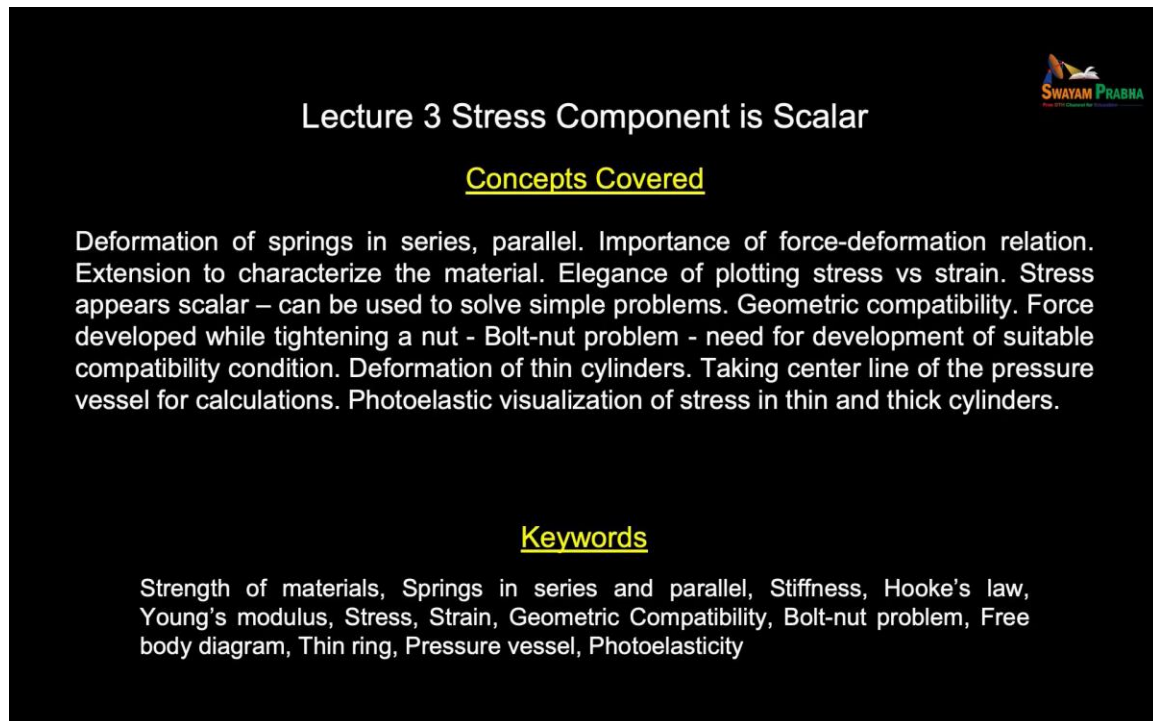


**Strength of Materials**  
**Prof. K. Ramesh**  
**Department of Applied Mechanics**  
**Indian Institute of Technology, Madras**

**Lecture - 03**  
**Stress Component is Scalar**

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**Lecture 3 Stress Component is Scalar**

Concepts Covered

Deformation of springs in series, parallel. Importance of force-deformation relation. Extension to characterize the material. Elegance of plotting stress vs strain. Stress appears scalar – can be used to solve simple problems. Geometric compatibility. Force developed while tightening a nut - Bolt-nut problem - need for development of suitable compatibility condition. Deformation of thin cylinders. Taking center line of the pressure vessel for calculations. Photoelastic visualization of stress in thin and thick cylinders.

Keywords

Strength of materials, Springs in series and parallel, Stiffness, Hooke's law, Young's modulus, Stress, Strain, Geometric Compatibility, Bolt-nut problem, Free body diagram, Thin ring, Pressure vessel, Photoelasticity

Let us move on to our next topic and understand little better, how do we comprehend deformation? Let us take certain simple problems and one of the main issue is, what is the compatibility condition? When you have connected members, when one member deforms, the other member also should deform appropriately. Only then, that compatibility of the system will exist, fine? So, we will take up certain simple problems and learn ourselves.

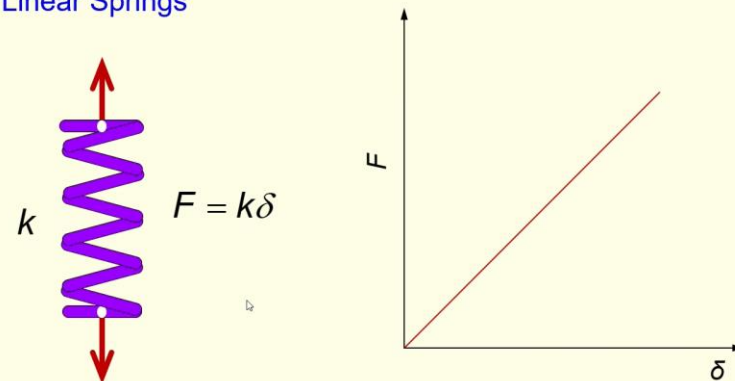
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Relation between Force and Deformation

**Mechanics of Deformable Bodies**

SWAYAM PRABHA

Linear Springs



The diagram shows a purple coiled spring with a red arrow pointing up and a red arrow pointing down, indicating force. The spring constant is labeled as  $k$ . The equation  $F = k\delta$  is shown next to the spring. To the right, a graph plots Force ( $F$ ) on the vertical axis and displacement ( $\delta$ ) on the horizontal axis. A straight red line starts from the origin and extends upwards and to the right, representing a linear relationship between force and displacement.

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We start with the simple linear spring. See, I have asked you to figure out from your background of rigid body mechanics, what an element of spring experiences as resistance; at least the force that is acted when I am pulling it. Anyone figured it out? If you have not figured it out, please go back and then find out what a spring is experiencing. It is not simple tension.

What aspect we exploit for our understanding of tension or torsion or bending from spring is, its linear relationship between force and displacement, and also its principle of recovery. Because spring, you can easily see, you can compress and release your hand, it comes back to its original position. Similarly, you extend it, it comes back to its original position. Because when you move from rigid bodies to deformable solids, your mind should be tuned to visualize displacements and deformations.

