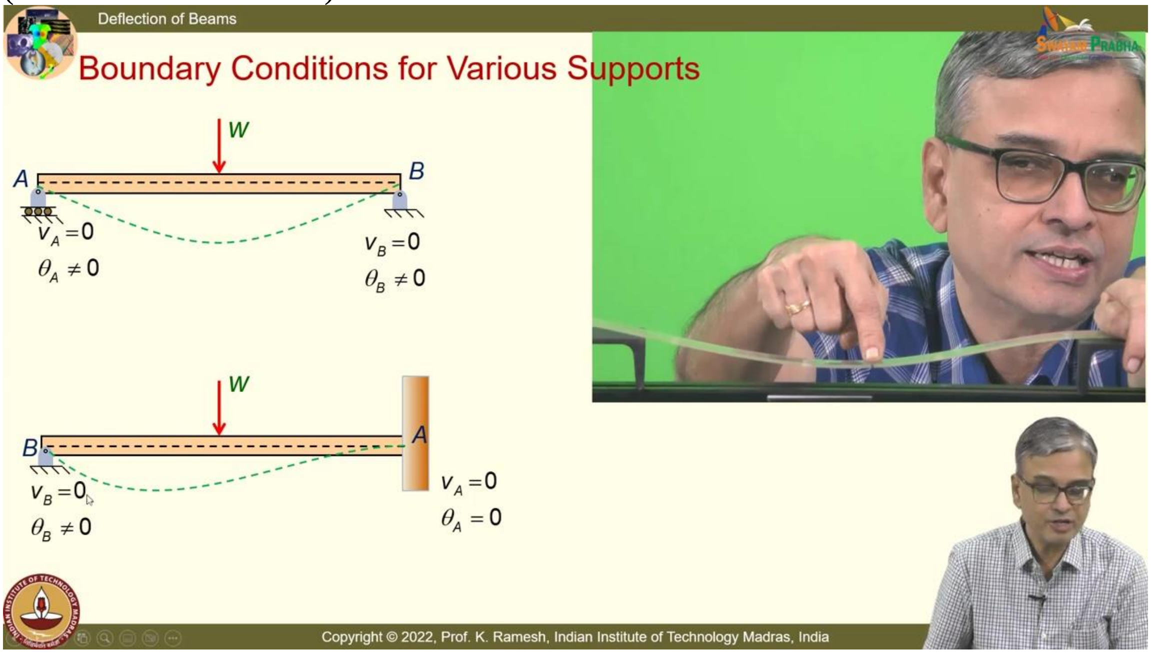
(Refer Slide Time: 00:20)



Let us continue our discussion on deflection of beams. In fact, we have looked at double integration method to find out the deflection. You can find out the deflection curve, you can also find out the slope. And you know, if you have the equation written, if I have multiple loads, it becomes difficult to do by double integration. People also have developed singularity integrals, which we are not going to look into it, because it becomes a mathematical exercise. If you are able to write one single equation for the entire length of the beam, the equation is valid and we have followed a sign convention, you get the slope and deflection as per the sign convention without any difficulty.

The same thing applies also for using the load deflection and then do the integration. And I said that load deflection equation which I have given is also valid for statically indeterminate beams.

(Refer Slide Time: 01:31)



And in deflection, what you need to understand is, how the beam can deflect depending on the support conditions. When I have a simply supported beam, you can have free rotation at the supports.

So, it will flex like this. When I go and clamp it, you have this slope as zero, whereas this is allowed to have its own rotation. So, rotation will exist here, which is also pictorially shown here. I have a simply supported beam. And you know, we have also solved this by double integration.

I have taken an equation in this segment. So, whatever the result I get is valid for this segment. For this segment, I have to take another equation and then do it because of symmetry, we have simply used the result. So, you should understand the difference. So, when I go to a cantilever beam which is fixed; see you should be sufficiently comfortable in drawing this deflection curve for known loadings like what we have seen in the class, but also for unknown loadings.

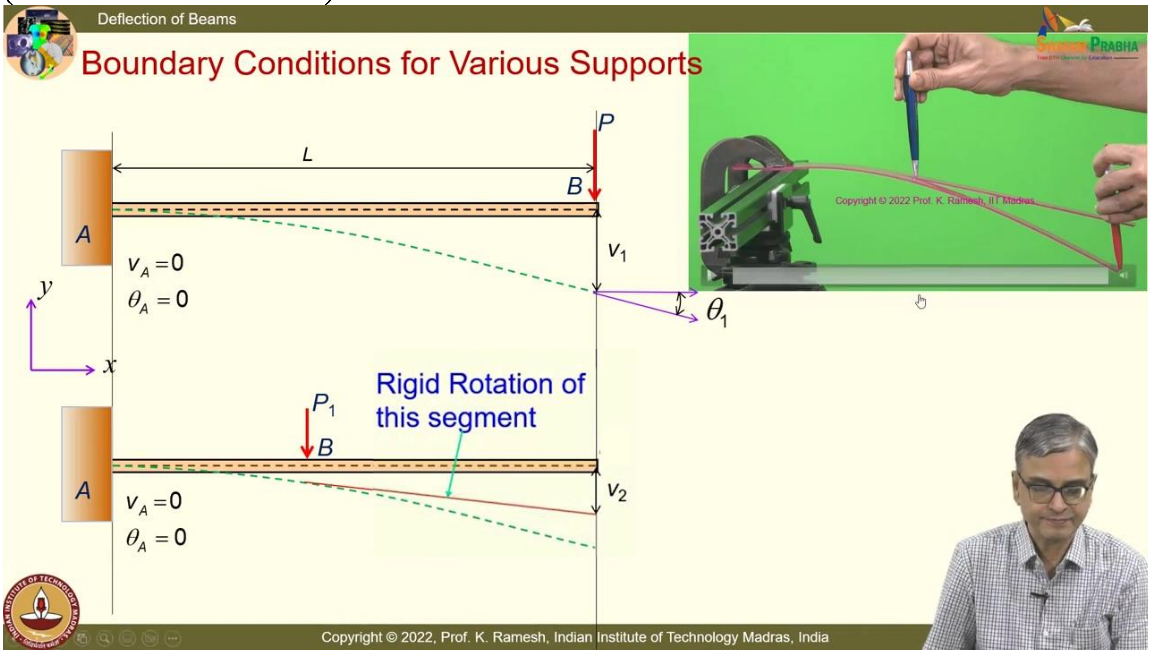
The issue here is how do you handle the support? What happens in a support? If I have a fixed support, I can have displacement; that is the deflection is zero as well as the slope is zero. And we have also seen slope is nothing but the rotation at that point in the beam. And you know, you also have the boundary condition which you can use

$$v_A = 0$$
$$\theta_A = 0$$

$$v_B = 0$$

And you should understand that θ_B need not be zero because that support will allow the rotation.

(Refer Slide Time: 03:31)



And when we go for the cantilever, I have a beautiful flexing like this. And I am also showing a cantilever loaded in between. So, what is shown here is up to that load, the deflection is similar to the other curve. After that, the entire length of the beam has only one rotation. Whatever is the slope at the load application point, the same slope is carried out. There is no further rotation of the beam which is beautifully captured in this experiment which I am also going to show in the diagram.

So, when I put the end load, I have a deflection like this. And at this point, you also have a slope. Both of these need to be determined because I have an interest. Later on, I want to do principle of superposition where I am going to have problem 1 and 2. So, keeping that in mind, it is put as v_1 and θ_1.

And the boundary condition is

$$v_A = 0$$
$$\theta_A = 0$$

Now, I do not have the load applied here, but I apply the load at some other point. So, the idea here is, when I apply the load, this load is not necessarily the same load as P, but some other load P 1. And what you have is; I have the deflection curve which is in green. And what you have as a continuation as a red line is that; like you have a slope here, here also you have a deflection plus a slope.

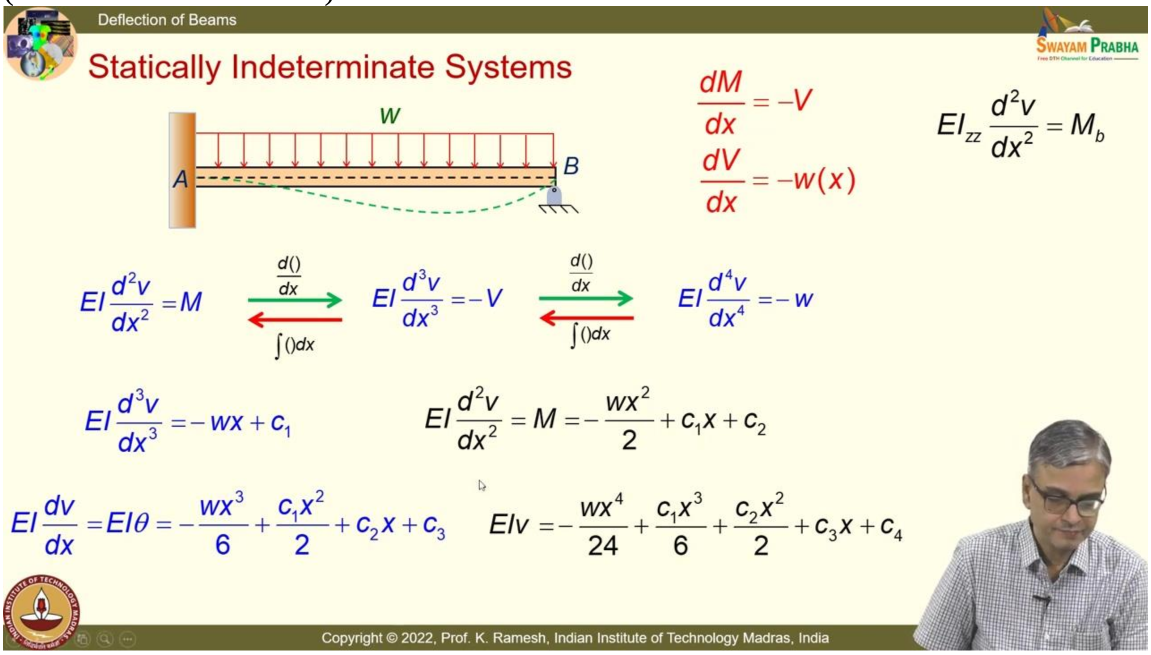
The same slope is indicated in the rest of the beam. The rest of the beam rotates with that slope. There is no further rotation on it. This rotates like a rigid body. And when I consider this as a rigid body, later on when we use the energy method, when you say rigid, it cannot store energy.

