Strength of Materials
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Lecture - 29
Bending 7 - Unsymmetrical Bending and Combined Loading

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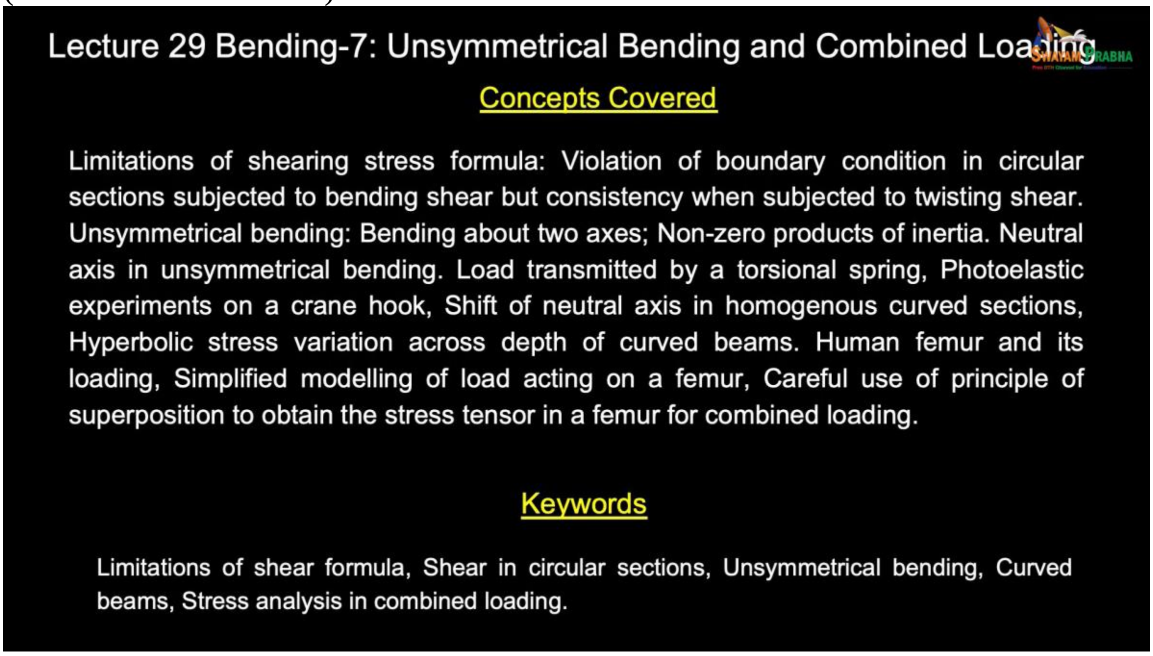

Let us continue our discussion on problems related to bending.

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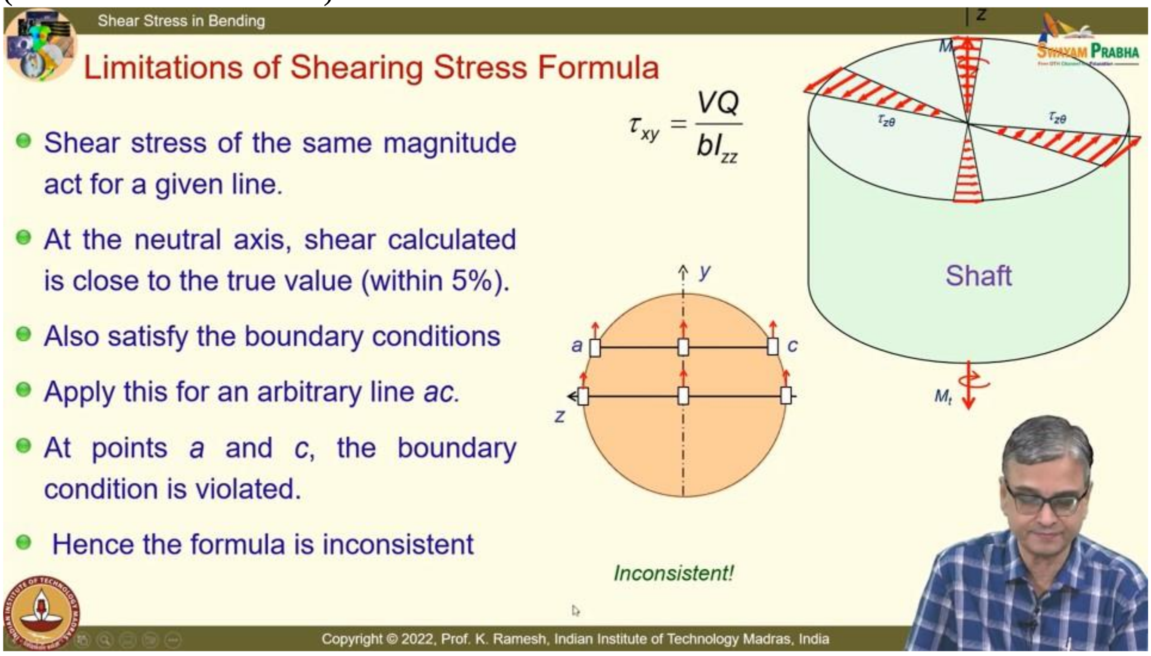

We would also see some very subtle concepts related to bending. What are the things that we know from our simple bending theory? What other aspects that you need to learn in advanced courses? See, one of the commonest example of a beam under bending is a shaft. A shaft transmits torque; because the weight is supported, it is also transmitting bending moment and shaft is a circular cross-section. And let us look at, we have got $\tau_{xy} = \dfrac{VQ}{bI_{zz}}$ as the expression for the shear stress. How far is this applicable to a circular cross-section? When you have this, the understanding is shear stress of the same magnitude act for a given line.

The magnitude does not change. So, if I take the reference axis like this and you all know the neutral surface carries the maximum value of shear stress. So, if I want to calculate the Q for this, I can get the shear stress acting on this line. And this statement says, whatever the magnitude that I have, this is constant for the entire length of the line.

And we have discussed at length, what are free surfaces? And you know from the loading of the beam, the complete outer boundary is free. Is the idea clear? That is a very important aspect. And when I have shear stress acting in this direction, this also satisfies the boundary condition. There is no issue because this is tangential to the surface at these points. There is no contradiction there.

And if you actually do the theory of elasticity calculation, the results show that whatever the results we get out of this is accurate within 5 %. So, that is a good estimate. Suppose I take an arbitrary line *ac*, I am going to use the same expression and I am going to get similar results for this line also. And you will have to identify where we have a catch. For me to get the Q, I have to calculate the first moment of area of this green portion.

And then when I do this, this expression gives me only the components like this. Do you find any contradiction at this stage? If you look at what is the tangent that you draw here, it does not coincide with the tangent. So, you cannot have a stress like this. So, that is the contradiction! The boundary condition is violated at points *a* and *c*. So, if the boundary condition is violated, we can only say the formula is inconsistent.

And what is the other situation that you have come across shear transmitted by a shaft? You have seen what happens in the case of a twisting moment. In the twisting moment, what happens? You have shear stress varying from maximum at the outer boundary to zero at the center and this happens all around the periphery. See because it is written in an isometric view, you do not see that this length is same as this because of the drawing, you know, requirement, this appears small. So, you have this tangential and this is tangential all around the boundary. That is why you call this is also $\tau_{z\theta}$.

So, there whatever the solution that we have got, this is the free surface. The free surface requirement is completely satisfied in our torsion development. There also you saw shear stress being transmitted by the shaft. Here the component is different. Component is τ_{xy} here and you find that it is violated at points *a* and *c*.

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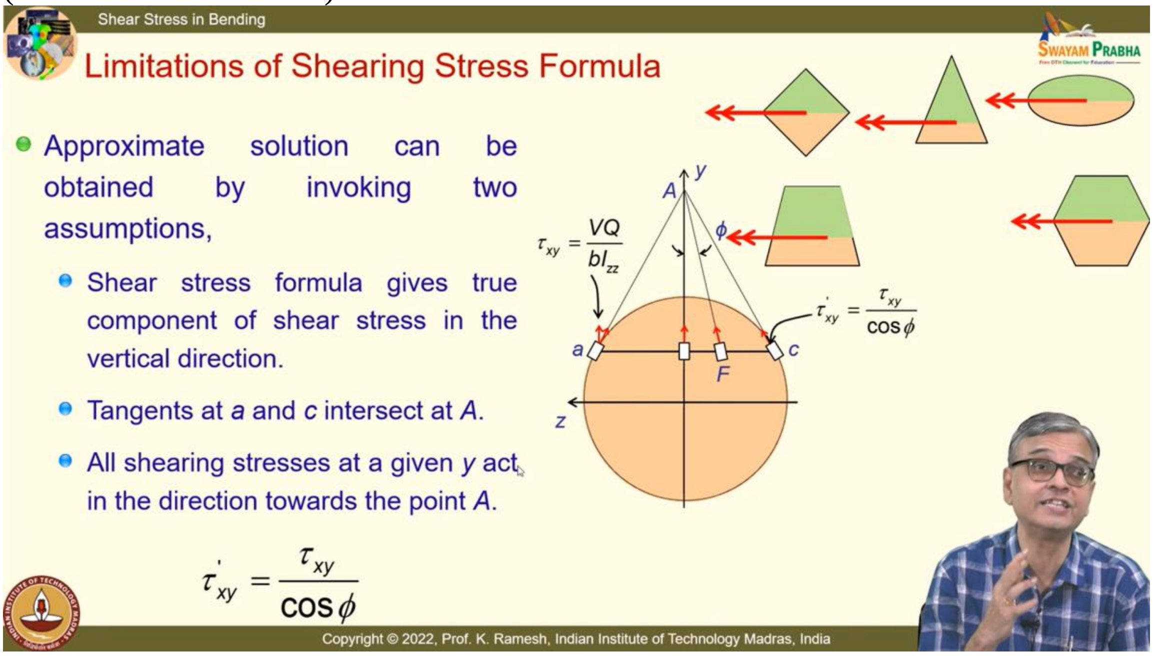

So, we have to wriggle out of the situation. How to wriggle out of the situation? So, we are going to look at an approximation. Shear stress formula gives two components of shear stress in the vertical direction. We can take it that way, but that is not what is possible here. To satisfy the boundary condition, it has to be tangential to the surface.

So, they intersect at *A*. All shearing stresses at a given *y* act in the direction towards the point *A*. So, this is what is the approximation that you make. So, instead of taking the value like this, this is what you get from your shear stress formula. You take the value that is tangential to that and you say that this $\tau'_{xy} = \frac{\tau_{xy}}{\cos\phi}$.

So, it is an approximation. See, the way you have to look at is, if at all we want to find out the shear stress importance in bending, it reaches maximum at the neutral surface. For the circular shaft at the neutral surface, the result is very close to what is predicted by theory of elasticity. The distribution is just to satisfy ourselves, how the force is distributed. For all our practical calculation, we will actually find out only at the neutral surface or the neutral axis, whichever way you do it.

So, there is inconsistency in the shear stress formula. We do not have to worry too much about it and we have also seen another aspect. If the beam is slender, the magnitude of shear stress away from the loading points are very small compared to bending stresses. So, from that count also, even if you make a calculation mistake in the estimation of shear stress, we can still carry on with our analysis, engineering analysis. From a mathematical perspective, there are inconsistencies.