See, for you to do the thought experiment, these displacements have to be big enough for you to visualize. In reality, the moment you see displacement, it is large deformation. If you perceive it with your naked eye, then it is large deformation. Because if you look at a beam that is supporting this roof, it is deflecting. Fortunately, you and I cannot observe it. If you observe, you will be the first one to get out of the room, fine? So, you have to understand that we talk about small deformation.

But for us to write the equations and develop our thought experiment, we exaggerate these. So, you have to keep that in mind. And spring is not simple. So, I am giving you sufficient time for you to figure out what it is, fine? Please exercise your effort to do that. And you

also have a very famous relationship relating force to the displacement and you call this  $k$  as stiffness.

So, if you know the stiffness of the spring, you understand what is the force-displacement relationship, or when you make the force-displacement relationship, you can also find out the stiffness. When you make measurements of the displacement, you can also go back and calculate the stiffness.

(Refer Slide Time: 03:37)

Relation between Force and Deformation

**Mechanics of Deformable Bodies**

Springs in Parallel

Force Equilibrium

$$F = F_1 + F_2$$

Compatibility Conditions

$$\delta = \delta_1 = \delta_2$$

$$\therefore k_{\text{eff}} = k_1 + k_2$$

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And this you would have done many times in your JEE preparation, I suppose. I have two springs in parallel. Now, I pull this in a manner that this horizontal bar remains horizontal. It does not deflect, ok? I am pulling it in a manner. So, when I pull it in this manner, what happens? I can look at the free body diagram of each individual spring. I can have a force acting on it. Now, you have to tell me, what is the way forces are related and what is the compatibility condition. Can you, can you just write it down in your notebook and then check what I am popping up in the PowerPoint? Because this is fairly straightforward.

I have also said what way I am pulling the two springs. So, what is the compatibility? It is very obvious, isn't it? The displacement happening in both the springs are same, even though their stiffnesses are different. See, I have given a schematic. Do not say that this is of the same size like it is depicted. That is the way we can do it when you are making a diagram.

But I have labeled that the stiffness of spring 1 is different, stiffness of spring 2 is different. So, the force equilibrium is straightforward. The force is shared by spring 1 and spring 2.

The compatibility is, because the bar remains horizontal, both the springs have identical displacements. And I can also go and say, what is the equivalent stiffness of the system, fine? Can you write down what is the equivalent stiffness? Because this you are exposed to.

I am starting from what you already know, fine? And the  $k_{\text{eff}}$  you can simply say, it is nothing but  $k_1+k_2$ . So, once you see springs in parallel, the next step is springs in series, fine? So, we will take up that also.

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Relation between Force and Deformation

**Mechanics of Deformable Bodies**

SWAYAM PRABHA

Springs in Series

Force Equilibrium

$$F = F_1 = F_2$$

Compatibility Conditions

$$\delta = \delta_1 + \delta_2$$

$$\therefore k_{\text{eff}} = \frac{k_1 k_2}{k_1 + k_2}$$

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I have springs in series and recollect how do you draw the free body diagram. Whatever you have learnt in the rigid body mechanics, you must employ them here. So, brush up your fundamentals on free body diagram and look at what happens when I pull it.

When I pull like this, what is identical between the two springs? We have looked at force, we have looked at displacement. Now, you have to look at which one you have to worry about; how the compatibility is, fine? The compatibility was very simple in the previous case. And when I draw the free body diagram, I have it like this. I have spring 1 is pulled by force  $F_1$  and spring 2 is pulled by force  $F_2$ . Because they are connected in series, it transmits the same force, ok? But each one will have different displacements.

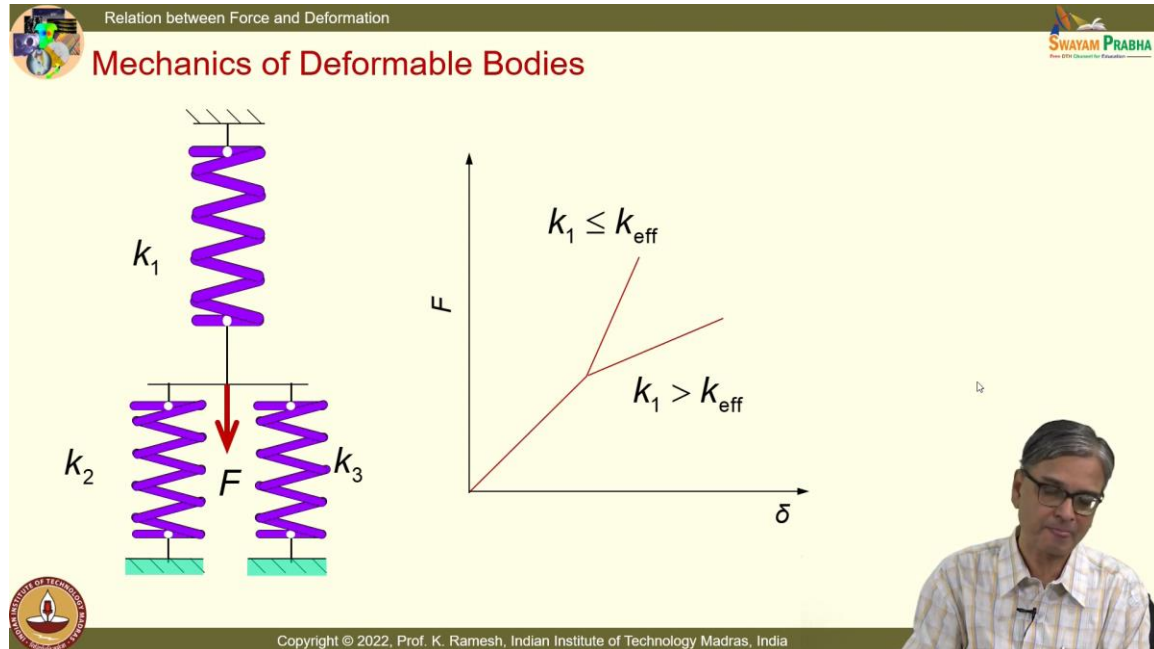
So, the force equilibrium is, each of the spring is experiencing the same level of force. But the compatibility is, I have  $\delta_1$  for spring 1,  $\delta_2$  for spring 2. And what happens at this end is

the summation of these two displacements. And how you calculate  $k_{\text{eff}}$ ? You will have

$$\frac{1}{k_1} + \frac{1}{k_2} = \frac{1}{k_{\text{eff}}}. \text{ So, if I simplify that, I get } k_{\text{eff}} = \frac{k_1 k_2}{k_1 + k_2}.$$

So, this is again straightforward. Now, let us look at one more system. You also have an equivalent problem in your tutorial.

