Strength of Materials
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Lecture - 28
Bending 6 - Shear in *I*-Beams and Shear Centre

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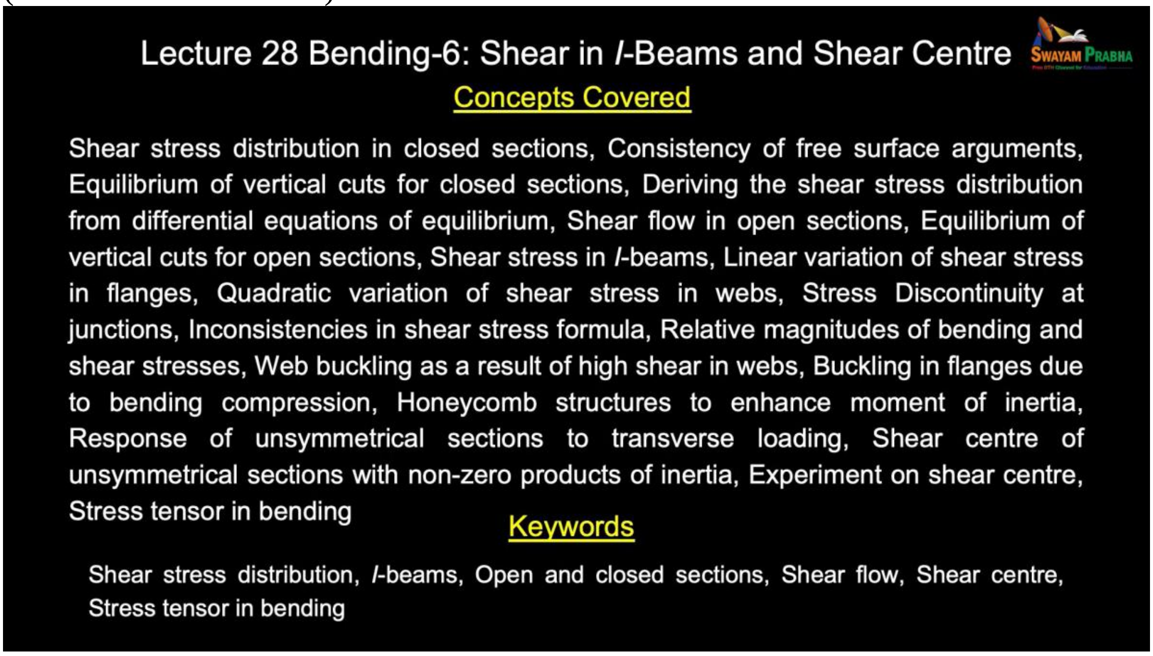

Let us continue our discussion on shear stress developed in bending.

You know, we are going to discuss certain subtle concepts, which are quite important for you to understand the limitations of the equations that we have developed, and also how to use them in solving actual real-life problems.

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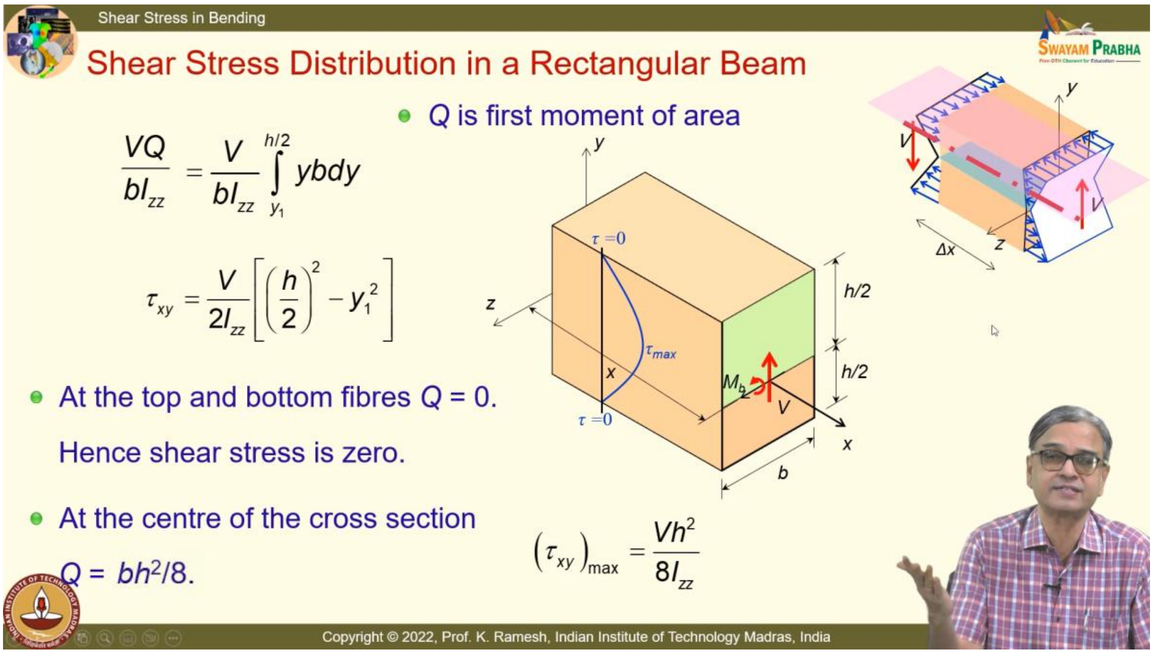

See, we started with shear stress distribution in a rectangular beam, where we had made a horizontal cut, and based on that, we have been able to get an expression for shear stress. And you should recognize that even when the beam transmits a constant shear force, we saw a variation in bending moment from cross-section to cross-section. And you know, you have learnt that you have to find out the Q, first moment of area that helps you to find out what is the shear stress acting on this surface, which is located at y_1. And we have also written down a generic expression in terms of integrals, so that when you have a non-rectangular cross-section, you can find out this Q comfortably, which you are taught in mathematics courses.

So, I get this as:

$$\frac{V}{bI_{zz}}\int_{y_1}^{h/2} ybdy$$

And when I integrate and substitute the limits, I get the expression $\tau_{xy} = V/2I_{zz}\left[\left(h/2\right)^2 - y_1^2\right]$. And obviously, when this area becomes larger and larger, Q also will become larger and larger, and hence your τ_{xy} will be larger. And you have this maximum, when this reaches the neutral axis.

And you also have, on the top and bottom fiber, Q is zero, which we have reasoned out from free surface discussion. See, whichever way you have the discussion, you should get one unique answer, only then, your mathematical analysis is correct. That is the reason why I go back and forth. And we are also going to determine the shear stress by solving differential equation; that also we will do. All of them will converge to the same answer.

So, what we understand is, top and bottom fiber should be zero. And as I told you that at center of the cross-section, Q is maximum and you get the shear stress variation like this. This is a parabola. I also emphasized, all along we have been seeing; shear stress varies linearly in the case of a torsion. And bending introduces axial stress, which varies linearly over the depth of the beam.

And the moment you come to shear, you find that it varies parabolically. So, that is the first observation. And when you make a horizontal cut, we found, when the bending moment is varying along the length, there was an unbalance; to keep the cut element in equilibrium, you need to have shear stresses developed. That was the idea.

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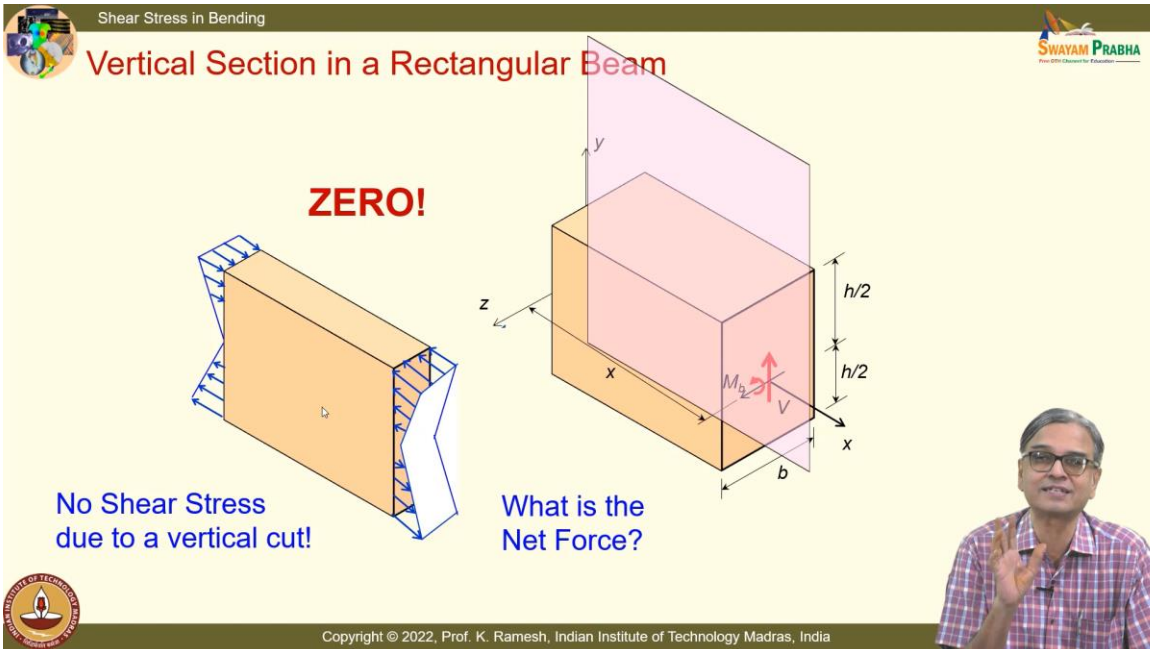

Then I gave you a homework; why not I make a vertical cut? Instead of horizontal cut, let me have a vertical cut and separate out the beam. Let us see, when I take the free body and list out what all quantities that act on it, we will get an idea how to handle it, fine. So, I have the cut portion taken out. Is the idea clear? I have asked you to cut like this. And bending moment varies along the length of the beam. So, I will have bending stress variation like this on the surface and bending stress variation like this on the surface.