From an engineering perspective, we can accommodate that later either in our factor of safety. And we have also looked at several other cross-sections because bending, what you require is, you require only an axis of symmetry. In fact, only along the plane of symmetry, we do all the calculations. We simply extend it to the thickness. So, for all these cross-sections, this inconsistency applies and in some of them, leaving the elliptical cross-section, even what you estimate along the neutral axis will also be questionable.

It will also have more error. But nevertheless, we always say that shear contribution is less when the beam is slender and we carry on with it, accommodate it in our engineering analysis.

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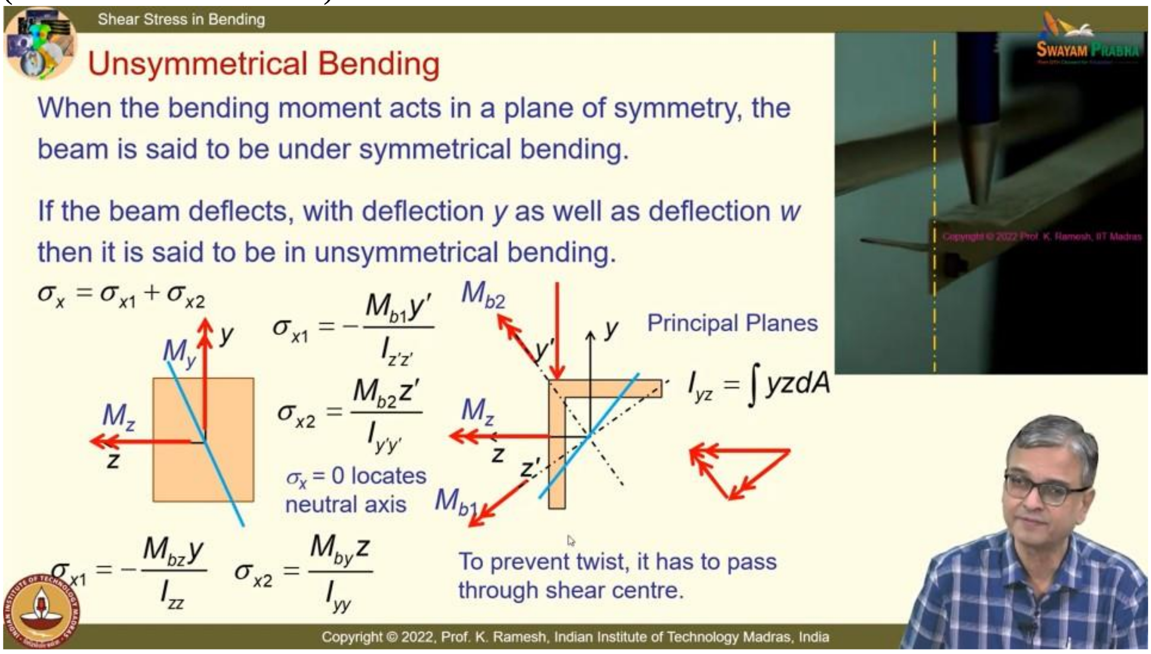

Then we also discussed, what is an unsymmetrical bending? See, I have taken a symmetrical cross-section. Even a symmetrical cross-section can have an unsymmetrical bending. The moment you say unsymmetrical bending, you may anticipate that I am taking a cross-section, which does not have symmetry and applying the bending load.

That is also unsymmetrical bending. Even when I have a symmetric cross-section, suppose I have a situation when the bending moment is acting on, you have M_{bz} as well as M_{by}. First, I have shown only M_{bz} and I say that it is under symmetrical bending because the cross-section is symmetrical. And once you have this, you know the flexure formula and I can find out what are the stresses introduced. The σ_{x1} due to this bending moment is given as $-\frac{M_{bz}y}{I_{zz}}$.

And suppose, I individually consider the bending moment M_{by}; and you should also recognize, when this is positive, what would be the stress in the positive *z*-direction? It is going to be tensile. So, I can also write σ_{x2} and when I write σ_{x2}, I would be careful in simply saying $\frac{M_{by}z}{I_{yy}}$ because I know we have already embedded sign of the stress also in the basic equation. And if I have both M_{by} and M_{bz} are acting, then I say the cross-section is subjected to unsymmetrical bending. And if I have to calculate the stresses, I will calculate the stresses by principle of linear superposition. $\sigma_x = \sigma_{x1} + \sigma_{x2}$; and you know, to locate the neutral axis, find out where σ_x goes to zero? It would be something like this.

It is depending on the magnitude, it would be something like this. So, a symmetrical cross-section subjected to bending moment in two perpendicular directions like what I have shown here, it will have an unsymmetrical bending. Now I have taken a cross-section which does not have a plane of symmetry. When I do not have a plane of symmetry, what is the first thing that you come across? If I have the reference axis as *z* and *y*, I would definitely have a magnitude for moment of inertia, I_{zy}. It will not go to zero or I_{yz} whichever way, I_{yz} equal to integral $yzdA$, this will not go to zero.

And I have a bending moment M_z acting on this. Even if I have a single bending moment acting on this, because the cross-section is unsymmetrical, this will have unsymmetrical bending. And you know when I have a unsymmetrical cross-section like this, I can also find, from moment of inertia point of view; principal planes and if I have the reference axis as y' and z', I would have $I_{y'z'} = 0$. These are the principal planes. So, if I have a bending moment M_z acting like this, I can visualize a component of this bending moment acting along the $M_{bz'}$ and $M_{by'}$.

So that means, very similar to this, I have bending moment acting on two perpendicular planes, that will lead to unsymmetrical bending. And you know, I have also shown a nice animation here, let us look at it very closely. I have only taken an unsymmetrical section. And when I apply it like this, you can prominently see the member twists, which we have seen; what is the need for a shear center. And shear center for this cross-section actually coincides with the meeting of these two angles, but you know while applying the load, I had difficulty because the pen was slipping out of it, so I put it just next to that for convenience. See, this is for illustration, so you will have to understand that this is for illustration, not a very carefully done experiment, but it illustrates certain basic aspects of it.

So, following the logic what we have done for the other case, I can also calculate σ_{x1}, I can also calculate σ_{x2} and then use principle of superposition and find out the values of the bending stress; and you will also find out, what is the neutral axis? When σ_x goes to zero, you call that as neutral axis. See, we have not looked at how this bending moment is created? Suppose, I put a load on this, this will also create a twist; that is what we have seen here. So, I should not put the load here for me to create only bending moment, I have to shift it to the shear center, if I put it along this, I get this twist as well as bending. And I have to put it along the shear center. I said that I had difficulty putting the pen. You know, this is made of thin aluminum, but it was very very stiff, very difficult to apply the load with the pen.

And I want you to make one very subtle observation. I will also assist you to visualize this, I have a line here, look at the pen, I will align it with this and then when I bend, see this bends, do you observe what happens? See, this has only bending. Is the bending in one plane or there is something else happening? I will repeat the animation, the animation will first have the twist, then you will have the bending, you have a reference line here for you to visualize.

See, if the beam bends, the pen should remain in this line, is it happening? The central line of the pen is slightly shifting, so what is happening is, the beam is bending like this as well as bending like this, is the idea clear? It is bending as well as bending like this. So, it is a very very subtle point which is captured in this animation and that is why this is called as unsymmetrical bending. Even though I apply a simple bending moment, when I have an unsymmetrical cross-section, the bending is complex.

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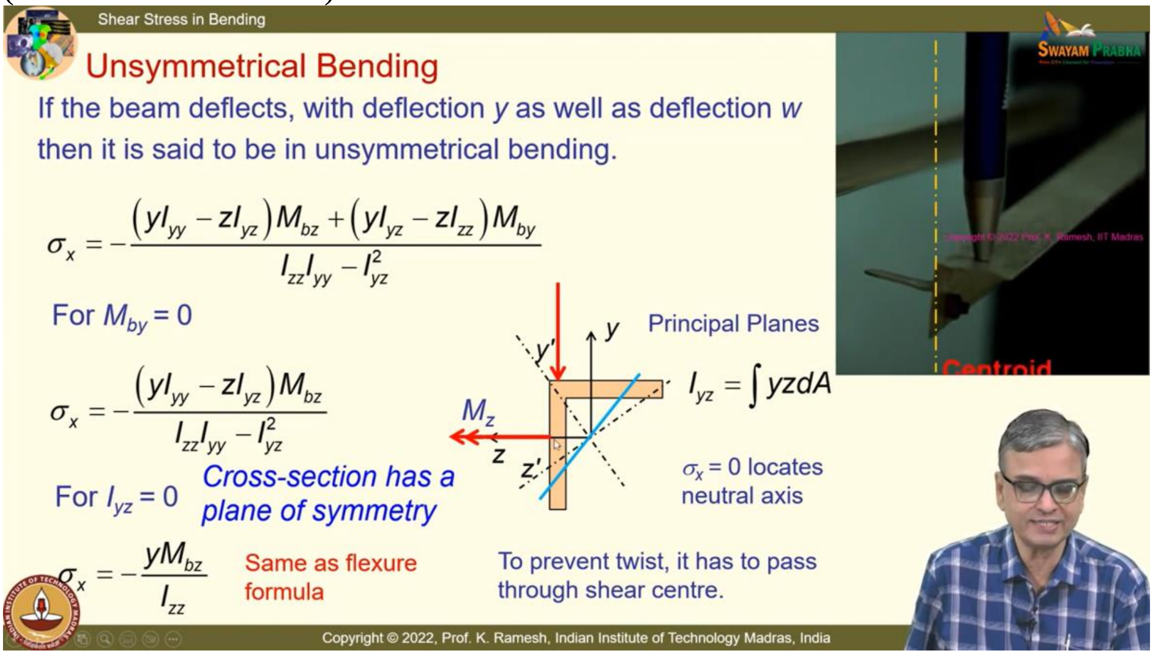

And you know, suppose I have unsymmetrical bending, I can also estimate the stresses directly without mathematical steps, I am going to give you the final expression, please write down the final expression. I have M_{bz} as well as M_{by} is acting, if both the moments are acting, I have the expression for σ_x that is given as:

$$\sigma_x = -\frac{\left(yI_{yy} - zI_{yz}\right)M_{bz} + \left(yI_{yz} - zI_{zz}\right)M_{by}}{I_{zz}I_{yy} - I_{yz}^2}$$

And once you go to an unsymmetrical cross-section, I will have I_{yz} non-zero and that will figure in the equations.

Suppose I have only one bending moment like what I have shown here, for $M_{by} = 0$, I get

$$\sigma_x = -\frac{\left(yI_{yy} - zI_{yz}\right)M_{bz}}{I_{zz}I_{yy} - I_{yz}^2}$$

See, one way of verifying whether these expressions are all right, we must also find out if I simplify it for a symmetrical cross-section, what happens? The moment I say a cross-section is symmetrical, I will have I_{yz} is zero. It is not meant for this problem, but for the generic case when $I_{yz} = 0$, cross-section has a plane of symmetry, we have seen multiple cross-sections. So, when I substitute this, I get this as $-\frac{yM_{bz}}{I_{zz}}$. In fact, this is what we developed as part of our flexure formula.