Only when you say deformable, it can store energy. So, you can have very interesting problems when you go to the energy where you should recognize whether this segment where it is not loaded it has a rigid body rotation. And that is what is indicated here. And here again you have the clamped end condition,

$$v_A = 0$$
$$\theta_A = 0$$

So, what is the purpose of all these experimental demonstrations is to help you to visualize; and it depends only on the support. If you know what way you have to handle the support, identify what way it has to be, then rest of them join by a smooth curve.

(Refer Slide Time: 06:36)



And I said for statically indeterminate system, if I have the load deflection curve, it is much easier for us to do. I have this as

$$EI\frac{d^2v}{dx^2} = M$$

I have

$$\frac{dM}{dx} = -V$$
$$\frac{dV}{dx} = -w(x)$$

So, when I substitute this, that is when I differentiate this and when I use this identity, I get this as

$$EI\frac{d^3v}{dx^3} = -V$$

And when I substitute this, you know I would essentially get this as *w* without a minus sign, but in this problem, *w* is in a negative direction. So, I have this as

$$EI\frac{d^4v}{dx^4} = -w$$

That applies to this problem when *w* is in the negative direction. This expression is derived with *w* as positive. There is a sign convention that we have used. See, this is a statically indeterminate problem because you do not know how to find out this reaction. From the principles of statics, you will not be in a position to do it. If you are not in a position to do it, then you cannot write what is the bending moment. So, I cannot use this equation directly. You can still do that by some other; if you find out the reaction and then do it, you can always employ that.

Right now, you do not have this, but with this, I can directly do the integration and find out by substituting the appropriate boundary condition, it ends *A* and *B*. I can find out the deflection curve as well as the slope. It is simple and straight forward. There is no great mathematics involved. So, when I integrate, I get the other equation.

And this is written for this problem. The final expression is applicable for this problem, but the generic expression is

$$EI\frac{d^4v}{dx^4} = w$$

You have to investigate whether it is positive or negative and put the appropriate sign. So, when I integrate it once, I get this as

$$EI\frac{d^3v}{dx^3} = -wx + c_1$$

And you know what is the boundary condition at *A* and what is the boundary condition at *B*.

So, I have $EI\dfrac{d^2v}{dx^2}$. When I integrate again, I get this as bending moment that is equal to

$$EI\frac{d^2v}{dx^2} = M = -\frac{wx^2}{2} + c_1x + c_2$$

And if I integrate it further, I get this as θ. See, we have been using the rotation as ϕ in our derivation, basic derivation. Books also use multiple symbols.

It is better that you get exposed to those symbolism. And instead of using ϕ, they have used θ. So, you can also recognize the slope as θ. I get this as

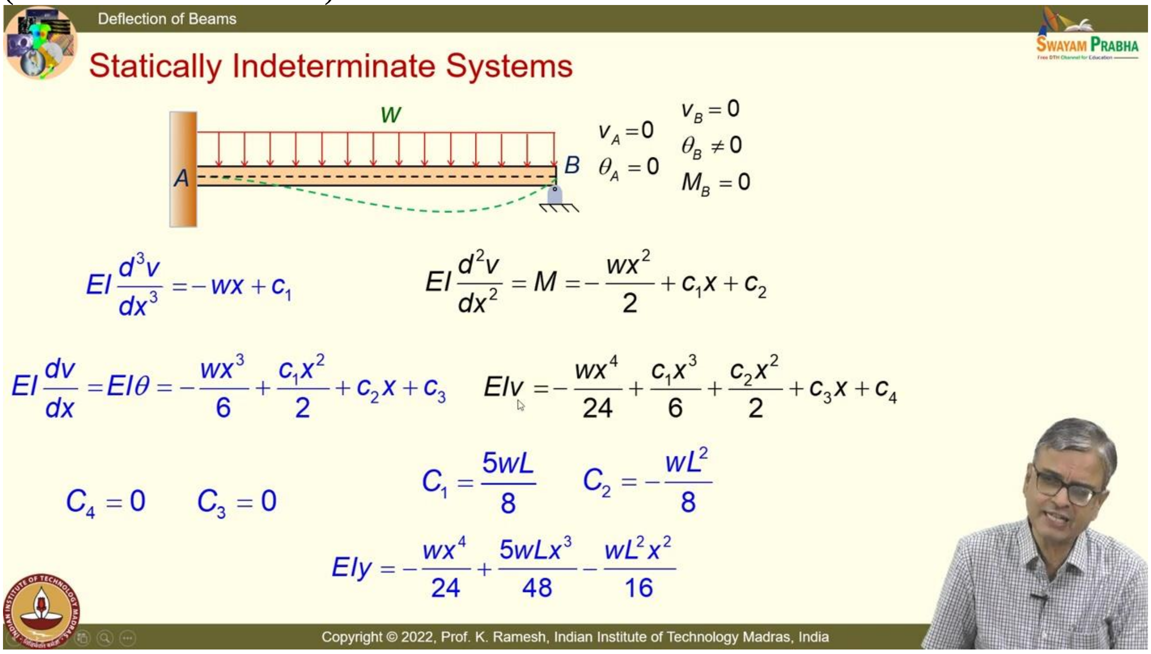
$$EI\frac{dv}{dx} = EI\theta = -\frac{wx^3}{6} + \frac{c_1x^2}{2} + c_2x + c_3$$

And I have

$$EIv = -\frac{wx^4}{24} + \frac{c_1x^3}{6} + \frac{c_2x^2}{2} + c_3x + c_4$$

So, how many constants do I have to find out? I have to find out four constants. So, I should have four boundary conditions for me to determine that, fine. Let us see what are the boundary conditions that we can apply. Have you been able to write this? Ok.

(Refer Slide Time: 10:29)



I have the same expressions given here. And then we have a convenient boundary condition

$$v_A = 0$$
$$\theta_A = 0$$

at the fixed end. That you should be able to recognize; that is why we have done the experiment to show that slope is indeed zero. Deflection zero, you will be able to visualize, but slope being zero, you have to specially understand when you have a fixed end, slope is also zero. So, this gives me $C_4 = 0$ and $C_3 = 0$. These two boundary conditions help me to knock off C_4 and C_3.

And at this end, at the end B, $v_B = 0$. That is very clear. And $\theta_B \neq 0$. You can have rotation because the support allows the rotation. So, now you go to the force, a pin joint cannot support a moment.

So, the moment has to be zero at that point. So, I have four boundary conditions that can be used. Theta not equal to; $\theta_B \neq 0$ is not a useful boundary condition. So, I have enough conditions and which you have to do the simplification. Please take your time at home and then evaluate

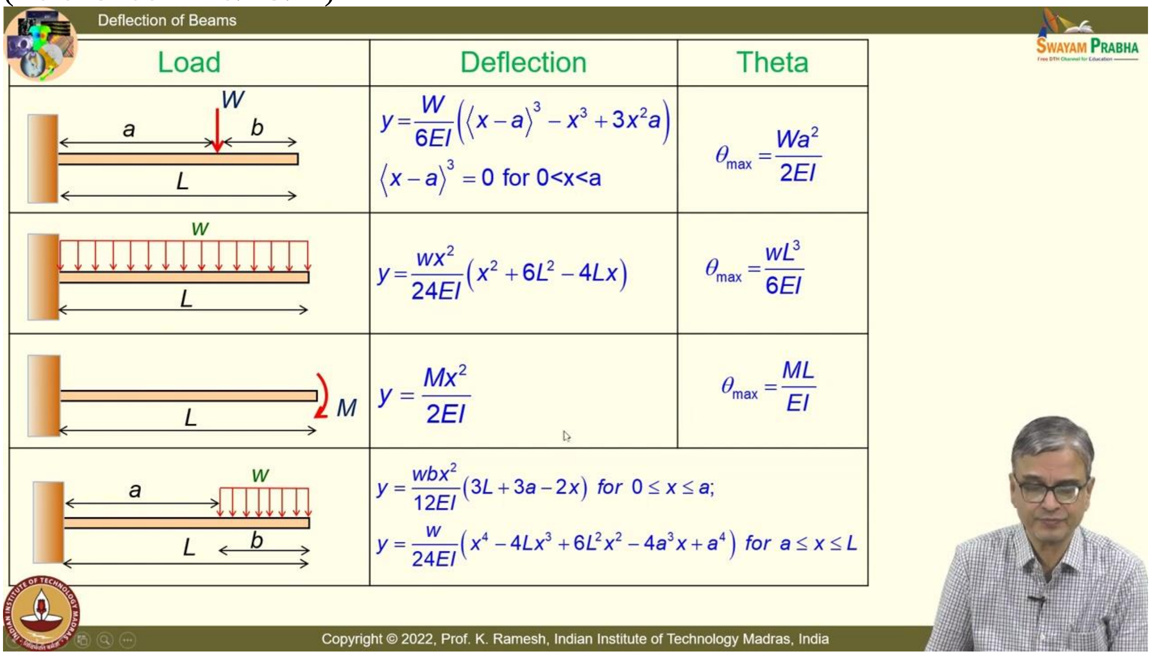
$$C_1 = \frac{5wL}{8} \qquad C_2 = -\frac{wL^2}{8}$$

See, I have also given you a homework where you have a uniformly distributed load and this end was simply supported, fine. I have the same basic expression; by changing the boundary condition, you can solve all of those problems. All those problems are solved in one shot. Is the idea clear? Suppose I do not have a support here, that problem also can be solved from this by applying the boundary conditions. So, this is lot more useful and I have this.