And you know very well, when I substitute the bending moment, which was smaller in magnitude here, I will get smaller stresses. I will get larger stresses on this face. I mean, this is difficult to draw, I have drawn it to the extent possible. If you can improve the figure further, you are welcome to do so. Now the question is, will this element, is in a state of equilibrium or not? Do you require any additional stresses developed; particularly on this surface? This is the surface that is cut, fine.

What is your first appreciation of your understanding? You will have to find out; what is it that you have to calculate? See, what is the net force acting on this face? What is the net force acting on this face? Net force is zero because I have taken a vertical cut. If I have taken a horizontal cut, net force is not zero. Is the idea clear? So, what is the net force? Because that is equal and opposite from the bottom to top, it is zero. So, you do not have any shear stress developed on this face of the element. Is the idea clear? Very subtle point! Then we move on to another cross-section.

See I said, whatever the bending equation that we have developed, they can also be applied to open sections; and move on to the open section. See, you will have to find out the shear stresses from fundamental understanding. It is not like you have your eyes blindfolded and simply use the equation; whatever the equation that you remember and then get the shear stress. It is not like that; because I am going to take a counter example. I have always been saying shear stress has been parabolic in nature.

I am going to show something different, fine. And I am going to show; when I make a vertical cut, I am going to have something interesting. So, it is problem dependent. You will have to understand how the unbalanced force comes into play.

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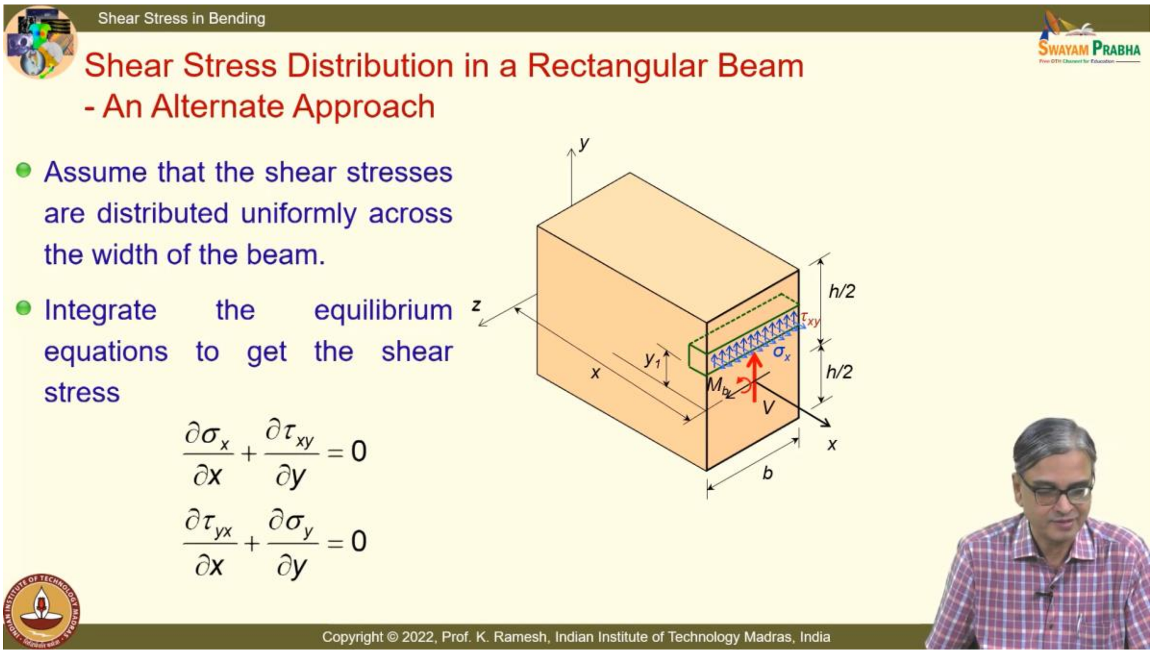

So, before we get on to that, we will also use an alternate approach to find out the shear stresses in the case of rectangular cross-section.

You know, in this course, we have not been utilizing the differential equations of equilibrium. We developed it in the first few lectures. Isn't it? We have determined what are the equations of equilibrium and then, we will not consider the body force here. And we will use those expressions to find out the shear stress. So, you will also get an idea that you can also find out by solving differential equations.

It gives a flavor of it because there is no great mathematics involved. It is very simple. If I have to solve this as a boundary value problem, then I have to develop lot of mathematics before I can solve. But this, I can get a direct solution. So, I have an element taken out.

It has a bending stress σ_x and we all know that you have shear stress acting on the surface like this. The idea is, how to get the expression for τ_{xy}. And you make an assumption and we have also determined it like this. The shear stresses are distributed uniformly across the width of the beam. And you can get the expression by integrating the equilibrium equations. Can you write the equilibrium equation when body force is absent and restrict it only to *x* and *y* components? Do you remember? Because when we do that in rigid body mechanics, you write $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$; you do that.

And similarly, we also saw, by using those expressions, a differential equation. You have that as

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0$$

$$\frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0$$

in the absence of body force. And our interest is to find out, what are the shear stresses? And we know from your flexure formula, what is the expression for σ_x. So, essentially if I handle this differential equation, I am in a position to get, mathematically, the expression for shear stress. But you have to use all the understanding that you have developed as part of interrelationship between bending moment and shear force, all that you have to know.

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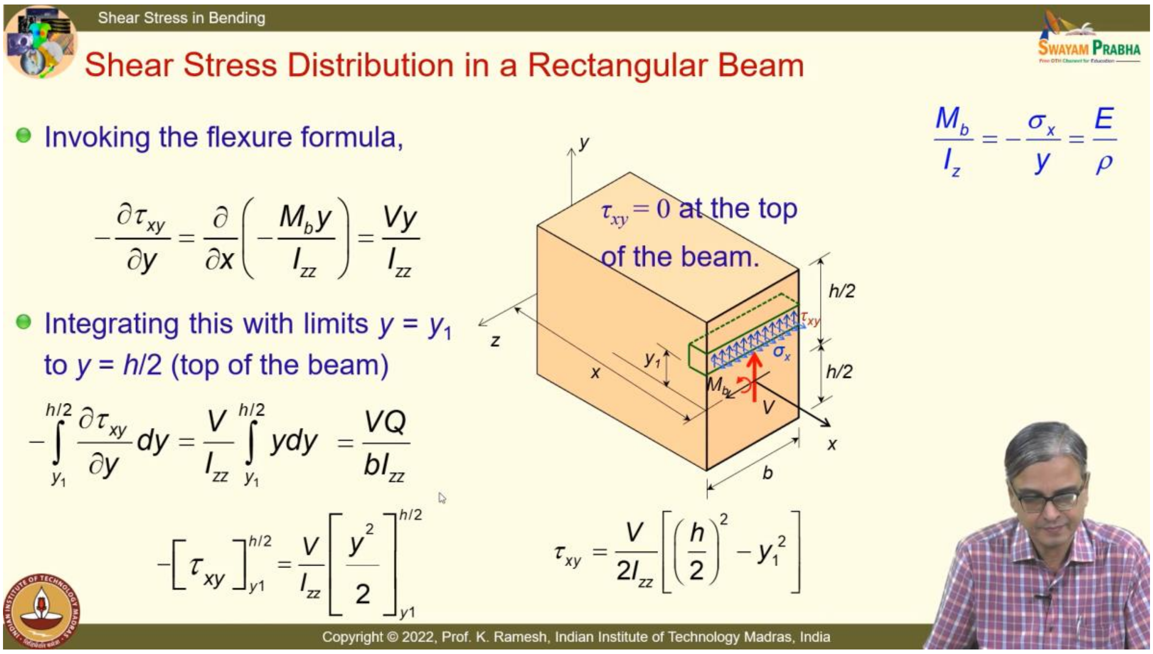

So, what we are going to do is, I have this as $-\frac{\partial \tau_{xy}}{\partial y} = \frac{\partial}{\partial x}(\sigma_x)$, I can replace σ_x in terms of what we have determined from flexure formula. You know, I keep repeating this flexure formula because if you know that, you can solve variety of problems related to bending. And you have this as $\frac{M_b}{I_z} = -\frac{\sigma_x}{y} = \frac{E}{\rho}$. So, I can replace expression for σ_x in terms of bending moment and the moment of inertia, I write it here; $-\frac{M_b y}{I_{zz}}$. And do you see, we can make another simplification because our idea is; relate the shear stress to the shear force; that we have written it as a force *V*, is it not? When you look at this expression, do you recall how we can handle this? I can have $\frac{\partial M_b}{\partial x}$, I can go back to the interrelationship and then write that as $\frac{Vy}{I_{zz}}$.

This you have to know, you know, whatever that we have developed, fundamentals you have to know; apply the fundamentals and simplify. So, once you have this, it is just a question of integration and then get it, that is all. And we have taken an element at a distance y_1 like what we have done in the other derivation. And we have to go from y_1 to $y = h/2$, that is top of the beam. And you have to write the limits and also substitute; very carefully do the mathematical step.

You will get the required expression. So,

$$-\int_{y_1}^{h/2} \frac{\partial \tau_{xy}}{\partial y} dy = \frac{V}{I_{zz}} \int_{y_1}^{h/2} y dy$$

We have already seen this *ydy* is nothing but a symbol *Q*. So, this is the same expression that we have got earlier also, VQ/bI_{zz}, that we have developed it based on slicing a portion and then finding out whether it is in equilibrium. So, it is a verification that such procedure was indeed correct.

And also have a satisfaction of having developed the differential equations of equilibrium, you should have used it at least once in a while in the course, fine. And this you have to look at it very carefully. What happens to τ_{xy} at $y = h/2$? So, that is what; you have to be careful. So, I have this as $\frac{V}{I_{zz}}\left[\frac{y^2}{2}\right]_{y_1}^{h/2}$ and $\tau_{xy} = \frac{V}{2I_{zz}}\left[\left(\frac{h}{2}\right)^2 - y_1^2\right]$. This was the expression that we were able to calculate based on VQ/bI_{zz}.