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I have a collection of three springs and I have pulled this. It has come to this stage. Until this stage, it is not really looking at these springs. Only then, these springs come in contact. If I pull it further, what way should I draw the diagram  $F$  versus  $\delta$ ? So, here I have two springs in parallel. So, now, this is getting compressed. So, there are two possibilities that can happen;  $k_1 < k_{\text{eff}}$  and  $k_1 > k_{\text{eff}}$ . Then you can have a diagram like this. It is still linear in segments. See, we will again use it in advanced studies. In this segment, it is linear.

In the next segment, it is still linear, but with a different slope. That is how engineers handle complex situations. They do not jump into non-linearity straight. They will always find out methodologies to make it as linear, so that it is amenable for comfortable mathematical development. And next, we go on to real materials, fine?



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Relation between Force and Deformation

**Idealisations**

- Small deformation
  - Apply the equilibrium requirements to the *undeformed* configuration.
- Material is Homogeneous
  - Elastic property is same at every point
- Material is Isotropic
  - Elastic property is same along any direction
- Material is an elastic continuum
  - No defects!

Concrete cylinder – heterogeneous with pores etc.

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Because now, when I look at the real materials, we say that we have to have these idealizations. I am going to pop up these idealizations again and again, because repetition is also very much needed when you learn a course. We have small deformation, we have homogeneity, we say elastic properties are isotropic and then material is an elastic continuum. You must never forget these idealizations.

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Relation between Force and Deformation

**How to Characterise a Material ?**

- Consider rods of different cross sections made of a homogeneous material

Area:  $2A$     Area:  $A$     Area:  $A/2$

Load

Deflection

Robert Hooke 1635-1703

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And when I take a real material, how to characterize a material? That means what? If somebody gives you a material, I must perform a test. From the test, I should say, this is how the material will behave in actual situation.

The test should be simple enough and minimum for me to characterize a material. What Hooke has found out? He found out that many materials have a linear force-displacement relationship. Now, I take rods of different cross-sections made of same material. The only difference is, if I say that this is cross-sectional area  $A$  in this segment. This is like you know in the clamps, we have a larger section so that the material does not fail at the clamps. And the area is divided by 2 and area is doubled here. Suppose I pull these and plot the force versus displacement, how do you anticipate the relationship? How do you identify? Will they all lie in the same line? First thing is, you expect it to be linear as long as your deformation is small. I am not pulling this material to the level of breakage.

I am just elongating it. When I remove it, it will come back to its original position. So, I am really stretching it to small amounts, fine? And even Galileo found out that it is proportional to the cross-section, independent of the length. So, if I have stretched this cross-section  $2A$ , which will have the maximum stiffness, then you anticipate, this will have a lower stiffness and this will have the lowest compared to the three cross-sections that we have taken. And if you look at; this is what Hooke was credited and you have also seen that he determined this in 1660, but revealed it to public only in 1678.

This was his basic contribution. That means what? If you have to design a particular cross-section for a given application, you should have this force-displacement relationship. Only then, you will know its behavior. But you find, same material is used with different cross-sections. So, is it a comfortable way of characterizing the material? Definitely not, because for each cross-section, it demands one additional test. And if you look at the whole of engineering, when you look at experimental data, how do you represent the experimental data has given birth to new concepts, deeper understanding.

So, plotting a graph is not trivial. Many times, you know, you come to the laboratory exercise usually in the afternoon, because morning time, we want to keep theoretical classes. You are also very sleepy and then you know even before you do the experiment, you anticipate what is the result and then you are all very good in cooking, fine? Whether you cook your food or not, that is the casual attitude you take. But in reality, if you are given with a challenge that you have to design and you are going to be responsible if there is a failure, then you have to carefully design an experiment, carefully make measurements and interpret the results appropriately.

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Relation between Force and Deformation

SWAYAM PRABHA

## Speciality of Stress and Strain in Simplifying Material Characterisation

Maximum elongation not greater than 0.1% of original length.

Area:  $2A$     Area:  $A$     Area:  $A/2$

Stress ( $P/Area$ )

Strain ( $\delta/L$ )

Jacob Bernoulli  
1655-1705  
Switzerland

Final paper of his life in 1705 revealed that stress and strain are better quantities to depict deformation.

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### Is Stress a Scalar Quantity ?

So, what people did was, it has taken quite a bit of time. See, it is not instantaneous. We are going to see a person's complete research in his entire life towards his end got this idea, fine? It is no other person less than Bernoulli. We will see that. Now, what I am going to do is, I am going to repeat the same experiment, but I would plot my data differently. I have the same experiment; I will pull it. I will plot, in the  $x$ -axis, change in length divided by original length, and in the  $y$ -axis, I will plot force divided by area.

That is the only change I am going to do. All the other measurements are identical. What is striking is, you have the data collected differently in the previous set of graphs. They all lie in one line. That means, to characterize a material, just one experiment is sufficient. It is a very fundamental understanding. See, the concept that why should I plot change in length divided by original length and force divided by area, it was prompted by this kind of a development. And you also see the photoelastic fringes that you have this as constant color. And we have already noted that it should be small deformation. This is valid as long as my deformation is 0.1%. And this was credited to Bernoulli and you know when he got this? That was his last paper in his life. Imagine, and you all have a Bernoulli-Euler hypothesis for your bending theory. We all give lot of credit to him. So, some of these developments that look trivial are not really trivial.