So, this indirectly confirms what I have got this as an expression can be correct, fine. You please go and verify and if there are any typographical errors, please alert me. And you have also seen this animation multiple number of times, how unsymmetrical bending actually takes place, fine. It is very interesting to see. So, this is what it says, you have this expression same as flexure formula and you also see the principal planes and the neutral axis.

See, this depends on what is the values of this individual bending moments, fine and also the sign. So, it is only an illustration.

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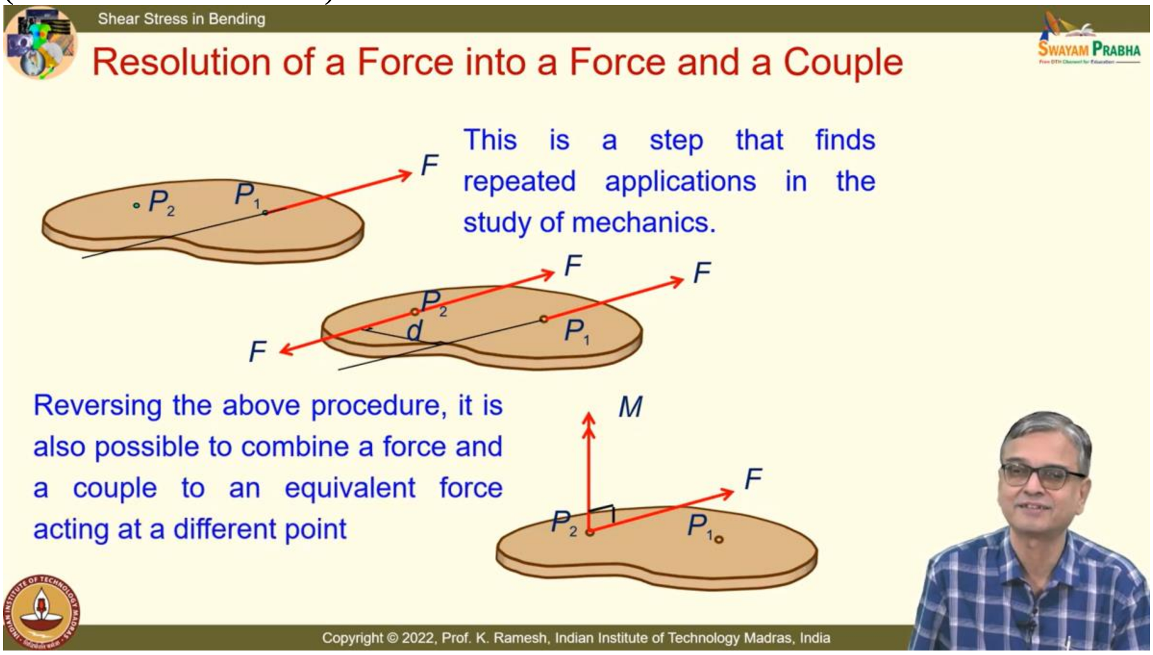

And you know, I have been emphasizing that resolution of a force into a force and a couple is a very very important concept. If you get that, many problems, you can easily analyze. I can move this force along this line of action, no issue.

The external effects are not changed. Suppose I want to move it from point P_1 to P_2, I cannot simply move it. One way of conjecturing is, I imagine that I also add another force. So, I have not done anything to point P_2 at this stage. But this system as a whole can be looked at from a different perspective that this force and this force forms a couple, replace it by a couple. And when I replace it by a couple, I have achieved the task that I have been able to move the force from this point P_1 to P_2.

And I find when I move the force from point P_1 to P_2, I have the force as well as a couple. You know, this is very very important. I am going to discuss.

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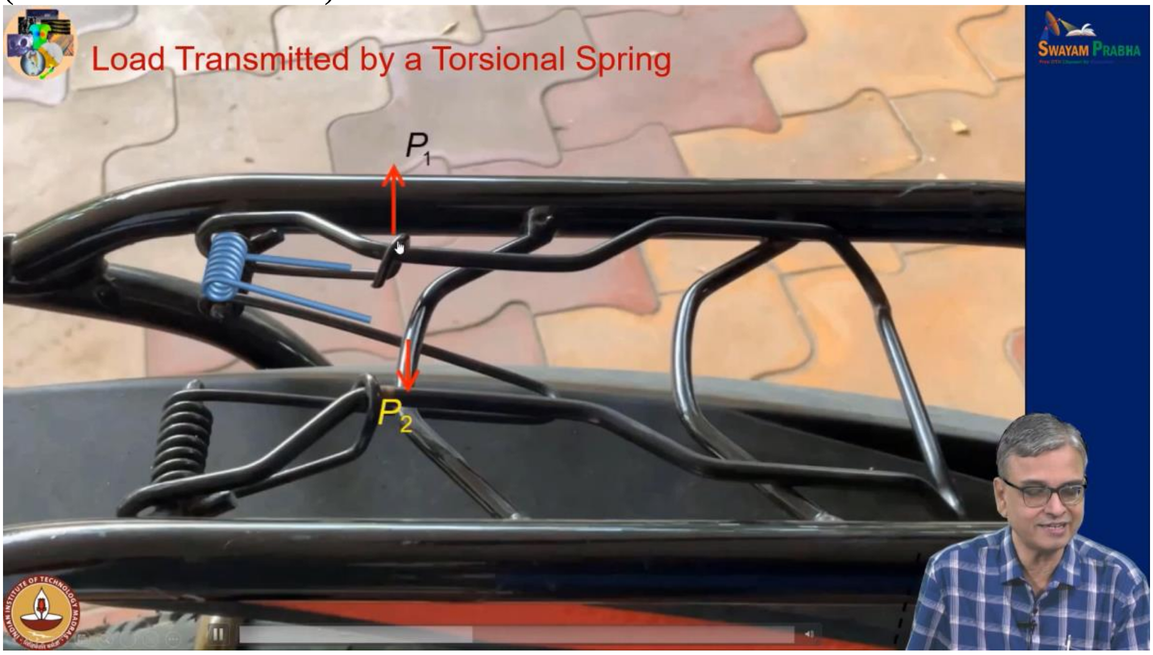

I have asked you to look at what happens to a cycle carrier. I do not know how many of you have had the inquisitiveness. See, you should have inquisitiveness in learning, fine.

If that inquisitiveness is put at the back burner, then learning is not enjoyable. So, you see, you are not done yourself, the experiment. I am showing the experiment, you are lifting the carrier. You see the spring here. Can you tell me, can I say that this is interacting here and this is a very long this one where I can replace this by force P_1 to P_2?

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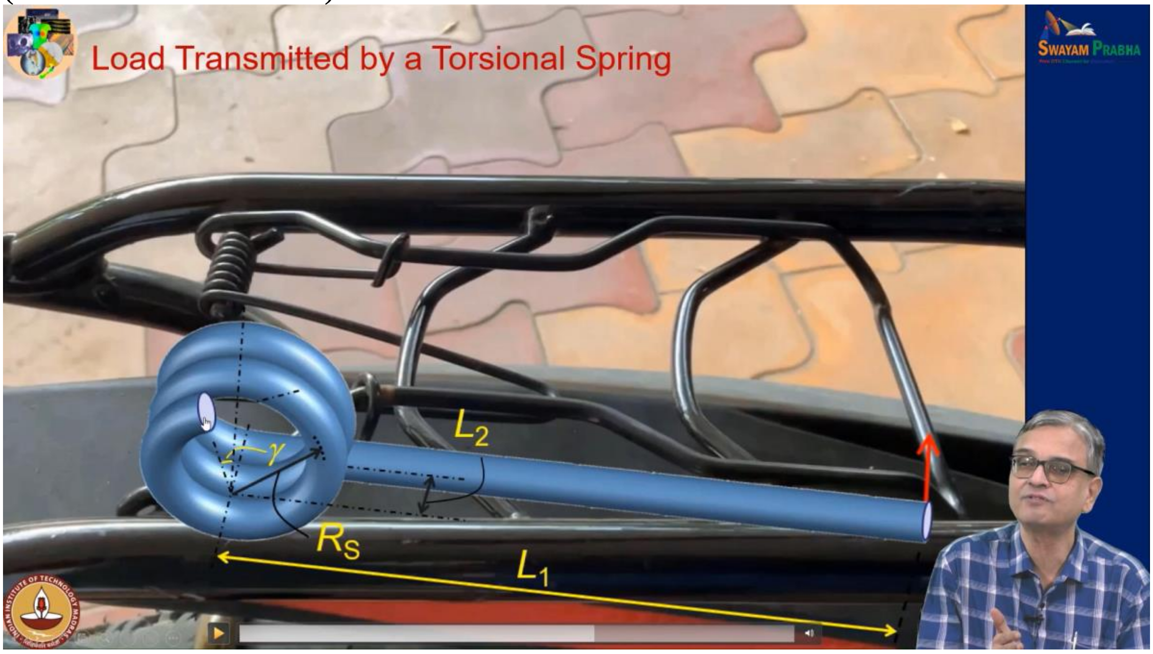

Is the idea clear? You have a spring like this and you have a bent like this, this is the torsional spring and you have a long arm.

So, to visualize, I have a long arm shown here, I have a force here. You just find out what is the effect of this force on this cross-section.

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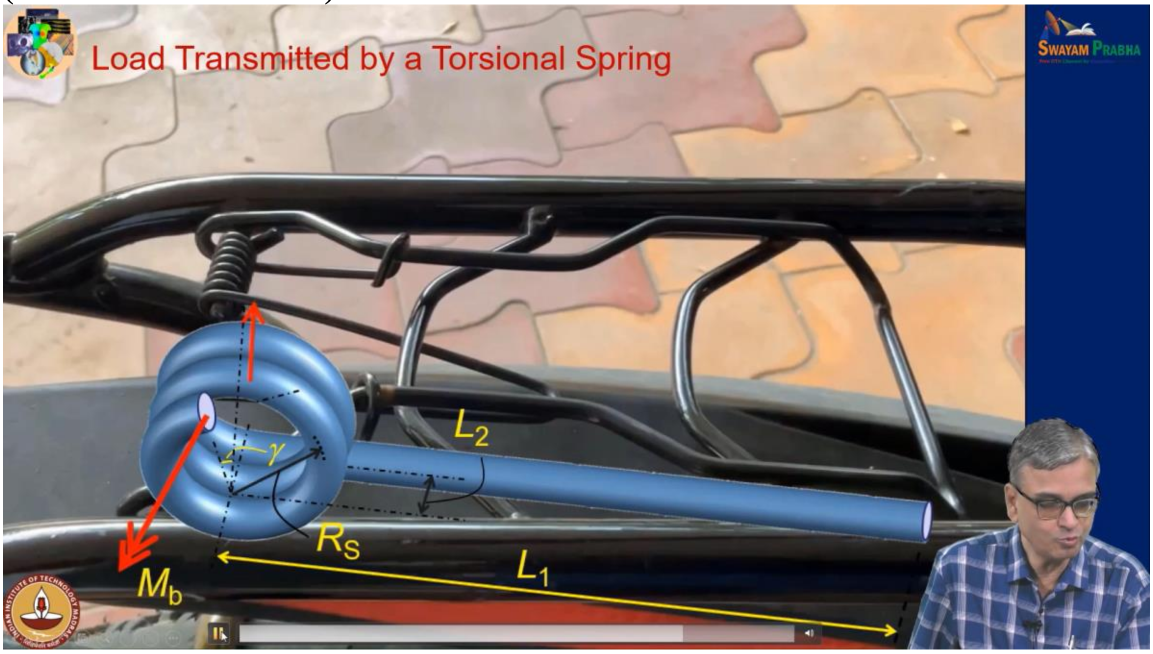

I just now discussed how to move a force from one point to another and what is the kind of loading. Can you try? Can you make an attempt? You make an attempt and then verify with my animation. What you will have to appreciate is, it is a simple day to day torsional spring that you come across in your cycle. And with your background, please make an attempt.

