Strength of Materials
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Lecture - 30
Review 1

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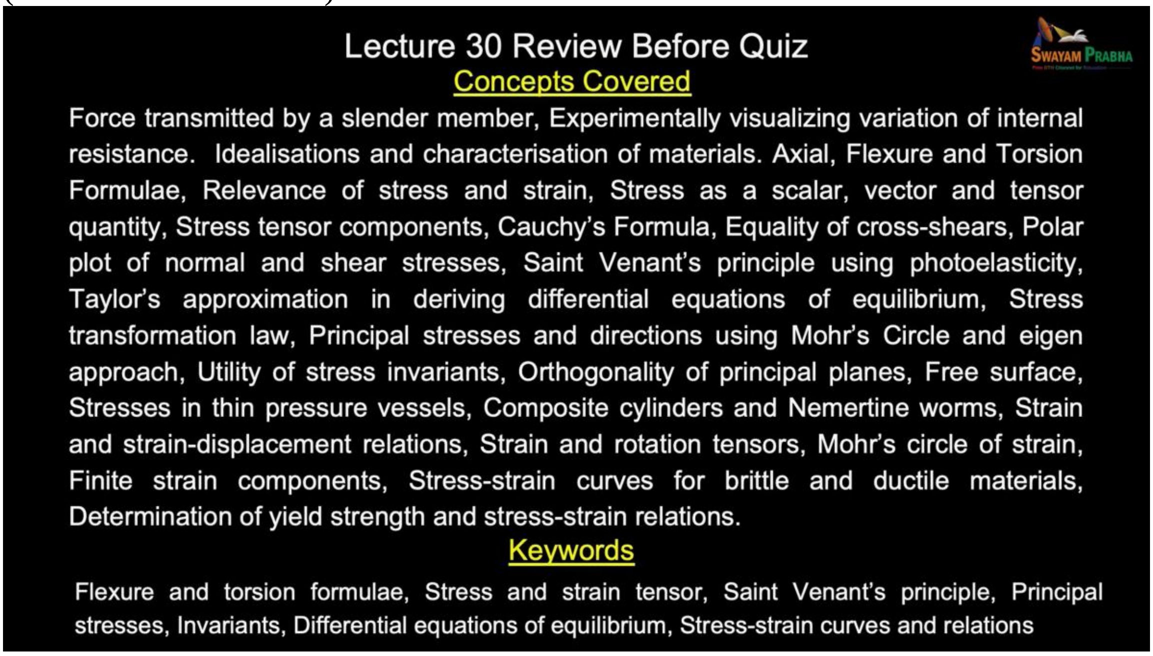

You know, this is a class on review and Strength of Materials is a subject where experimental information forms the basis for the development.

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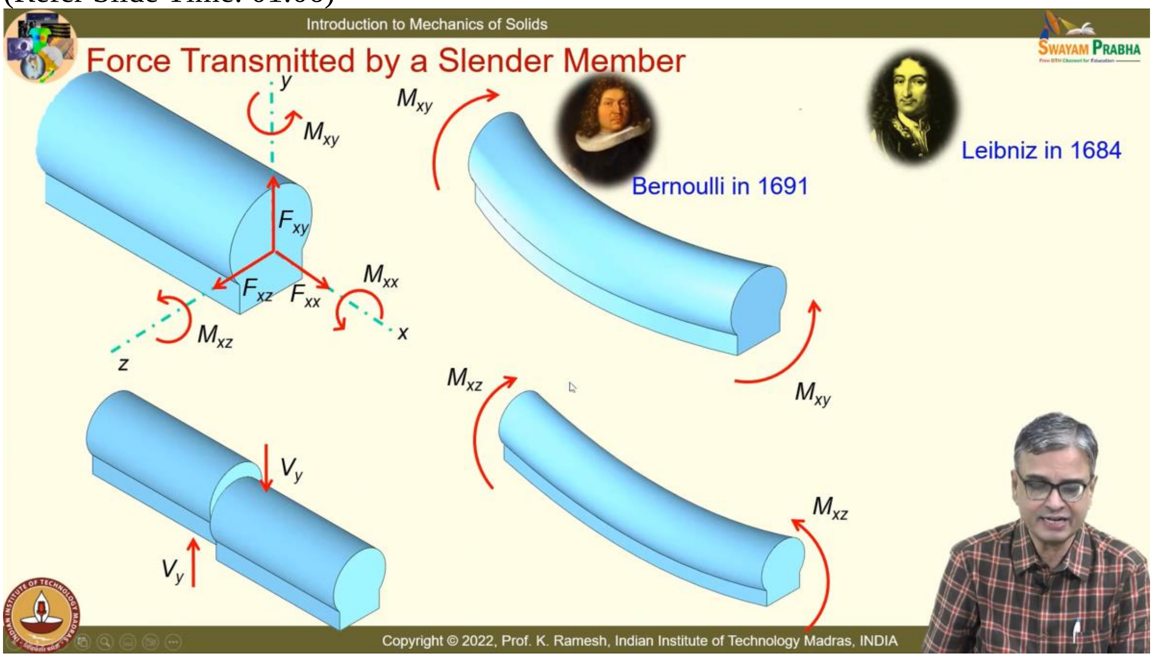

You know, with the modern multimedia facility, we are in a position to integrate the experimental information along with theoretical development. So, you have seen thought experiment; actual experiment all along the course. And even though we discussed several other things as part of the course, you should keep track that we confine our mathematical development only to slender members. You should never lose track of it. And a slender member, in general, can transmit three forces and three moments.

And if you have a force like this, you call that as a shear force. And then if you have a moment like this, you call this as a bending moment. Now, you know what is the kind of distribution that you have in the member and the values of the magnitude of shear force, bending moment and also the bending moment in the other plane. All that you get from your course on rigid body mechanics.

If we want to find out how this cross-section develops a resistance for supporting this shear force or a bending moment, we have to come to deformable solids and then find out what the variation is. And we have also seen that you can have a shear force in this direction. And you can have an axial force and we have also done twisting. And you essentially do in your rigid body mechanics, a shear force diagram, bending moment diagram, axial force diagram or twisting moment diagram. All this is to find out, along the length of the member, which cross-section is transmitting the maximum of these values and which cross-section is the worst affected. So, for that cross-section, you have to find out the stresses and then look at the theories of failure and find out whether the cross-section that you have decided or the material that you have chosen can bear this. And you know, to recognize that a slender member will transmit force and a moment, even though they have not considered all these six components, the basic idea goes to the credit of Bernoulli and Leibniz; they were all around 1700 A.D.

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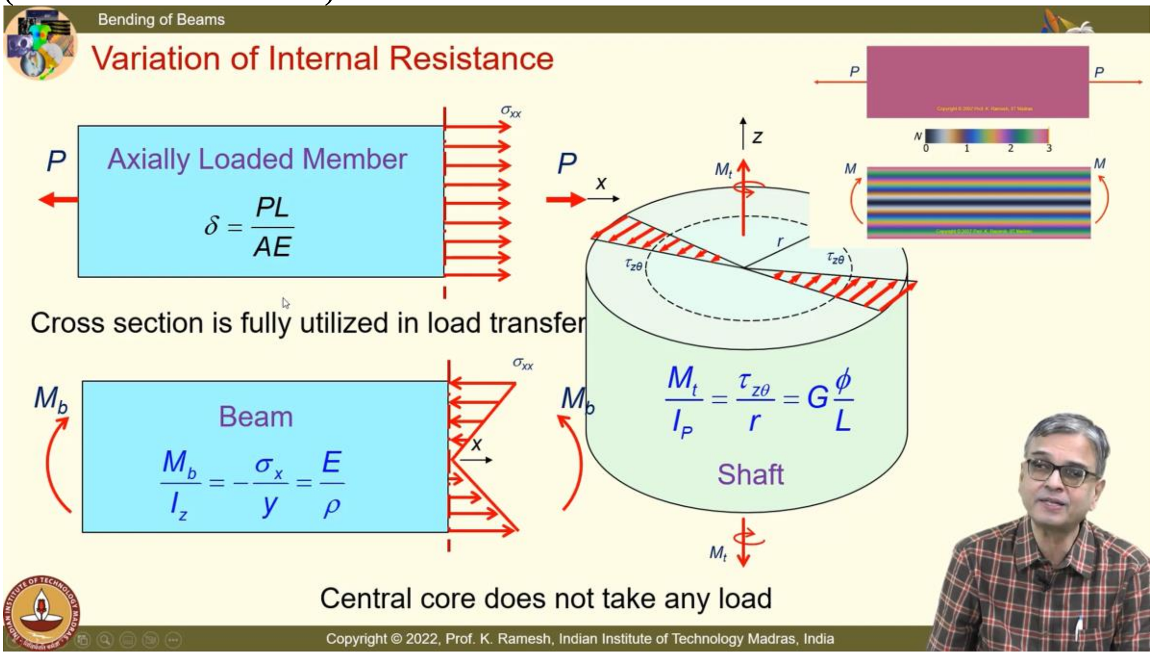

And you know, we asked the question - what is the typical variation of stress in these members? And even before I do the mathematical development, I have shown by inference, by looking at the photoelastic fringes, when I have this color as constant over the width, we said that this is transmitting a uniform force; the resistance is uniform. And the moment you come to bending, you have fringes which are equidistant and we said that it is having a linear variation. And before we looked at bending, we saw the torsion. We found that the shear stress is varying linearly over this section; that is very important; see, this surface is free. And the moment you come to bending, we also find that stress is varying linearly. For all these three problems, we have looked at uniform cross-section, it is transmitting only one of them, if it is transmitting axial force, it is transmitting only axial force.

If it is transmitting twisting moment, it will transmit on a shaft of constant cross-section. And then for a beam, we have also looked at; among the several ways of beam bending, we carefully chose the beam under pure bending, so that it transmits only the bending moment and we have also developed certain important relations. You have this as a torsion formula. So, you have now, the knowledge what is

$$\frac{M_t}{I_P} = \frac{\tau_{z\theta}}{r} = G\frac{\phi}{L}$$

This is valid for a constant bending moment and a constant value of the cross-section. So, similarly you also know the flexure formula.

In fact, your next course on design of machine elements can be done only with these two expressions. And once you look at the axially loaded member, the celebrated relationship is, what is the extension?

$$\delta = \frac{PL}{AE}$$

These are all celebrated expressions. You need to remember them. You will not have time to derive this from first principles. And you know, we have effectively used this when we wanted to analyze the hoop. So, $\delta = \frac{PL}{AE}$ is a very, very famous expression.

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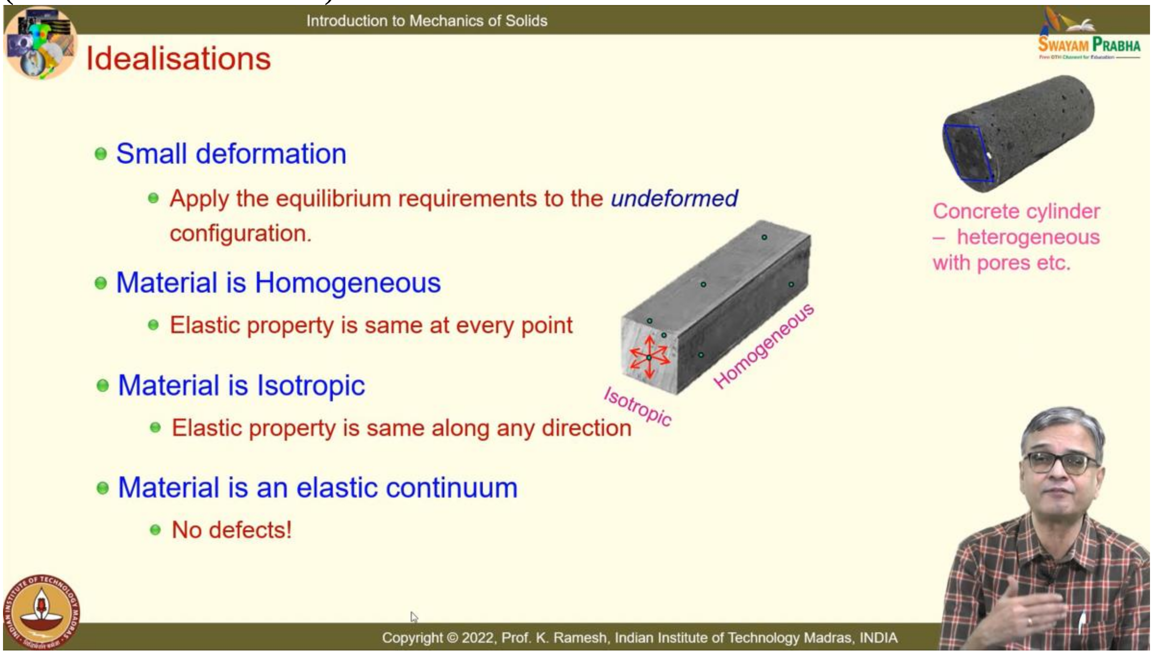

And we have been able to do this with important idealizations; we have said it is small deformation. The moment I say small deformation, we imply - apply the equilibrium requirements to the undeformed configuration; that is very, very important. Material is homogeneous, elastic property is same at every point, that is why I have shown multiple points; it is homogeneous; properties are same at every point. And you understand this with a contrasting example. If I have a concrete cylinder, it is heterogeneous, if you go and see how they mix concrete, you will know they will have aggregates, they will have cement, they will have sand; it cannot be homogeneous. Because we find it difficult to handle the heterogeneous material, we also idealize it as homogeneous. And I have said with modern developments in experimental analysis with rapid prototyping and three-dimensional photoelastic analysis, you can analyze even a heterogeneous material, may be come out with better material models. And we have also made a very interesting and important idealization - material is isotropic.

Elastic property is same along any direction. I have repeatedly said, extrusion - a simple metal processing process, it introduces alignment of the grains. So, it disturbs whether it is being isotropic or to an extent orthotropic, but we close our eyes and then say that is easy for me to develop the mathematics if I consider this as isotropic. And may be do a tension test by taking the specimen along the fiber direction, where the grain is elongated; use that as a property for predicting its failure. So, we have found a way to wriggle out in practical situations. And another very important aspect is, we have made a very fundamental and important idealization - material is an elastic continuum, otherwise you cannot say $\Delta x \to 0$, you cannot do that mathematical step.

And in reality, you have voids, you have cracks, all that is taken care of in another development of mechanics called fracture mechanics.