So, you have the identical expression and you have the satisfaction that you have also used the differential equations of equilibrium. And also gives a confirmation that what we have done are consistent mathematically.

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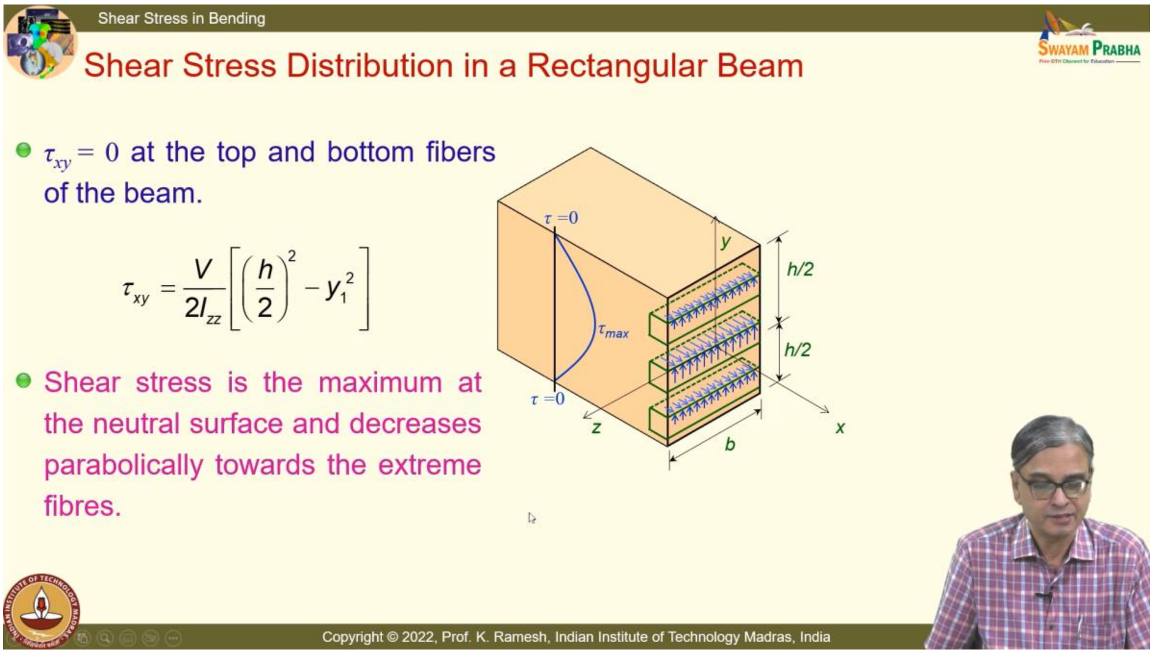

And we also have a distribution pictorially represented in a nice manner. I have different elements shown and then you have a variation. This is smaller here and you also understand on which surfaces, you have the shear stress.

You will have a complementary shear, fine. We have actually determined only this by your mathematics and this will be the complementary shear. And near the neutral axis, these magnitudes are higher and far away from the neutral axis, the magnitudes are smaller. See, what do you understand in bending is, when bending stress is maximum, shear stress is zero. When bending stress is zero, shear stress is maximum with one exception.

When I have a load application point like in a three-point bending, shear stresses are very high near the top surface. It is not parabolic; it is non-parabolic which needs to be taken care of while constructing the beams. For your analysis, you may say I stay away from the load application points and I solve my basic mathematical requirement. But when you go for construction, you have to recognize that these are the variations. So, you have this varying parabolically. The same expressions are repeated. If you have not written it down fully, you can correct it.

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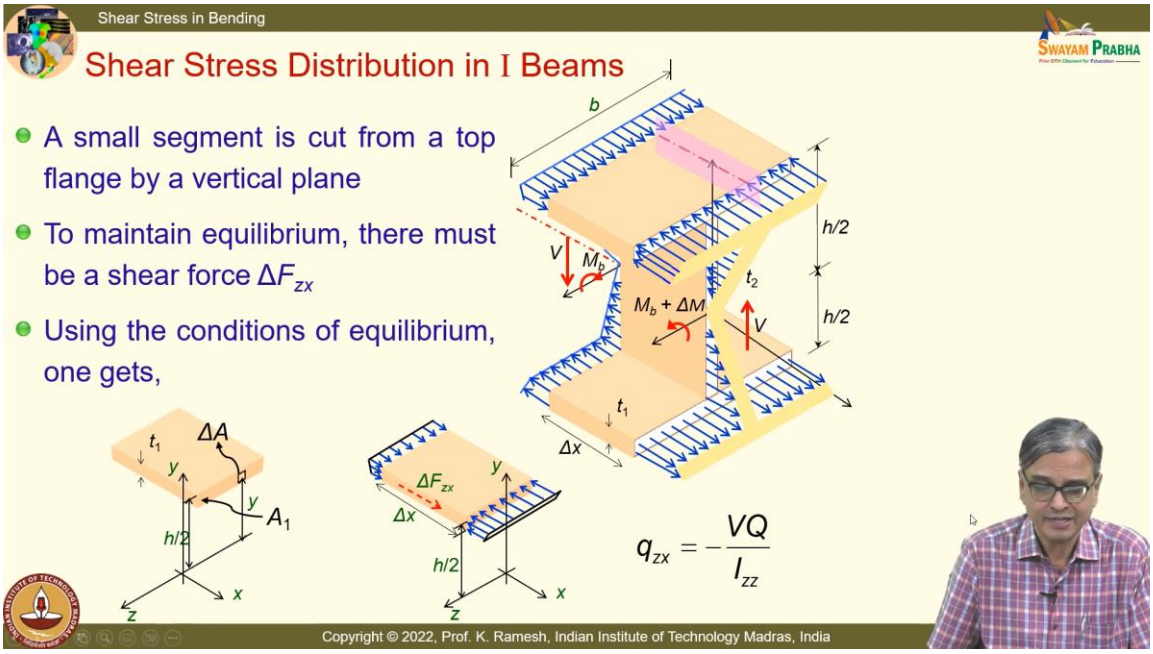

Now, we shall go and see what happens; how to find out the shear stress in the case of an *I*-beam? See, I will have to make a horizontal cut. I will also have to make a vertical cut. Only then, I will be able to find out the shear stresses in all parts of the *I*-beam. Even before I go and do the calculation, let me show what way I would calculate this *Q*.

The same *Q* is going to come in some other form, fine. Let me see whether you look at it and you are able to guess what is happening. These are all the geometrical details. You know, when you write it in isometric view, the distances appear like this and I have a shear force acting. I have to find out the shear stress variation on the flange.

This is called a flange; this is called a web and this is again called a flange. And for me to find out what happens if I put a horizontal cut, you can anticipate that horizontal cut will give me a variation of shear stress over the depth of the beam, fine. We have already seen what should be the variation. We have seen that the variation should be parabolic which is different from what you have looked at in torsion or bending; only the axial stress. We will also have to find out what happens on the flange? Suppose, I take a vertical cut, what do you anticipate? When I make a vertical cut, will that vertical cut remain in equilibrium or will it have un-equilibrium and some stresses have to be developed? See, when I make a horizontal cut in this or when I make a vertical cut at the central plane, the story is different.

When I make a cut here, you will have stresses increasing. Suppose, we assume there will be an unbalance. So, you will have stresses created on this kind of a surface. And I will also have to calculate Q and then I will be calculating Q like this. I have M_b here and $M_b + \Delta M_b$ here and you have the bending stress developed.

Very nicely drawn sketch, try to take it out as comfortably as possible. Now, I am going to make a cut here, vertical cut. So, do not jump to conclusion, when I get a vertical cut, the result is zero. That was applicable for a rectangular cross-section or any closed cross-section, when you take a vertical cut, you can do that. When I have an open section, you have to investigate the problem afresh.

So, you have to go back to the fundamentals; how we have started calculating the shear stress distribution. So, I am going to make a vertical cut. I am talking slowly, so that you have opportunity to write and take a record of this diagram. So, I have taken a vertical cut and then you are shown stresses on this surface, stresses on this surface. For your benefit, the board is not smart right now, I will make the board intelligent.

You make a sketch, neat sketch of it to the extent possible, so that you get the idea. I have a vertical cut here. And when I make a vertical cut on the flange, this portion of the flange will not remain in equilibrium. And when I make the board smart, it understands that it is not in equilibrium. So, it will move and this has to be in equilibrium because the whole beam is under the equilibrium. So, you will have stresses developed on this surface. Is the idea clear? So, you have to fundamentally investigate when I make a cut what happens.

Particularly, when you want to understand shear, you have to go to the fundamentals. Even though we may use the readily available equations for our calculations, how those equations have been developed, those steps you need to understand and apply carefully. Otherwise, you can really make a very complicated problem or a simple problem may get complicated for you. So, you have to understand, it is not just use of expression. How the expressions have been derived is equally very important.

And you know we have a sign convention, so this is a positive or negative surface? It is a positive surface. So, on a positive surface, I will put positive direction as positive. So, you should recognize in the case of an *I*-beam, when I make a vertical cut, I get this as ΔF_{zx}. So, that is a very important learning. And once you have done this, other steps will be very similar.

So, using the conditions of equilibrium, one gets $q_{zx} = -\frac{VQ}{I_{zz}}$. I am skipping the steps, so you can go back and redo the steps, very similar to what we have done for the horizontal cut. And we have also discussed what is the meaning of shear flow. This is force per unit length; when you divide by the width, you get the stress. And you know, you have this area; these are all very nicely drawn sketches.

So, you integrate it and then find out what is Q. And now, you have to tell me, I will also show you this. This is the area A_1. So, when I have to find out Q, I have to actually find out for this area. Is the idea clear? So, the basic expression looks very similar, but the Q; how it varies when I want to do it on the flange?