Have you observed what is the load that is coming on this? It is nothing but translate this force acting here to this point. This is located away in a horizontal distance like this and there is also a depth here. Is the idea clear? So, I will have to move it horizontally, then from the depth wise I have to move. So, I have to do two movements. What do these two movements, introduce the couple? Depending on the nature of the couple, we will leave it as a bending moment or a twisting moment.

If it is along the axis of the cross-section, you call it as twisting moment. If it tries to bend, then you call it as a bending moment. Is the idea clear? Please make an attempt, make an attempt. See, only if you make an attempt and make mistakes, it is learning. Otherwise, it is only cramming up whatever that we discuss and then you just vomit that in the examination and get what is called as a grade which is useless.

You will have to get the fundamental understanding of what is happening. So, I am going to have two movements that I have explained. It will move horizontally and then from the depth, it has to come to the front and this animation will be little faster. So, please observe this. When I move it horizontally, what happens? When I move it horizontally, this horizontal movement also creates a couple. What sort of a couple is this? It is trying to bend and you have a very long arm, isn't it? So, your bending moment is of a very high value and I have to move it through the depth.

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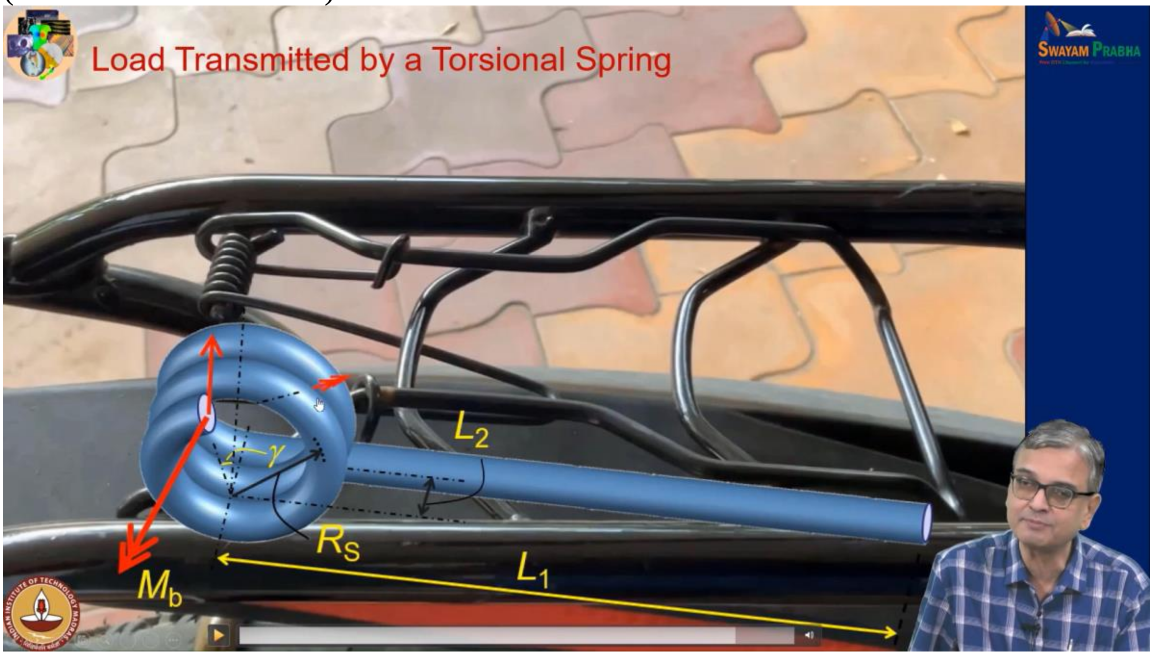

I have to move it through the depth; that is a very short length and I have a twisting moment.

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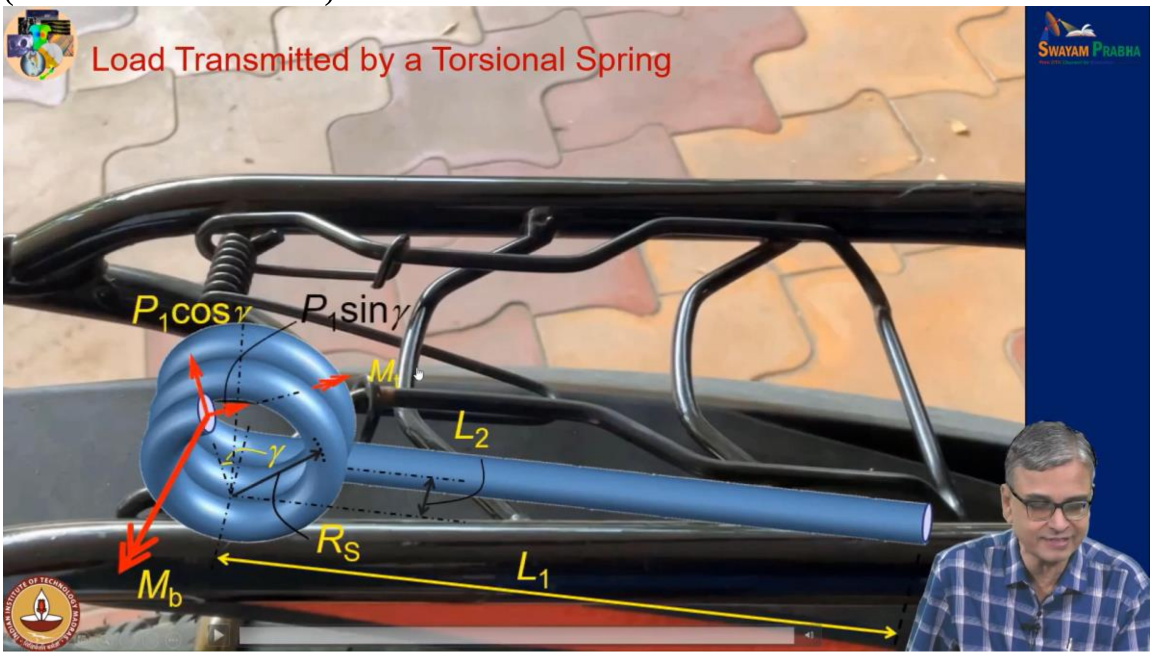

Now, I can resolve this force into tangential to the surface and perpendicular to the surface. Is the idea clear? I will do that. So, when I have a force acting here, that force is felt by this cross-section as a very large bending moment, a small twisting moment and you have a shear stress and an axial stress.

It is a very nice application of basic idea. A cross-section can transmit forces in three directions and moments in three directions and in this case, it transmits force in two directions and transmits moment in two directions. Now, the other question is, do you have the background to analyze this beam? See, engineers always look like this. I have also helped your visualization that this has the largest value. So, I have shown it as a very prominent vector. For a simplistic analysis, we can ignore the effect of all of this and consider only major load as bending moment.

See, it is very funny, you say it is a torsional spring, but it transmits bending load. And the question is, do you have the background to analyze? You have to recognize that this is an initially curved section. It is not a straight beam.

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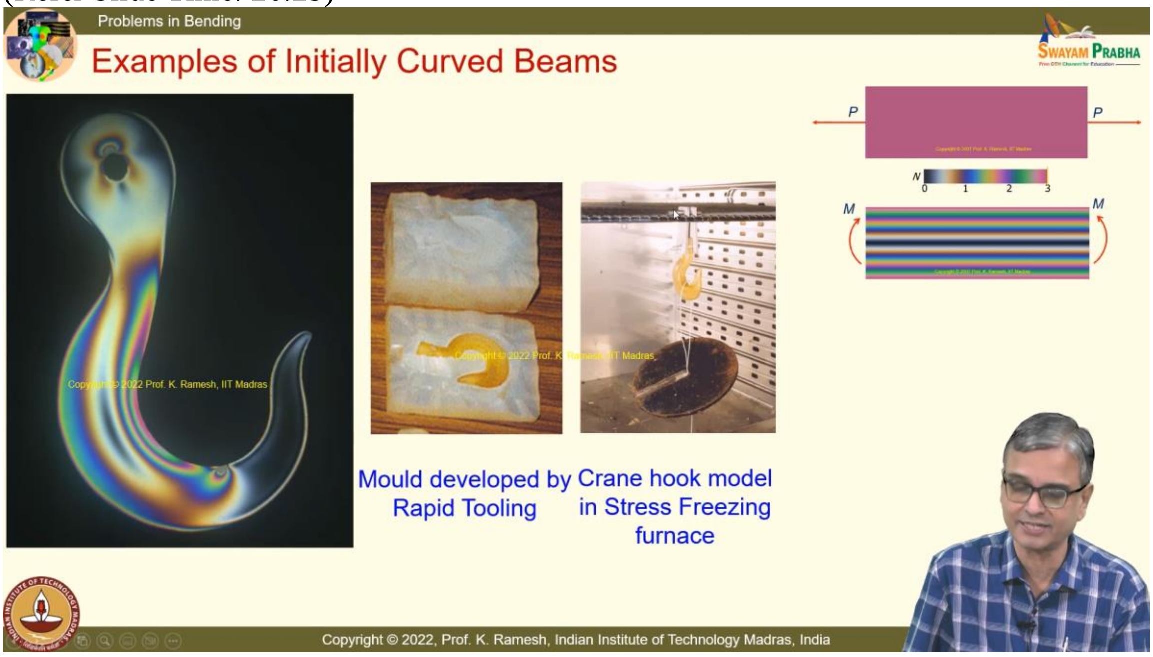

And what is the other famous example? You have a crane hook and this is beautifully done in our lab here. You have the fringe pattern for a two-dimensional model and I have sufficiently trained you by looking at the fringe patterns in pure bending.

So, you have the similar features, I have the black fringe depicting the neutral axis and you see something different in the depth. You find the fringes are of equal distance here. How do you find the fringes along this direction? They are not of equal distance. This is also subjected to bending even though the moment-arm is very small because the weight of this has to carry is very large. So, you have a significant bending moment and in reality, you will not have a crane hook like a two-dimensional object.

It will be a three-dimensional object; it will be very heavy. And you know you also have a three-dimensional model of the photoelastic hook, it is created by rapid tooling. You have silicon mold, the mold is prepared based on actual steel crane hook. Then you have the mold cavity generated by rapid tooling and you get the hook done and there is also a special process by which by applying the load whatever the effect of the load can be frozen into the model, this is called stress freezing.