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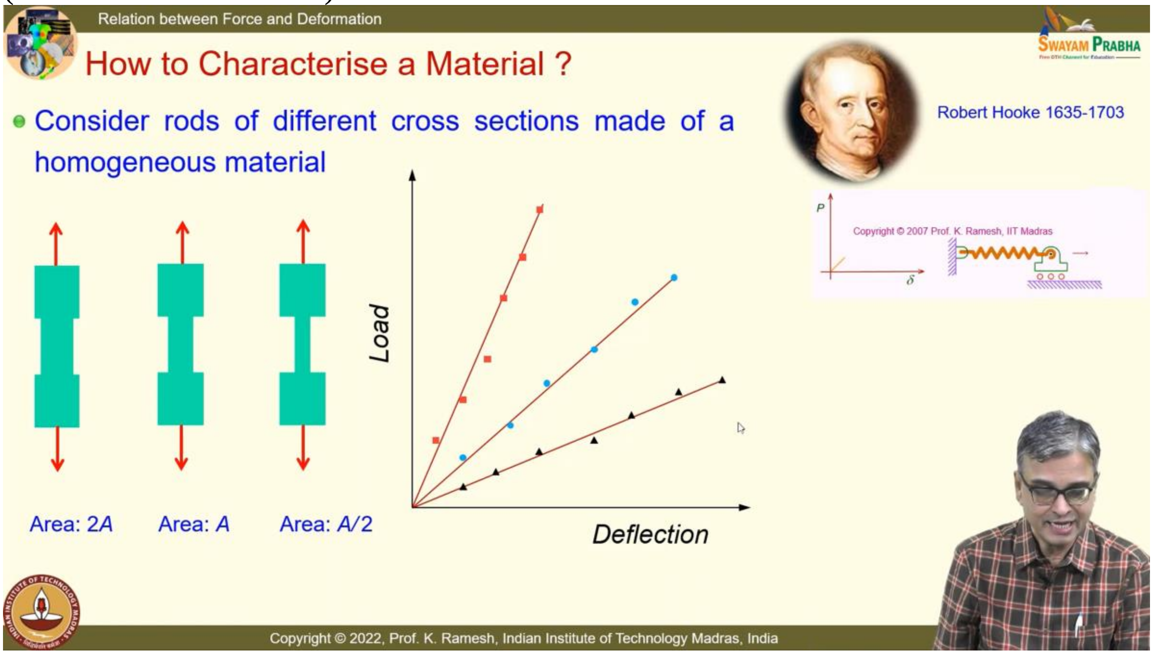

So, idealizations are very important; without idealization we will not be in a position to do. And in order to do all this, we started with a very simple experiment of taking a material and then elongating it with different cross-sections. And we plotted the load deflection, you are getting one graph for each of these materials. And even though this is an observation done by Robert Hooke, the fundamental convenience of this is, by looking at a spring, he was able to visualize that all the engineering materials also have elongation, because you have to graduate from rigid body mechanics to deformable solids.

So, from ideological point of view, this thinking was very very crucial. But plotting this load versus deflection and observing this as linear was also very important, but inadequate. Because when I have to characterize a material, I have to have thousands of tests, in fact infinite number of tests, you may have to do, which is impractical. You will have to have minimum number of tests, so that you can characterize the material.

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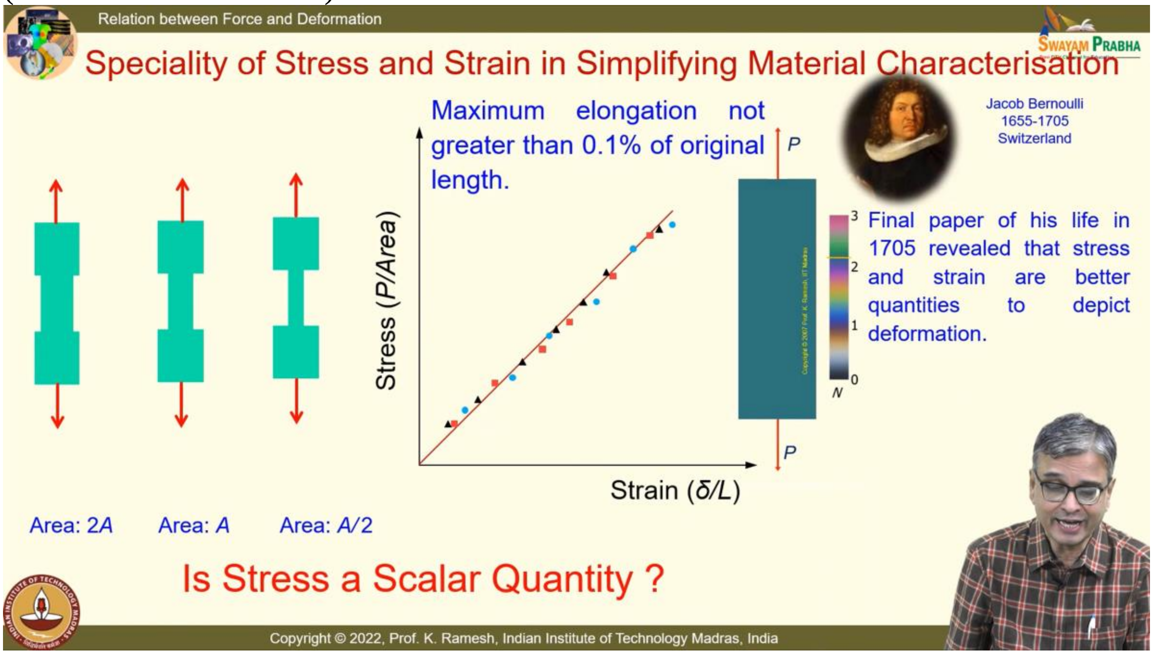

And we have also looked at; that credit was given to Bernoulli and for the same three different cross-sections, if you plot the graph differently; by plotting change in length divided by original length and plotting force divided by area, for all these three different cross-sections, you had only one graph. And you should also understand that this is restricted to only 0.1 % of original length. If you have really taken a material and done a tension test, you want to mathematically express it for the complete range, Strength of Material would not have progressed. You have to look at, from the practical point of view that our operating conditions require - we apply very small load, I cannot live in the comfort of rigid body mechanics, I have to bring in deformation. So, as a first baby step, bring in small deformation; you could solve a variety of simple problems. And you should also appreciate, a simple change like this, it took his entire life time; towards this was his final paper; Bernoulli, he is also credited with the beam theory.

So, some of these developments look very simple now, but they are very difficult and you have to look at; people have burnt their fingers, looked at practical situation, looked at nature and you need to have mathematics also developed. So, things have happened in different points in time. And when they were doing this development of strength of materials, photoelasticity was not available to them. It was also happening parallelly, it became very prominent only in the 1930s. Now, you have the advantage, I have used photoelasticity as a vehicle to appreciate various concepts through inference.

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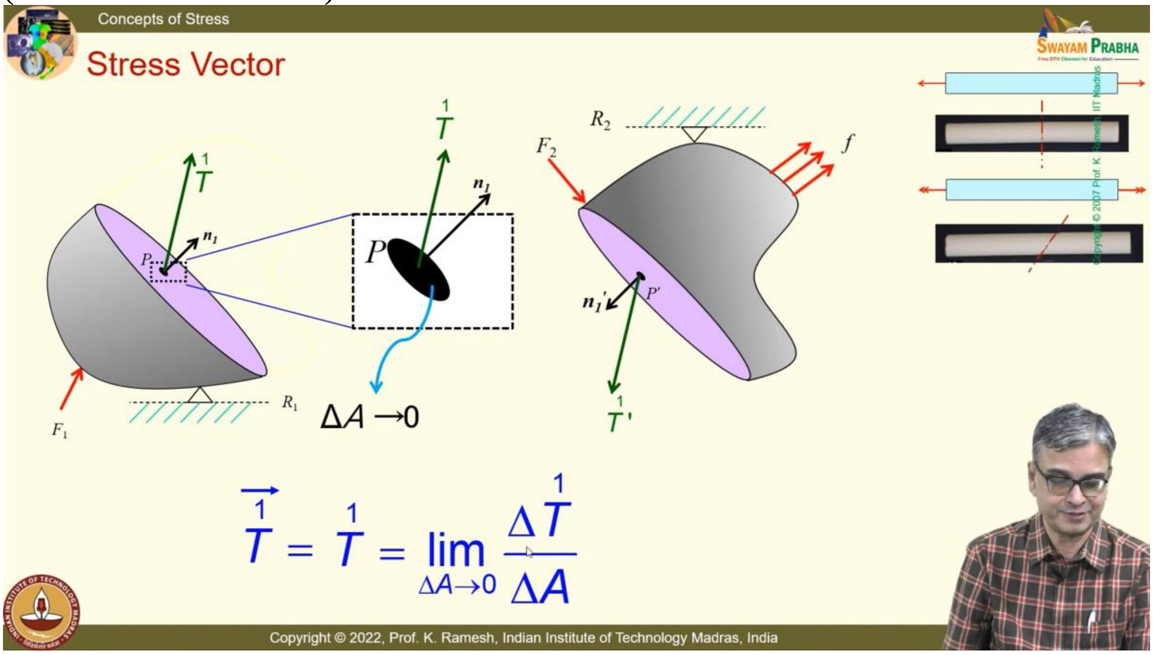

And in order to develop the mathematics, we also developed new mathematical entities, we developed what is known as a stress vector. That stems from the fact that when I break the chalk, the chalk breaks at different planes as if chalk is intelligent.

Depending on the load applied, failure is on different planes. So, what happens on a particular plane is very important and fundamental. And then we took a point and a small area surrounding this point, that is the definition. And we found there could be a resultant force. And following what we have looked at in Robert Hooke's experiment and also the following improvement by Bernoulli, we defined a quantity force divided by area for the small area around it, in the $\lim \Delta A \to 0$. So, we coined a new symbol

$$\vec{\overset{1}{T}}$$

when I say 1, it is referring the plane 1, plane is defined by outward normal. And then it is desirable that you put a vectorial symbol, but it may be difficult to write.

So, we also said that the moment I put a cap, you understand that this is a vector. If I have any subscripts, then you look at this as component, easy for us to handle while writing the equations. And the definition is, the stress vector acting on plane 1 is

$$\lim_{\Delta A \to 0} \frac{\Delta \overset{1}{T}}{\Delta A}$$

And you know, you have this material because we have assumed this as an elastic continuum and then you have equal and opposite forces happening. So, this was very fundamental for you to develop all the related concepts.

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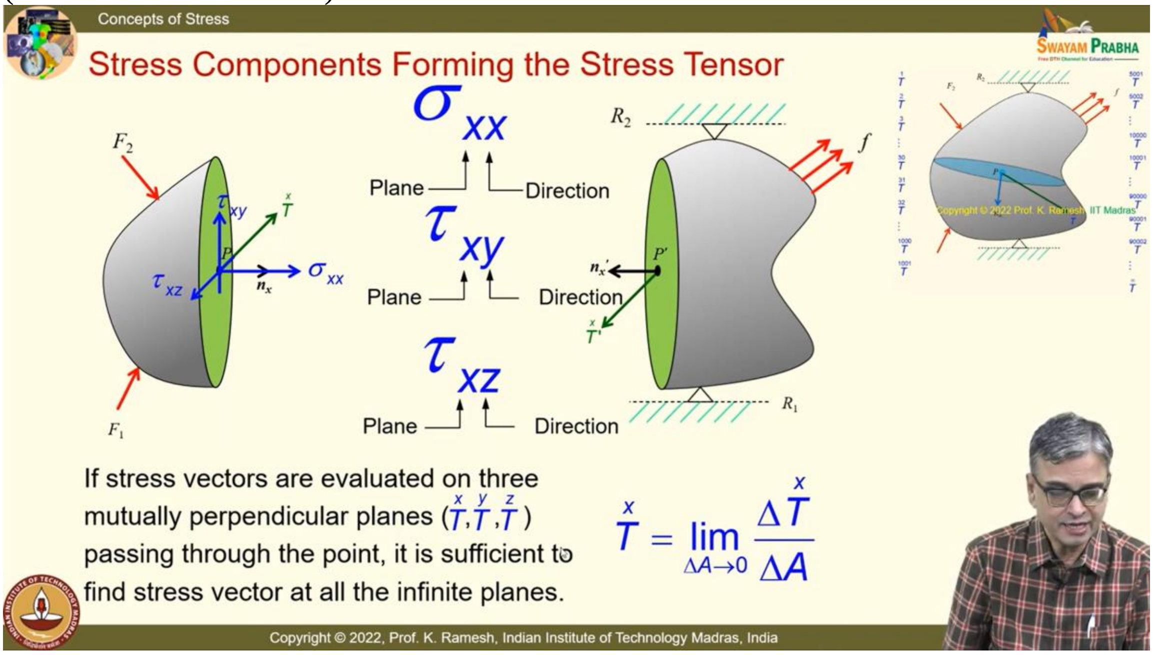

And we said that you have to get stress vector on all the infinite planes passing through the point of interest, only then this is complete. And we have also said, I will cut a special plane which has outward normal coinciding with the *x*-direction. And we have defined new components σ_{xx}, when I say σ, it is denoting a normal stress. You have two subscripts, the first subscript refers to the plane on which it is acting, the second subscript refers to the direction in which it is acting. And you know, in books, they may also call all of this as τ.

You have a special symbol, when I put σ, it is always denoting a normal stress. And when I have τ, this denotes a shear stress acting on the surface. So, you have the plane and the direction. And at this stage, it appears as if it is too complicated, you need to get infinite number of quantities. And Cauchy showed that it is enough that you get the stress vector on any three mutually perpendicular planes, $\overset{x}{T}$, $\overset{y}{T}$ and $\overset{z}{T}$ if you get, you can get stress vector on any of the possible infinite planes passing through the point of interest.

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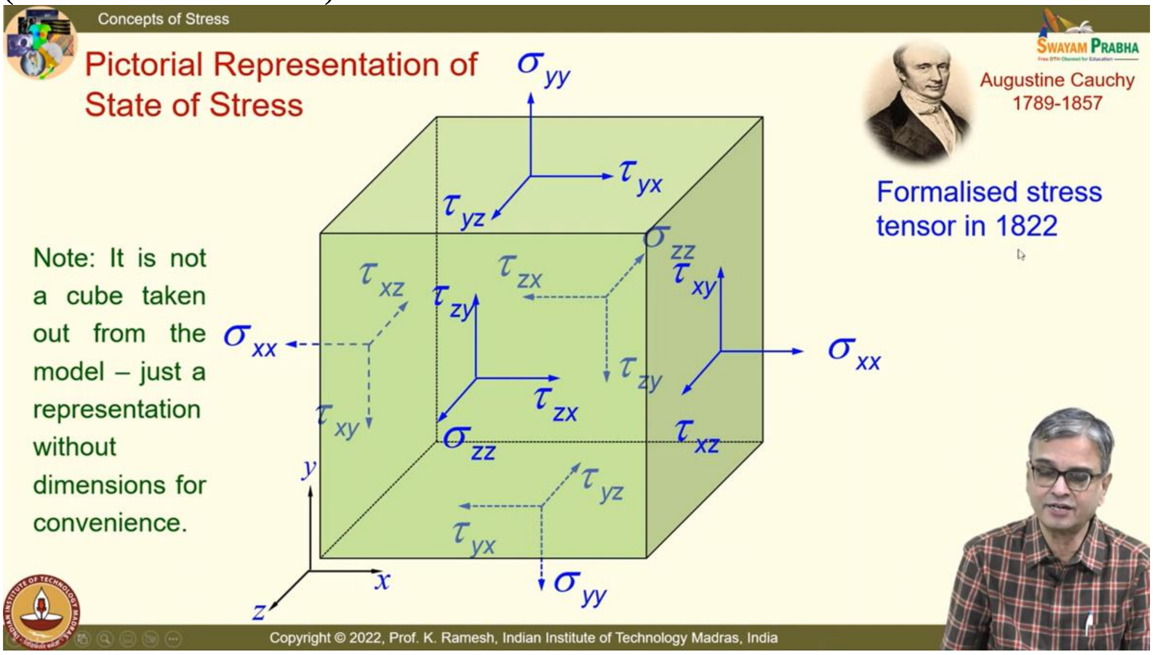

That was the very important mathematical development. And you know, you have these components represented in a cube. And I have also said, this is for representation purpose, the cube has zero dimensions. And you should know how to write these quantities. And we have also discussed, on a positive plane, positive direction is positive, on a negative plane, negative direction is positive.

And this goes to the credit of Coulomb. And I have also summarized that this has zero dimension. It is a representation, what happens at a point of interest. And so, formalised stress tensor in 1822. So, we are in the 200th year, we are learning this course.