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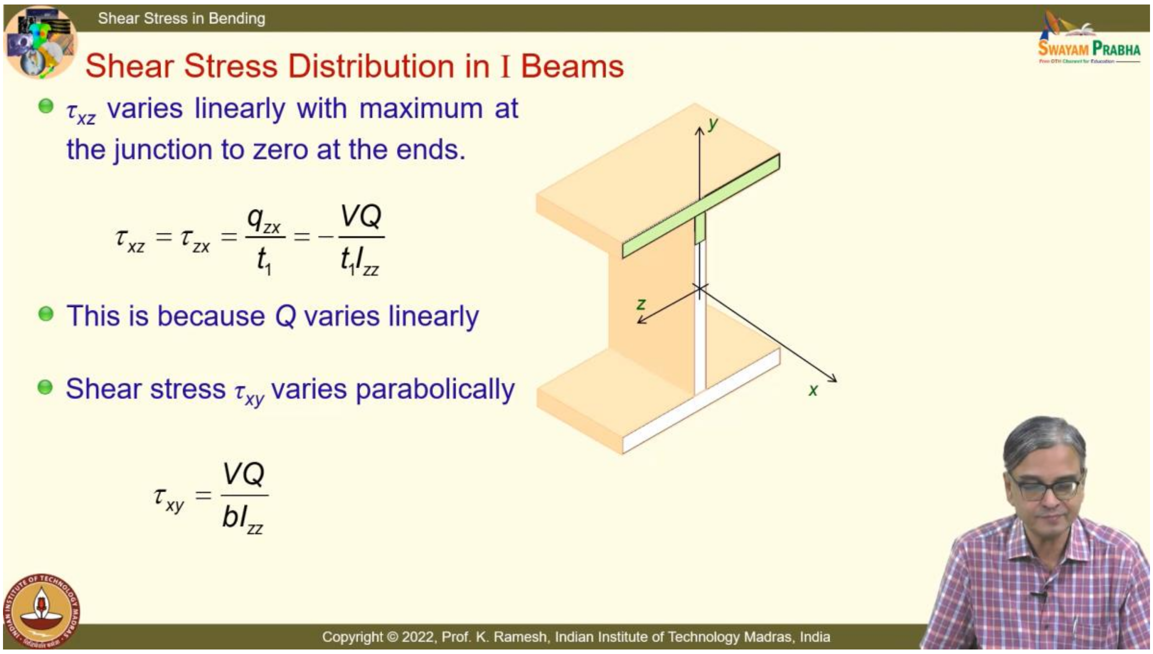

I go to the flange and when I pictorially represent Q, let me see how many of you guess what is happening. See if I want to find out what is the stress on this surface, I will have to take this much area and then calculate the Q, am I right? Suppose, I want to come to a portion here, I will have to have a larger area.

How do you anticipate Q to vary? Mathematically, is it linear, parabolic or cubic? Apply or go back to your mathematics. See, I am showing the area, I am showing the area and then it is increasing like this; ydA. That is all. What remains constant here? y remains constant. So, you have to appreciate shear stress can also be linear in a beam; where it is linear, you have to look at.

Everybody thinks that shear stress varies parabolically, fine. It varies parabolically in some portion of the beam. You can also have a shear stress component τ_{xz} that varies linearly with maximum at the junction to zero at the ends. It is very interesting. So, unless you understand how the equations have to be derived, you cannot use the expressions for shear stress blindly.

You have to apply your thinking. So, I will have this τ_{xz} equal to τ_{zx}; equality to your cross shears. That turns out to be $-\dfrac{VQ}{t_1 I_{zz}}$, where t_1 is the thickness. And even though I have expressions like this, see normally when you get an expression, you see that it is applicable everywhere and there are inconsistencies. We will also have to discuss the inconsistencies.

Particularly, this junction is going to be a trouble spot. This junction is also going to be a trouble spot. Recognize that these are trouble spots, but find out a way around to get over it. That is what engineers do. So, you have this varying linearly because Q varies linearly. On the other hand, if I take the horizontal one; when I do this, my y also will change.

So, the change is parabolic for the shear stress τ_{xy}. So, you have τ_{xy} as well as τ_{xz} exist in the case of an I-beam. You may have to investigate when you take a channel. You should look at how it is? How it is loaded? If you take an L-angle section, you will also have to find out, what is the way that you have to make a cut. So, the moment you come to open sections, you have to understand the fundamentals, how the shear stresses are developed.

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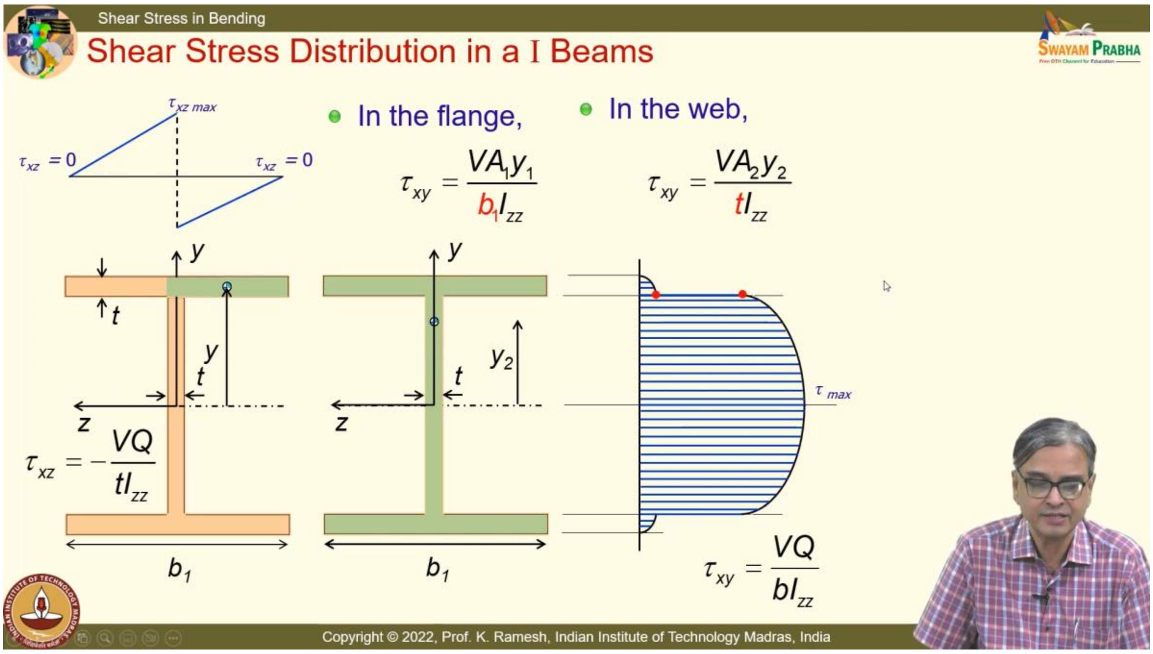

And you also have a nice 2-D picture. I have this shown. This you can easily draw. This you can easily draw and then make your ideas clear. So, I have $\tau_{xz}=-\dfrac{VQ}{tI_{zz}}$. When I want to calculate this, I will calculate only like this. I have a small area and have this y and then the area increases and then I have this y remains constant.

If I do this, the y remains constant, area keeps increasing. So, the shear stress will be zero here and it will linearly vary, reach a maximum in the junction and die down. Since we have put this as negative, this is what is shown here. τ_{xz} is zero, reaches a maximum here and then goes to zero. So, shear stress has two different distributions. If I look at τ_{xz} based on the reference axis that we have taken, it varies linearly.

This is what happens in the case of the flange. And we also have the expression when we have τ_{xy}.

$$\tau_{xy} = VQ/bI_{zz}$$

See, b is the width and you also have a sudden jump in the junction. The width was b_1 and suddenly it reduces to t.

Is the idea clear? For the same area, you will have two different points. One corresponding to the flange and another corresponding to the web. And we know that this varies parabolically. I will just put the sketch. So, I have this. This point I can find out that is; please write down; $\frac{VA_1y_1}{b_1I_{zz}}$ because the width is this much.

And when I am making a cut here, I will have; the shear stress on the flange will be like this. But if I consider same point forming part of the web, I will have a value as $\frac{VA_2y_2}{tI_{zz}}$. It goes up. Then again, the whole thing gets replicated parabolically.

So, I will eventually have a curve like this. You will have a maximum value at the neutral surface; neutral axis. So, I have this variation like this. So, once you know how to interpret this, any cross-section given, suppose I give a T-section, you can also draw schematically, fine? You do not have to go to a computer to plot the graph.

You can get that schematically. So, what you find is, in the plane like this, when you are looking at τ_{xy}, it varies parabolically like this and you have this varying linearly on the flange.

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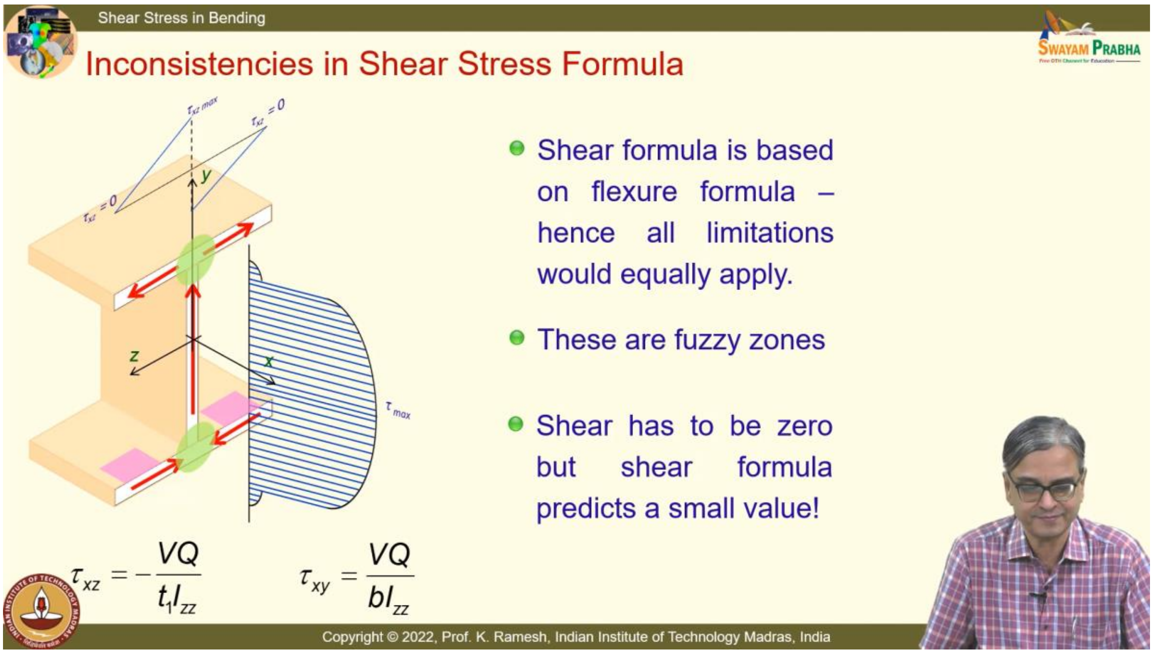

I have also shown it in various different ways to drive home the point and we will also look at what are the inconsistencies in shear stress formula. We have the expression $\tau_{xz} = -VQ/t_1 I_{zz}$ and I said that; why you call it as a flow. You have this as a fluid flow coming, joining together, move up, and then separate out.