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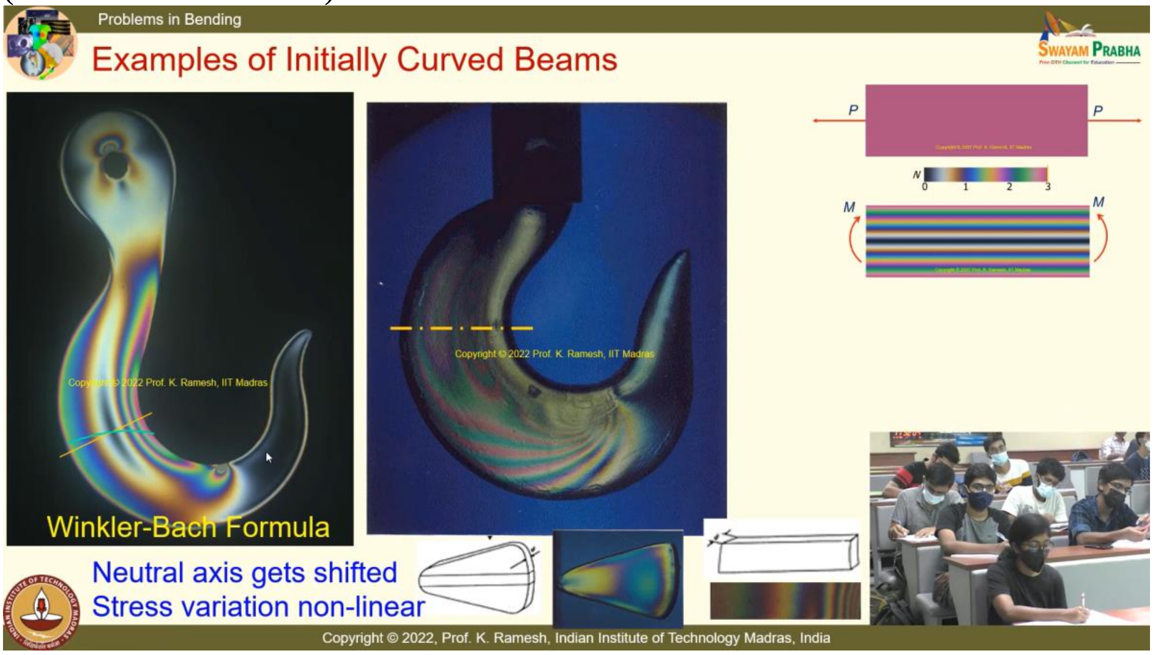

It is all done by my students and when you have the fringe pattern, this is the integrated fringe pattern because it is a very thick cross-section and if you take a cross-section here, the cross-section will be like trapezoidal. This is a beam of uniform strength where it has the higher number of stresses, value of stresses you have this as thick and you take a slice out of it, you see the fringe pattern. And you take a sub-slice; what you see as true line is the cut from this and this is seen perpendicular to that. So, you see the fringe pattern; they are dense here, they are of different distance here, this depicts that there is bending taking place here. And if you look at what happens on this cross-section and if you plot the actual stress variation, this is non-linear.

So, the moment you go from a straight beam to initially curved beam, there are two things happening. One is neutral axis gets shifted. See, in the case of a straight beam only if I have multiple materials, the neutral axis gets shifted, otherwise it coincides with the centroidal axis. In an initially curved beam, even though it is made of a single material, please make a note of it, these are very, very subtle learnings, the neutral axis gets shifted and the second important observation is stress variation is non-linear. In fact, it is hyperbolic. It is a very very important aspect. See, even though you use your cycle daily, with the second level course, first level course is rigid body mechanics, the second level course lifts its hand and says you do not have the expressions to analyze, fine. You are still doing baby steps and whatever the expression that you have is known as the Winkler-Bach formula, hopefully, we will study in the next level course.

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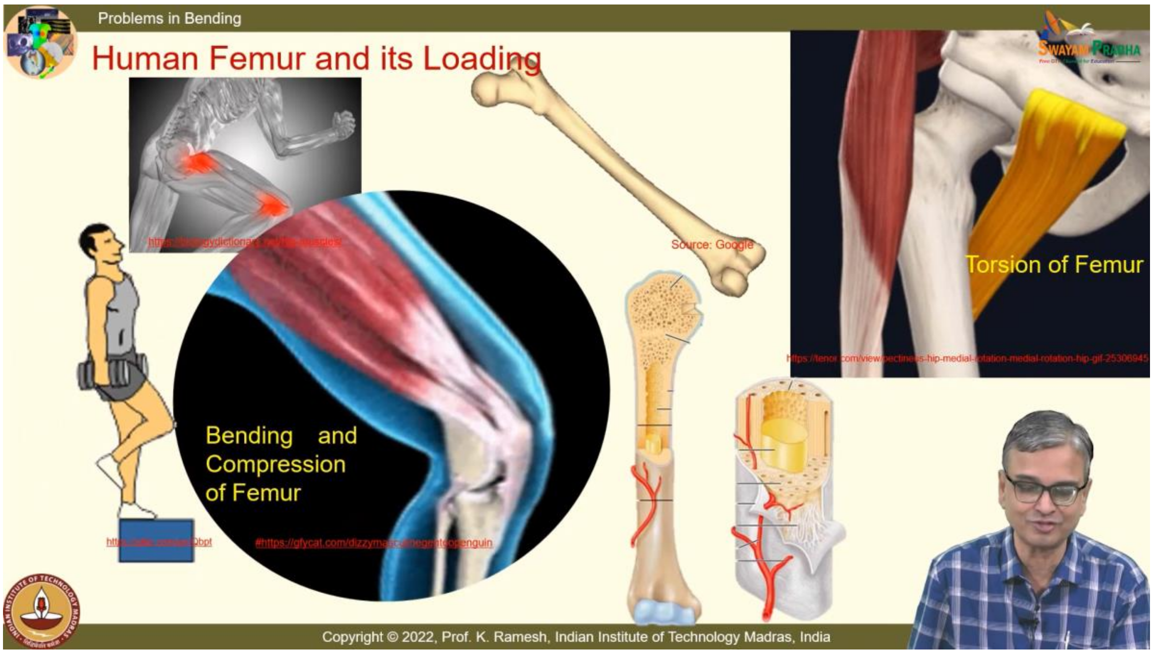

See, the future is biomechanics; you know there are lot of issues in biomechanics that's not solved and it requires engineers to apply their minds and then do it. And another area where this is also becoming important is in the area of prosthetics. See, earlier we were not having very high speeds permitted in our highways. Now, we are also going to international standards where you are going to have 140 kmph, I think 140 kmph or 130 kmph, I do not know, they are going to increase the speed from 100 kmph. So, the moment you are going to have high speed, you are going to have collisions, you are going to get injuries out of it and you have to rehabilitate the people who are caught into the accident. And you need to understand prosthetics. And you should appreciate how God has been very clever in designing your body.

If you look at; when you have action like this to your legs, the femur, the longest bone in your body is subjected to both bending and compression. And if you look at femur in all its splendor, you find it is not a straight beam, actually it is curved, not only curved even the cross-sections change from section to section, it is very very complicated and you have another movement of your own body which can produce torsion of the femur shaft. So, the worst-case scenario is I may have bending and compression as well as torsion of the femur. See, we have seen beam theory took 400 years and we understood that inner core does not transmit load. Your body was there since millennia and God has understood your bending, he has understood your torsion and if you cut the bone open and you see inner core is your soft marrow which produces the hemoglobin, outer core is your bone and we have also discussed what you see as the bone which is of porous structure here, this is functionally graded adding another complication to this.

See we have only looked at a steel plate attached to an aluminum channel. Then we had a situation of steel rods embedded into concrete, even that mathematics was back-breaking. And if I am going to look at the femur which is functionally graded along its length, cross-section changes and it is not a straight beam initially curved; you will only say let somebody else analyze the problem. That is not the way engineers operate, engineers take it as a challenge and then simplify to the extent; with the available knowledge what you have been exposed to, we analyze the problem. And whatever we have not accommodated, put it in a factor of safety. This is the basic approach in engineering, you must also have the courage to do that.

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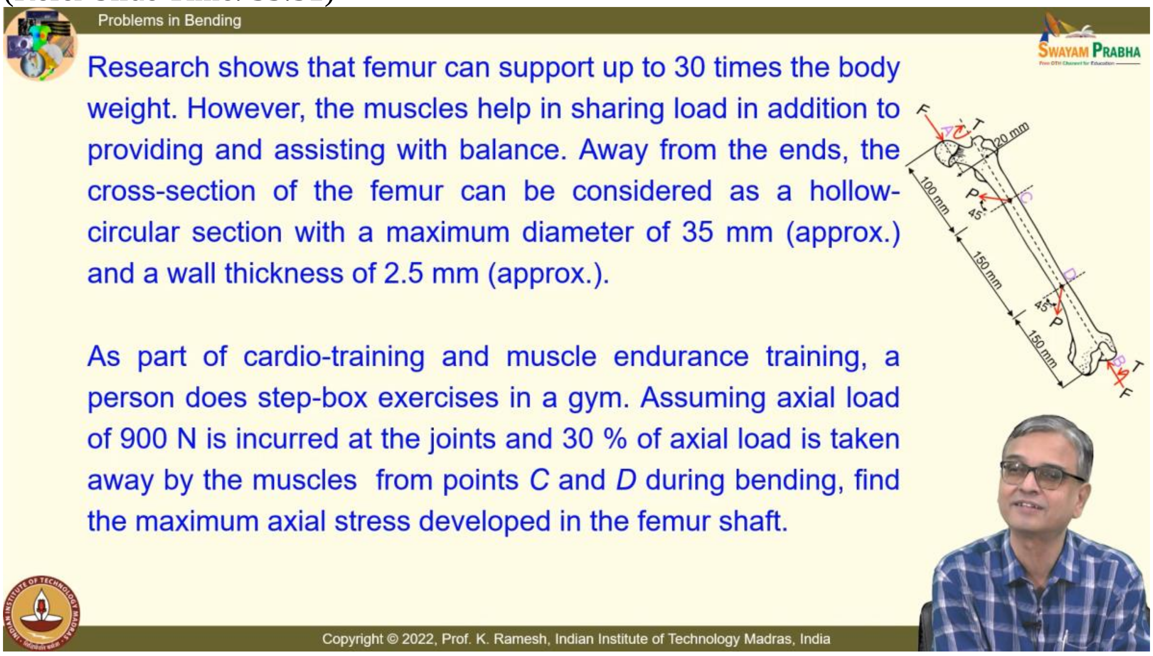

So, the problem says like this; research shows that femur can support up to 30 times the body weight. So, God has done all your stress analysis and he has found out humans are going to create cars and it is going to travel at 140 kmph and there is a collision this fellow should say be safe, otherwise why you need your femur to support 30 times the body weight, you do not need that, fine. With all that factor of safety inbuilt, if you get injury, God only should save you! There is also another thing I am not sure, you know these days see we are in a tropical country, in tropical countries, people have found out that we all have vitamin D deficiency. And you know vitamin D is vital for absorption of calcium in your system and calcium is necessary for strength of your bones. So, if you do not have sufficient vitamin D it also leads to other complications; one complication is also in the bone and you know particularly for women when they become 40 or so, they are prone to get what is known as Osteoporosis. See, when you design a nuclear power plant, the irradiation damages and deteriorates the material. So, when I put any component into the nuclear power plant for it to be constructed, the designers take into account material deterioration as a function of time.