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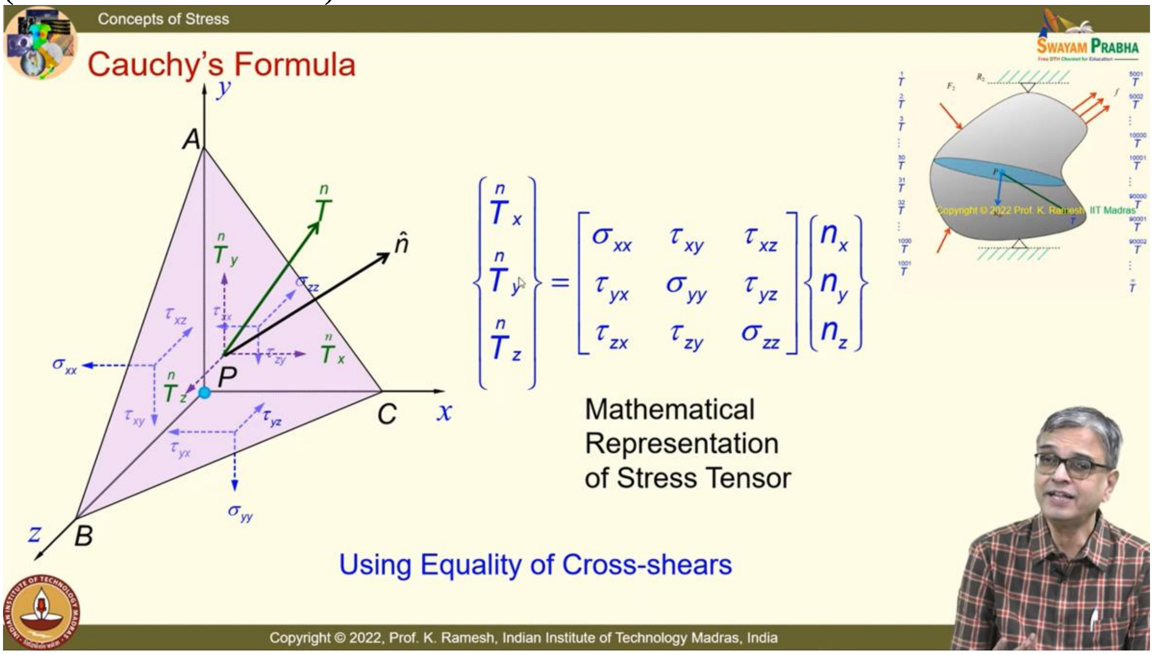

And the Cauchy's formula takes a tetrahedron and then in the limit, shrink it to zero. And we have developed it by invoking the equilibrium conditions. In fact, we have got this as with respect to this column, but we wrote it like this because it is easy for me to write it in a matrix form. And we have utilized the concept of equality of cross-shears to write this. See, in strength of materials, it is very difficult to develop the subject without looking at what is the future development.

Because it is not in a sequence, certain concepts have to be introduced even before we derive and prove it. But we have definitely shown by looking at the moment equilibrium that you have τ_{xy} equal to τ_{yx}. So, the idea is, for any plane of interest, I can find out the stress vector. To do this, you get this and this you call this as a stress tensor. Stress tensor is a consequence of our requirement to find out stress vector. And stress vector is the one which is going to help you; why the chalk fails at different planes.

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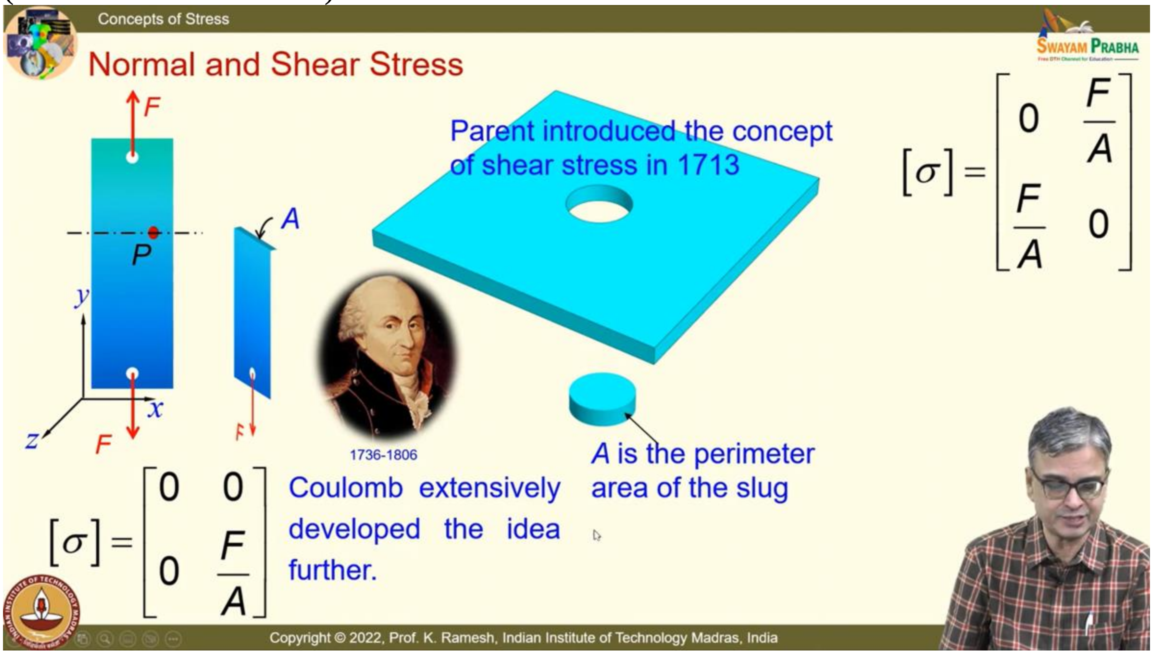

And then we also recognize that stress tensor is tensor of rank 2 and all that. And you know, normal stress was something easier to perceive. So, people were able to understand what is normal stress, which we have looked at from a simple tension specimen. And we have also defined force divided by area as this value.

And I have always emphasized, in the mathematical development, we idealize the deformation picture and directly evaluate the component. It will appear only like a number, but you should have the practice, how to recognize this as a tensorial quantity; always write it in a matrix like this. That is a training, because I have this as a y-axis, I have F/A as the component which is σ_y component here. And you know, people found it difficult to appreciate what is shear. And one simple experiment is you punch it, so you have this outer surface.

So, I have force divided by area will give you this shear. And this was introduced by Parent in 1713. And we have also seen it was extensively used by Coulomb, developed the idea further. And when you have this basic stress tensor, I will have this shear component shown in the diagonal elements. Like I have shown this F/A here, area definition is different.

This is the area contributing to the resistance of axial force and the complete area forming the perimeter is forming the area resisting the shear. The idea of what is shear, you can very well comprehend by this punching experiment.

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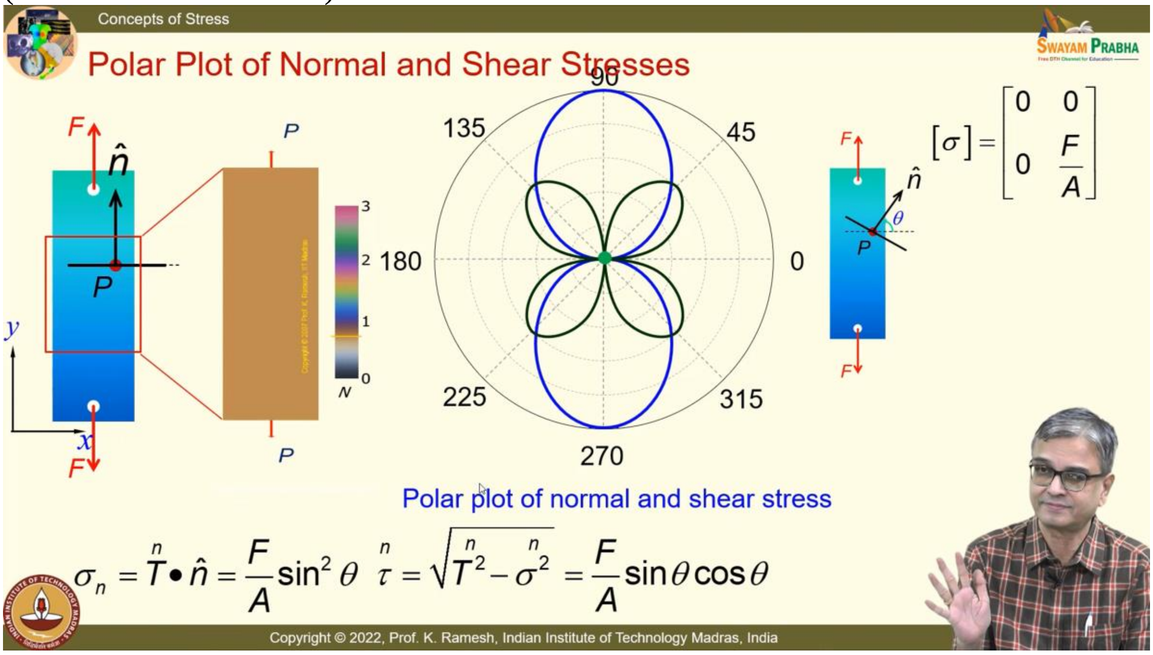

And we have also looked at how do I represent this graphically, because in early days, graphical information was very very strong. People did not have calculators and computers, so they were making assessment based on graphical appreciation. And one of the simplest ways to represent them is a polar plot.

And we have seen what is the polar plot for normal and shear stress. You have the expressions $\frac{F}{A}\sin^2\theta$, and you have this for each of the planes, how the normal stress is plotted. It looks very beautiful, but it is difficult to draw, fine. And shear stress appears like a beautiful butterfly. And you can also correlate with respect to the different planes.

So, this is one simple way of doing it. Even for a simple axial load, you have such beautiful patterns. But later on, we realized that the stress state can be beautifully and conveniently represented in the form of a circle, that we call it as a Mohr's circle.

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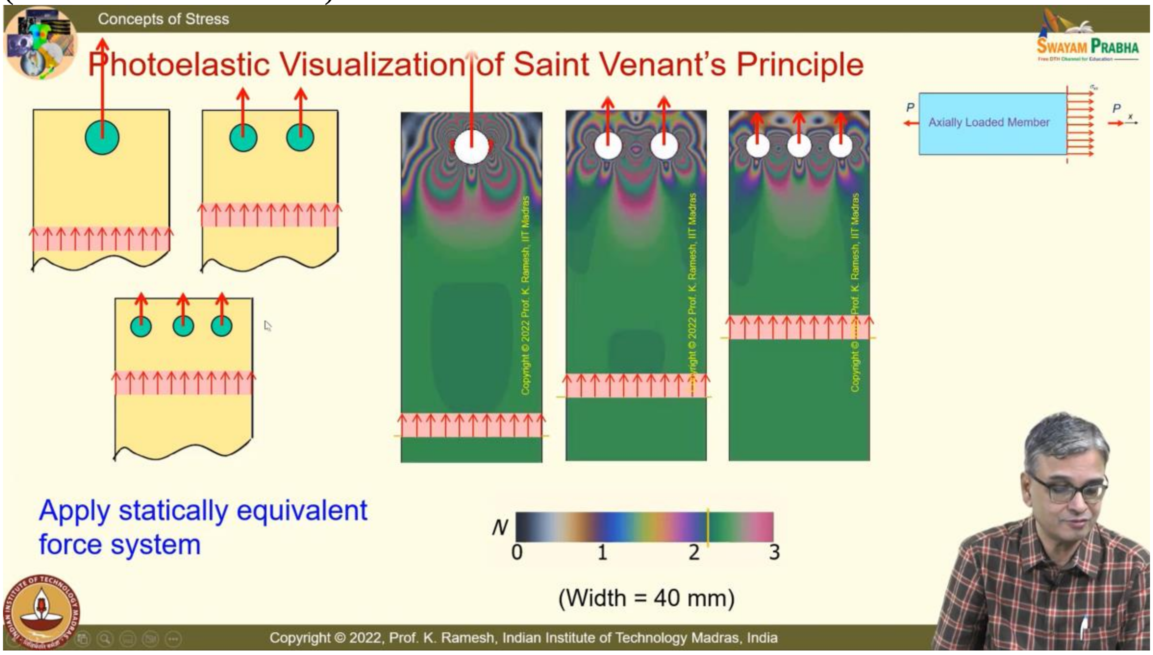

And in the development, we have also discussed a very important principle which is usually talked in the side line in books. So, you have to appreciate what is the Saint Venant's Principle. If I apply an axial load, I can apply it by either a single force or two forces or three forces or even a uniform loading if I am capable of having a grip and pulling it.

So, when I go from this to this, I am going more and more uniform. So, what happens is, this is a photoelastic fringe pattern. So, when I apply the load, you find, that becomes a uniform color only at distances away from the load application points. And this points get shifted when I make the load more and more similar to the anticipated distribution. So, what the Saint Venant's Principle says is, if the applied load is statically equivalent, away from the point of loading, your computation will match the theoretical conjecture.

And you do not have this readily available in books. With photoelasticity, we are in a better position to show. So, it takes some distance. So, it becomes finite only when it is distributed three times. The resultant is same as this force, never forget that.

So, very important principle that we have utilized it now. Even in your future courses, you will utilize this Saint Venant's Principle.

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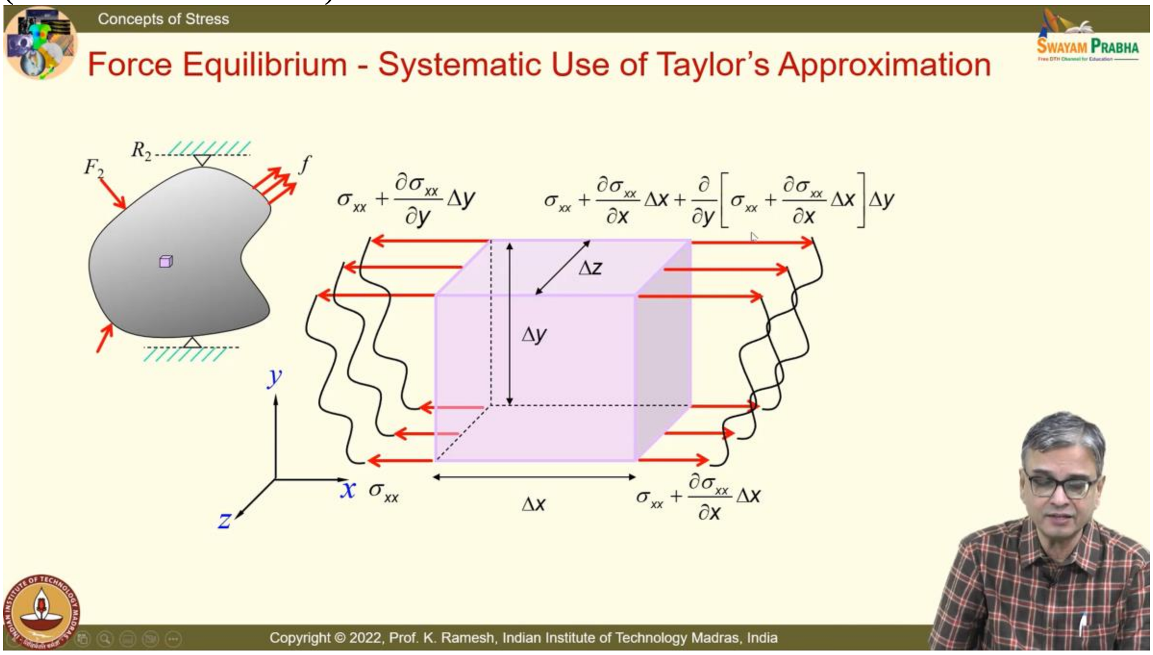

And you know, I have also emphasized that you should have a smooth transition from mathematics to engineering. So, when I say that stresses are functions of x, y and z; suppose I say for discussion purpose, it is a function of x and y for me to write simple diagrams. When I say σ_x in this face, it will vary as $\sigma_x + \frac{\partial \sigma_x}{\partial x}\Delta x$ when I move by Δx.