See, this change of symbolism is used repeatedly because some books use this deflection as simply as *y*. So, I have written this as

$$EIy = -\frac{wx^4}{24} + \frac{5wLx^3}{48} - \frac{wL^2x^2}{16}$$

(Refer Slide Time: 13:27)



And you can also find out; you know it is better that when we want to go and do the principle of superposition by direct integration, double integration or quadruple integration, I can find out for a variety of cases. And usually this is available in a table; that is how engineers operate. In the field, they do not have time to do the integration or derivation, they simply use the available expressions.

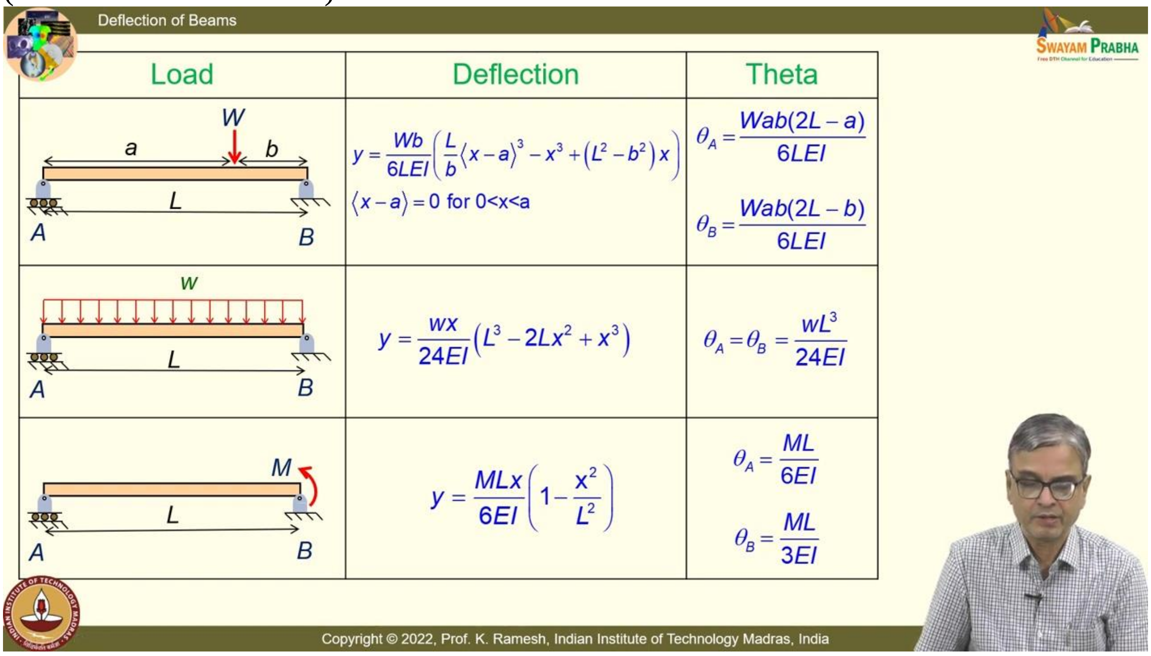
So, in this it is put only as; deflection as put as *y*, it is used as *y* as well as *v*. So, understand that symbolism and you may take a photograph of this slide once it is complete. And I have these expressions given for deflection and you have for theta. So, instead of ϕ, it is all put as θ, *y* and θ is put in this table. So, I can have a bending moment applied, this

$y = \frac{ML^2}{2EI}$ is what you are getting and $\theta_{max} = \frac{ML}{EI}$.

And I have a part of the beam is having a uniformly distributed load. And you have the interpretation, see you should know how to interpret this equation, that is also given here. When I put a bracket like this, this is how singularity integrals are used. So, this is defined as equal to zero for $x < a$.

So, that is easy for you to interpret. So, I can find out use this for you to construct principle of superposition, fine. You can take a snapshot of this. And this table would be provided in your examination when you have to use principle of superposition, you do not have to remember.

(Refer Slide Time: 15:00)



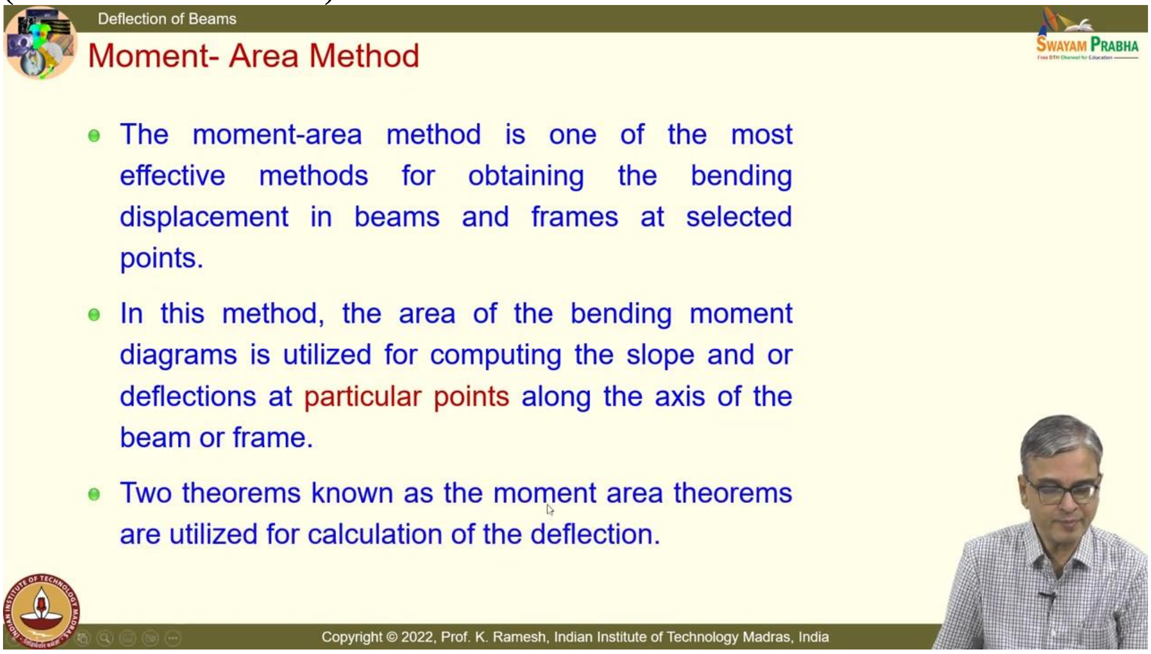
Focus your energy on understanding the subject. So, I have this three-point bend beam where the bend; where the load is applied at an offset point and you have it uniformly distributed, this is what you should have got.

You have this and anyone has calculated what is the deflection, maximum deflection? That is again a very famous expression normally we remember. It will be

$$y_{\max} = \frac{5wL^4}{384EI}$$

It is again a very famous relationship. So, you have this table also you can take a photograph.

(Refer Slide Time: 16:04)

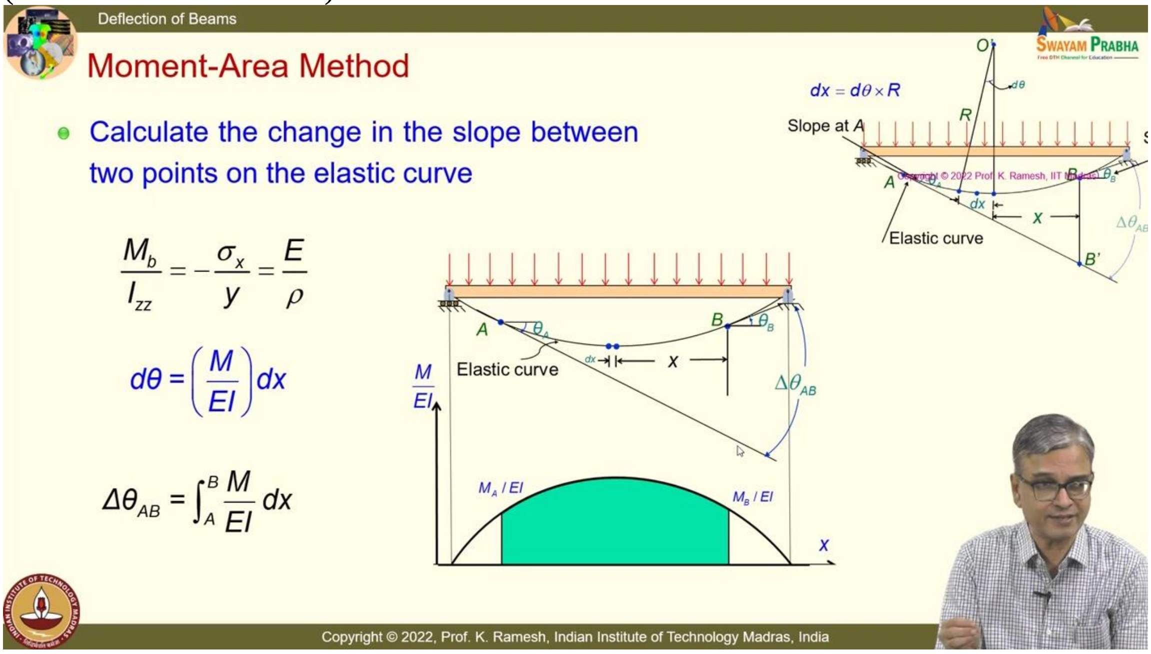


Now, we move on to graphical integration. I said that graphical methods are very widely used in the early development of engineering and physical sciences. And it is one of the most effective methods for obtaining the bending displacement in beams and frames at selected points. That is a key information. In this method, the area of the bending moment diagram is utilized for computing the slope and or deflections at particular points along the axis of the beam or frame. So, you have to find out; the name itself says moment area.