I think God has done that; he has done that Osteoporosis is going to affect you. That is why he had designed the basic femur which can support 30 times the body weight. So, you know, you should have outdoor activity which helps you to absorb D vitamin even in tropical countries, this is the problem. And what it says is you know he has done this exercise after the several times; you know, you do not follow rules you know. The body has to be aligned in a particular manner. If you misalign that leg movement, it can cause a twisting moment.

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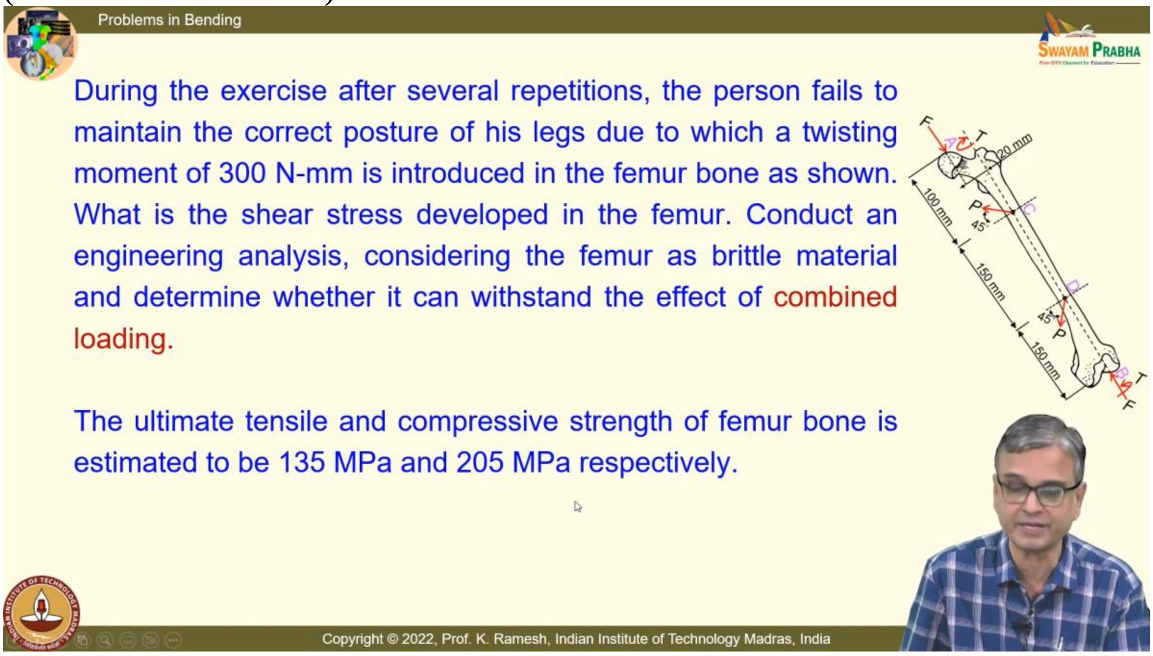

So, this is a problem which is artificially coined to give a flavor of how do we handle a real-life problem which is very challenging. So, the femur shaft is subjected to compression, bending and twisting. And you have to verify whether it can withstand the combined loading. So, I am going to use this problem as a basis to illustrate many many concepts. And you are also given these are all real values taken from research papers.

Ultimate tensile and compressive strength; that is tensile strength is 135 MPa and compressive strength is 205 MPa. This is taken from research paper; I hope they have done the measurements little more carefully.

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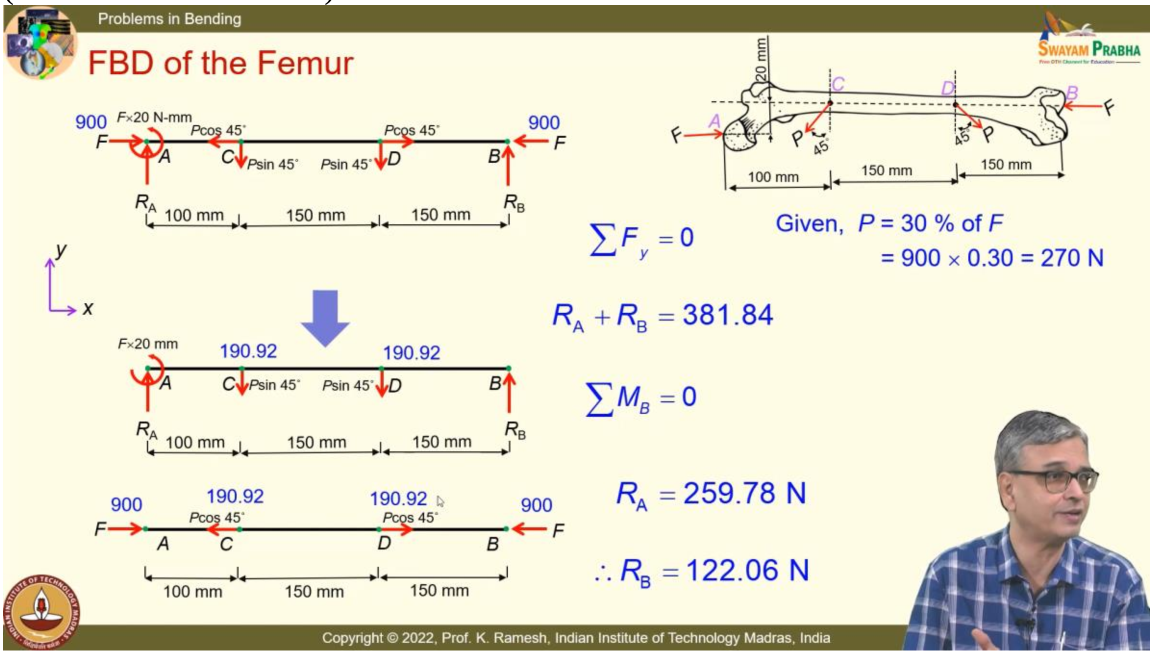

So, the first approximation what they do is; we take a convenient axis which is more or less horizontal and then we ignore the initial curvature of the femur and we will solve the problem in three stages. The first stage we would take only the combined bending and torsion, I mean bending and axial compression. I can replace this as addition of two sub-problems because we are living in linear elasticity.

One is causing bending moment, other one causing only axial compression. And you also have two muscles attached at points *C* and *D*. They carry some amount of load and you are given that this takes 30 % of *F* and you have shown the direction of the force. So, if I have this as 270 N, if I have to find out what happens at *C* and *D*, vertically it is 190.92 N, this is also given at 45°, horizontally also it will have 190.92 N. So, I have one problem which deals with bending, another problem deals with compression. Once it is given, the problem is very simple for you to solve. First is we have to find out the reactions, so I have $\Sigma F_y = 0$ and this gives me $R_A + R_B = 381.84$ N. See, we have also done a simple idealization, you have this as ball and socket joint at the hip and the other joint is also some sort of a revolute joint.

So, we simply replace it by a horizontal and vertical force and horizontal force is already given as part of the problem. Only the vertical force, we have to find out as part of the solution that we have to do. And you know by taking the bending moment $M_B = 0$, you can find out, this is from your rigid body mechanics, please make a note of it, make a note of it.

I have this reaction as 259.78 N and R_B as 122.06 N. So, once you have these numbers you can find out the bending moment diagram. Why do you draw the bending moment diagram? What is the need for drawing the bending moment diagram? From a design perspective, I want to find out what is the maximum bending moment; that is the purpose of it. You have learnt to draw bending moment diagram not to get grades in your previous course, but to analyze the beam in a later course meaningfully.

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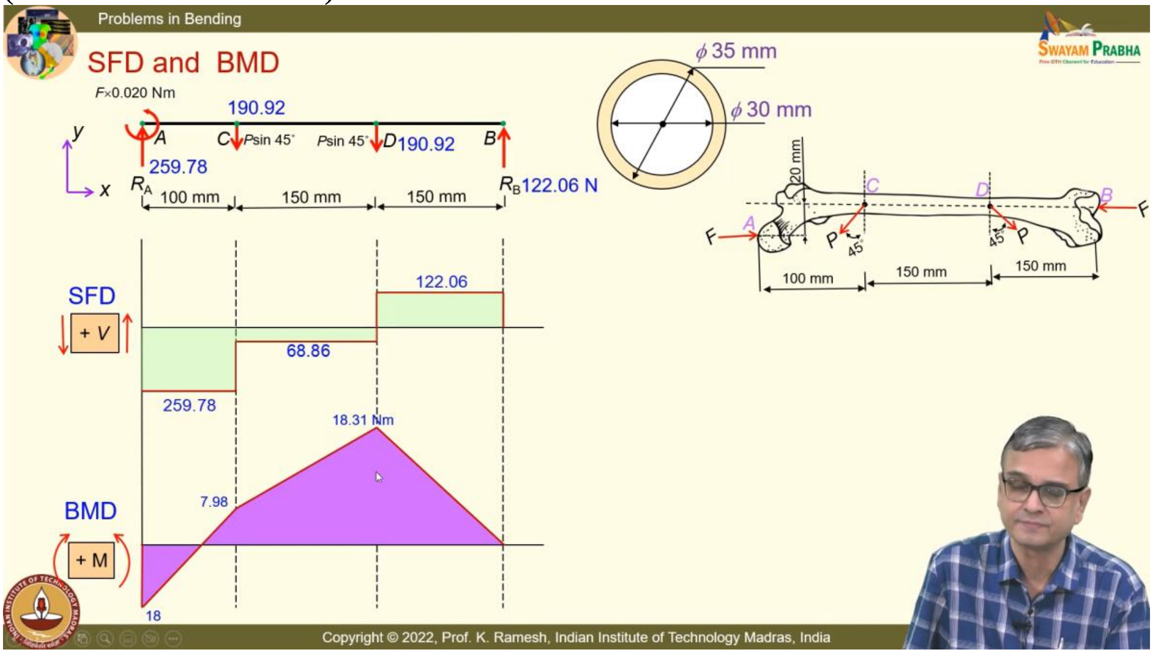

So, I go to the bending moment, shear force and bending moment diagram and I have already told you that you should attempt to draw this by inspection from right to left. So, when I have this reaction, I have the shear force go up like this, it remains constant and comes down by 190.92 N, I will go like this and go down, come like this and go up, fine. I have shown you, how I am drawing the shear force that is what I am going to show here, you must pick up that speed. You have come to the second level course and you do this from right to left, when you do this from right to left, it satisfies the sign convention and it is nothing but moving this force to the different locations. When I move this load, the load will also have a couple and that is nothing but a bending moment; that is what is reflected as in your bending moment diagram. And I have this as a hollow section, outer diameter is 35 mm, inner diameter is 30 mm which is all given in the problem. Since I have circulated the problem sheet, I have not read the problem here. And when I do this, I get this as 18.31 Nm and I find this cross section has the maximum level of bending moment, fine.