When I move by Δy, I will have this. But when I move by Δx and Δy, I will have this kind of variation, fine. But you know, product of small quantities is much smaller, fine.

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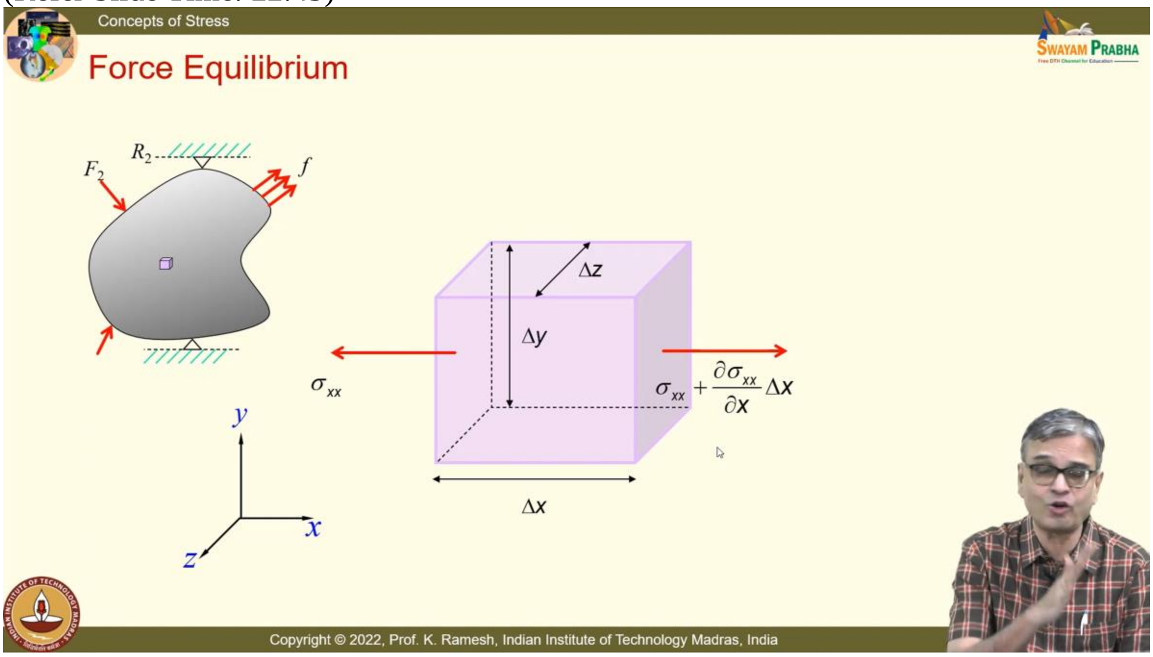

So, when I replace this by an average that is acting on this face, I can represent it like this and I can also take this as an average and represent it like this. Even though in reality, this is what happens on this face and this face, since our interest is to find out the equilibrium along the direction, it is convenient if you drop off these two terms which is not going to affect your mathematics in any form. This is easier to write and this is how we wrote it for all the other directions when we had all the stress components acting.

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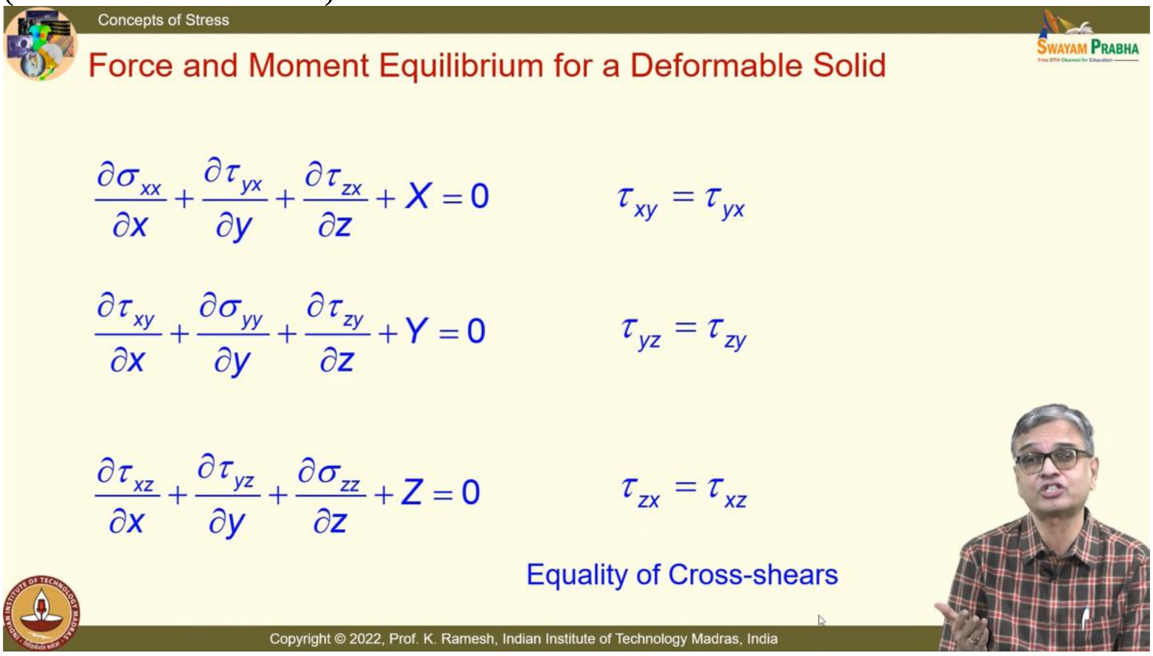

And we did develop equilibrium conditions and the equilibrium conditions appeared as differential equations mainly because this is a deformable solid. So, I can have infinite possibilities of subsystems and if I solve the differential equation, the equilibrium condition is completely satisfied. So, I have this as a force equilibrium condition and this is the moment equilibrium condition and the moment equilibrium condition established equality of cross-shears. This property we have used already while writing the Cauchy's formula for finding out the stress vector. Some back and forth is unavoidable. And you know stress is a tensor, it is not a vector.

So, if I have to find out stress tensor with respect to $x' \; y'$, initially I will have to get the stress vector on plane $x' \; y'$; that is easily achieved by your Cauchy's formula. So, by using Cauchy's formula, you get this vector, then transform this vector by vectorial transformation.

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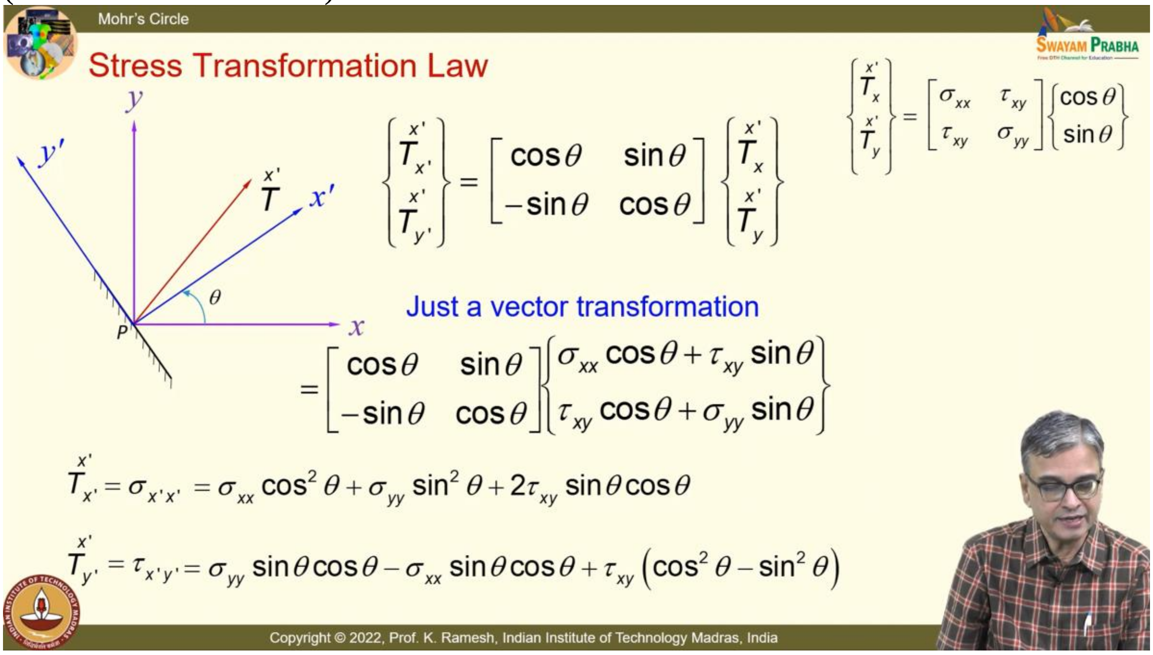

So, you have to do that transformation two times. And we have this stress vector $\overset{x'}{T}$. And this is simply your rotation matrix. Rest of it, mathematics is simple, you should write the rotation matrix properly. And this is how you have done the tensorial transformation, the second stage is a vector transformation.

And when you do this, you get long expressions and I have also said how to get these without going through matrix multiplication; just by indicial notation how to write this, it is very convenient; you do not have to remember provided you write the rotation matrix very carefully. The question is whether you have to have $-\sin\theta$ here or $-\sin\theta$ there, that is where the confusion will come. So, if you simply take $x\ y$ axis and $x'\ y'$ axis, take a point and you find that x' is longer, so it will be positive; and y' is shorter, so it will have negative; that is one way of remembering it. And it is very important.

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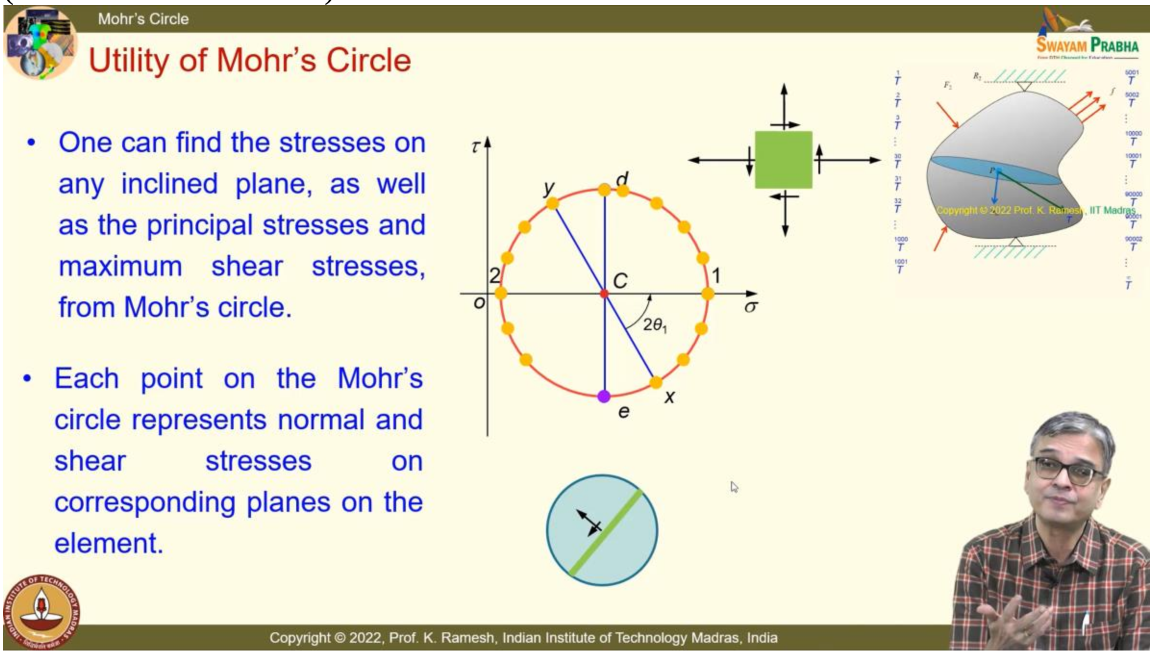

Once you have seen that stress transformation law, we have also looked at how this can be written in the form of a circle. And there is a procedure by which we adopted, we have plotted the graph between σ and τ .

And this is the stress state given pictorially and I have the positive shear stress. For the positive shear stress on the *x* plane, we plot it downwards. The reason is whatever movement I do in the real plane, I should do in the same way in the Mohr's plane. So, if I rotate clockwise, I should rotate clockwise in the Mohr's circle also. If I rotate anticlockwise, I should rotate anticlockwise also. That is achieved only when I follow the sign convention for plotting the *x* plane. Once you plot this plane, you can join them and get the center and with this center, you draw the circle.

And the circle contains the information of state of stress on all the possible infinite planes passing through the point of interest, at least in this plane. And you can also find out two points which have only normal stress, there is no shear stress that is why they are called as principal planes. And you can also identify two other points where you have shear reaches a maximum, and I said, in general, when shear reaches a maximum, you will also have normal stress. If you go to only to pure shear stress state, you will only have maximum shear stress, you will not have any normal stress. And I have also said that I have taken a stress state so that this axis is not coming in between.

And every point in the Mohr's circle is representing a stress vector on a particular plane. And I have also shown it pictorially, how the stress components change. I can have different values of normal stress and shear stress. And as I come to the principal plane I have only normal stress, I do not have any shear stress and likewise this goes. And when I go to the maximum shear stress state, I have shear is maximum, but I still have some value of normal stress. And likewise, we understood every point; so graphical representation was much more informative and convenient for you to comprehend the totality of what is the meaning of stress tensor. So, this is very convenient to see that.

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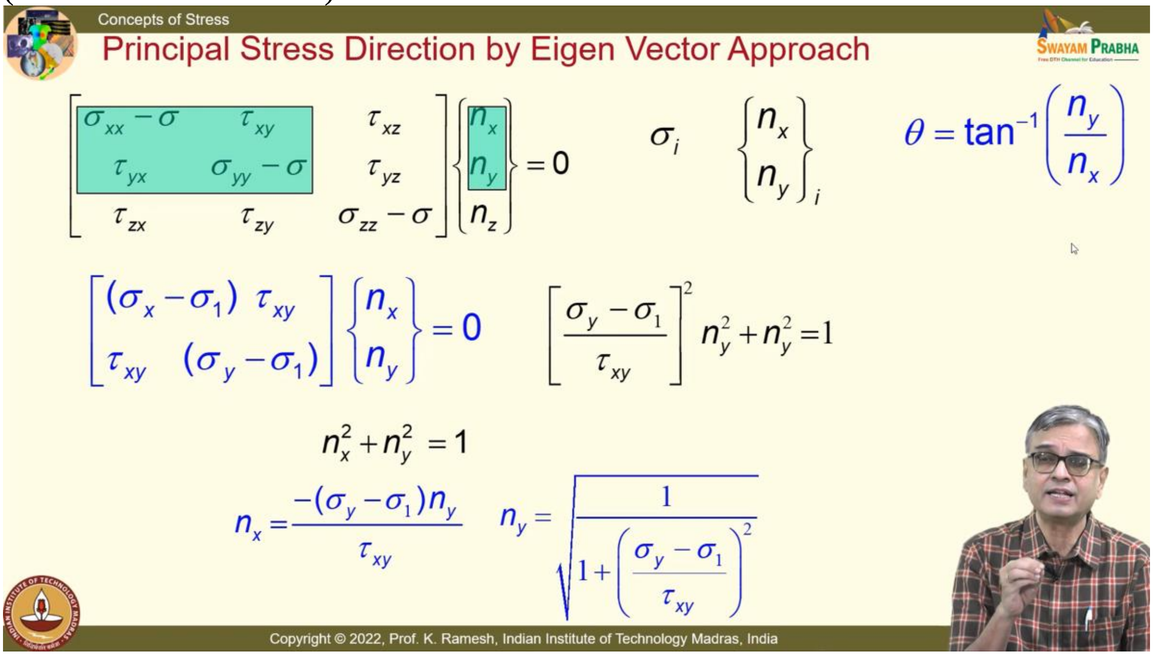

Then we have also looked at what are principal stresses and their directions. You know you can also coin this problem as a problem of eigenvalue and eigenvector. And if I solve it as eigenvalue and eigenvector problem, I have no difficulty in finding out the associated directions. Because I have the eigenvectors as σ_1 and σ_2; eigenvalues as σ_1 and σ_2. And eigenvectors I can find out n_x and n_y for each of this value.