So, that is what is shown here. $\tau_{xy} = VQ/bI_{zz}$. So, I have this and it gets separated out. These kinds of pictures you will see in the books also. And the important discussion is you have a junction here, these junctions where there are inconsistencies in our expressions. Recognize the inconsistency, you do it with care. And in this, you know, I have shown that these are sharp corners.

In practice, it will not be sharp corner. You will have a beautiful fillet there because you would like to reduce the stress concentration. And I have the shear stress varying parabolically. I have drawn it for the plane of symmetry, fine. And you have the shear stress τ_{xz} varies linearly.

And see, shear formula is based on flexure formula. Hence, all limitations would equally apply. Whatever the limitations that we have discussed on flexure formula would apply because I have a plane of symmetry here, fine. And you have this junction. I have junction 1 and junction 2 and we found, these are fuzzy zones.

I have two different values which I am saying. And if some of you are very alert, you would have also raised one more question. See, we had done exhaustive discussion on free surface. I have a top surface. There is no load is applied; bottom surface.

Similarly, I have the top surface of the flange, bottom surface of the flange. But when I look at my shear stress distribution, I show this is zero on the top surface; satisfies our free surface requirement. But when I come to the bottom surface, that is, this is not seen, but this surface is seen. If you look at the expression, you have a value. I have just shown a small portion as red.

This is applicable for the entire length of the flange. You find shear has to be zero, but shear formula predicts small value. You have to accept that. There is a contradiction, fine. How do I resolve the contradiction from an engineering point of view? We have already seen the magnitudes of shear stress are much much smaller, at least 20 times in the case of a rectangular cross-section.

So, if I have a slender beam, so that is what it says. You have to apply all this to a slender beam. Shear formula is based on flexure formula. So, when I have a slender beam, shear stress determination from these expressions is reasonably accurate. If you violate slenderness of the beam, then you have to look for other formulations. You have a Timoshenko beam and then improve the flexure formula itself as well as the shear evaluation. Those are postponed for your higher studies.

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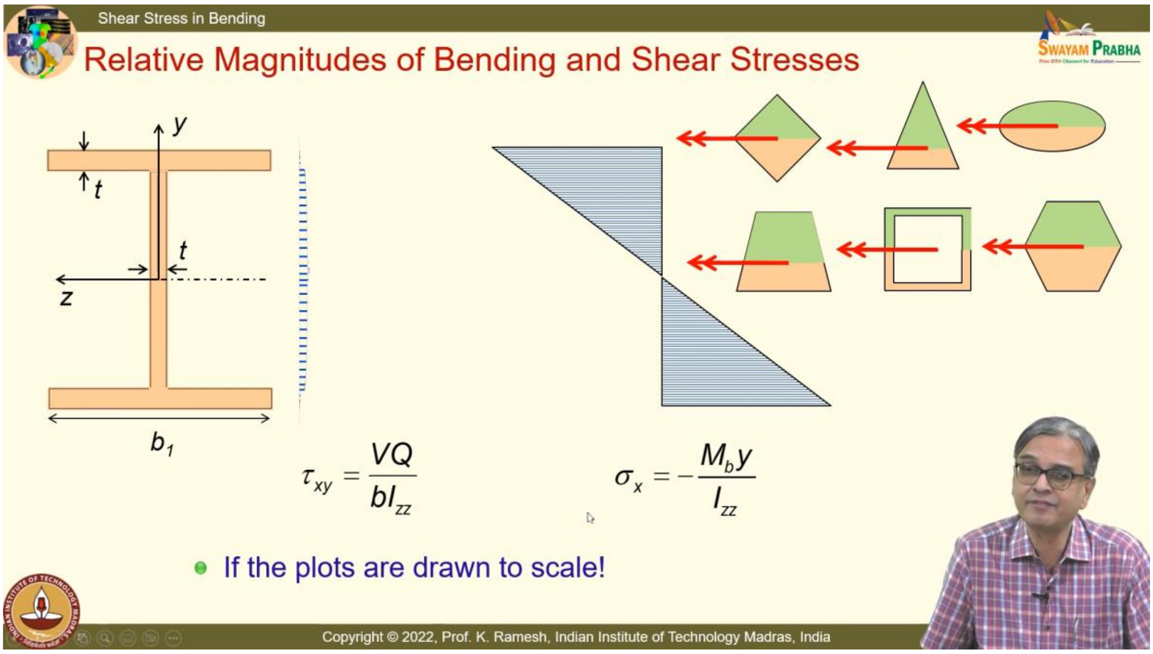

You know, in order to drive home the point, the relative magnitudes; because one focus is to illustrate what is the variation. The variation is also very very important. Other aspect is what are the actual magnitudes? Normally, books stop here. You know, they show the bending stress variation which varies linearly, shear stress variation which goes parabolically and it will have very nice curve. And I said, these are all looking very nice, but there are inconsistencies, fine.

And if you really look at what is the magnitude, it is hardly anything here. So, even though my formula gives me some number, it is very close to zero. So, it satisfies what should happen in reality.

So, that is why we do not worry about it. When it matters, we worry about it. When shear stress magnitude matters like at the load application point, we immediately take corrective action, fine. If the plots are drawn to scale which we have also discussed in the context of rectangular section, I am also discussing in the context of I-section to drive home the point which is missing in many of the books. See, like we have applied flexure formula, the only restriction was I should have a plane of symmetry. So, I can find out the bending stress using this expression.

Look at the bending moment diagram and get the bending moment and simply apply it. Look at the shear force diagram, get the value of V, get the value of shear stress. You can do your design course comfortably. And for you to get the shear stress, you know, you also have to find out Q; and Q is nothing but you progressively find out what happens at each level and then plot it. It will be parabolic plot for all of these cross-sections, parabolic plot.

But the story does not end here. There are also inconsistencies. Even if I take a simple circle; circular cross-section, we will also discuss what is the way the shear stress has to vary. So, what you will have to appreciate is, as long as the beam is slender, from a design point of view, you will know how to calculate the axial stress developed due to bending and the shear stress developed due to transmitting of shear stress or variation of bending moment, whichever way you want to do that.

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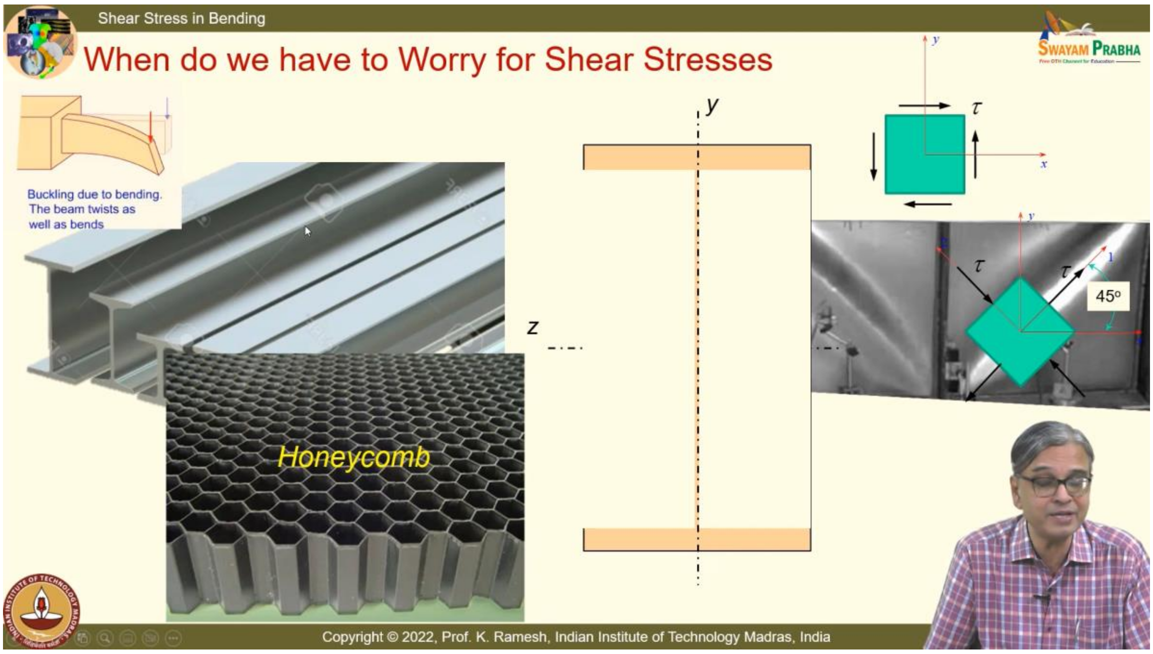

So, I said where we have to worry, we have to consider shear stress. Let me ask, I take a rectangular cross-section. Now, you are all taught, it has to transmit essentially bending load, the inner core is not participating and you want to be a very optimal engineer.