Area is nothing but integration. That is why it is; instead of doing a physical integration, mathematical integration, we do it in a graphical sense. And when I go in a graphical sense, only if I am in a position to find out the area which becomes simpler, the bending moment diagram becomes simpler when I have concentrated load. When I have distributed load, I am going to have second degree curve or third degree curve depending on what kind of load variation I have. So, we would also solve, while developing the theory, we will take a distributed load, but while solving we will apply it for concentrated load.

You have to know that this is also another method. That would be the focus in our development. And you have two theorems. They are known as moment area theorems which are utilized for calculation of the deflection.

(Refer Slide Time: 17:45)



And here again, you know, people use instead of rho, they also use the symbol *R* as a radius of curvature. And what you have here is, on the deflected beam, I have a length *dx* that can be expressed as $Rd\theta$.

That is what I am going to use. And the first theorem is like this - Calculate the change in the slope between two points on the elastic curve. So, whenever you have a problem which has a zero slope at some point, it is very easy to solve a moment area method. If you do not have a zero slope, it is difficult. So, we will also take up such simple problems to appreciate another method for calculating deflection.

And you have the flexure formula. I have written it in terms of ρ and the same ρ is used as *R* in several books and instead of ϕ, people use θ. So, there is mixture of symbols. It is better that you know that these are available. And what you have is, I have an elastic curve which is how it is deflected and I will have, because I have a distributed load, I will have a bending moment diagram like this. And on this elastic curve, I have selected point *A* and draw a tangent to that.

So, that gives the slope at point *A* and take another point *B* and draw a tangent. Please draw your diagrams in this sequence, then you will appreciate the method. What I can at best get from this is, I can find out the change between this. I can find out $\Delta\theta_{AB}$, I can find out. How do I do that? I take a small distance *x* and if I want to see from point *B*, I measure *x* and then I have a small distance Δx.

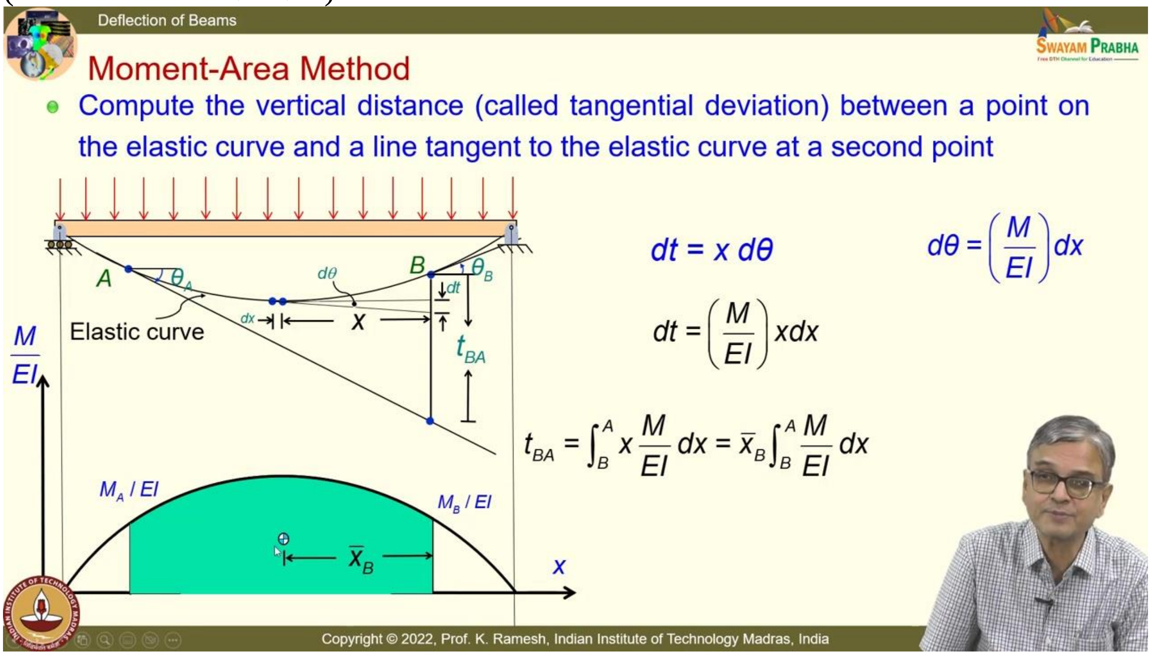
And we know from this diagram, Δx or $dx = Rd\theta$. And from this, you know, you know the ρ is nothing but *R* there. So, I can write this

$\rho = \frac{M}{EI}$ and I have said, you know, to simplify your writing and you know, you are working with axis x along the length of the beam, z is perpendicular to that. You drop off these two symbols to make your life simple in writing.

So, I have $d\theta = \frac{M}{EI}dx$. So, when I integrate, if I want to have $\Delta\theta_{AB}$, when I integrate from A to B, I get this as $\frac{M}{EI}dx$. So, this integration is nothing but the area between the two points. What it gives? It gives the change in the slope between two points on the elastic curve. It gives only the change. So, if I choose the point where the slope is zero, I get the absolute slope at the other point.

That is how you use the method. So, it can at best give, if we find out the area between the points of interest, that area can be interpreted as what is the change in the slope between the two points. See, when we had the direct integration method, we had a sign convention and you got the final expression and you substitute the values, you get all the quantities as per the original sign convention without any difficulty and mathematics is perfectly correct. But once you come to a method like this, you have only suggestive sign conventions and it is better you draw the elastic curve, calculate the quantities as modulus and then finally assign the sign. That would be a better strategy in handling a technique like moment area method. That would be my recommendation. But we will also see what is the sign convention that is being talked about.

(Refer Slide Time: 21:56)



And you can also calculate the tangential deviation. It is a very important aspect. I cannot find out the deflection, I can find out only the tangential deviation between a point on the elastic curve and the line tangent to the elastic curve at a second point. So, it cannot find out deflection directly, I can find out only a tangential deviation.

So, you should understand what is the tangential deviation and I have an animation. So, follow the animation and then draw the diagram. So, you will understand what is meant by this tangential deviation. And you draw this M/EI diagram; just a scaling up of the bending moment diagram. So, you have a bending moment diagram like this and I also plot the elastic curve, fine.

And you know I take a point *A* and I have what is the value of M/EI at that point; that is noted here. And I draw the tangent and I have a point *B* and at point *B*, I know what is M_B/EI. And you know, I have also drawn the slope, but what is meant here is, the tangential deviation is between a point on the elastic curve, the point *B* is on the elastic curve and a line tangent to the elastic curve at a second point. The second point you can call it as *A*. So, this is what you can find out and this is known as a tangential deviation.

And what you have is, suppose I have two points which are taken at a distance of Δx or *dx*, I draw the tangent at these two points, I will also find out what happens in this. This is $d\theta$, this is at a distance *x*. So, I can write this distance as *dt* and *dt* you can write it in terms of *x* and $d\theta$, is the idea clear? So, I can write *dt* as $xd\theta$ and you know how to write $d\theta$, we have seen in earlier discussion as $\frac{M}{EI}dx$. A very circuitous way of finding out what it is, but application is very simple. Application is, in one case you just find out the area between the two points, in another case you find out the first moment of area.

If you find out these quantities comfortably, if your mathematics is simple, then this is the method for you to use. Suppose I have the bending moment diagram as a non-linear curve and finding out the area under the curve is so difficult, then this method is not useful. And do you appreciate what is the meaning of a tangential deviation? And this is given as

$$dt = \left(\frac{M}{EI}\right)xdx$$

And you know if I want to do it for the entire length, if I want to find out what is this entire length, then I have to do the, I have to vary *x* from point *A* to point *B*.