If I substitute for σ_1, I get the value for n_x. I also get the value for n_y and my principal stress direction is given as $\tan^{-1}\left(\frac{n_y}{n_x}\right)$. Mathematically perfect and no ambiguity.

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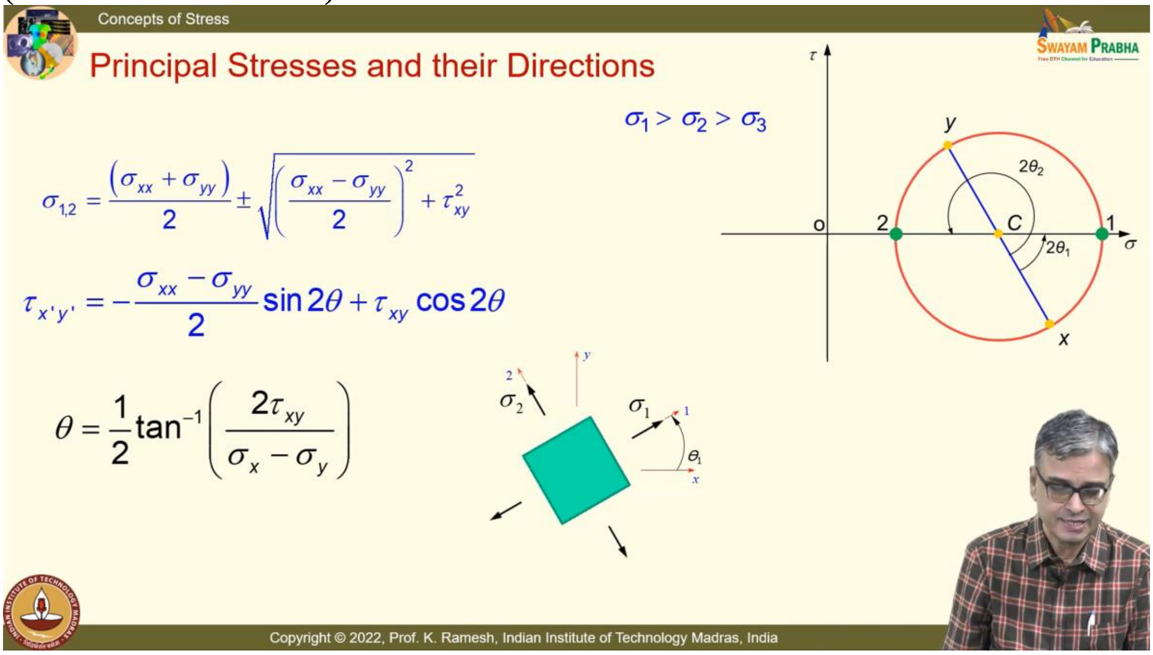

Whether I have θ_1 is associated to σ_1 or σ_2, their ambiguity will not come. On the other hand, for hand calculation, we always calculate σ_1 and σ_2 using this basic expression. And once you calculate this, you should find out and label which is σ_1, which is σ_2.

We have understood that σ_1 is always algebraically greater than σ_2 and σ_2 is always algebraically greater than σ_3. That is how we have developed all our mathematics and also the failure theories. Failure theories use basically the principal stresses. And if you want to find out the orientation of the principal stress direction from stress transformation law, you get $\tau_{x'y'}$.

By the definition of principal planes, we say that this is zero. So, this gives you

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)$$

This is multi-valued problem. So, what you get as θ is not θ_1, what you get as θ, the second value is not θ_2. You will have to go to Mohr's circle and then find out from the Mohr's circle, how you locate the θ_1 and θ_2, what is acute angle and what is obtuse angle.

Only then you can fix. On the other hand, if you go by the eigenvalue, eigenvector route, you do not have this difficulty.

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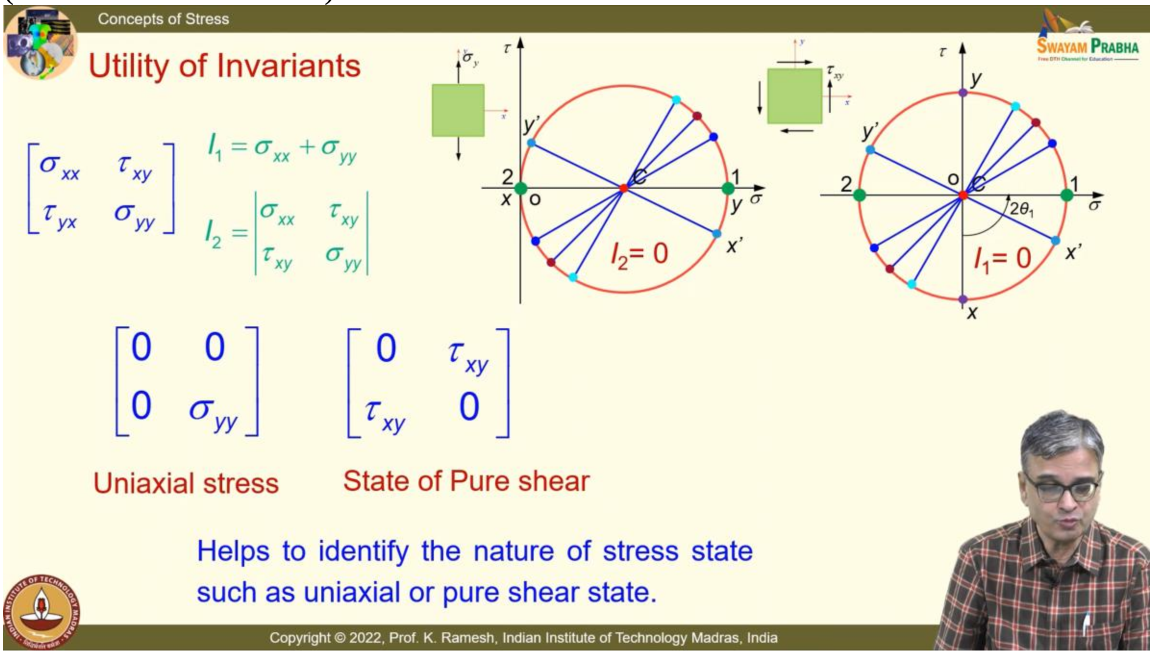

And we have also looked at; there are invariants. For a two-dimensional representation of stress tensor, I_1 is $\sigma_{xx} + \sigma_{yy}$; I_2 is nothing but the determinant of this matrix. And I have this uniaxial stress and if I plot Mohr's circle, the Mohr's circle is like this. And you know the same stress state can be represented in the tensorial matrix with non-zero numbers. The question is, when some stress state is given to you, how to investigate whether it is uniaxial or have pure shear.

These two you can identify from the invariants. And I can represent this same stress state with non-zero values of σ_{xx}, σ_{yy} and τ_{xy}. I can do it like this. So, if I calculate what is I_1 and I_2, I am in a position to do that. And when you have I_2 is zero, you have this as uniaxial stress, I_2 is zero here.

And I_1 is zero, you say that this is a pure shear stress state. It is a very powerful method of doing it. So, you should understand how to utilize these invariants. And you know when you do this as eigenvalue and eigenvector problem, eigen vectors are mutually perpendicular mathematically, fine.

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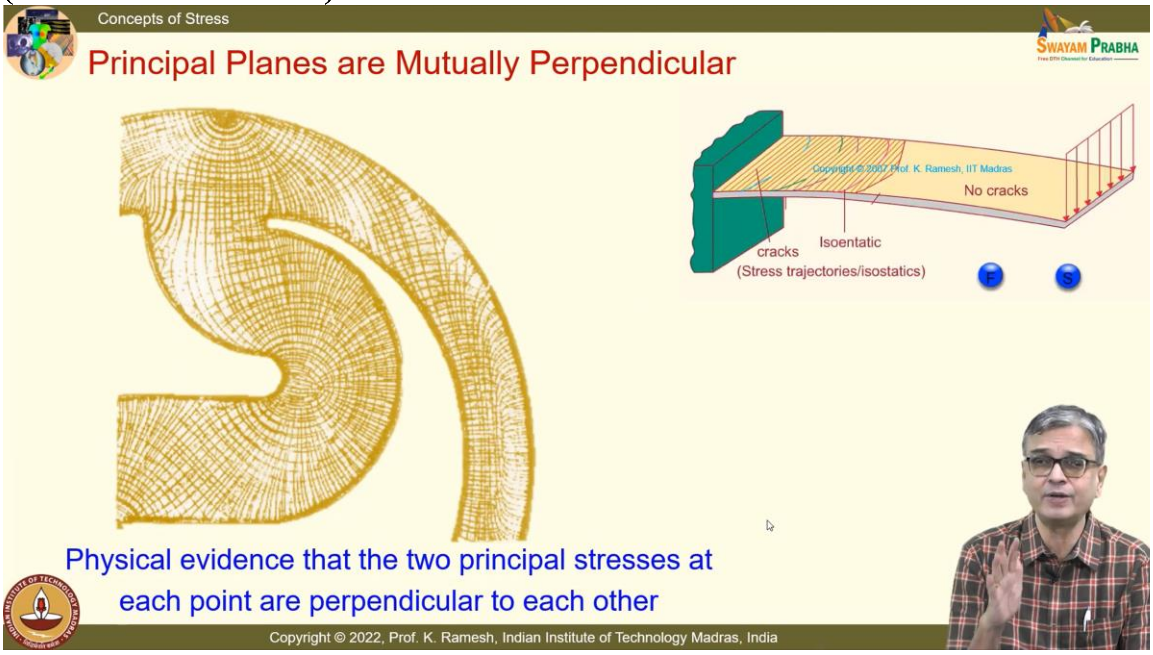

But people do not stop there, you know you have a very interesting experimental technique where you coat the specimen with a brittle material. So, when the stresses increase, I have said in the chalk experiment also, it is a brittle material, it fails by maximum normal stress.

So, you indirectly get the direction of the principal stress direction. And a very careful experiment was done by Professor Durelli and he has shown that you have two families, one is like circle, another is like a radial line. So, that established experimentally also, that principal planes are mutually perpendicular. It is very useful information.

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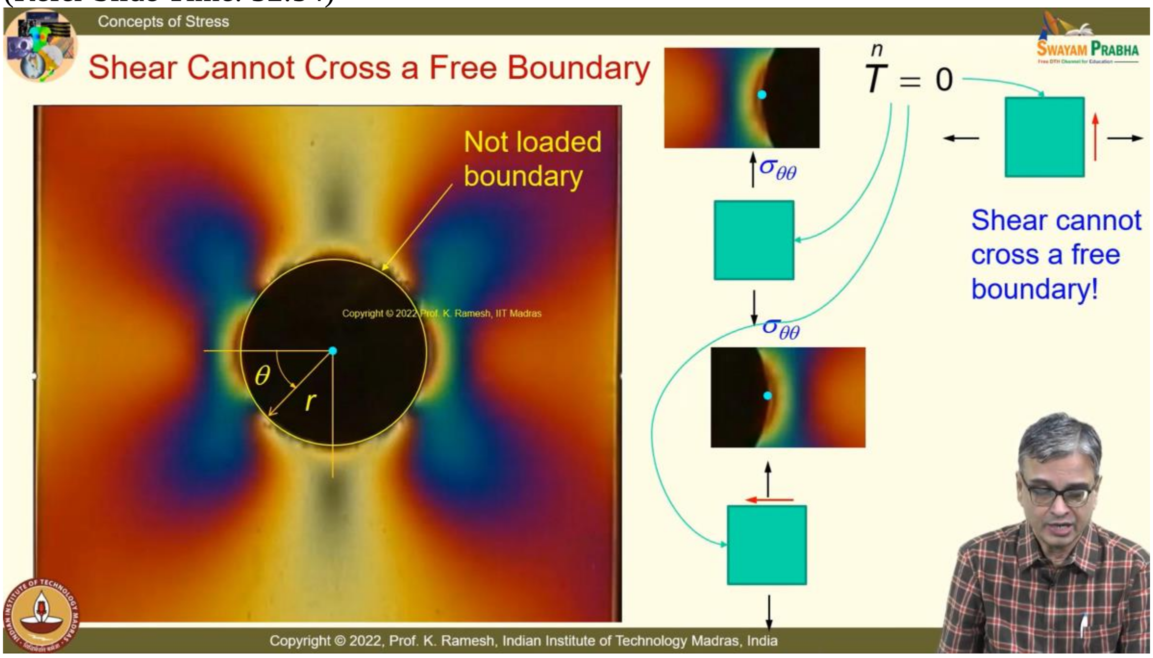

And we have also looked at shear cannot cross a free boundary. We have taken the problem of a plate with a hole and you know this has a very high stress concentration. And we have taken a point on the boundary and we have said that shear cannot cross a free boundary.

And with this knowledge, we have looked at and investigated what is happening at a point which I am taking on this. We have taken two points, one is this point and another is this point and it is not a loaded boundary. When the boundary is not loaded, you say it is a free surface and on a free surface, what can exist? And I have taken this small segment, I have taken a point here. And we have shown that what component of shear stress cannot exist. On that basis, we have said that you cannot have shear stress on that boundary, but you can definitely have a normal stress. You can definitely have a normal stress and when I have this, the circle is easily representable by a polar coordinate.

So, when I do it in a polar coordinate, what is this component of stress? This component of stress will be $\sigma_{\theta\theta}$. So, $\sigma_{\theta\theta}$ will exist and you will not have σ_{rr} and $\tau_{r\theta}$, they go to zero. And that is what we have seen on this point and we have seen it on another point. You can also repeat it for any point on the boundary.

These are all very important concepts. You know, books do not pay attention on all this. The concept of free surface is very essential when you want to write the boundary conditions. And when you want to solve the problem numerically using finite elements, you need to apply the boundary conditions correctly.

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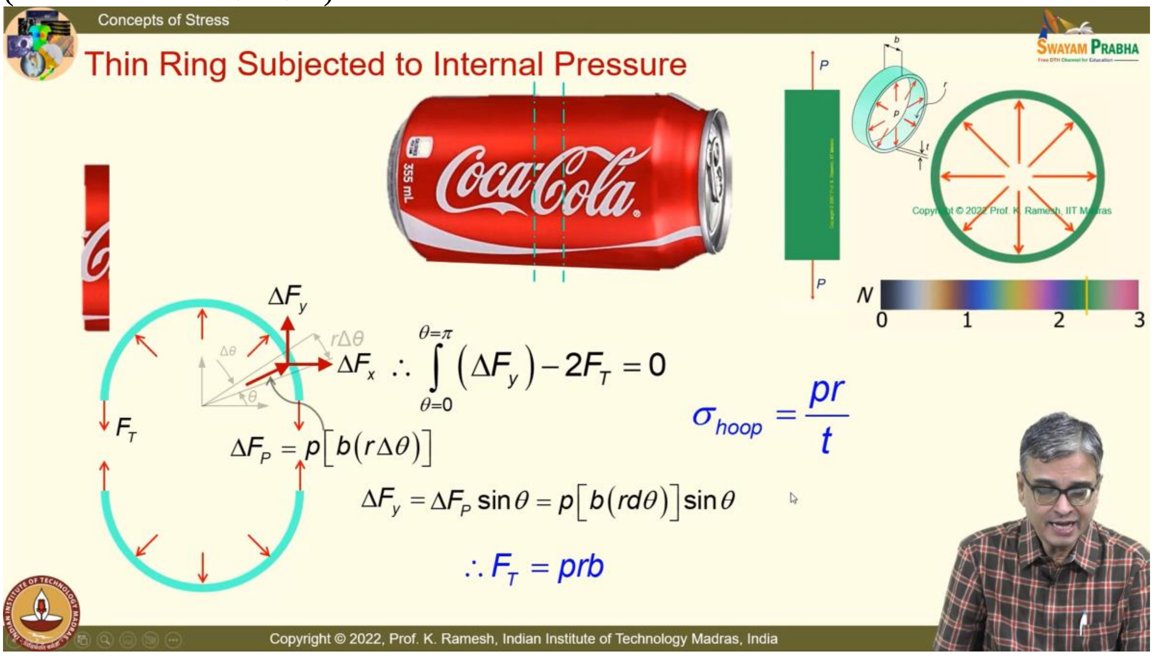

And we have also taken a problem of a pressure vessel which is very common. And you could do this from your strength of materials knowledge itself. And we have got the expression as $F_T = prb$ and when you divide by the area, you get the stress and we call this as hoop stress pr/t. And even by inference, I have shown, I have a thin hoop subjected to internal pressure and this is a specimen subjected to axial tension. You have constant color in this, you also have constant color seen in the hoop. So, even though the pressure vessel looks complicated, the stress transmitted by this is very simple, it is constant like what we have in uniaxial tension which you had seen it by inference.