People have struggled. Entire life is spent in understanding. He has contributed many other great inventions. But this is also like that. But what is the idea it gives? I have  $P/Area$ , we also labeled as stress. And when you say  $P/Area$ , it looks like a number, isn't it? But we have already said that in the chalk experiment, it is failing at different planes.



Unless I bring in the plane information, my understanding of the resistance is not complete. We will postpone that for the time being, fine? We will solve certain simple problems with this understanding. So, you should understand the final paper of his life in 1705 revealed that stress and strain are better quantities to depict deformation. It is again need based, because when I want to characterize a material, I would like to have least amount of effort in conducting a test. No matter what cross-section, you are able to do just one experiment and get the results out of it.

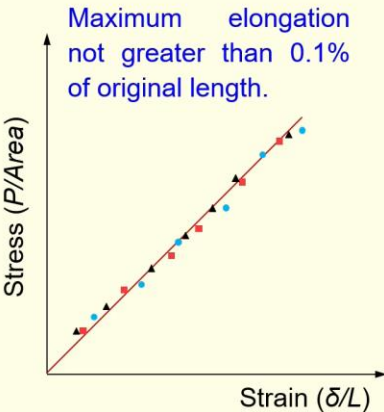
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Relation between Force and Deformation

SWAYAM PRABHA

### Speciality of Stress and Strain in Simplifying Material Characterisation

Maximum elongation not greater than 0.1% of original length.





The slope of this line is called the *modulus of elasticity* and usually denoted by the symbol  $E$ .

Proposed by Euler in 1727

$$E = \frac{P/A}{\delta/L}$$

$P/A$  is the average stress across the area  $A$  and  $\delta/L$  is the average strain along the length  $L$ .

$$\delta = \frac{PL}{AE} \quad k = \frac{AE}{L}$$



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And this was also summarized and people have, like we have  $F = k\delta$ , people also have put this;  $E = \frac{P/A}{\delta/L}$ . And this is famously known as Hooke's law. But if you look at history, this was actually proposed by Euler in 1727. And you call this  $E$  as Young's modulus, whereas Thomas Young got this understanding only in 1807. See, how the scientists have given credit to certain others is a puzzle to me, fine? If you look at the history, somebody has invented it, but in his honor, we all call this as Young's modulus.

And we also say that this is the Hooke's law. But it was actually proposed by Euler in 1727. And we will use this; I have this displacement as  $PL/AE$ , is a very useful relationship. In an axial pull, the deformation, the change in length is nothing but  $PL/AE$ . We will use this to solve certain simple day-to-day problems. And you can also find out the stiffness as  $k = AE/L$ .

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Relation between Force and Deformation

NATAL STRESS ANALYSIS

Overview of Mechanics of Solids 01

SWAYAM PRABHA

### Tensile test for a Mild Steel specimen

play stop

Elastic Region  
Yielding Region  
Strain Hardening Region  
Necking Region

Stress  $\sigma$ , MPa

Strain,  $\epsilon$  (in %)

UPPER YIELD STRENGTH  
LOWER YIELD STRENGTH  
ULTIMATE TENSILE STRENGTH  
FRACTURE STRENGTH

Back to main

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And in this course, I will again and again show this tensile test and today you must make a neat sketch of it. See, the idea is, you must also draw the axes and label them. I am drawing this till 28% of strain, that is very important. And in the y-axis, depending on the material, you know, this is about 275 MPa is the yield strength.

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Relation between Force and Deformation

NATAL STRESS ANALYSIS

Overview of Mechanics of Solids 01

SWAYAM PRABHA

### Tensile test for a Mild Steel specimen

play stop

Elastic Region  
Plastic Region

Stress  $\sigma$ , MPa

Strain,  $\epsilon$  (in %)

UPPER YIELD STRENGTH  
LOWER YIELD STRENGTH  
ULTIMATE TENSILE STRENGTH  
FRACTURE STRENGTH

Back to main

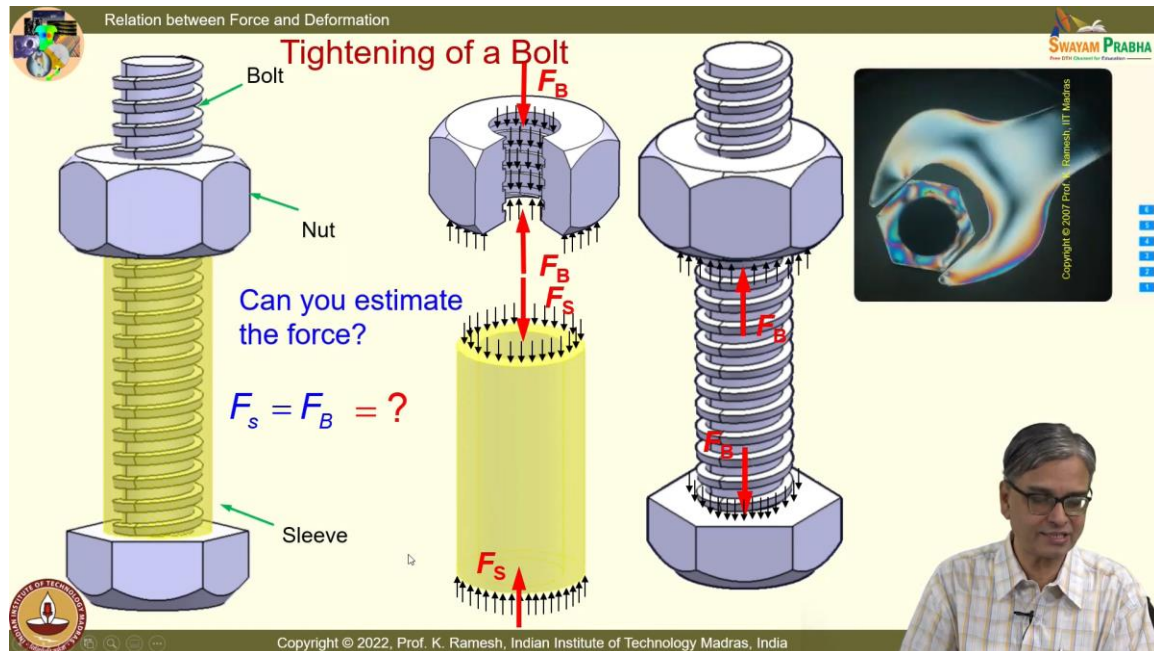
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So, what you do is, I have the final graph where I have red region and yellow region; that graph you make a plot. Please make a sketch, please make a neat sketch. Because what I find is, many of the books when they talk about engineering materials also, they do not label the axes; they simply say stress versus strain and draw the line without any numbers and you see only a straight line. And I want you to appreciate, if you take any of the engineering materials, when you do a tension test as per standards, you will always end up getting a non-linear relationship. And you have a red region which is very very small, which is the region where the material elongates and when I remove the load, it will come back to its original position. It remains like an elastic body, like a spring coming back to its original position. So, that is why this is put as an elastic region and the plastic region is very large.