That is why you should not do optimization to the hilt. God has given you two kidneys. He has not given you one kidney. He was very careful. Suppose I want to remove the material and then make an *I*-section, you all know, inner core is not participating. What is the limit? Can I keep on going like this, remove all the material? You should not do that.

What do you anticipate? I have given you the clue and then this also says, worry; shear stresses. What happens? Where this happens? This happens at the central core. So, what would happen when I have shear stress? How do you visualize it from our earlier discussion? We have been discussing that a shear stress; pure shear stress can be looked at as tension and compression. The moment I have compression, what will happen? I can have buckling. I can have one form of buckling which is recorded in literature.

You have this buckling and when I put this at 45^o, you see there is a compressive stress developed. So, do not handle shear stress as innocent. It can be very damaging to you if you do not recognize it’s another avatar because we have seen, when you look at the Mohr's circle, you should recognize the same stress tensor can appear in different ways. So, it can precipitate failure in one form, but you can also have failure in another form. I do not know how many of you have noticed it.

See, the moment; why people fear bending, I will have tension as well as compression. Even if I take a simple rectangular beam, so before we go into that, you also have honeycomb structures. In all space structures, they use honeycombs to make the web portion lighter and then have the flanges so that I will have good inertia. That is how it is done. That is one way of practically handling this. And then even if I take a rectangular beam, because I have tension at the top and compression at the bottom, your flange also can buckle.

Forget about the web. Here I talked about the web; extreme condition. I showed that it is almost like a thin plate. I can also have flange buckling and the flange buckling is, you know, it can make the beam to bend as well as twist. Is the idea clear? So, you will have to worry about; when you are deciding the cross-section, find out the extreme situations. Do not simply reduce to have maximization of one parameter.

You will have to holistically look at the problem and take a decision.

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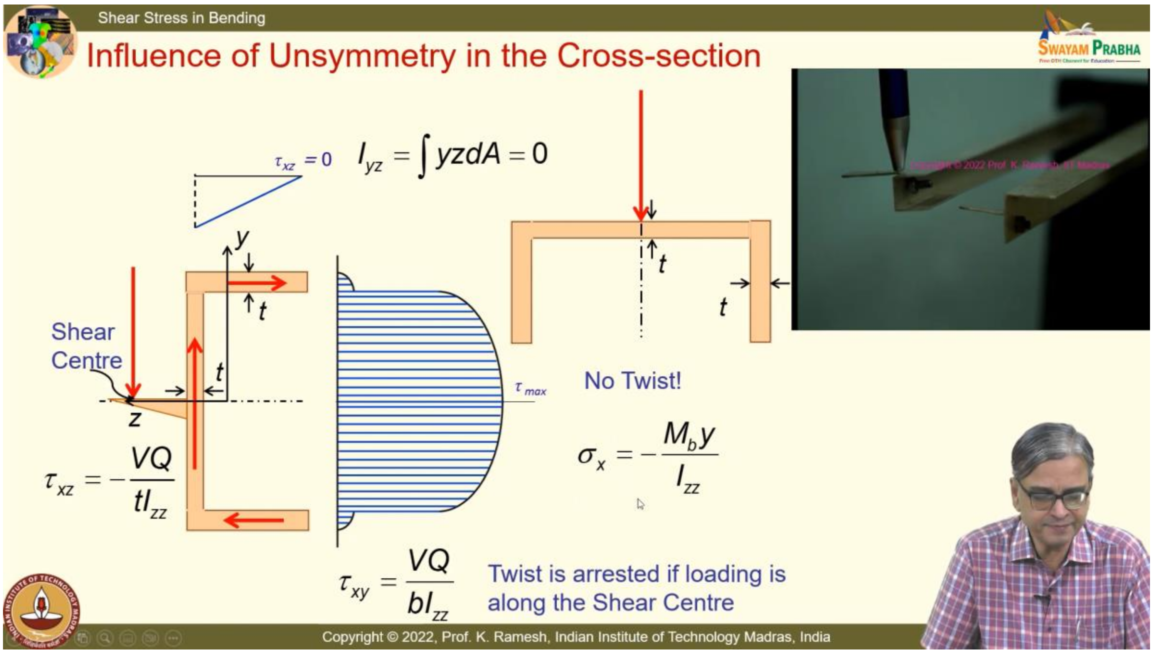

You know, I have an *I*-section and then we have already looked at how to put the shear flow. And I can construct a channel out of this. Suppose I knock off this, and knock off this, I can construct a channel out of this. Do you anticipate something? Because the diagram is drawn with arrows.

Suppose I knock off this and knock off this, cut this off. Do you anticipate something happening to the beam on the strength of what is shown as arrows here? Anybody looks at it? I could see some hand movement, that is correct.

The hand movement should translate into words. It will also twist. Very funny. This twisting is different from our buckling. Earlier I said that rectangular cross section can have a buckling, because of that it can have a bending and twisting. But here, if I put this and make my board smart, it is going to twist. It is exaggerated.

See, to drive off the point, what do I do? I have to tell you that there is unbalance. So, this has to twist. And I have always emphasized that we should look at the experiment and convince ourselves what happens. Is the idea clear? So, we will also look at the experiment.

And you have a concept called shear center. Please write down. You have to write down this. If I apply my bending load, whatever the load that causes bending, not along the centroidal cross-section, but along a point like this; shear center which is away from this, you can find that these two form a couple. It opposes the couple here. So, it will not twist, it will only bend. I am telling you theory.

I will demonstrate by an experiment, because you have always seen everything by verifying with an experiment. Is the idea clear? We have just not taken on the face value of simple discussion. So, I have this. One of the quantities that I am going to have this is, because it is not having an axis of symmetry, I will have non-zero I_{yz}.

We have taken a cross-section with symmetry. In all our calculation, I_{yz} was zero, but you will have a non-zero value now. And that is what is shown here. I do not know whether you are able to see. I have also applied it with the shear center. If I do it with the shear center, it will not twist.

Can you see that this line is not straight, it is getting twisted? I will also show a bigger picture. Suppose I apply it outside, it will only bend. That is like putting it here. For clarity, I have shown a large distance, but in reality, it is small.

And we will see that also in a bigger diagram. So, I have $\tau_{xy} = VQ/bI_{zz}$. And then, I have τ_{xz} also. And when I take the same channel like this, for this case, I_{yz} is zero, because this has a vertical plane of symmetry. My loading plane is like this. And it has a plane of symmetry.

The plane of symmetry comes when you look at what happens I_{yz}. See, this is also a channel, this is also a channel. I can put the channel like this and apply my bending theory and then all the other calculations, I can do. I can also do the calculations here. But if I apply the load along the *y*-axis, I will have this also getting twisted.

I should apply it slightly away. We will see this in a bigger diagram. You have to appreciate that twist is arrested if loading is along the shear center.

It is a concept. It is a conceptual development. There is something called shear center. Funny name, that is what you have to look at. I mean, you have put that along that axis, if you apply the load, it is not going to twist. Somebody coined it as shear center, it got stuck. So, you can still use the expression for normal stress from bending comfortably here.

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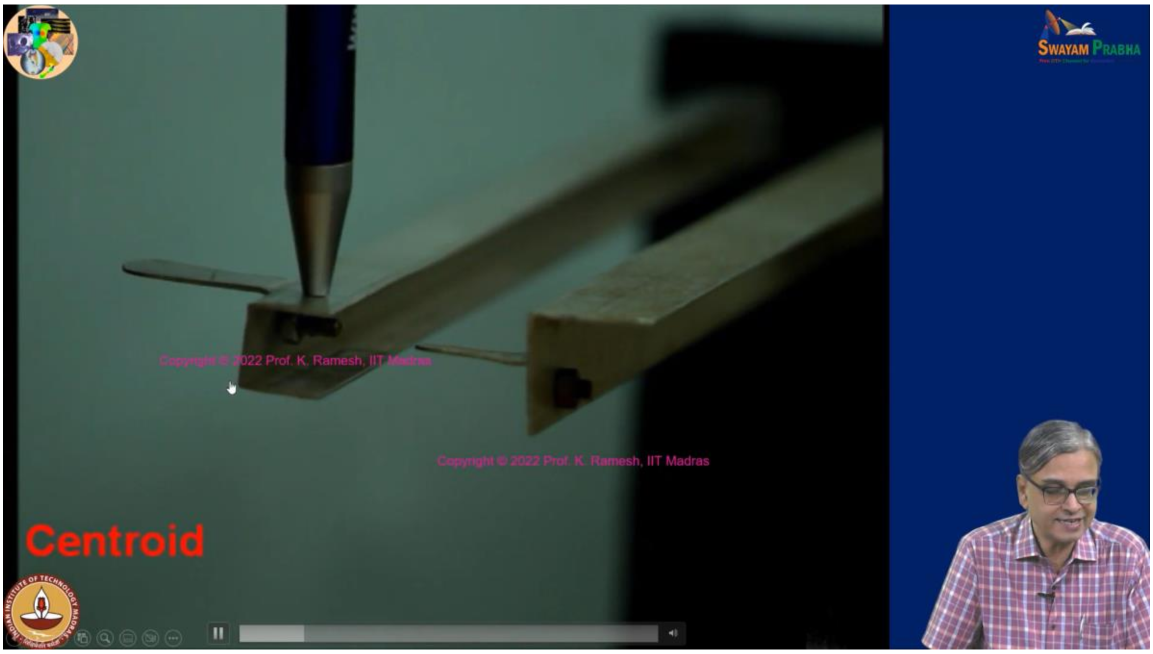

And let us see the test again. You please make an observation what is happening. Do you find that this is inclined? It is very clear in this animation. It was so difficult for me to press because this is made of aluminum, not of plastic.