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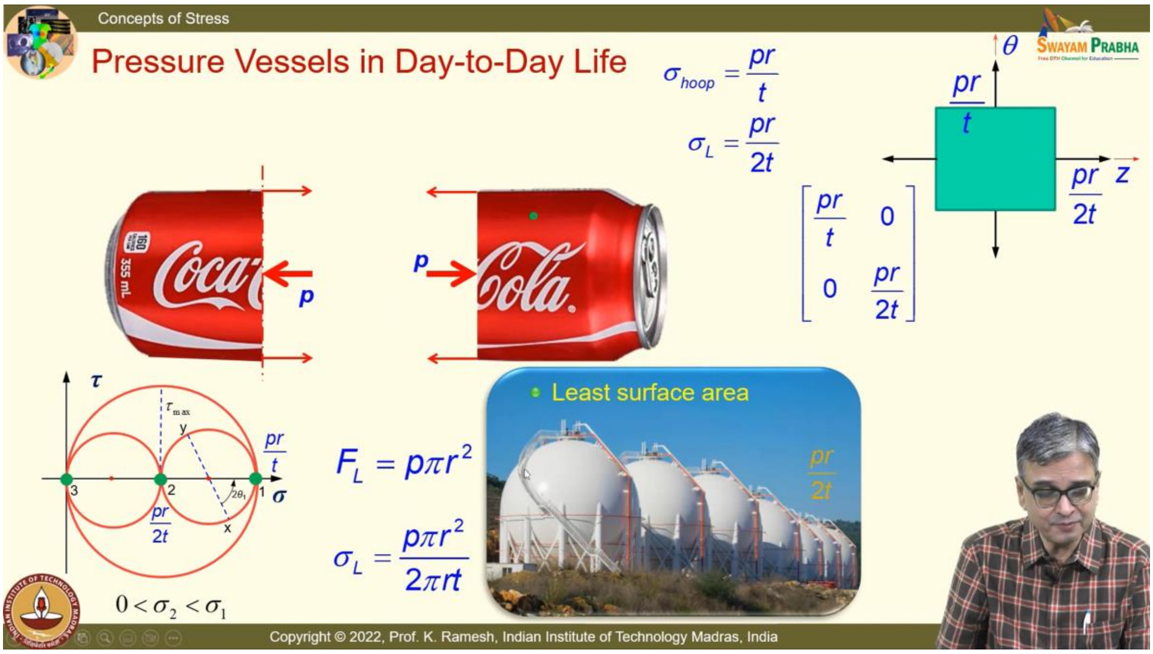

And I said that this is a closed pressure vessel. When I have a closed pressure vessel, I should also worry about what is the force in the other direction. And we have taken a generic cross-section and this is $F_L = p\pi r^2$. And I get $\sigma_L = pr/2t$ and we have got $\sigma_{hoop} = pr/t$. And if you take any generic point, it is a true case of a two-dimensional state of stress.

It is because if you look at I_1 and I_2, they are not zero. And I have this very famous stress tensor. And I have also said when you are plotting the Mohr's circle, you should also be careful if σ_3 is zero, its nuisance value has to be appreciated. So, if you draw the Mohr's circle for this and if you say that this is $\tau_{\max}$, then you are mistaken, your metallic can will fail even when you take your Pepsi. You do not want that to happen because this is a ductile material, it fails by shear. So, you have to recognize the nuisance value of σ_3 and the $\tau_{\max}$ is actually twice this value.

So, you have to be very alert. So, one of the commonest examples has both the principal stresses in the same direction. In all those cases, whether it is positive or negative, you should recognize the nuisance value of zero. If you have the theory of failure differently, it gets automatically taken care of; that we will see later. Right now, you know only maximum shear stress. So, if you use maximum shear stress, do not call this as maximum, the real maximum is double this value.

That is what we have alerted from this. And you know, you must also have seen, when you go in the train, you will see spherical tanks all over the place. Have you noticed it? Why do they use spherical tank? That is also you are having a fluid at high pressure, fine. It is difficult to construct, but if you do mathematically, if you take any section, see here we have taken a section perpendicular to this, your area is $p\pi r^2$, area is πr^2 and the pressure is p. So, the force is $p\pi r^2$. So, on a sphere what happens? Anywhere you cut, you will have the, because it is a sphere, it is only πr^2.

So, everywhere here you have pr/t and $pr/2t$. There, it is $pr/2t$ and $pr/2t$. What is the advantage? Do you see that it merges into a point when you want to draw the Mohr circle? And you will have to worry only about this $pr/2t$ and 0 and your maximum shear stress is one half of what is there in a cylindrical pressure vessel. So, the spherical pressure vessel is difficult to construct, but experiences less load. That is one advantage. Another advantage is, you know, they all store, you know, below the normal temperature, fine. I mean gases which are in liquid form.

So there, sphere is one shape which has the least surface area for a given volume. So, heat transfer is less. When heat transfer is less, the fluid will not get pressurized. So, from all that practical consideration, even though higher cost in construction, spherical vessels are very widely used. So, you have to look at the application and find out whether you want a cylindrical pressure vessel. Cylindrical pressure vessel also used, spherical pressure vessels also used.

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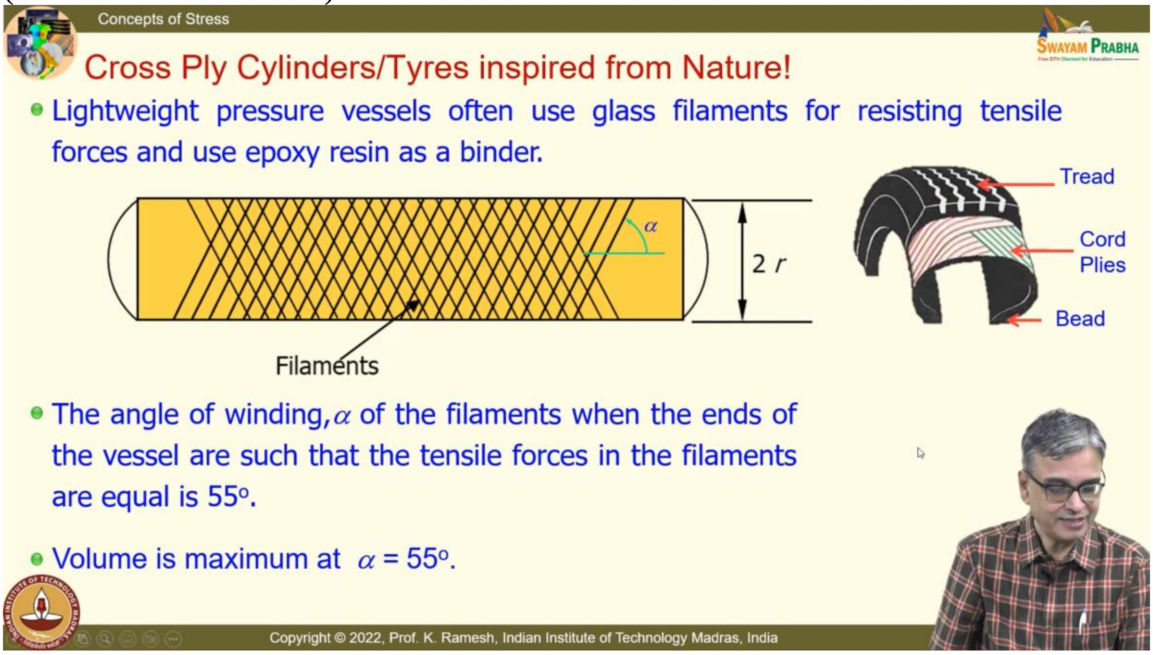

And we have also looked at, if you want to reduce the weight of the pressure vessel, you can have this fiber reinforced. And one of the important knowledge that you have to gain is, what should be the angle of this fiber? And we have also noticed that even in the tyre, normally you ignore the tyre; after all it is a tyre, cycle tyre may be like that, but the same tyre is also supporting an aircraft which is landing and taking off. So, it has a very, very complex mechanics involved. They also have cross-plies. And I have showed a very interesting; whether scientists are great or nature is far superior than scientists.

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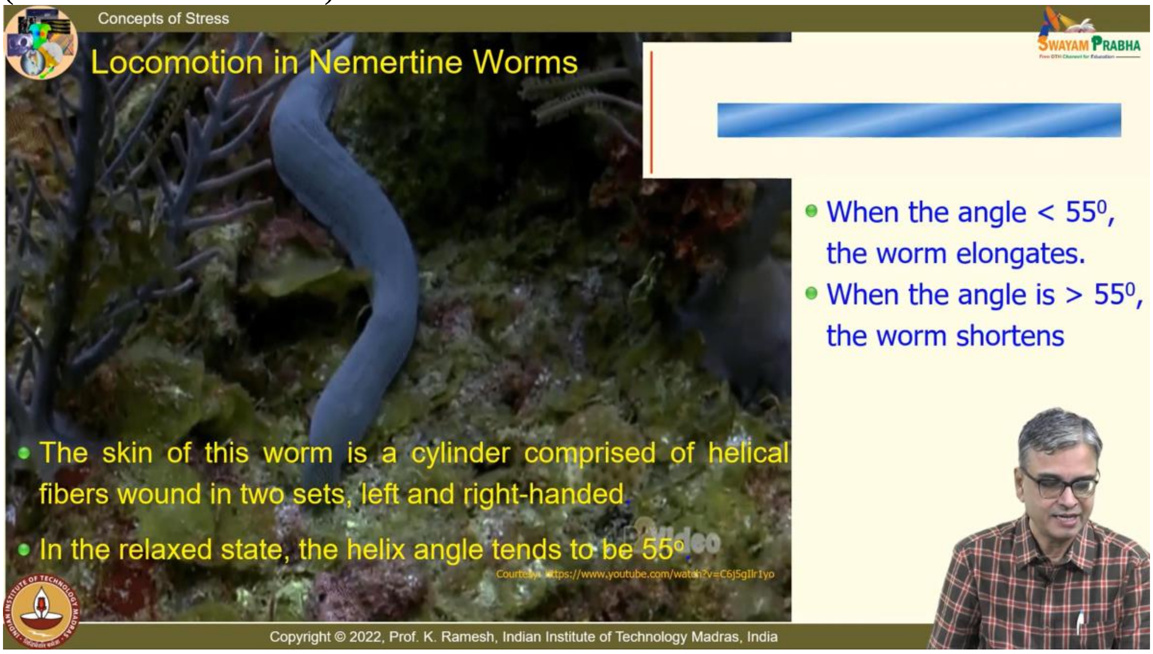

You find a simple worm which is living under the sea, has a body which has fibers which are oriented helically and it manipulates this for its locomotion also. It bulges and then stretches and that is how; it does not have legs. You and I have legs and we can run. So, it has to move. So, it utilizes this. So, it is inspired from nature. So, nature is far superior than human intelligence.

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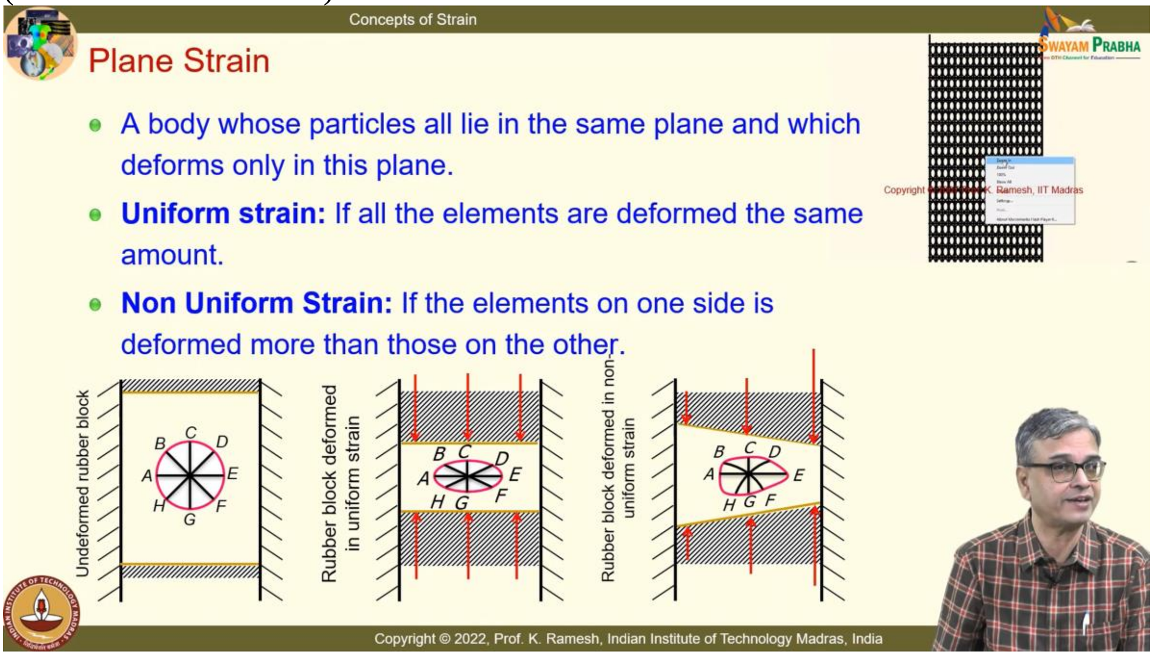

And we moved on to strain. And the important learning in strain is, you know, if I apply strain in one direction, if I apply a load, strain is there in all the three directions. And we have looked at uniform strain and we have also looked at non-uniform strain. So, you have the circle deforms in the circle when there is uniform strain and if there is a load that is not uniform and in order to get an appreciation of this kind of deformation, we need to develop new quantities. So, we said the change in length divided by original length is normal strain. Change in the original rectangle, we called it as shear strain.