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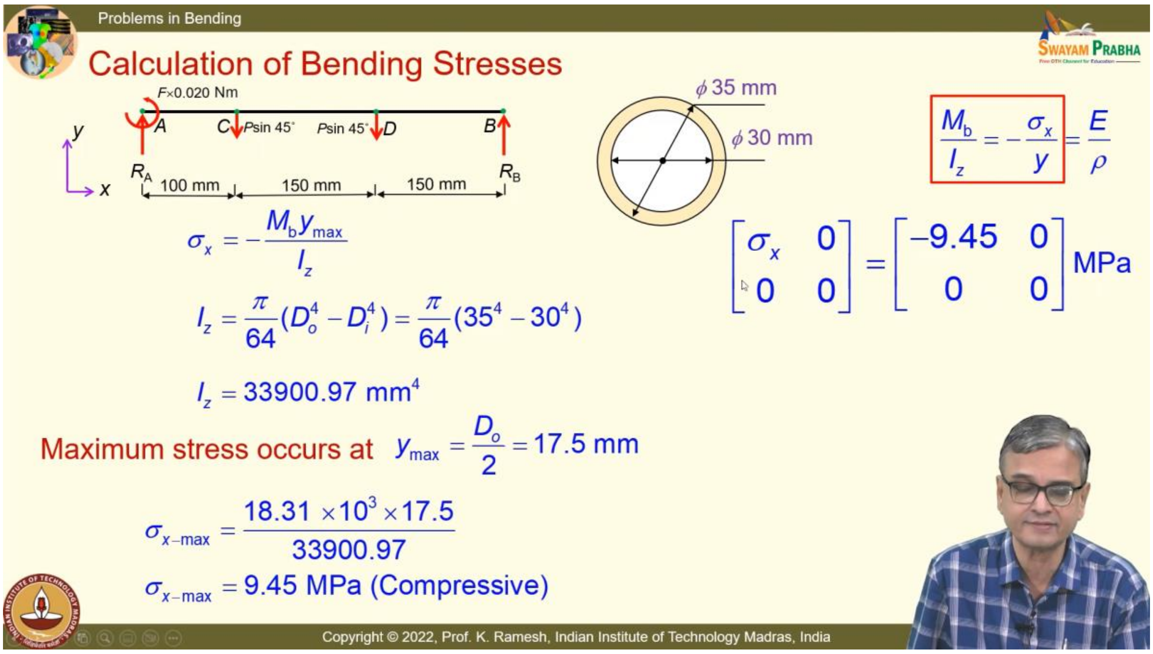

And you know from your flexure formula, how to find out the bending stress. See, this also, I am repeating it again and again. See if you keep looking at the expressions, it also helps you to remember the key aspects. So, I find out the maximum bending stress and you are given the cross-section, so I need to find out the I_z; I_z depends on the cross-section and this is a hollow tube. You all have the basic expression for I_z

$$I_z = \frac{\pi}{64}(D_o^4 - D_i^4)$$

and this you get a number like this.

So, when I substitute, I get the $\sigma_{x-\max}$ as 9.45 MPa, compressive. See when the femur bends, it has a tensile stress as well as compressive stress. My idea is to find out, what is the combination of, you have axial compression, so I have looked at the compressive side, fine. You can also calculate for the tensile side what happens. Now, I have calculated the value and this is the reference axis, the *x*-axis is along the axis of the beam. How do I write the stress tensor? It is nothing but $\begin{bmatrix} \sigma_x & 0 \\ 0 & 0 \end{bmatrix}$, so I can substitute this and say $\begin{bmatrix} -9.45 & 0 \\ 0 & 0 \end{bmatrix}$ MPa. So, I have got the stress tensor in the case of bending.

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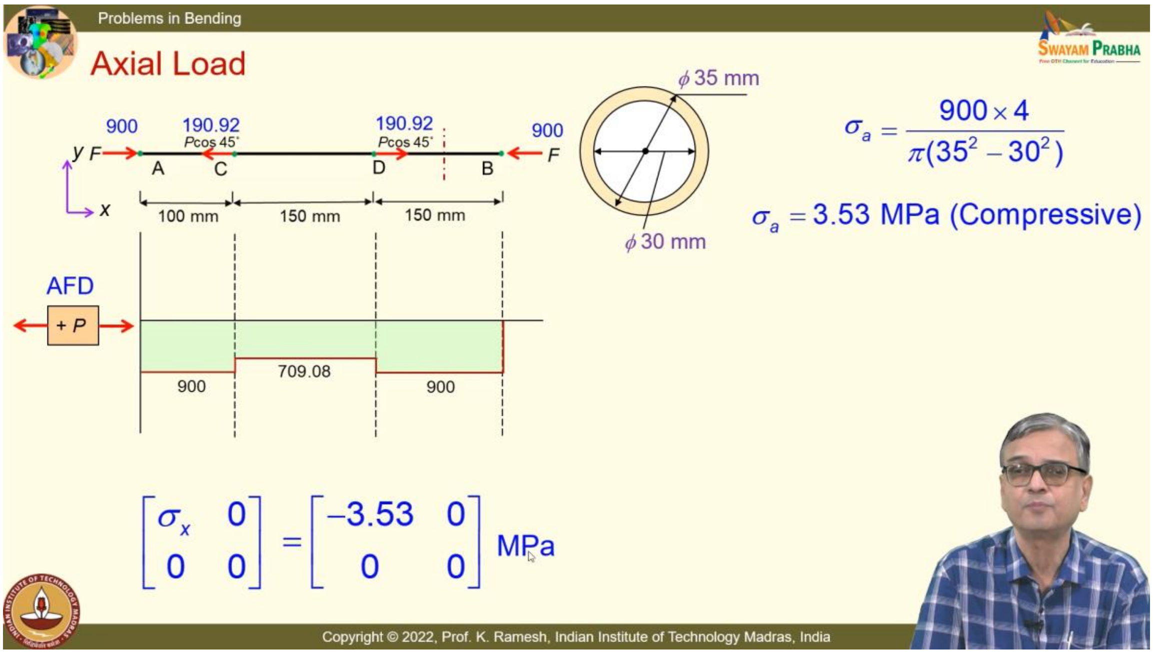

Now let me go to the axial load and it is easy for you to draw the axial force diagram. You can take sections and do it, but you can also do it directly like this and here again the point *D* is the one which has the maximum axial load. From the diagram, you can read that this is 900 N and you have the cross-section, so I need to find out the area and you should also know, what is this axial stress component? When I have the reference axis as *x* along the axis, this is nothing but σ_x.

So, the stress tensor is again $\begin{bmatrix} \sigma_x & 0 \\ 0 & 0 \end{bmatrix}$, so I can also get this as $\begin{bmatrix} -3.53 & 0 \\ 0 & 0 \end{bmatrix}$MPa. So, we have determined; what is the stress tensor caused by bending; what is the stress tensor caused by axial compression. Now I said after doing the exercise several number of times, he changed his posture, the change in posture has resulted in additional torsional load and we will analyze torsion separately.

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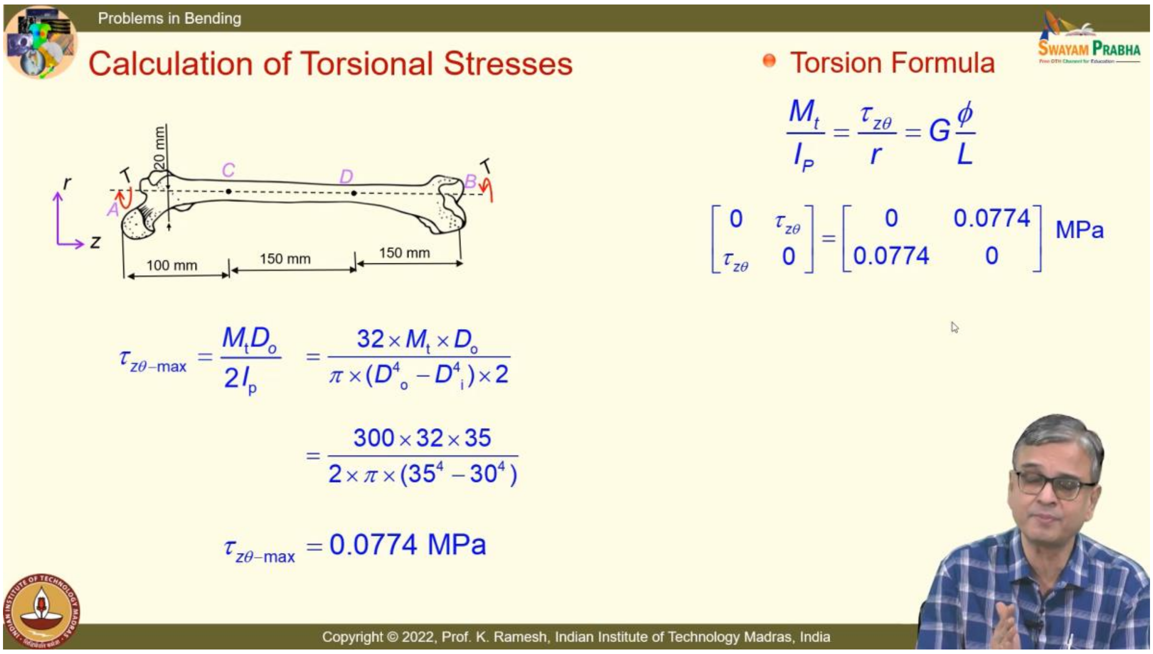

So, you are given the torsional load like this and we have also developed torsion formula, using what as a reference axis? Our entire discussion in torsion was confined to circular cross-section, whether it is solid or hollow. So, it was convenient to depict a circle by $r-\theta$ coordinates and we have taken the axis as *z*.

So, you have the torsion formula;

$$\frac{M_t}{I_P} = \frac{\tau_{z\theta}}{r} = G\frac{\phi}{L}$$

So, I can calculate, from the twisting moment, what is the shear stress $\tau_{z\theta}$, is the idea clear? I have this as $\frac{M_t D_o}{2I_p}$ because the maximum stress occurs at the outer radius that is $\frac{D_o}{2}$. So, when I have this, the number comes to be like this; $\tau_{z\theta-\max} = 0.0774\ MPa$. I have always emphasized that do not look at the components, you try to express this as a stress tensor and I want to do it in two dimensions. I have $\tau_{z\theta}$, so it is prudent for me to write it like this; $\begin{bmatrix} 0 & \tau_{z\theta} \\ \tau_{z\theta} & 0 \end{bmatrix}$; I have used equality of cross shears. And you have the stress tensor and I have filled in the numbers that I have got.

I have got this as 0.0774 MPa, but when I write it in tensorial form, I have it like this. See, I said that we live in the luxury of linear elasticity and individually these loads can be analyzed, you can sum them up because they are all linear.