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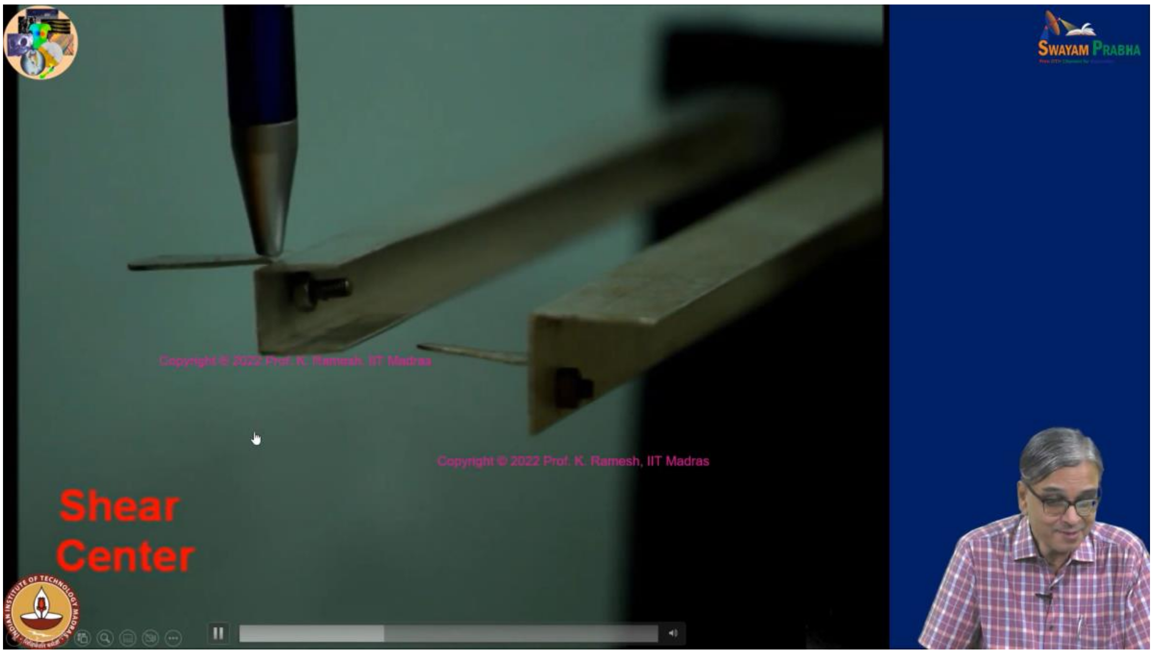

You see a marked difference; don't you see that? So, you appreciate there is something called shear center. I can just make an open section like this to only bend, not twist and *L*-angle.

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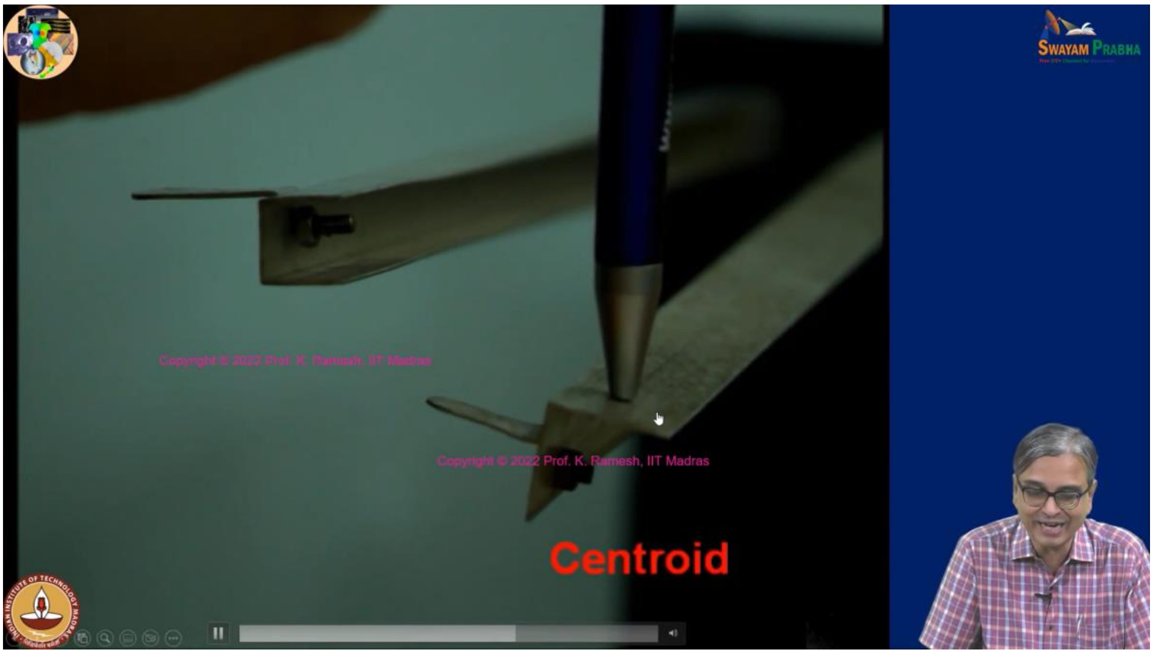

You can see this is beautifully getting twisted because it is not as stiff as this.

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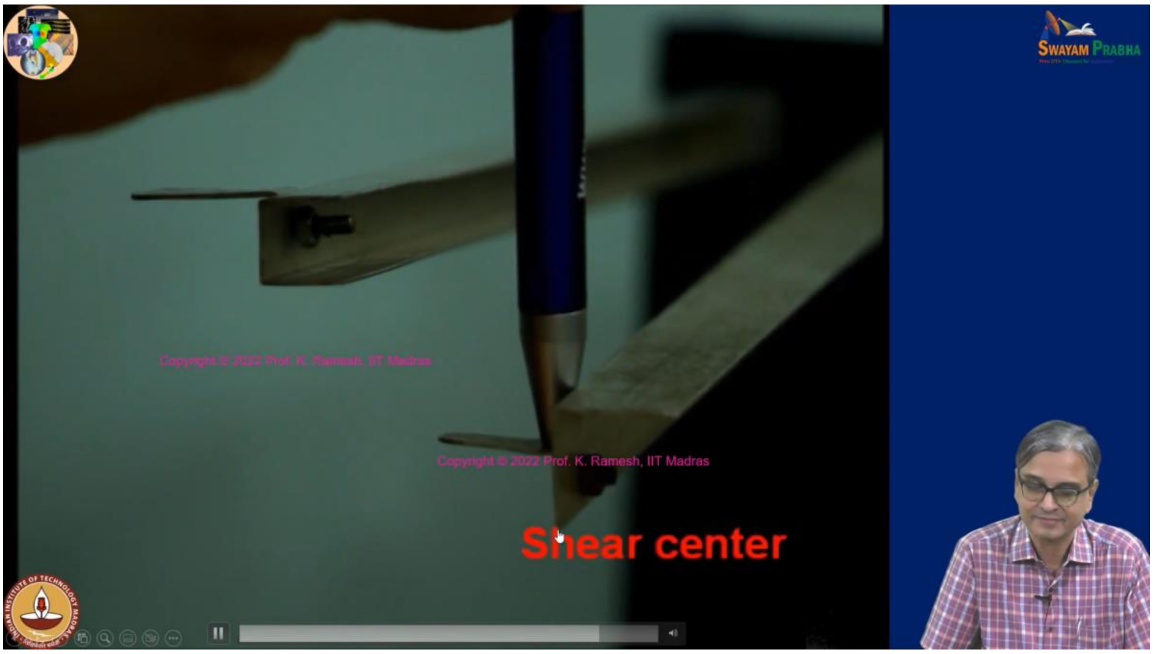

So, I could show the twist better and shear center is very close to this and you find that this is moving straight.

And now let us take; I want you to do it yourself because that will give you clarity. I have always emphasized, make a sketch of this.

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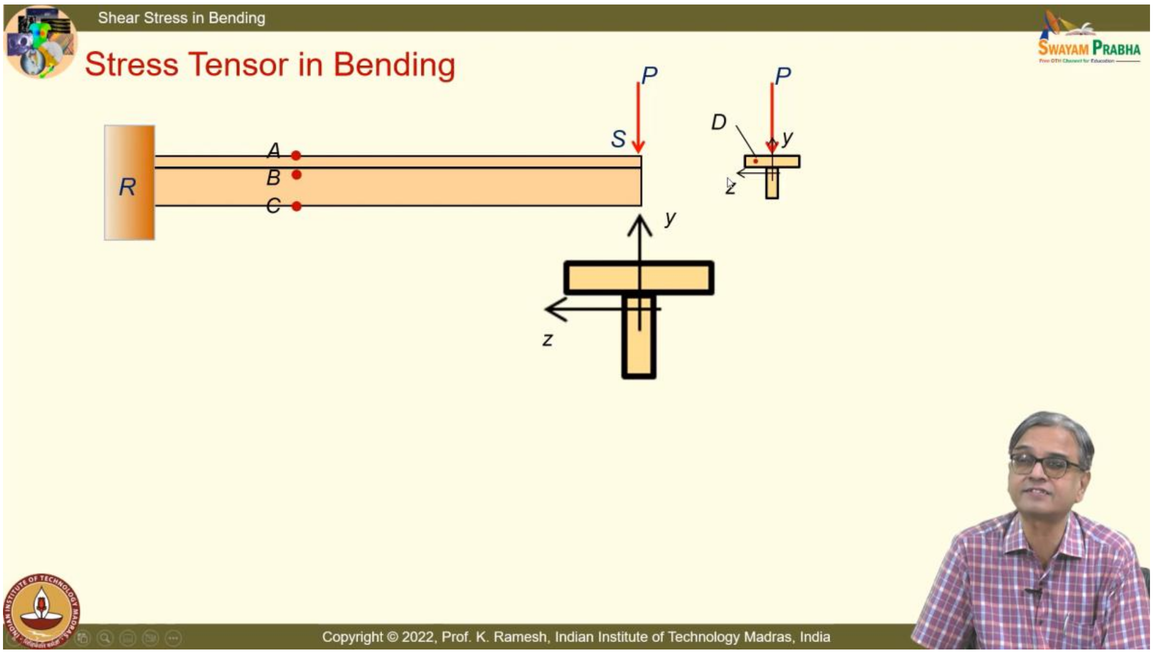

I have a *T*-beam. I have always emphasized that in our calculations, in our derivation, we only evaluate the particular component. But you should also get the practice how to recognize the stress tensor at the point of interest. I have given you the axis. I have this as *z* and *y* and you have the *x* axis along the length of the beam and I am taking three points. Point *A* is on the top surface of the beam, point *B* is on the centroidal plane.