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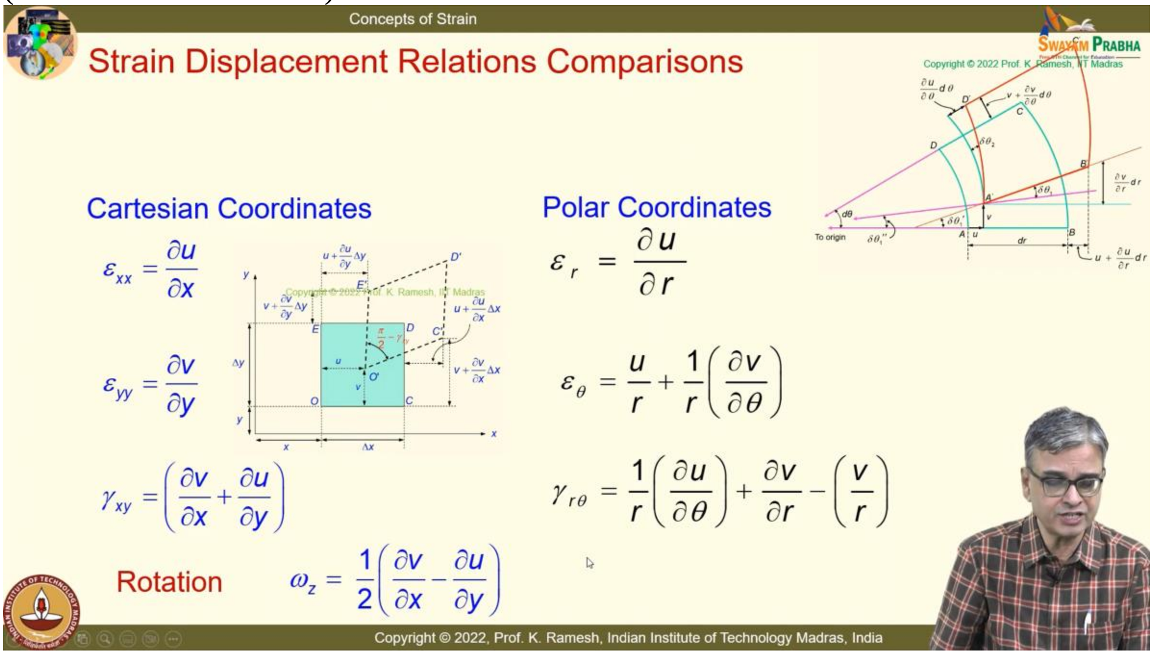

And you know, we have also seen it in Cartesian coordinates, we have also seen it in polar coordinates and then we have looked at how by using Taylor's approximation, these quantities can be written. So, we have seen parallelly

$\varepsilon_{xx} = \frac{\partial u}{\partial x}$ and $\varepsilon_r = \frac{\partial u}{\partial r}$

$\varepsilon_{yy} = \frac{\partial v}{\partial y}$ and I said whenever you have a θ, you will have $\varepsilon_\theta = \frac{u}{r} + \frac{1}{r}\left(\frac{\partial v}{\partial \theta}\right)$

but you should know how to remember this u/r. And similarly, you have

$\gamma_{xy} = \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$; the opposite differentials, whenever you have θ, you have $1/r$, this you can write, these two terms you can write; the third term minus $-v/r$ you have to know. $\gamma_{r\theta} = \frac{1}{r}\left(\frac{\partial u}{\partial \theta}\right) + \frac{\partial v}{\partial r} - \left(\frac{v}{r}\right)$

And you also have what is known as rotation. See, in fact, when we moved to bending, we saw this rotation. All comes from the displacement quantities, gradients of the displacements used suitably.

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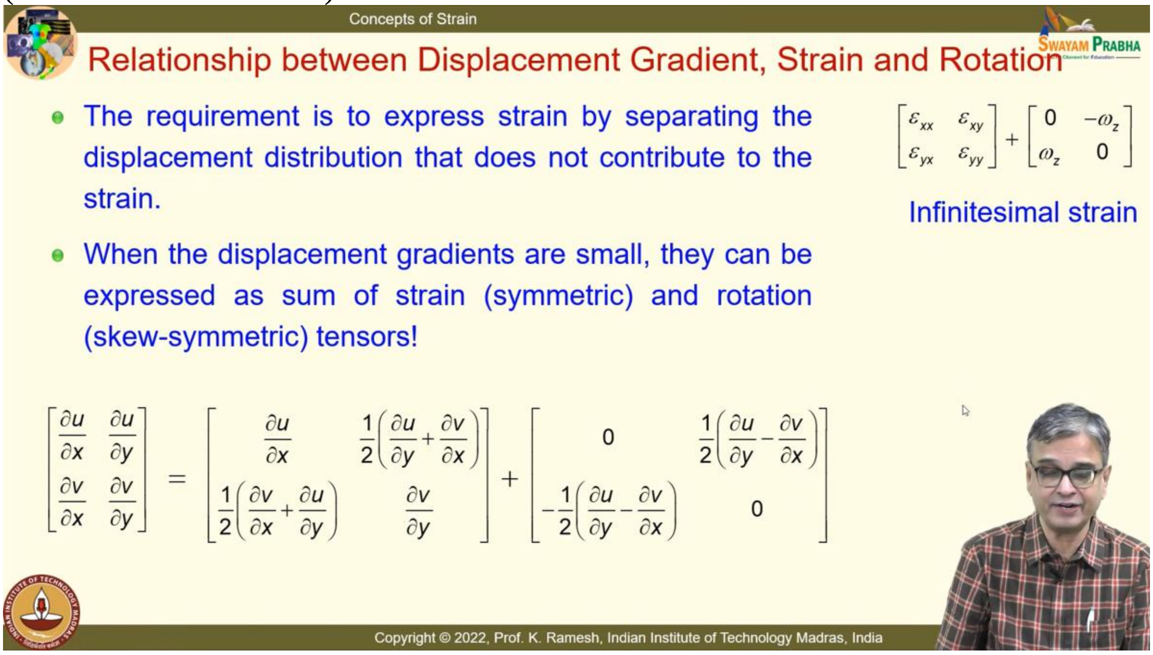

And we have also noted a very interesting relationship between the displacement gradient, strain and rotation. When you have small deformation, this can be put as sum; that is sum of a strain tensor and a rotation tensor. And we have also looked at, this is called infinitesimal strain, because we were looking at small deformation and we have looked at infinitesimal strain.

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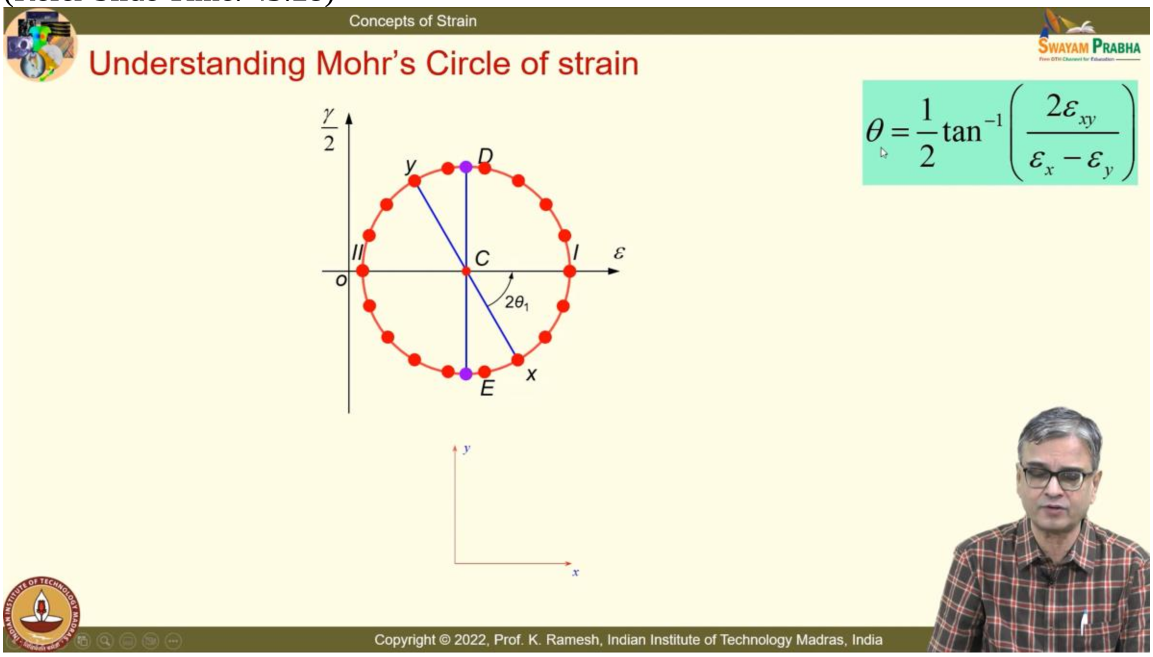

And you also have similar to Mohr's circle of stress, you have Mohr's circle of strain, the principles are same. Only thing is you have to put $\gamma/2$. I said whether you like it or not, people have developed many concepts using the shear strain as γ, but tensorially only ε_{xy} transforms. So, when you write it as a tensor, you write it in terms of ε_{xy}, when you do certain other discussion, we do it in terms of γ. The only difference is I have to put $\gamma/2$. Here again every point represents what is the strain happening along that direction. And the greatest advantage in the case of isotropic material is the principal planes of strain and principal planes of stress are identical, that makes our life enormously simple. And you know we have also looked at, what is the expression for the principal strain; same expression like what you have got for the principal stress, you also have this for principal strain.

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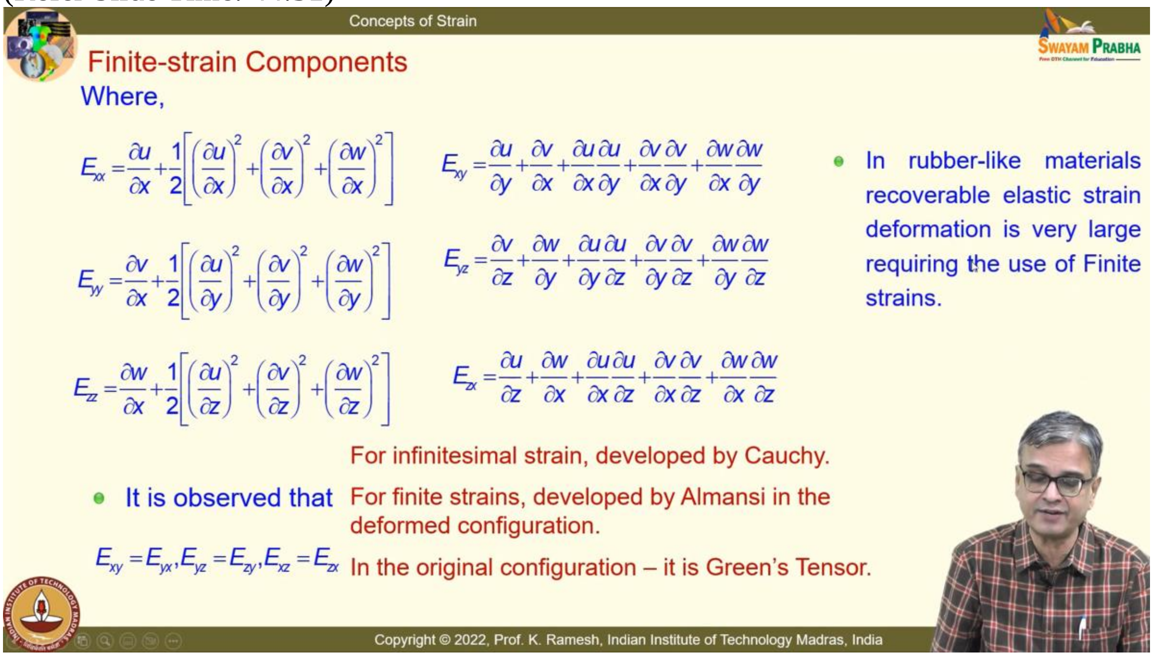

And we have also looked at finite strain components and we have said that the literature says that these are the quantities are exact. These become necessary when we want to go for large deformation. And this has been developed by several people; Cauchy, Almansi and you have Green tensor. And you have this in metal forming, because we started strain by looking at the forming of gudgeon pin which is what is used in the IC engine. Here you see the deformation so visibly, that deformation you see visibly, it is large deformation, only finite strain can help. And other applications, see, the future is biomechanics, in rubber like materials, recoverable elastic strain deformation is very large requiring the use of finite strains.

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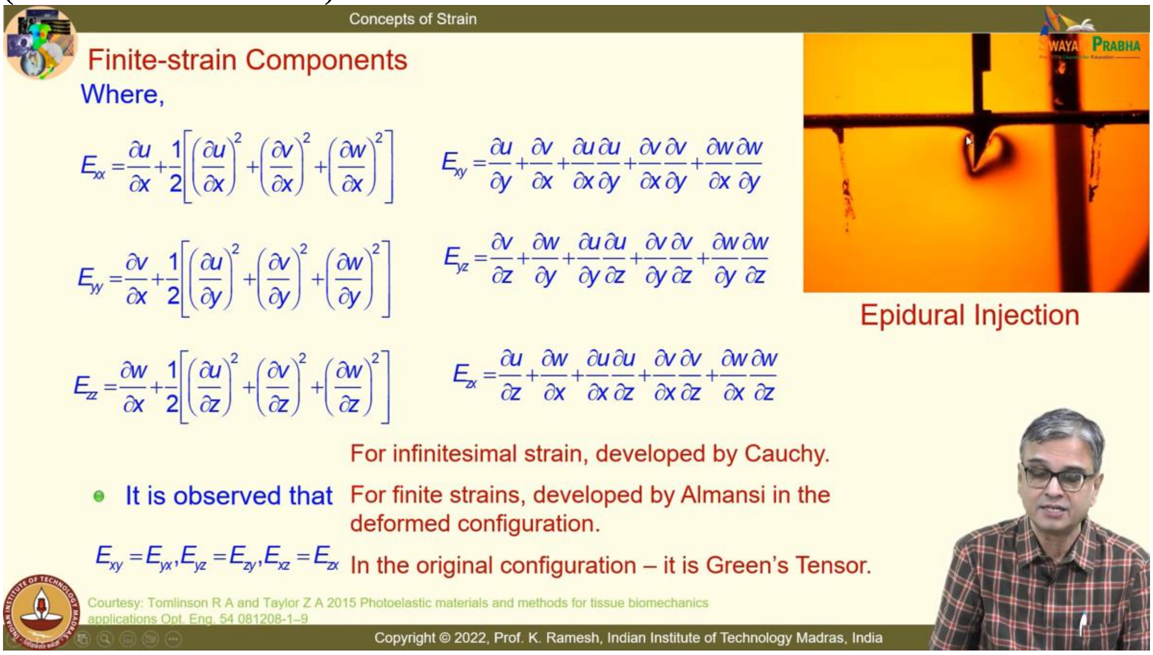

And I have shown use of photoelasticity for epidural injection and you have this needle is inserted deep into your skin. So, how the stresses are developed, if you want to mathematically analyze, you need to go for the finite strain quantities.