This would be required if I have to design a crash bar. I would like to have a good plastic region. And many of the engineering components, they work together. So, they have to remain in an elastic region. So, from a practical perspective also, we will live in very small deformation in many of the applications. In some applications, we will have to go to large deformation, large plasticity and so on; but those instances can be handled as special cases.

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So, now we move on to another interesting problem. You know, we have seen in the earlier classes that you tighten a nut using a spanner, which you would have done in many of your toys, at least Meccano said they would have given you when you were young. If your parents would be happy, my son or daughter is going to become an engineer, fine? There you have done this tightening and we saw what happens to the spanner and what happens

to the nut. In this, what we want to ask the question is, what happens to the sleeve? I have a sleeve and I have a nut which is tightening this and I would like you to draw the free body diagram. This is nothing but an exercise on free body diagram. Can you put what is the force interaction for this? How do you expect? This is the sleeve.

I have a sleeve here and when I have the nut, I go and tighten it. You understand the problem? And when you tighten the nut, what you feel physically? You feel the resistance. That is number one. Other one, do you also feel that there is an axial movement of the nut down the bolt? You have observed it, fine? You have observed it. Can you draw the free body diagram for this? Can you draw the free body diagram? Quickly! Free body diagram is the bread and butter for all mechanical and naval architecture engineers, fine? So, I can visualize that there is a distribution of forces on this surface.

Similarly, there is also distribution of forces on this surface. They could be represented as a resultant force as  $F_S$ . And what happens to the bolt? Bolt also experiences the same thing. You will have forces on the bolt head. I will have forces acting on the bolt head. I will also have the forces acting on the nut, fine? And you can also find out the resultant. I would have this as  $F_B$ . And what happens to the nut? Nut has threads. And from your force equilibrium, you can only say  $F_S$  equal to  $F_B$ . And what happens to the nut if you look at? It is cut open and then shown as threads. So, I have forces acting on this and then you will also have resistance developed on the threads.

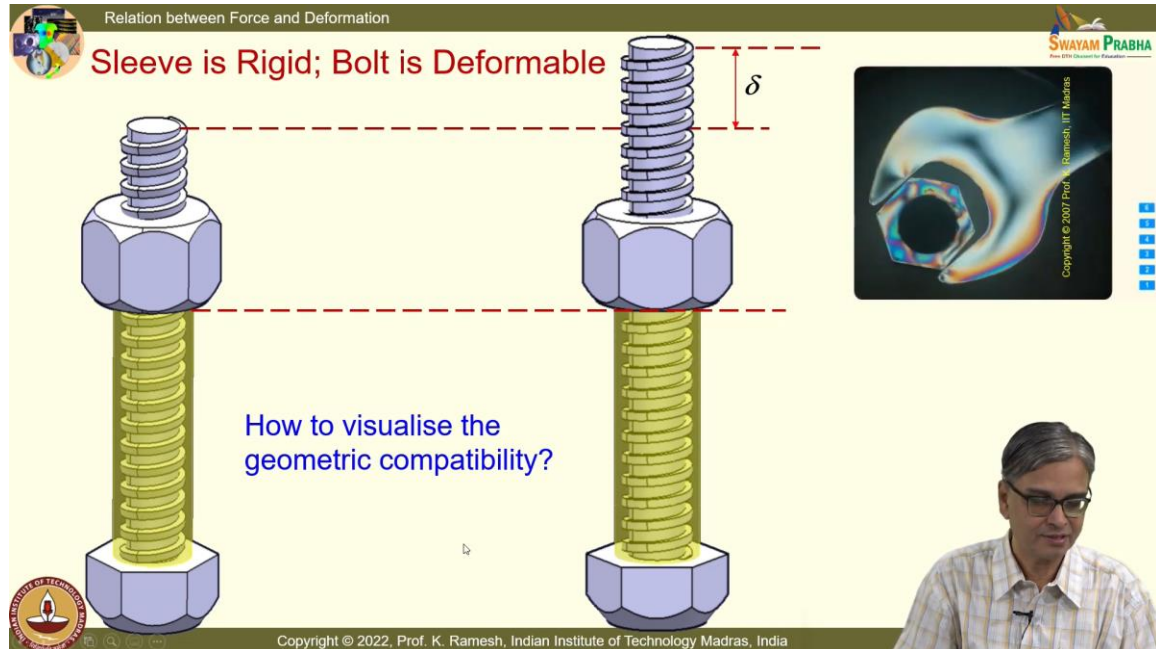
So, this also you can visualize this as  $F_B$  and this is what happens when you look at all the sub-systems. What is missing here? I have not put one important force. There is no torsion. We are only looking at after I rotated it, what is the situation? I am not looking at while I am rotating, how the forces are developing. No, we never do that mistake. We always say step 1 and then step 2. We look at the system after some time. In a free body diagram, what are the things that you have to take care? You have to look at the surface forces as well as body forces. See, the body forces if they are very small in comparison to the applied forces, you have the freedom to neglect it.

So, we have removed that here. We have not put any body force. Now, the question is, you have done one full course on rigid body mechanics; can you find out what is the force acting on the sleeve as well as the bolt? This is part of your tutorial problem. I am giving you hints. So, obviously, for a simple day-to-day problem, unless you bring in the deformation into the picture, you cannot proceed further. Is the idea clear? Now, how do you write? In the case of springs, it was quite obvious. It was quite obvious, springs are parallel, springs are series, you are able to do it.