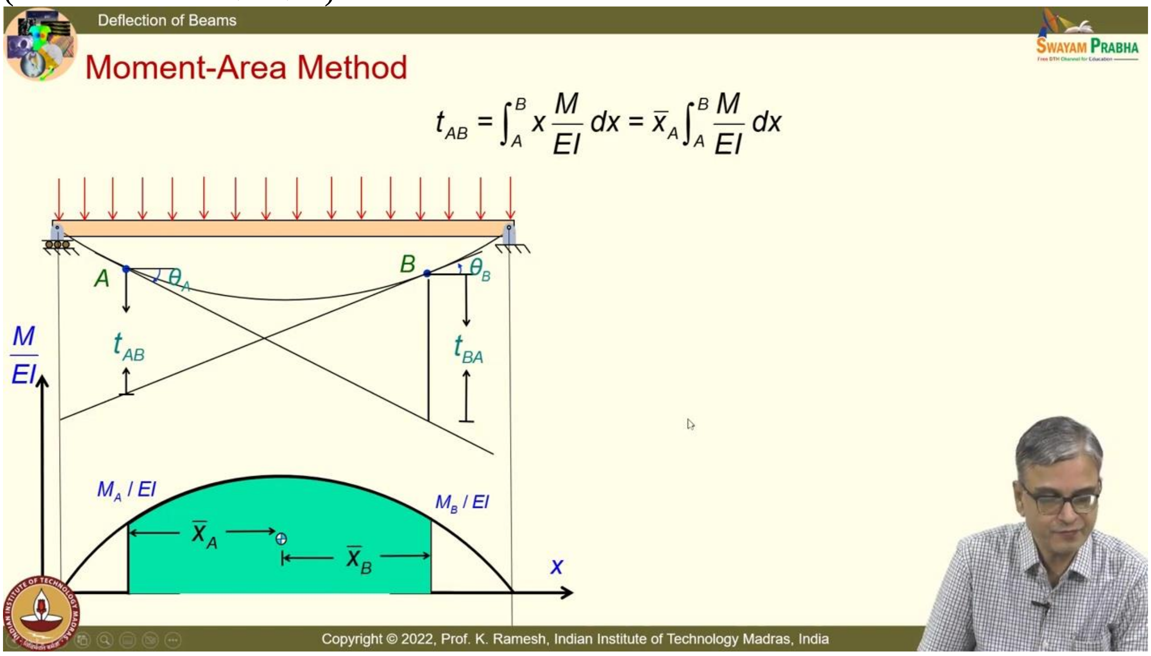
That means, I have to integrate over this, ok. And this you call it as t_{BA}. So, you should also understand what is the symbolism? *B* is the point on the elastic curve, *A* is the point from which you have drawn the tangent. So, t_{BA} only finds out; only this tangential deviation; it is not the deflection at that point. You should use your mathematical skills and geometry to find out the deflection. And when I want to do this, I have to do the integration.

I have to go from *B* to *A*. This is again very important. When you calculate the area, it does not matter. Whether you do it from *A* to *B* or *B* to *A*, it does not matter. But when you have to find out this quantity, because I am multiplying by the quantity *x*, from where I do the multiplication is very important. I go from *B* to *A* and I have already said *B* is the point on the elastic curve. And from *A*, we have drawn the tangent and we have defined the tangential deviation as t_{BA}.

The first symbol denotes the point on the elastic curve, second symbol denotes the point on which you have drawn the slope. And this you know; I can take outside and then say this is nothing but $\bar{x}_B$ into area under the curve. So, I have shown this as $\bar{x}_B$. Is the idea clear? So, by looking at the bending moment diagram, if you calculate the area of the bending moment diagram between two points of interest, I can find out the slope. I can also find out the tangential deviation, if I have the first moment of the area.

Here I have shown how to do it from *B* to *A*. Suppose I want to do it from *A* to *B*, what I should do? I should extend this slope and then I should find out what is the deviation from point *A* to this slope. For that, what will you do? You will take the distance from this side for you to do the first moment of area. We will see that in the next slide.

(Refer Slide Time: 28:20)



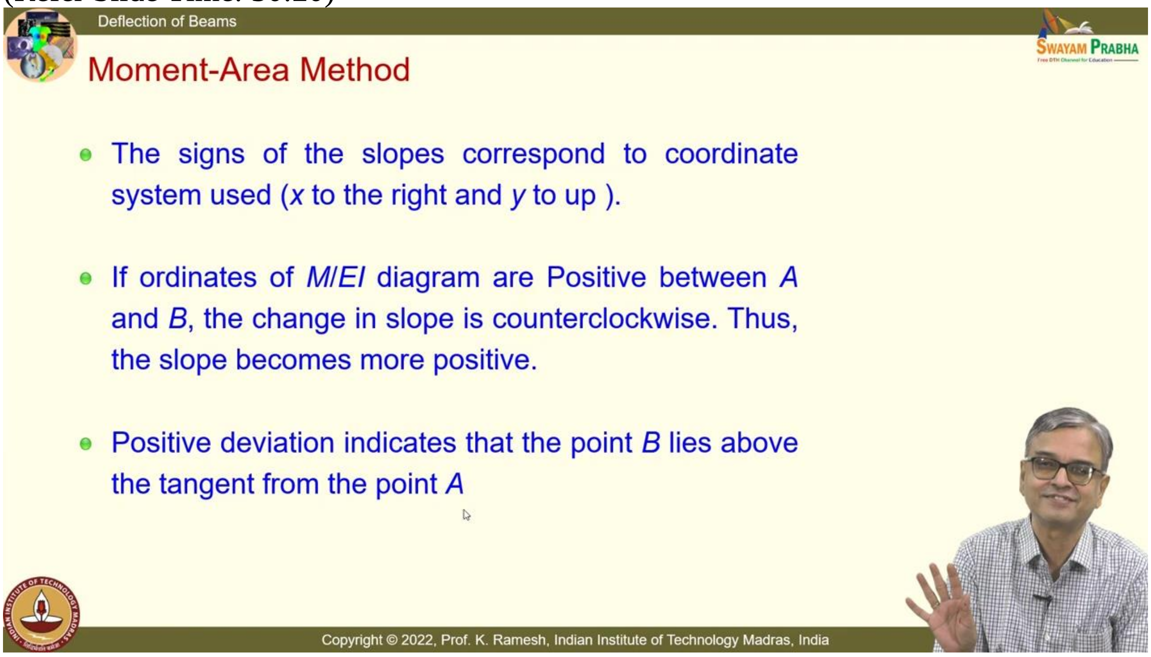
So, I have the loading diagram, I have the M/EI curve, elastic curve, I have point *A* and then I have drawn the slope there.

At point *B*, I draw the slope and I define this as t_{AB}. Please draw the sketch because the sequence I have followed will give you clarity on how to interpret the tangential deviation. The first symbol shows that this is the point on which the elastic curve I am looking at and second symbol is the one where I have drawn the tangent to the elastic curve. So, I can, at best, find out only this tangential deviation. I cannot find out the deflection. And then I go to the area and this is nothing but you calculate the area and then have this distance for your first moment of area.

So, if I have the point located, either I can find out t_{AB} or t_{BA}. That is all you can get from your bending moment diagram. That is, I find out the area between the two points and find out the bending moment; I mean first moment of area of that. Either I can select from one point or I can select from the other point. There you should interpret, what is the deviation that you are calculating it? And I have already said that this is t_{BA}.

For that, you have to use $\bar{x}_B$. If you understand this, then the whole method application is very simple and straight forward.

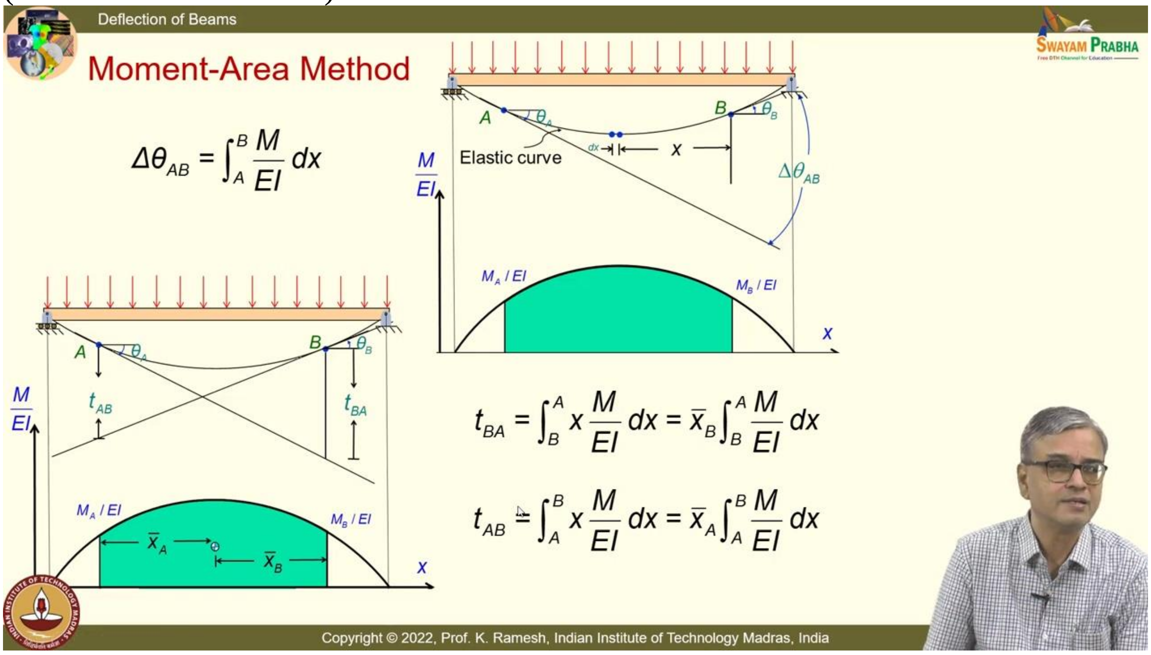
(Refer Slide Time: 30:20)



You have all the points has come up in one shot. The signs of the slopes correspond to coordinate system used; *x* to the right and *y* to up. If ordinates of M/EI diagram are positive between *A* and *B*, the change in slope is counterclockwise.

Thus, the slope becomes more positive, very very difficult to understand. When I solve a problem, I will show how it is. I will also give a via media, how you should handle it when you solve a problem. Positive deviation indicates that the point *B* lies above the tangent from the point *A*. That is all you can say. It cannot say whether it is a deflection downwards, deflection upwards, all that it cannot say. It only says when there is a positive deviation, you have the slope is below the point *B*. That is all.