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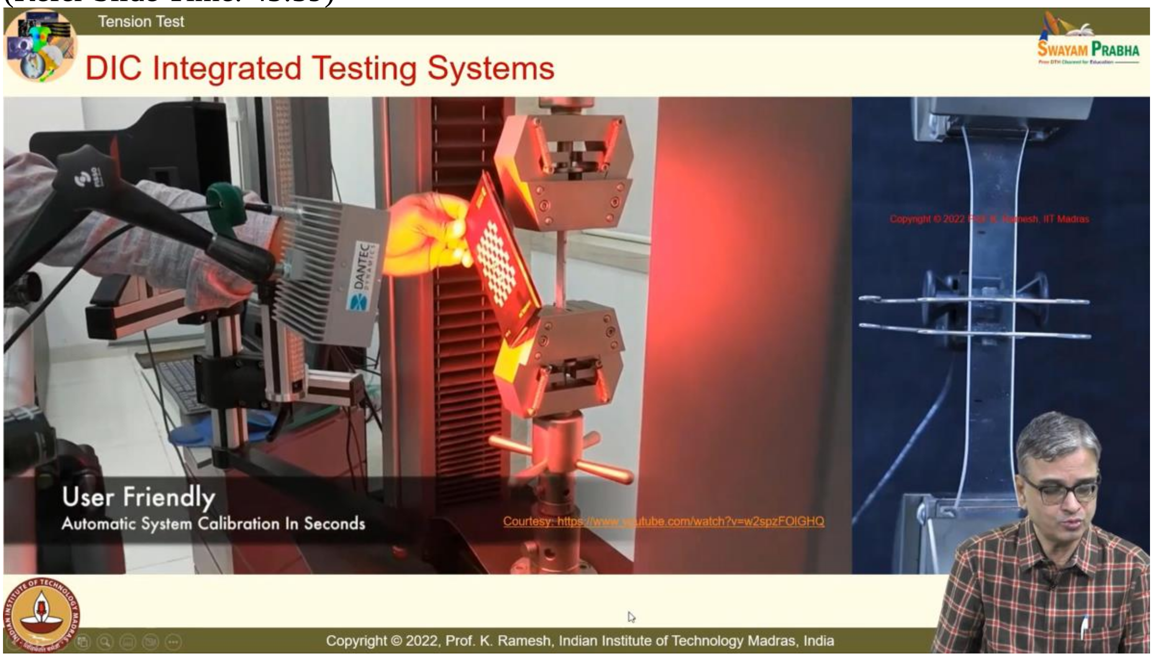

And we have also looked at use of DIC along with tensile testing machine for you to get the data and you can also get true stress and true strain.

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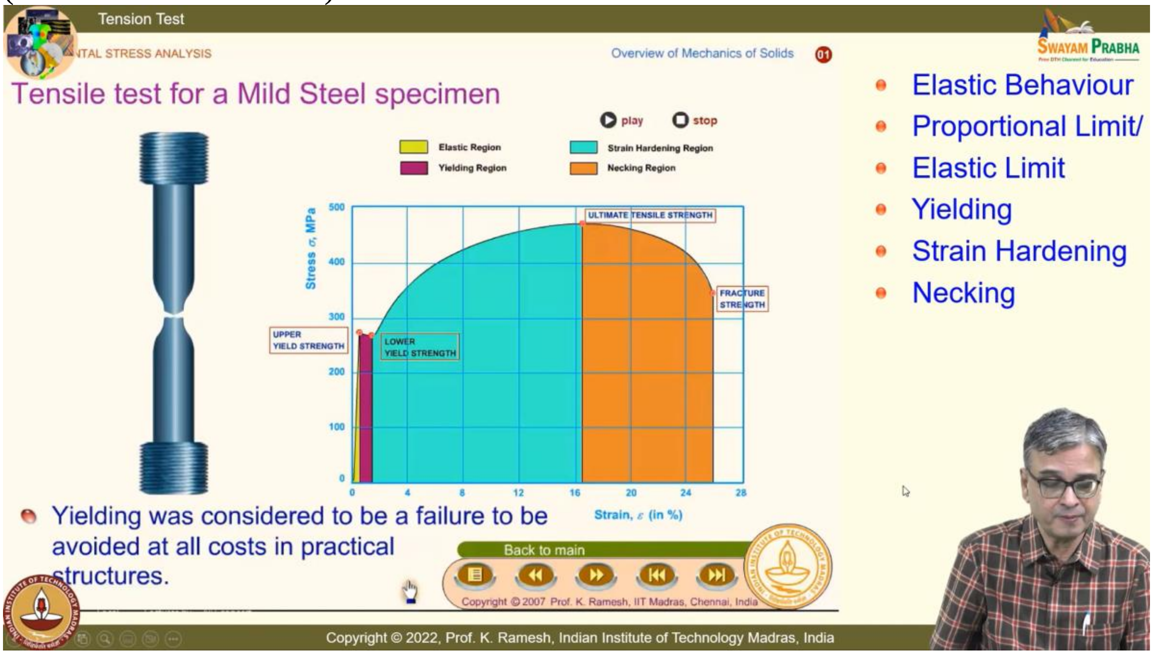

Now, when you have the simple tension test, there are many many things we have looked at elastic behaviour, we have looked at proportional limit, we have looked at elastic limit. And yielding is very prominent in the case of mild steel. You have a peak and then it drops immediately. And we have looked at the usable range is very very small and we said that this is about 0.2 % strain, all that is very important.

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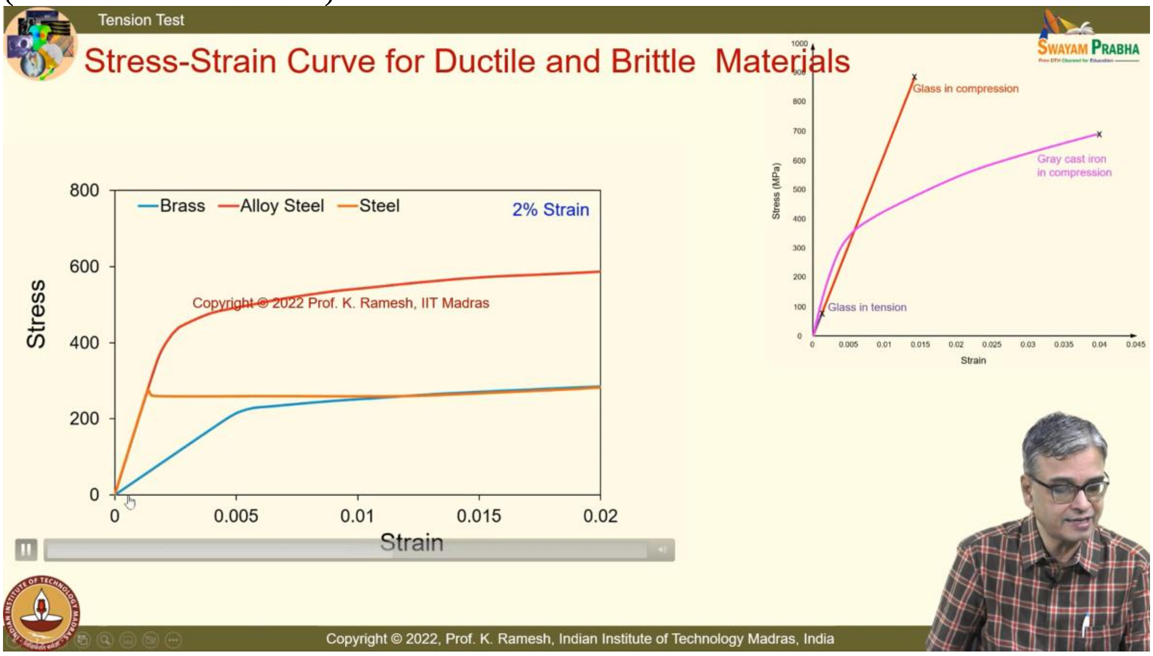

So, this is how you have the material fractures and we have also looked at what happens when I plot the same graph for ductile material with different percentage of strain, all your material models can be seen; elastic-perfectly plastic and then bilinear elastic and then elastic-strain hardening; all these are captured in the data. When you look at the strain levels that we want to work, these are applicable. And in contrast, what you find is, in the case of brittle materials, the tensile and compressive strengths are largely different; they are very strong in compression, very weak in tension. And also the strain levels, what they can reach maximum are also considerably small compared to ductile materials.

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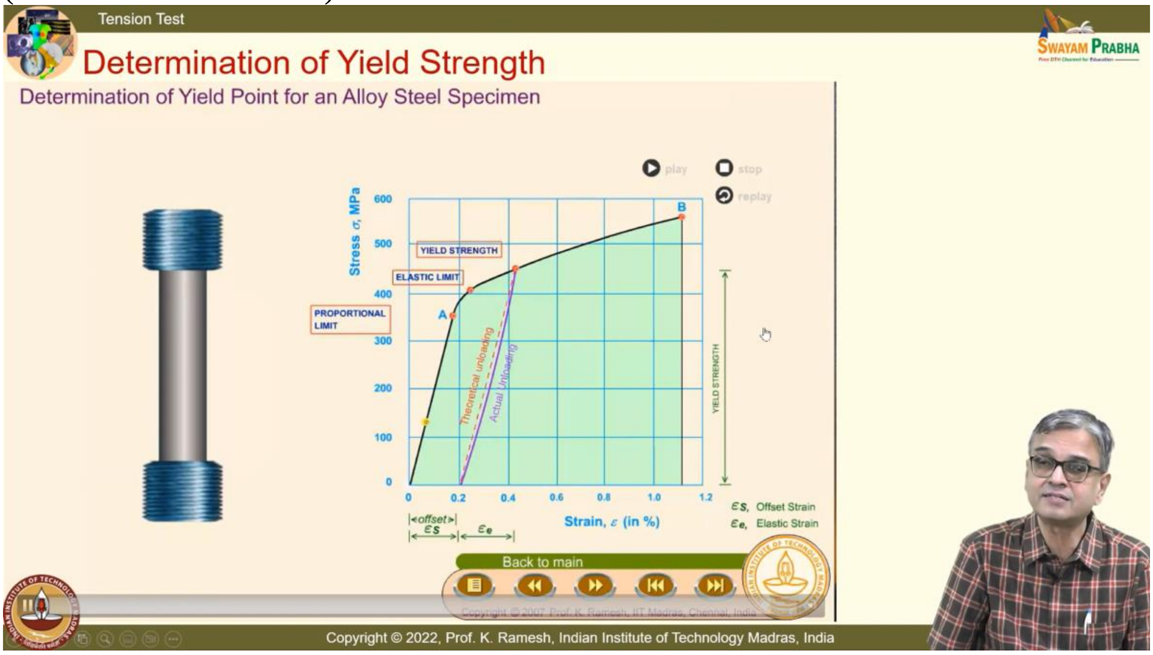

And the other important aspect is, if I do not have a clear depiction of the yield strength, the way to do is, you have at 0.2 % strain, you draw a line parallel to the original slope and then find out where it hits and you take that as the yield strength; this is an accepted practice. And when you have unloading, unloading will happen only here, when you reach this plasticity. So, you have elastic limit and proportional limit; all this is illustrated in this.

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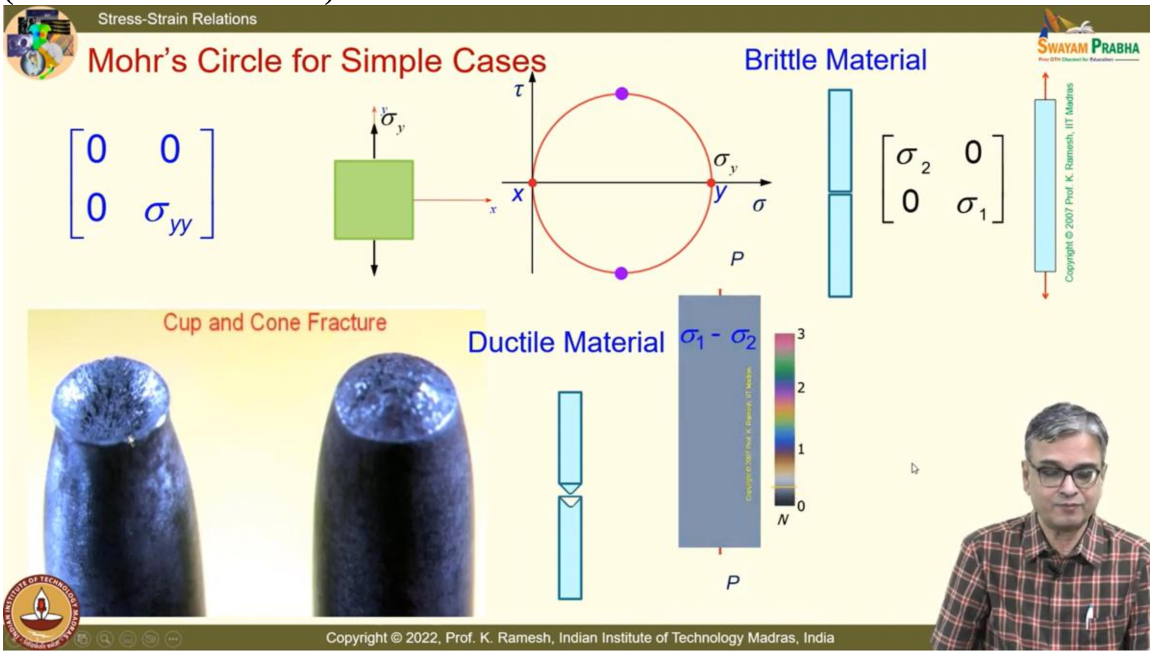

And in a ductile material, if it yields we say that it is yielding and we have also looked at for brittle material, how the failure happens and you are in a position to do that. We have said that this is maximum tensile stress theory and ductile material, it is at 45°; you recognize from Mohr's circle. So, failure happens at 45°, that is why you have a cup and cone fracture.

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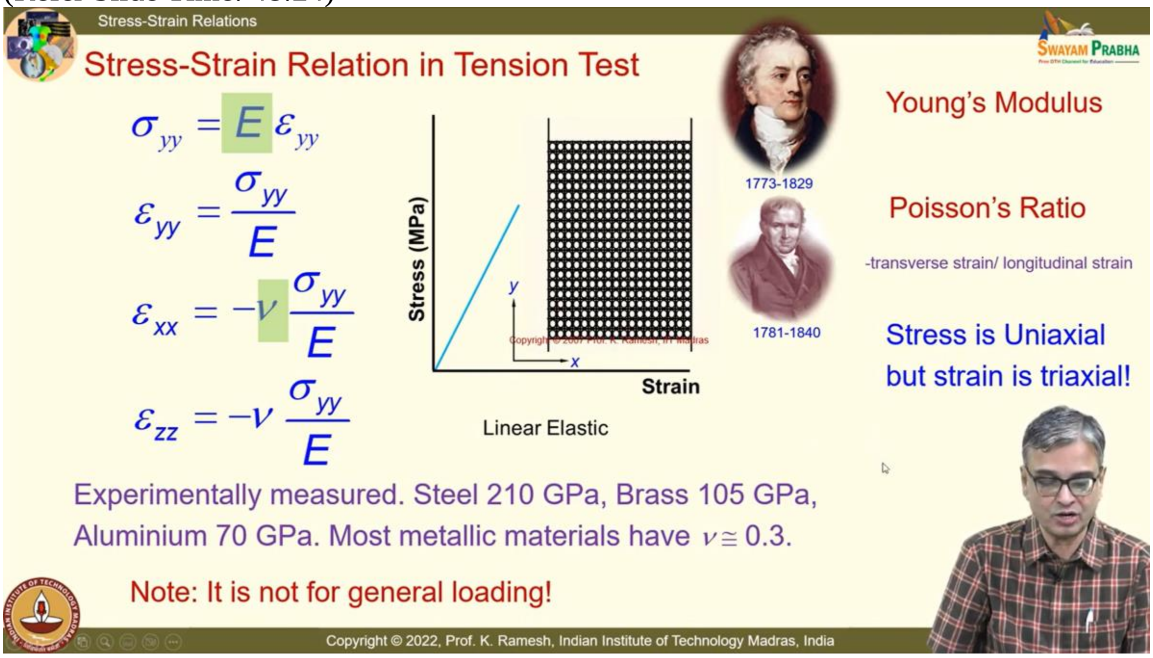

And we have also said how to label σ_1 and σ_2. And you know we have also learned what is elastic constant, Young's modulus, we have also looked at what is a Poisson's ratio.

When I say Poisson's ratio, it is $-\frac{\varepsilon_{\text{transverse}}}{\varepsilon_{\text{longitudinal}}}$. And we have also emphasized that stress is uniaxial, but strain is triaxial. And we also know the standard values of Young's modulus. In this class, you know we have looked at what is the basics with which we started, because some of the topics that we have discussed are very recent; probably you remember them better.

And the concept that we discussed earlier, we had a bird's eye view of how the subject has been developed. And we work on small deformation and slender members, never forget that.