Here, I have sleeve and bolt. The best way to proceed is you make one of the components as rigid, other component as deformable. Then visualize that component is rigid and this

component is deformable. If we do that, we can do a thought experiment. Yes or no? Ok, we will do that. So, the question is how to estimate the force?

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Now, we will consider sleeve is rigid and we will consider only bolt is deformable. See, engineers solve a problem and remove the second order effects implicitly. You should understand. Now, we will worry only about the sleeve and the bolt, we will not worry about the nut, fine? Ok. And I have this. So, when it is rigid, you must have observed that while tightening the nut, there is also linear movement of the nut.

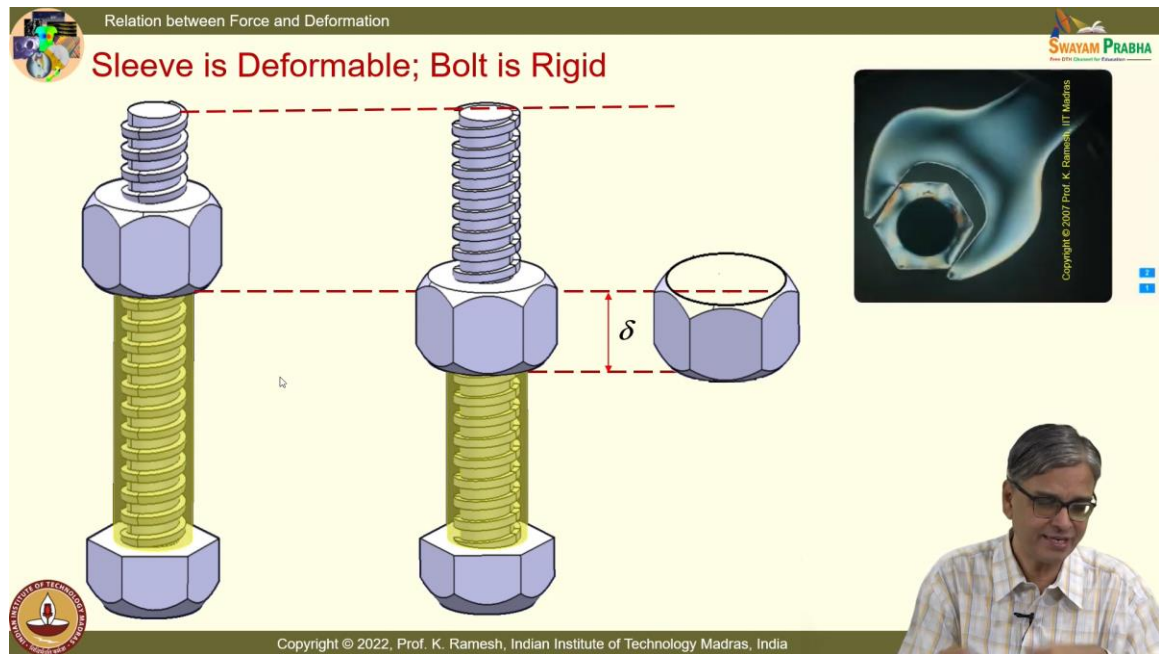
Where does this deformation come in? Suppose, I say the sleeve is rigid, it cannot deform. So, all that movement has to be perceived only in the bolt. Is that right? And I am showing very exaggerated displacement. See, these displacements will not happen physically.

For us to visualize, we should see them as big quantities. So, that is what you see here. The bolt has elongated by a distance  $\delta$ . Is it convincing to you? Because something has to go, because the nut has travelled a distance  $\delta$ . From where it has travelled distance  $\delta$ ? If a sleeve is rigid, it can travel only if the bolt has elongated in the process.

Is the idea clear? Now, we will do the reverse of it. We will make the sleeve as deformable and the bolt as rigid, ok? So, sleeve is deformable, bolt is rigid.



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So, here what you will see? Here again, you know, sleeve is deformable means when I rotate it, the nut would have moved down, fine? So, we will visualize that. So, it would have moved down and the complete deformation will now be borne by the sleeve.

That is also acceptable. But in reality, bolt is also deformable, sleeve is also deformable. So, what happens is while you are tightening it, the sleeve is getting compressed and bolt is also getting elongated simultaneously. What happens to the nut? We are treating that as rigid for our convenience. Even if we do not treat it as rigid, look at the height of the sleeve, look at the height of the bolt.

In comparison to those lengths, the nut is very small. So, even if there is something happening, it will be very small. As engineers, you simply take that as rigid and carry on with it, because engineering is approximation; intelligent approximation that validates your observations. It is not that you take an approximation and hold on to it blindly, fine?



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Relation between Force and Deformation

Both Sleeve and Bolt are Deformable

Geometric Compatibility

$\delta_1 + \delta_2 = \delta$

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So, both sleeve and bolt are deformable. So, what happens is there is an elongation of the bolt and there is a contraction of the nut. What is the compatibility condition now? The nut has moved a distance  $\delta$ . Why  $\delta_2 - \delta_1$ ? Why should you have a minus there? I have given you, for you to visualize physically also similar proportions. Geometric compatibility is,  $\delta$  is  $\delta_1 + \delta_2$ . See, these are the subtle things.

I have given you a via media for you to visualize. Please go back and ponder about it. Get yourself convinced, fine? And if you have a new situation, you may have to bring in deformable solids. Do not jump on to it. See whether you can make certain things as rigid, certain things as deformable. Stage by stage you try to analyze. So, unravel what is the connectivity. Is the idea clear? Ok. And this is part of your tutorial problem.

(Refer Slide Time: 31:02)

The slide illustrates the concept of internal pressure in a thin ring. It features a Coca-Cola can, a wooden wheel with a steel rim, and several diagrams. One diagram shows a thin ring with internal pressure  $p$ , thickness  $t$ , radius  $r$ , and width  $b$ . Another diagram shows a cross-section of the ring with internal pressure  $p$  and thickness  $t$ . A color scale for stress  $N$  is shown from 0 to 3. A video inset shows Prof. K. Ramesh speaking.