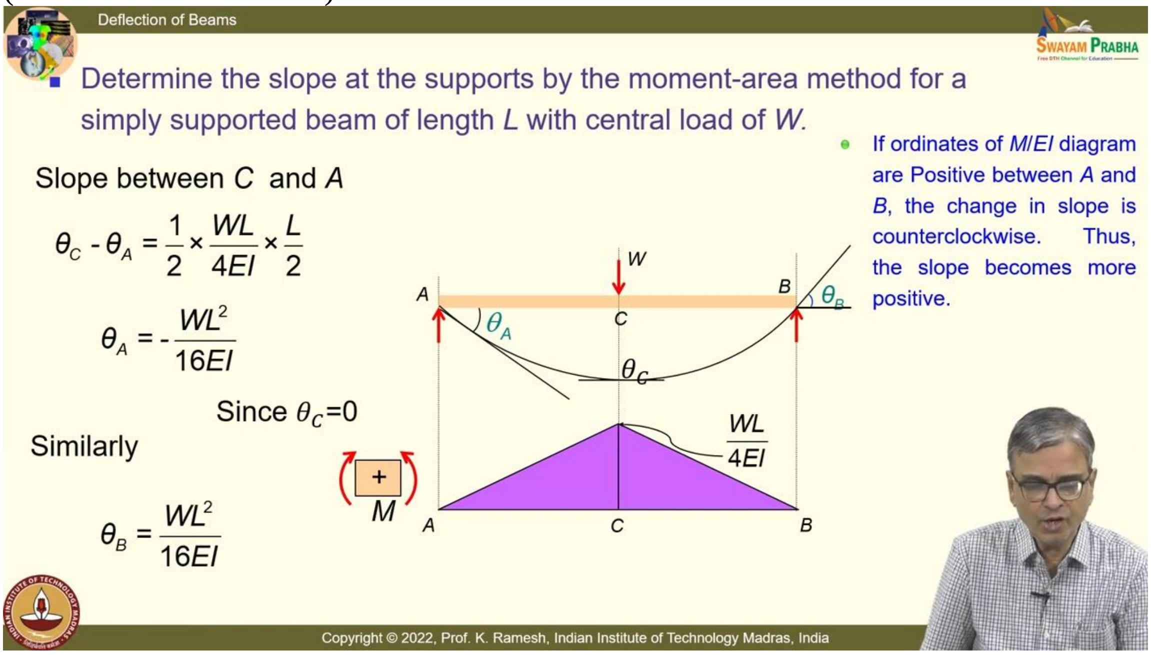
(Refer Slide Time: 31:15)



And this is a sort of a summary. I have θ_{AB} is the area under the curve. And I have t_{AB} and t_{BA} is nothing but first moment of area from this distance $\bar{x}_A$ and first moment of area from this distance. Area of the curve between the two remains same, but the moment arm changes and I get either t_{AB} or t_{BA} .

You can understand this easily when you solve a simple problem. We will take a problem with a concentrated slope.

(Refer Slide Time: 31:49)



So, I will take a three-point bent specimen. This we know the answer already, isn't it? What is the deflection at the load application point? What is the maximum deflection at the load application point? We have solved it I think last class.

$$\frac{WL^3}{48EI}$$

So, we have to get the same thing by this method also. So, I have θ_A here and then θ_B here. And I have deliberately taken a problem which is easy to handle because I have a zero slope because of symmetry, which you should appreciate from the way we have done the experiments, we have shown how the beam deflects. And moment area method is convenient to handle when I have a point on which you have zero slope. You have a bend; you have a deflection curve which has a zero slope is easy to handle.

And you need to have only the M/EI diagram, fine. And I have also labeled *ACB*; the same points here. It is nothing but the bending moment diagram. You scale it up, scale it down; I have this value as $\frac{WL}{4EI}$. You have the product *EI* coming down.

So, that is how you do the scaling, essentially a bending moment diagram. So, I know the essential quantities, fine. So, I have to find out what is the slope here? and what is the deflection here? That is the focus. How do I go and find out? I can find out this slope based on my moment area method because I have; if I take the point of interest as *A* and *C*; because the slope at *C* is zero, in one shot I can get what is the absolute slope at θ_A because the area gives only difference in slope.

It does not give absolute value. Because I have taken one point as having zero slope, I can. So, the idea is what points you should select for you to start solving the problem. That is where you have to apply your mind. I have from moment area; I have to get the area of this triangle. So, you have the base; height is given as $\frac{WL}{4EI}$ and then $\frac{1}{2}bh$ gives me the area. That gives me $\theta_C - \theta_A$; and you have

$$\theta_A = -\frac{WL^2}{16EI}$$

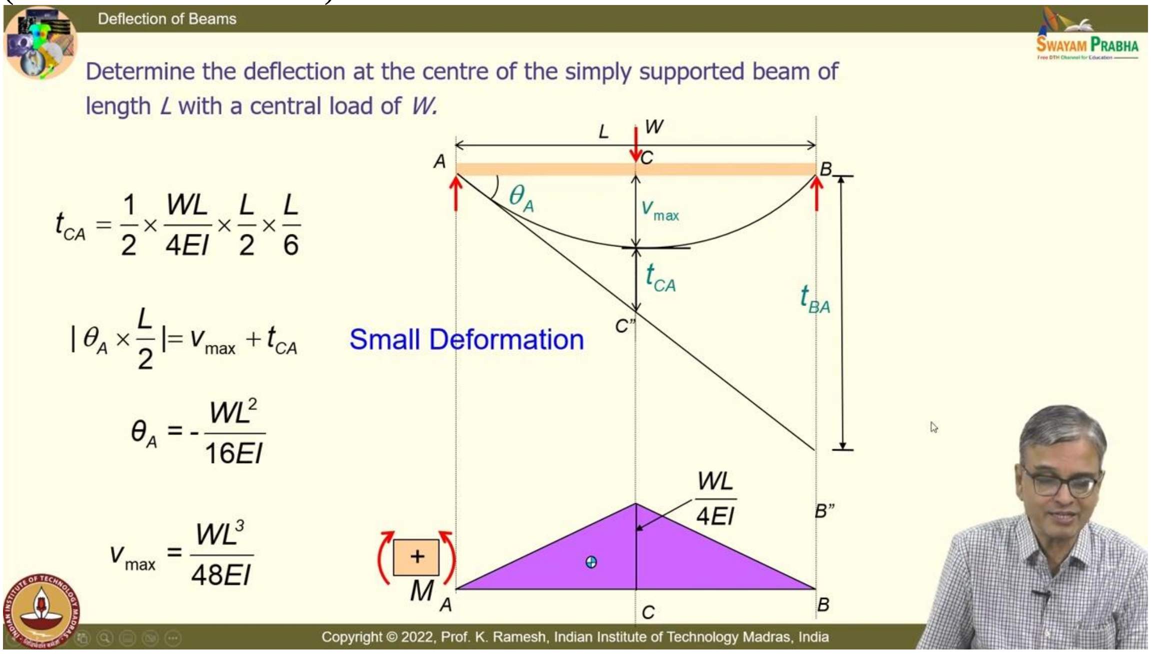
and so fortunate that you have also got the θ_A with the correct sign in the first shot itself, fine.

It is an incidental advantage, that is what I would say. Since $\theta_C = 0$, I get this no problem. Similarly, I can also find out θ_B; $\frac{WL^2}{16EI}$. I have put this as positive sign. See, if you look at this bending moment diagram which is scaled by *EI*, this is completely positive with the sign convention that we have used. And what is the sign convention which we have said? If ordinates of M/EI diagram are positive between *A* and *B*, the change in slope is counterclockwise.

Thus, the slope becomes more positive. That is all the theorem says, which I can give an interpretation here. See, I have a slope as negative. This negative slope has become positive here and this positive slope has become more positive here. That is how you interpret what is the sign of this M/EI diagram.

But it is very difficult to handle this way. My suggestion would be, you just calculate the area and then you draw the elastic curve. You get all these; because you know area has no sign. So, assign the sign based on the physics of the problem. That is the simplest way to do it. Is idea clear?

(Refer Slide Time: 36:26)



Now, let us see how do we go and find out the deflection at this point. Again, I have to do a circus. I have already determined θ_A. So, I will use that to advantage. I have this diagram here; M/EI diagram.

I have written it far away so that you get the slopes drawn decently. I draw the slope at point *A*. I also know θ_A right now. And if I use the second theorem and *C* is a very advantageous point because I have the slope as zero, I can find out only this distance. Do you recognize that? But my interest is to find out this distance. So, I have to bring in; I have to bring in all my approximations I have done.

And I have repeatedly been saying that whenever I do this, I exaggerate. So, when I write the expression, you will not get convinced whether the expressions are correct. So, I can get only t_{CA} from my moment area method. For that, I need to know what is the centroid of this cross-section.

It is very simple. You have this $h/3$ is what you have from the base. So, I can find out t_{CA} comfortably.

$$t_{CA} = \frac{1}{2} \times \frac{WL}{4EI} \times \frac{L}{2} \times \frac{L}{6}$$

Is the idea clear? That I can easily find out. That is not at all a problem. And we have already said t_{CA} means, the point on the elastic curve is *C*, I draw the tangent at *A*.

I get this distance t_{CA}. So, I have to take the moment from point *C*. So, that is why I have this as $L/6$. If you do that differently, then you are in for a surprise. Do not do that the other way. And I have to find out $v_{\max}$. How do I find out this? And you should recognize that we are working on small deformation.

Do not get duped by the diagram I have drawn. Imagine that this angle is very very small; less than 0.5 degrees. If it is less than 0.5 degrees, what is the way that I can write this? Can I write this

$$|\theta_A \times \frac{L}{2}| = v_{\max} + t_{CA}$$

Can I write that? That you can write, isn't it? The diagram is deceptive. But if you agree that we are discussing a small deformation, I am permitted to write this.