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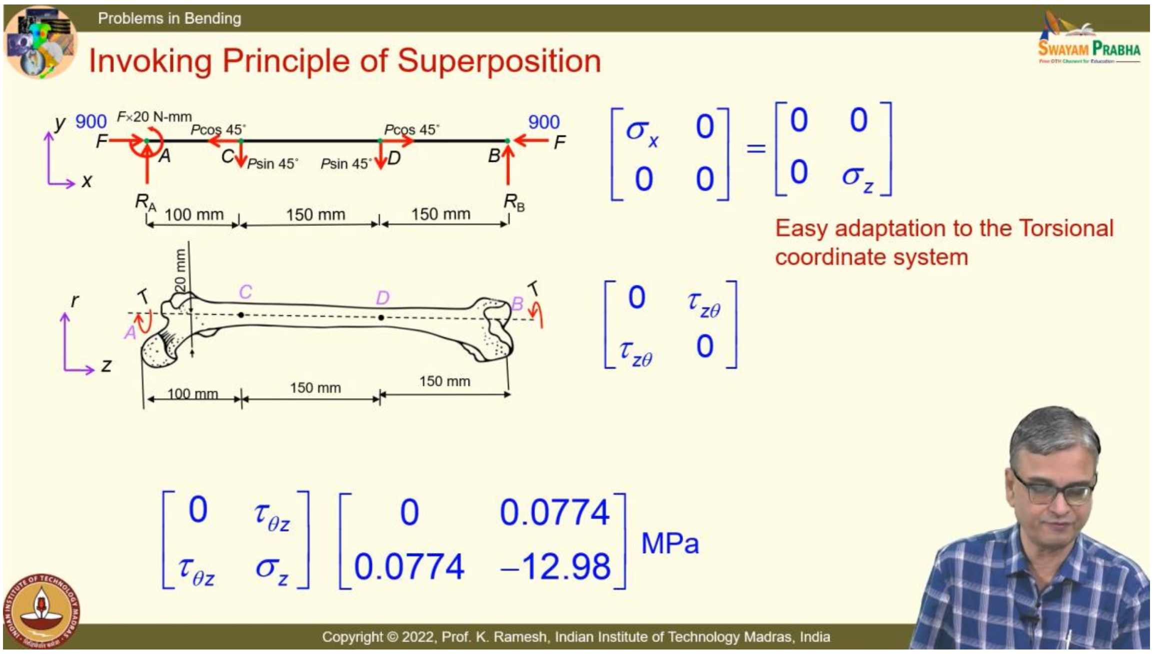

So, now I am going to invoke the principle of superposition. What is the catch here? I have analyzed this beam with one reference axis *x*-*y* and I have got the stress tensor and we will see that. We have analyzed torsion with another reference axis. Something is common, something is not common.

I have got in this $\begin{bmatrix} \sigma_x & 0 \\ 0 & 0 \end{bmatrix}$, I have got in this $\begin{bmatrix} 0 & \tau_{z\theta} \\ \tau_{z\theta} & 0 \end{bmatrix}$. One way of doing this is, apply stress transformation, reduce them to Cartesian coordinates, add them up and solve it, that is one way of solving the problem. What is the other easy way of solving the problem? If I make one small change, I have referred the axis of the femur as *z* in this, I have referred the axis of the beam as *x* in this. Suppose I replace whatever the stresses I have got in beam into *z* axis, then my assembly becomes much simpler. You can write this as $\begin{bmatrix} 0 & 0 \\ 0 & \sigma_z \end{bmatrix}$, easy adaptation to the torsional coordinate system. And what is the resulting stress tensor? I have, this is $\begin{bmatrix} \sigma_\theta = 0 & \tau_{z\theta} \\ \tau_{z\theta} & \sigma_z \end{bmatrix}$.

Is the idea clear? I have invoked principle of superposition, I have looked at the axis and I made a very simple adjustment; by a simple adjustment, I have been able to get this.

And I have this as $\begin{bmatrix} 0 & 0.0774 \\ 0.0774 & -12.98 \end{bmatrix}$ MPa.

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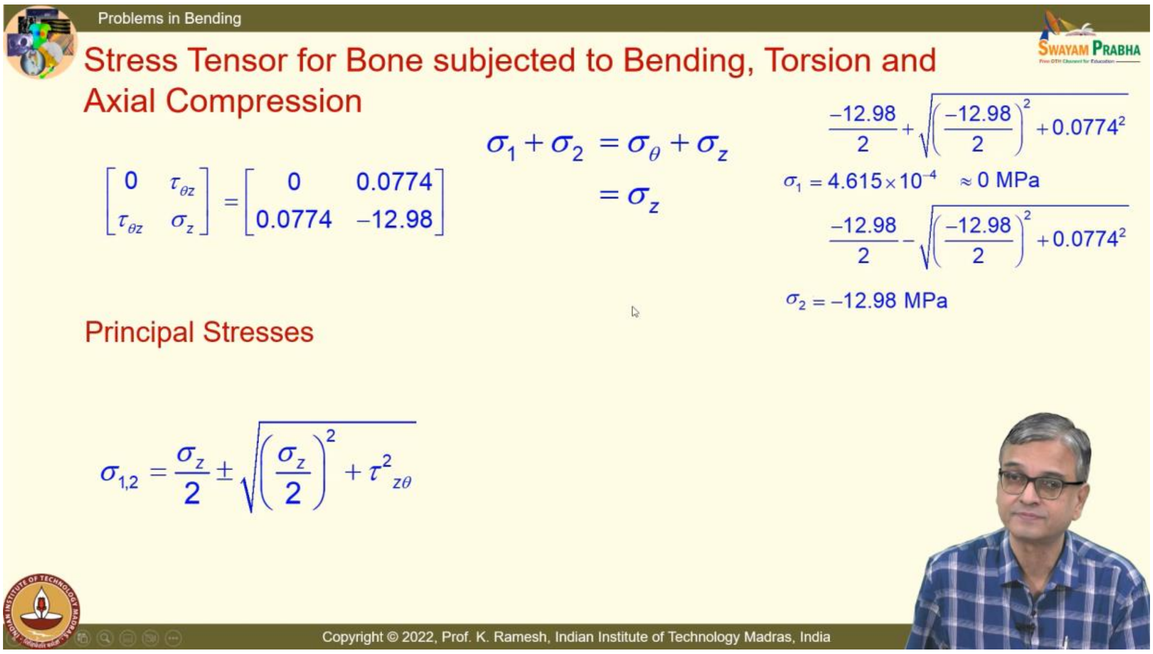

So, I have the stress tensor subjected to bending, torsion and axial compression. And I can have the principal stresses, I get this as

$$\sigma_{1,2} = \frac{\sigma_\theta + \sigma_z}{2} \pm \sqrt{\left(\frac{\sigma_\theta - \sigma_z}{2}\right)^2 + \tau^2{}_{z\theta}}$$

What is σ_1 here? What is σ_2 here? What are the common mistakes people do? They simply say σ_1 as + sign, σ_2 as - sign. What is the way we have labeled σ_1 and σ_2? Algebraically maximum is σ_1, algebraically smaller is σ_2.

So, you have no clue how to use; it may be incidental, in this problem it is incidental. So, I have to substitute this, I have σ_θ, so I can also have a much simpler expression. I have

$$\sigma_{1,2} = \frac{\sigma_z}{2} \pm \sqrt{\left(\frac{\sigma_z}{2}\right)^2 + \tau^2{}_{z\theta}}$$

I have to calculate, what is happening for the positive sign? What is happening for the negative sign? Then depending on the numbers, I should label, which is σ_1 and which is σ_2. So, which is σ_1 here? Zero; zero is σ_1, because algebraically 0 is greater and your $\sigma_2 = -12.98\,\text{MPa}$.

But I have always said when you calculate the principal stresses, do not stop there. You must also verify whether it satisfies the first invariant. First invariant is $\sigma_1 + \sigma_2$, which should be equal to $\sigma_\theta + \sigma_z$. In this case, because the torsional effects are so small, it is not influencing anything. So here, we have verified that this identity is also satisfied.

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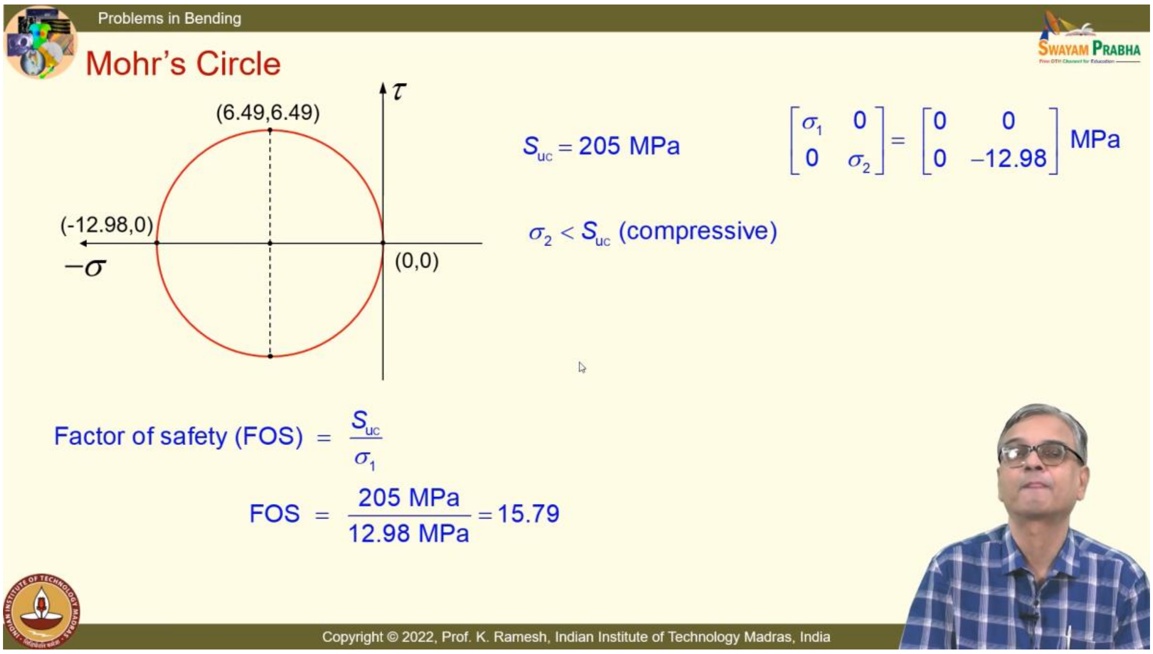

And I can also draw the Mohr's circle; I can draw the Mohr's circle and also find out what is the factor of safety in some sense. So, you get a Factor of Safety (FOS) of 15.79.

So, in this class, we have looked at certain subtle aspects of bending, the inconsistency of shear formula for analyzing a circular cross section, we have seen. Then we have also looked at, what is unsymmetrical bending. Even a symmetrical cross-section, if you have bending moments acting in two different directions, it can be labeled as unsymmetrical bending. And we have also seen if you have an unsymmetrical cross-section subjected to even bending in one axis, it has unsymmetrical bending. Then we have also looked at the problem of a simple torsional spring in your cycle.

We have learnt that it actually transmits a bending load. And at this level of the course, you do not know how to handle a curved beam to start with. And we have noted that in a curved beam, the stress variation can be non-linear and even for a single material, the neutral axis can get shifted. These are two important learnings.

Then we analyzed the problem of a femur, a very crude approximation. The future is biomechanics. You can bring in all complexities of solid mechanics in solving, bringing in finite strain, bringing in functionally graded material, bringing in variation of cross-section along the length. So, there is vast scope and you should be ready to capture that. Thank you.