And now we move on to another simple problem which is again easy to handle at this stage without further development of concept of stress. Many of you would have taken a Coca-Cola can and that is what you see these days. And it is a very thin sheet of metal. Once you have taken the liquid out of it, you can squeeze it with your hand. It is actually stable because of the internal pressure. So, it is a very thin cylinder. And I have already given you a clue. I have shown what are the fringes here and what are the fringes in the thin cylinder.

I have shown it as slightly thick for you to see. I cannot have a simple line and then ask you to see the color. Only if I show some width, you will be able to see it. Even before we go and solve the problem, you have a clue. Can anybody say what are the stresses developed? What way you anticipate the stresses developed in the cylinder? Is it uniform? Because that is what you can see.

Because if I have uniform color in tension, I see uniform color from photoelastic fringes. Now, we have to go to mathematics and then get it done. If you actually solve this, it is not a thin ring but it also has closed ends. I have taken this because that is what you see these days. Ideally, I should have taken a problem like this.

You know, when I grew, we used to see horse driven carriages like this. They will have a wooden wheel and if they want to have long life for the wheel, they will put a steel rim. So, what they do is, they heat the steel rim and then put it on it. So, it is a, when the temperature is cool, it grips the wooden wheel. It enhances the life of the wooden wheel. In fact, we are solving a problem like that. I have a thin ring subjected to internal pressure.

I have shown this because this is what you are able to see these days easily. May be in old movies, you will see a carriage driven by horses, ok?

(Refer Slide Time: 33:21)

Relation between Force and Deformation

**Thin Ring Subjected to Internal Pressure**

Symmetry demands  $F_R = 0$

By Force Equilibrium

$$\sum F_y = 0 \quad \therefore \int_{\theta=0}^{\theta=\pi} (\Delta F_y) - 2F_T = 0$$

$$\Delta F_y = \Delta F_p \sin \theta = p[b(r\Delta\theta)] \sin \theta \quad \therefore F_T = prb$$

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So, we want to solve this. So, we will go and do the free body diagram and we also visualize that when there is pressure, the cylinder expands, fine? Now, I draw an imaginary line along the horizontal axis and I say that this radial displacement as  $\delta_R$ . My interest is to calculate  $\delta_R$  with the background that we have developed so far. We have not developed the concept of stress or anything like that in great detail, but this is sufficient to solve this problem. So, can you draw the free body diagram? You start drawing it. I will also show it on my screen and you can verify whether you are able to. Because when I put an imaginary line and then cut it half, when I remove that section, you should put appropriate forces on that to be determined. That we have also shown. Suppose I have a member, if I have an imaginary section, what all can happen? Please make an attempt, make mistakes, no problem.

If you do not make an attempt, you will never learn it. You will have to make an attempt. Then see what I have got on my screen, then your understanding improves, ok? And taking proper notes is also very important. So, I have shown that when I have this, in general, I can have a force and a moment and all that. Now, because of simplicity, I am considering only two forces, one is  $F_T$  and another one is  $F_R$ , fine? And if I look at this, you know, we have been talking about symmetry. First of all, this structure is symmetric, loading is symmetric and we have assumed the elastic properties are symmetric.

That is what we said as isotropic, fine? So, everything is symmetric here. See, if one of these is not symmetric, you cannot bring in symmetry argument. You should have the structure should be symmetric, boundary condition should be symmetric, loading should be symmetric, elastic response should be symmetric, then you can bring in symmetry arguments. By symmetry arguments, what you can do? I can remove the radial forces because it is trying to push it down here, it is trying to pull it out at the bottom.

And we followed Newton's third law. If I assume this, this is automatically action and reaction pair. Once I have this, the problem is solved. Can you take a small segment here and then try to write the equilibrium of that? So, you follow whatever you have learnt in your rigid body mechanics. All that we will follow here. We will also have to bring in deformation picture for us to have additional equations, fine? Can you take a simple section? I have removed these radial forces and I have shown this reference axis.

I have taken an element at an angle  $\theta$ . These are all drawn very big for you to appreciate. So, this is  $r\Delta\theta$  gives this arc length and we have also taken a thin ring which is of width  $b$ . I have a force represented here, I call this as  $\Delta F_P$ . You have to write what is  $\Delta F_P$ . I have the internal pressure is given as  $P$ , ok? And we have taken a thin ring of width  $b$ , ok? So, I have this as pressure into area; that is the force. So, now what I have to do? I have to integrate from  $\theta$  varies from 0 to  $\pi$ . If I balance this segment, I have the necessary expression for what is the force  $F_T$ . So, I can put this as  $\Delta F_x$  and  $\Delta F_y$ . And I can write the force equilibrium,  $\sum F_y = 0$ , and I have

$$\int_{\theta=0}^{\theta=\pi} (\Delta F_y) - 2F_T = 0$$

Is the idea clear? This is very straightforward. We have taken a simple problem and then you substitute what is  $\Delta F_y$  and this is nothing but  $\Delta F_P \sin\theta$ . So, I get this as  $p[b(rd\theta)]\sin\theta$  and you can easily do the integration. I get this as  $-\cos\theta$  and put the limits, I get the famous expression  $F_T = prb$ . In fact, people find out the stress, I can also find out the stress, stress is nothing but force divided by area. Here, if I have the thickness as  $t$  so I will divide it by  $bt$ . So, I will have this as  $pr/t$ , and  $pr/t$  is a very very famous expression. But our ultimate goal is what? We want to find out what is the radial expansion of the pressure vessel, thin cylinder, ok?

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Relation between Force and Deformation

**Thin Ring Subjected to Internal Pressure**

**Force-Deformation Relation**

$$\delta_T = \frac{F_T 2\pi \left( r + \frac{t}{2} \right)}{(bt)E} = \frac{2\pi pr^2}{tE} \left( 1 + \frac{t}{2r} \right)$$

**Geometric Compatibility**

$$\delta_R = \frac{\delta_T}{2\pi} \quad \frac{t}{2r} \ll 1$$

$$\delta_R = \frac{pr^2}{tE} \left( 1 + \frac{t}{2r} \right) \sim \frac{pr^2}{tE}$$

$\delta = \frac{PL}{AE}$

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So, I have this  $F_T$  acting like this and if I completely unwrap this; for the paucity of space, I cannot show you the complete length.