And I have also shown that I am going to only worry about the modulus. I am going to worry only about the value. That is the best way to handle moment area method.

So, I can find out

$$v_{\max} + t_{CA}$$

Now the problem is solved. I know what is t_{CA}. I have to find out $v_{\max}$. I have this. So, when I substitute these quantities that

$$\theta_A = -\frac{WL^2}{16EI}$$

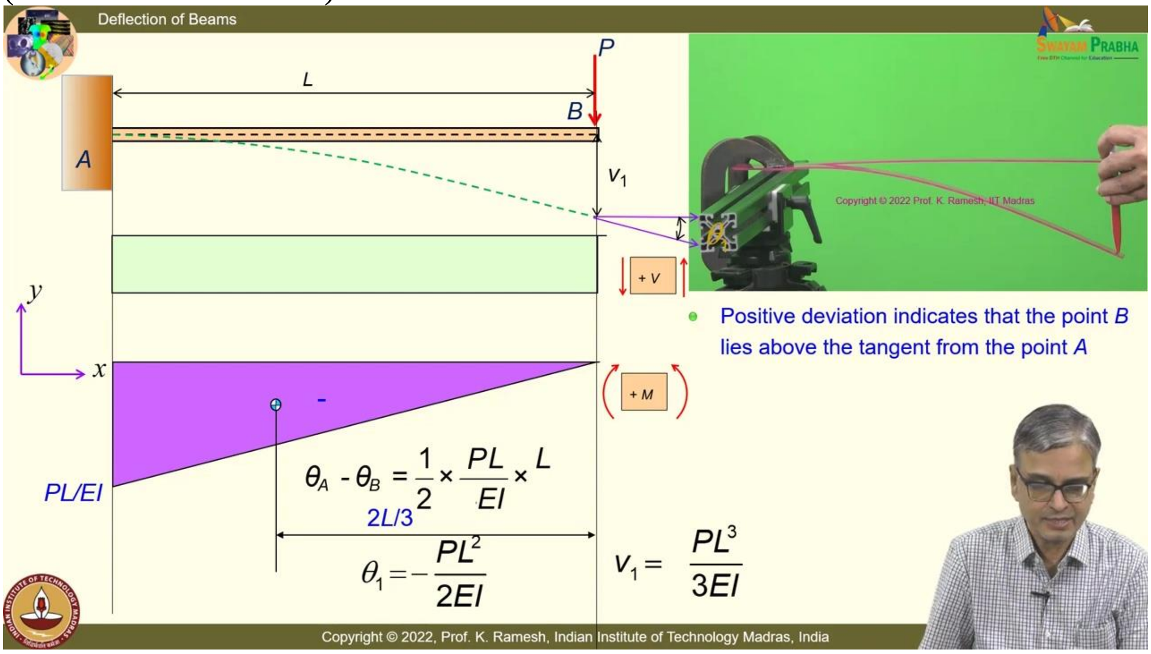
I get

$$v_{\max} = \frac{WL^3}{48EI}$$

I have not got the sign. Sign you put it or you label it and then put this deviation as this much. That is the only way to handle when you have deflection problems. It is better; It is better that other than the double integration, it is better that you put on the diagram, what is the distance.

So, we have used the tangential deviation as well as the slope determination in a judicious way plus our basic area small deformation, we have been able to find out the deflection. It is not that straightforward. But nevertheless, when calculus was not developed or something like that, I do not know what was the reason for them to use moment area method.

(Refer Slide Time: 40:19)



There must be some advantage. I do not know the advantage as such. So now, let us also solve this problem by moment area method because here again, I have a convenient point A where the slope is zero. So, I have selected points which are easy to handle and also the area is easy to find out. First moment of area you can easily do that. All that you can find out.

I have this $PL/_{EI}$ is what you have and I can find out

$$\theta_A - \theta_B = \frac{1}{2} \times \frac{PL}{EI} \times L$$

So, the area is easy to determine and the moment also you can find out. I get this as

$$\theta_1 = -\frac{PL^2}{2EI}$$

θ_A is 0. I put as θ_1, θ_B as θ_1 I have put because we are going to use principle of superposition later.

So, this is the centroid and I can find out this. This is $2L/3$. So, I get this as

$$v_1 = \frac{PL^3}{3EI}$$

I have not got the sign. If you do by direct integration, I will get this as $-\dfrac{PL^3}{3EI}$. So, I am sensitizing you.

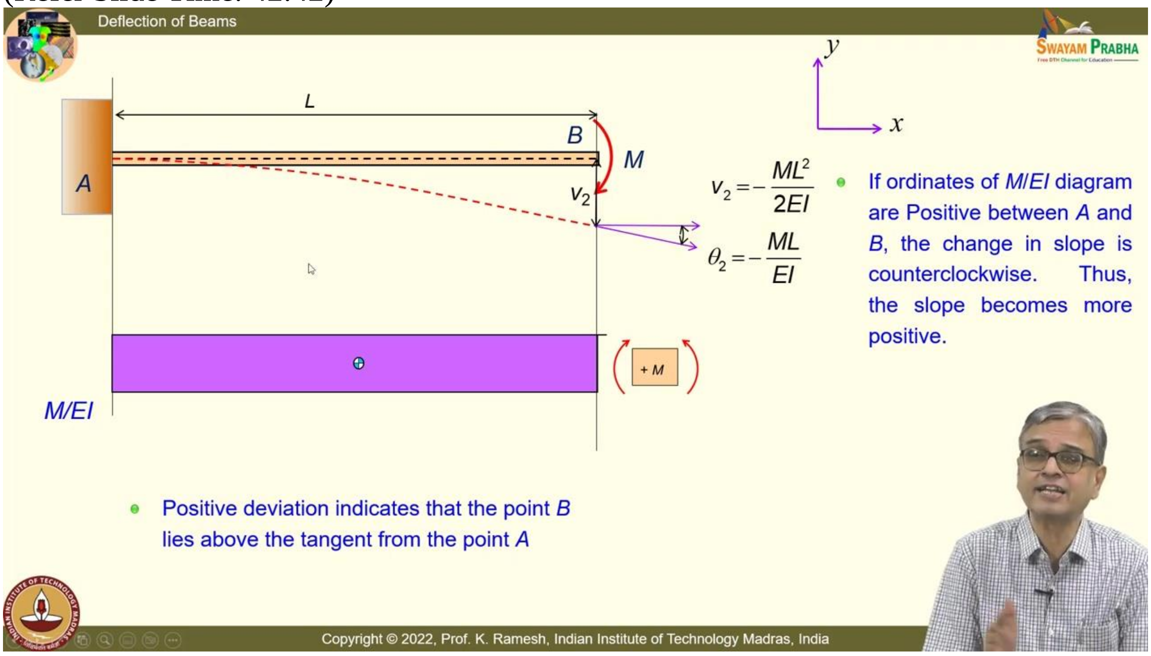
It is enough from our analysis point of view. I get the magnitude. I know physically how the beam is deflected. So, I can always label and say that this

$$v_1 = \frac{PL^3}{3EI}$$

I have already shown what is the deflection. Deflection is shown. So, this is what it says, positive deviation indicates that the point *B* lies above the tangent from point *A* which is seen.

That is what it gives. The sign convention in moment area method is not very convenient for you to interpret. So, it is better that you determine the area and the first moment of area; write the elastic curve and associate the sign or label it directly on the diagram. Even if you label it directly on the diagram, the problem is done.

(Refer Slide Time: 42:42)



Now, we go to another problem where I apply a constant bending moment. Can you do that? Can you do that moment area method? Find out what is the slope? Find out what is the deflection? Because these are all very simple problems I have deliberately taken where you can find out the area and first moment of area and get the deflection as well as slope, fine.

I have a simple bending moment acting on this. So, it is not difficult for you to process the information. First, I draw the anticipated deflection curve for us to have a visualization and I put this as deflection v_2 and this as θ_2; and your bending moment diagram and M/EI diagram are similar. You only do the scaling. Here again, I remind you that if ordinates of M/EI diagram are positive between A and B, the change in slope is counterclockwise.

Thus, the slope becomes more positive. Here, I have this bending moment diagram is negative. So, I get this deflection as; first the area, I have

$$\theta_2 = -\frac{ML}{EI}$$

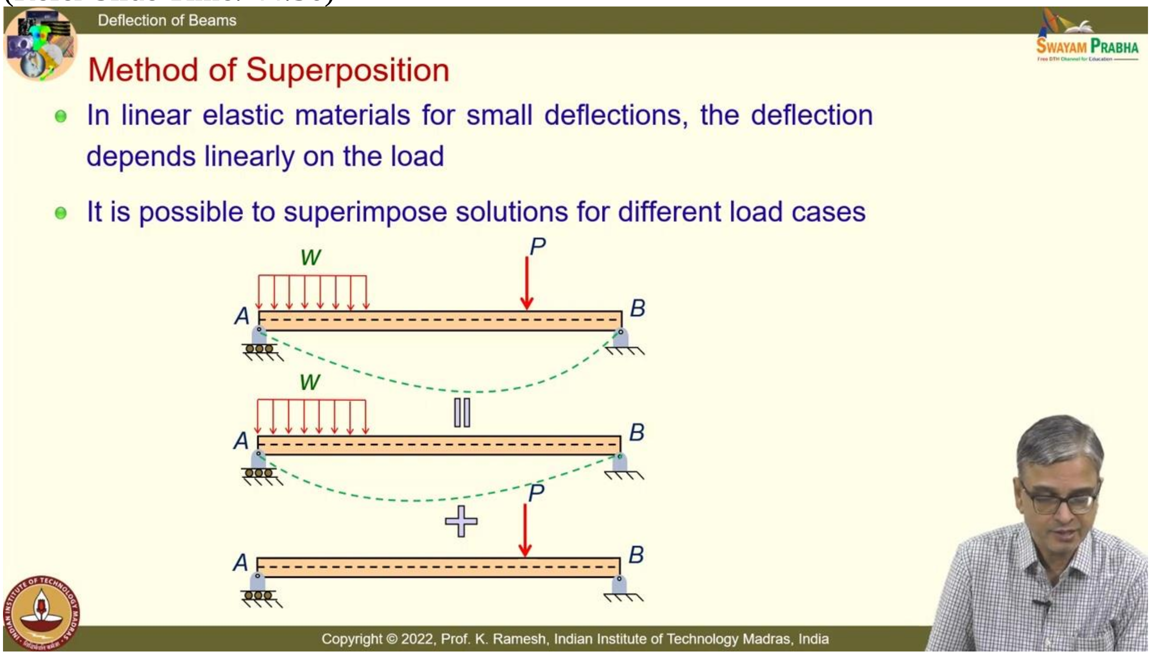
and when I go to the deflection, I get this as

$$v_2 = -\frac{ML^2}{2EI}$$

The positive deviation indicates that the point B lies above the tangent from point A and here you know the tangent and this one lies on the same level. So, it is very difficult to fully get the sign from the basic sign convention mentioned in the development of the method. So, it is better that you put this and you are in a position to get this.