So, I have put a cut here and this is of the length of the complete circle. So, I will have this as  $2\pi r$ ; to make it little more accurate,  $2\pi(r + t/2)$ . But engineers will always knock out the last term, fine? Since you are not very much into engineering, I want to put step by step and finally knock it off, ok? So, what we are doing is, we are taking an equivalent radius that is at the centre of the thickness. So, this is nothing but what? I have a simple strip pulled in tension. Is the idea clear? And tension you know, we have already seen what is the expression for  $\delta$ . I can find out the axial deformation, what we have seen? What is the expression we have seen earlier? Famous expression as  $PL/AE$ .

So, substitute for  $P$ , here it is  $F_T$  here, ok? What is the length? Length is also given. What is the area? Young's modulus, everything is known. So, I can find out what is the elongation of this, ok? I put that elongation as  $\delta_T$ . So, force is  $F_T$ ,  $L$  is  $2\pi(r + t/2)$ , area is  $bt$  and then Young's modulus  $E$ . So, if I substitute this, I get this as

$$\delta_T = \frac{F_T 2\pi \left( r + \frac{t}{2} \right)}{(bt)E} = \frac{2\pi pr^2}{tE} \left( 1 + \frac{t}{2r} \right)$$



But this is not what we wanted, we wanted the radial expansion. What is the interrelationship between axial; the circumference is increasing by this, how do I find out the radial displacement? Simply divided by  $2\pi$ , fine? So, you have to bring in all your analytical geometry.  $\delta_R$  is nothing but  $\delta r/2\pi$ . So, I get this expression. I get this 
$$\delta_R = \frac{pr^2}{tE} \left( 1 + \frac{t}{2r} \right),$$
  $r$  is so very high, thickness is very small, I can put this as  $\frac{pr^2}{tE}$ , again a very famous expression. See, we say  $t/2r$  is very very small. So, I can neglect that, ok? And you know, as a utility for this, you have a very interesting problem. There are many ways you can make a pressure vessel, fine? And let me have a look at it.

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Relation between Force and Deformation

**Tutorial Problem**

- A water pipe is made of longitudinal wooden staves held together with circumferential steel bars of 25.4 mm diameter, as shown in the sketch. The pressure of the water in the pipe is 689.5 kPa. Because of the danger of leakage between the staves, the diameter  $D$  of the rod centerline cannot be allowed to increase more than 0.762 mm due to water pressure. Estimate the maximum allowable longitudinal spacing  $s$  between the circumferential rods.

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We will use this knowledge to solve a practical problem. See, if you go to a book, the book problem will be only like this. So, if you read this, a water pipe is made of longitudinal wooden staves. I have a wooden stave like this. You know, I want to make a pressure vessel with local available materials and it should also be strong enough to withstand whatever the liquid that is pressurizing this. And what is done is, you assemble all of them, that is not very clear from this, which is very clear from this animation, fine? That is the advantage when you have this, ok? So, I have individual wooden strips and what is done is, you tie by a steel ring like this. The question is, what should be the distance between the two successive steel rings, which is essential for me to transport the liquid without any leakage. And what the problem says? The problem says, what is the size of this steel rod, that is 1 inch, that is 25.4 mm diameter. Pressure in the water pipe is also given as 689.5 kPa. And



from a design perspective, what is the allowable increase in the central line? Not more than 0.762 mm. That is given as a design parameter.

We have got the expression for  $\delta_R$ . So, now, what is given here is the central line difference. So, I have to find out what is  $\delta_R$ . So, I know all these parameters. So, now, what you have? You have a fluid which is going inside. What essentially you have to look at is, as if wood is not supporting any load, it is only the steel ring that is supporting the load.

That means, what? You have to find out, take  $s$  as part of your problem. So, you have pressure acting over a length  $s$  that is transmitted to the steel ring, fine? See, as a designer, you have to find out whether it should be 0.762 mm or something else. That you do not have to do, that is given in the problem. So, it is a very nice problem where whatever the knowledge that you have gained as what is the nature of resistance developed in the pressure vessel; here again, we have a nice thin ring. And you also have expression for what is the radial expansion. With this, you will be able to solve the problem, fine? It is part of your tutorial, ok?

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Then we move on to an observation. You know, I have always said that photoelasticity is very user interactive and you have lot of understanding is possible. Can you look at what happens in a thin cylinder? That is what we have looked at. We have got the fringe pattern as constant and we have also got the mathematical expression to be constant.

It is not a function of radius. It was independent of the radius. On the other hand, when I have a thick pressure vessel, what do you see? What way you can relate? What are the

comments that you can make about the resistance developed in the thick ring? What is that you see? Like you have seen in the case of beam, you see all the fringes here. Nothing is vanishing. It is not one color; it is different colors. And nature is also very nice. In the case of beam, you have parallel lines, fine? You may find it difficult to draw parallel lines because these days you do not use mini drafters, you directly get into AutoCAD. You do not have the feel of how to draw a parallel line comfortably, isn't it? You escape that. So, nature loves beautiful geometrical patterns and here what you find? These fringes are concentric circle. Can you make one more observation? Here I have shown 0, 1, 2 and 3. I have these fringes. What are the distances between the fringes? Is it uniform like what you had seen in the case of a beam? It is not uniform.

So, it is different. So, it is non-linear. In fact, if you have to solve this problem, you have to go to the second level course where you will have, it is called a Lamé's problem. You have a stress function and then you solve it. But even before you could get into complicated mathematics, it is easier for you to find out from this inference, what way the stresses could be, ok?

And with this, we have a bird's eye view of what happens in a thin cylinder and we have also understood what is the force-deformation relationship. And if you have to go and handle any new problem, make some of the aspects as rigid for you to visualize. From that, you find out what is the interrelationship and carefully write the compatibility condition. You cannot write it arbitrarily, ok? Thank you.