So, you need to specialize in drawing the deflected shape for any unknown loading also because it is modulated by the supports. So, we have looked at all the three kinds of supports, simply supported, free as well as fixed.

(Refer Slide Time: 44:56)



So, we would focus more on method of superposition. So, you have linear elastic materials for small deformations, that deflection depends linearly on the load; that is very important. It is possible to superimpose solutions for different load cases. I have a load case; can you draw the anticipated deflection curve? Because I have the support, this is the roller support; this is the pin joint.

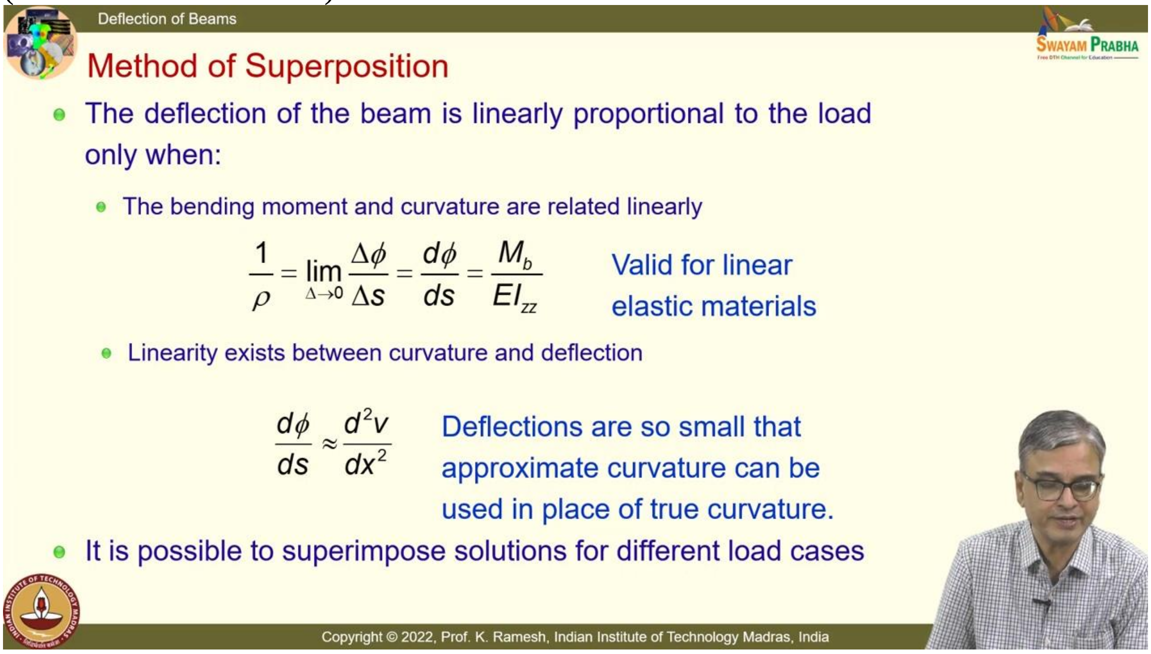
I have unsymmetric loadings, the loading is not symmetric. Have a training, you make a mistake, no problem, you draw the deflected curve. So, you also learn; how do you learn because you know I can have any slope here, this also permits any slope. But the deflection is zero at these two points, that is what I know. And another one is, I have a loading which is not symmetric. So, I will have a deflected shape which is also not symmetric, that much amount of recognition you should have.

Can I draw the diagram something like this? Because I have a load here, this is pulled down here more, fine. And this deflected shape is not symmetric, do not draw it symmetric. So, when I say principle of superposition, I can consider this as contributed by the distributed load as one problem; contributed by concentrated load as another problem. So, I have the first problem like this, can you draw the deflected shape? Draw the deflected shape, you draw the deflected shape; draw the deflected shape because you have to be trained on visualizing possible deflected shape of the beams. That is the focus when you learn method of superposition.

And even in your moment area method, if you draw the deflected shape, you are better off in assigning the appropriate sign to the quantities. So, this will be unsymmetric and it will have zero deflection, it can have different angles. See, I have shown deliberately, the slope is different from what the slope is here. And when I have the other load, here again you make an attempt on drawing the deflected shape because that is very important training.

As engineers, you should be able to visualize, what way the beam can deflect under the given type of loading. So, I have this as unsymmetric.

(Refer Slide Time: 47:45)

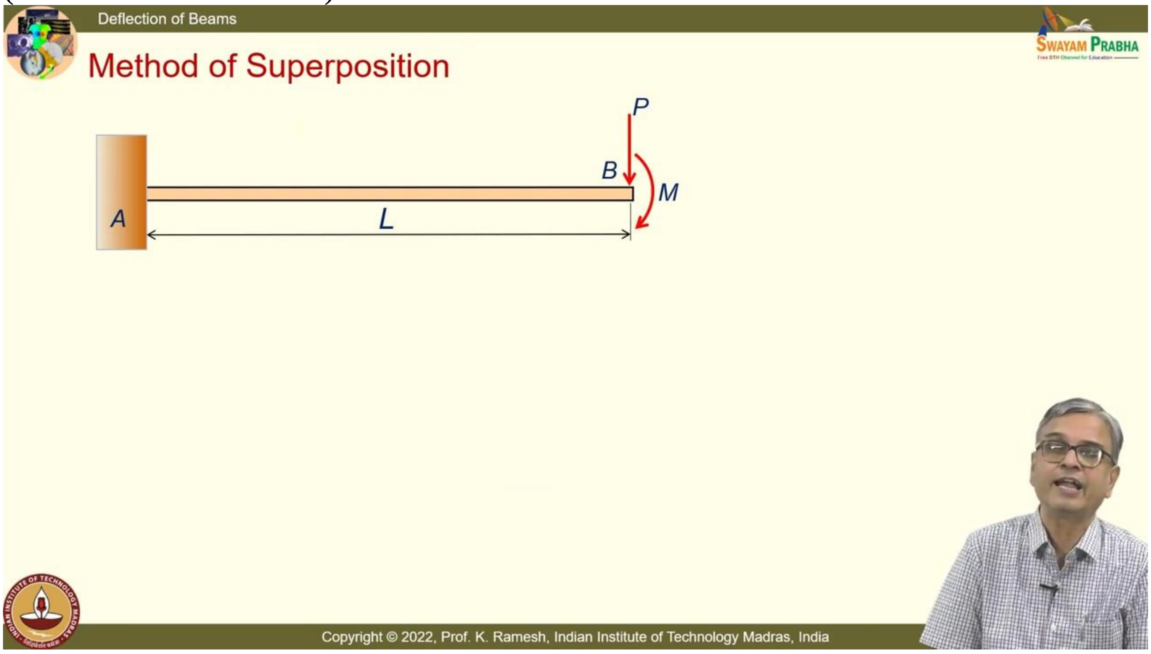


So, these two contribute to this. So, I can solve this as an individual problem, solve this as an individual problem and then do it. So, we go back and then solve our problem. Now, I have; and you also have other subtle point; the deflection of the beam is linearly proportional to the load only when the bending moment and curvature are related linearly.

And we have already said that we have an approximate expression for the curvature valid for linear elastic materials. And this, we have said that this is an approximate relation. And we have already said when the angle ϕ is less than 4.7 degrees, the error you make is only 1 percent; 1 percent is very small, 1 percent is very small.

Deflections are so small that approximate curve can be used in place of true curvature. It is possible to superimpose solutions for different load cases. So, you should know what are the limitations. You cannot have large deformation and use principle of superposition. Small deformation you can do it.

(Refer Slide Time: 49:08)



And I would appreciate that you try to do this problem as homework. We already have solution for the end load and solution for the bending moment. Even before you start the problem, you know what the solution is. It is a linear superposition of these two results. So, in this class we have looked at load deflection curve integration. We have integrated it four times and then we have been able to find out, for a statically indeterminate problem, what is the deflection curve as well as the slope? Then we discussed threadbare, what is the moment area method.

And in the moment area method, you find out the area of the bending moment; M/EI diagram between the points of interest. Find out the area, find out the deviation in the slope. Find out the first moment of the area, you get the tangential deviation. It is not deflection. You have to find out deflection appropriately using the geometry of the problem and the other mathematical principle. Then we have also looked at how to do method of superposition. Thank you.