I mean this is also the neutral surface and *C* is on the outer surface and you have a point *D* on the flange. So, you have to find out what is the stress tensor? And you will also have to tell me what is the magnitude of the stress quantity. Whatever the expression that we have developed, it is more of a recapitulation.

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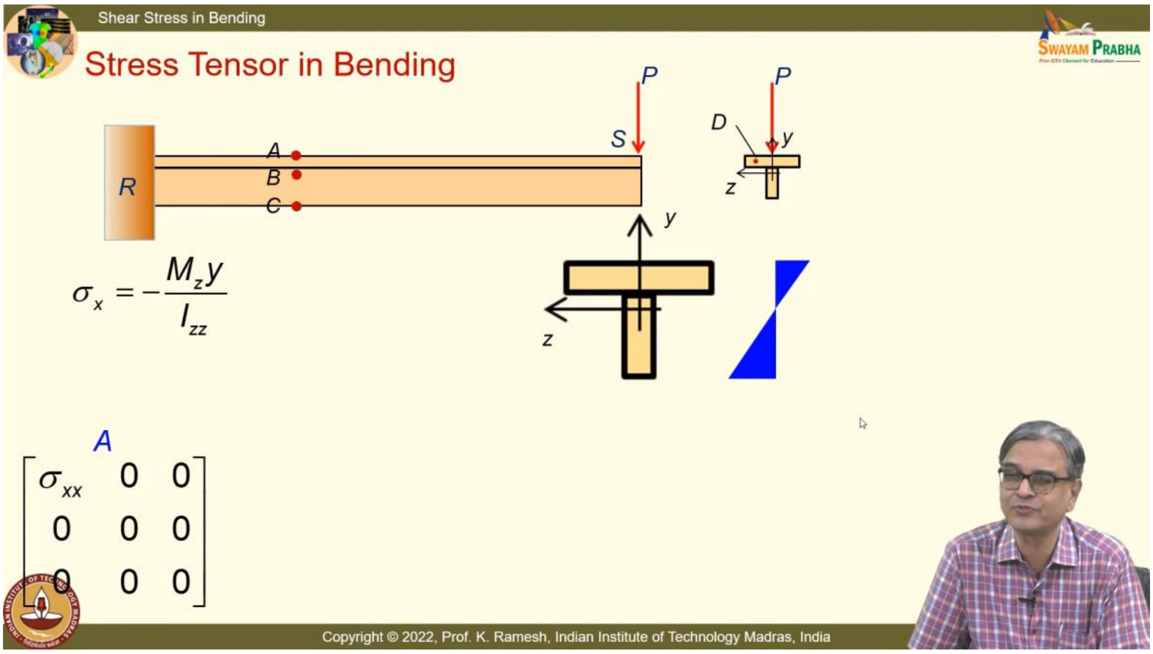

And what happens at point *A*? Point *A* is on the top surface. What is the stress that is developed there? What are the stress components that can exist there on the top surface of the beam? Only? I am not getting it.

I am not getting the answer. Can you repeat it? Only normal stresses. So, I have this as σ_{xx} and all other quantities are zero. And do you have an expression for normal stress? You have the flexure formula and can you write down the expression for normal stress? So, we have followed a symbolism so that I have put for a positive bending moment. We will have a compressive stress on the top. With that symbolism, this expression is written and you all know that this varies linearly.

This is made of one material. It is not made of two materials. So, your centroidal plane coincides with the neutral surface. And can you sketch shear stress variation? When I say shear stress, the dominant shear stress here is τ_{xy}. You put the τ_{xy} variation. Can you put the τ_{xy} variation? Let me know how many of you make a reasonable guess of it because you should; you are right!

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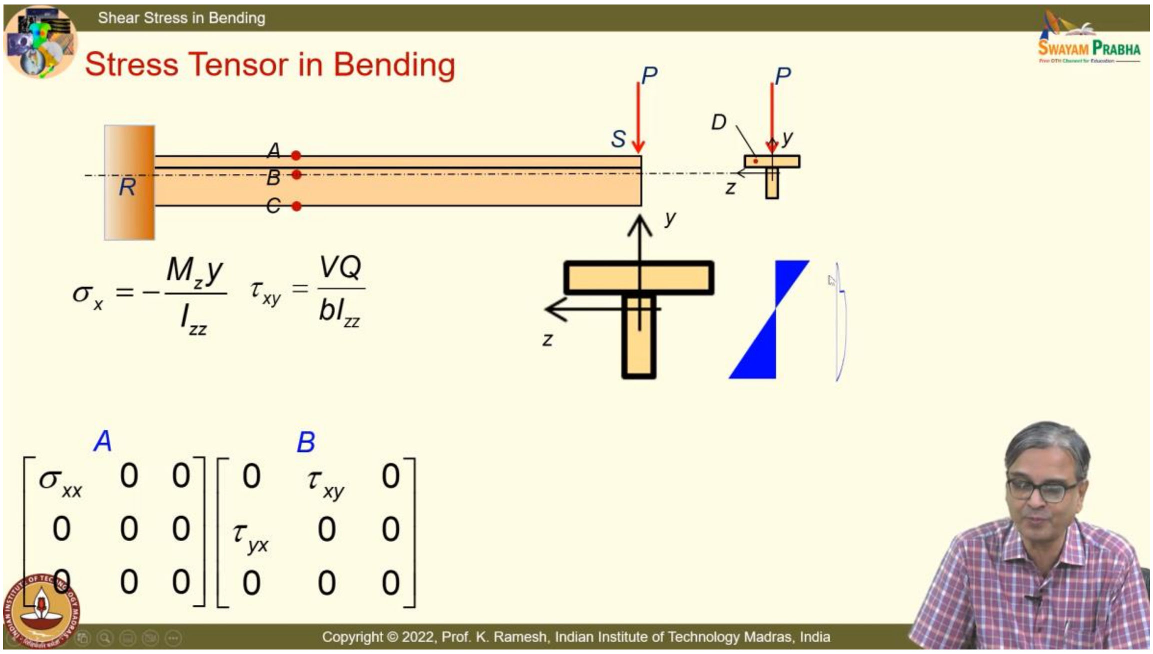

So, I have the point *B*. The point *B* is in the centroidal axis.

So, in the centroidal axis, what is the stress that you anticipate? What will be the stress tensor? Yeah, I get good answers because you make mistakes. The class is the right place where you have to make mistakes, not in the exams. What is the stress that is going to be there? See, I have changed only the cross-section. We had discussion on what happens in bending, what happens when it acts like a cantilever.

I have only a constant shear force. So, at point *B*, I anticipate only shear stress. There is no normal stress, fine, τ_{xy}. And do you have an expression for it? You have an expression for it.

$$\tau_{xy} = {VQ}/{bI_{zz}}$$

And I have asked you to draw this and some people have got this.

You know, this is also hand drawn. So, you will also have to accept me; that it reaches as a maximum in the centroidal plane and that dies down parabolically. And if you really look at the magnitudes, I have drawn it slightly smaller. It can even be much smaller than this. To drive home the point, that values are very close to zero in the flange.

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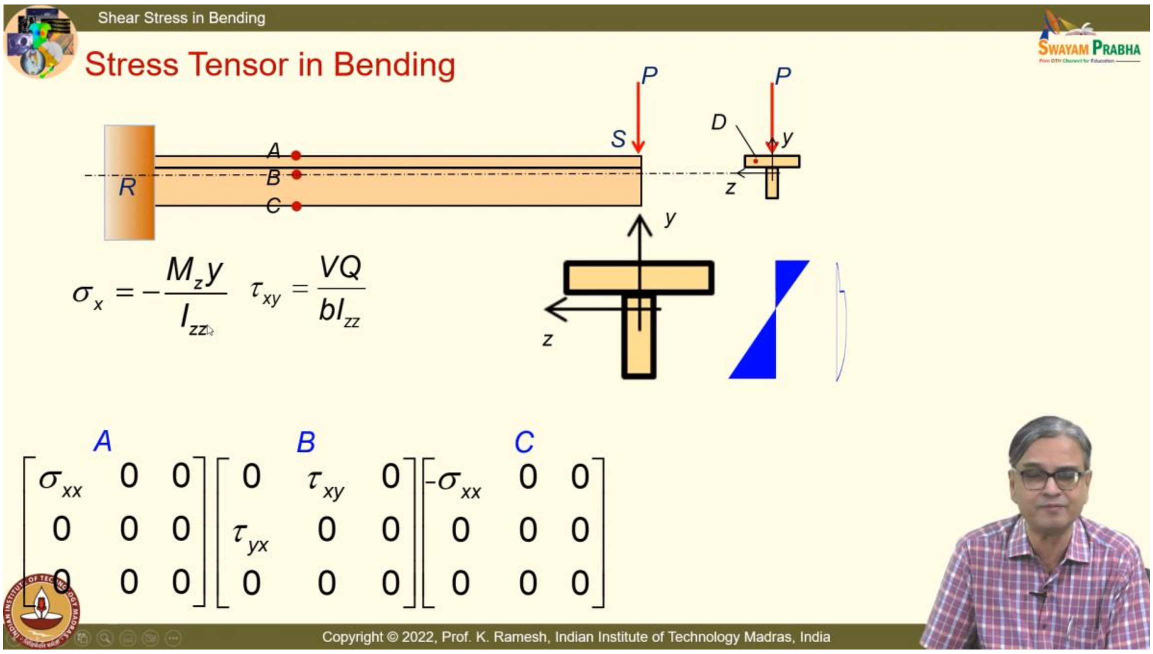

And point *C* is again very simple. What is the stress that you have? You have only normal stress. Because I have put this as tensile, I have put this as compressive or you can say that my mathematics will give me. Either way you can take it.

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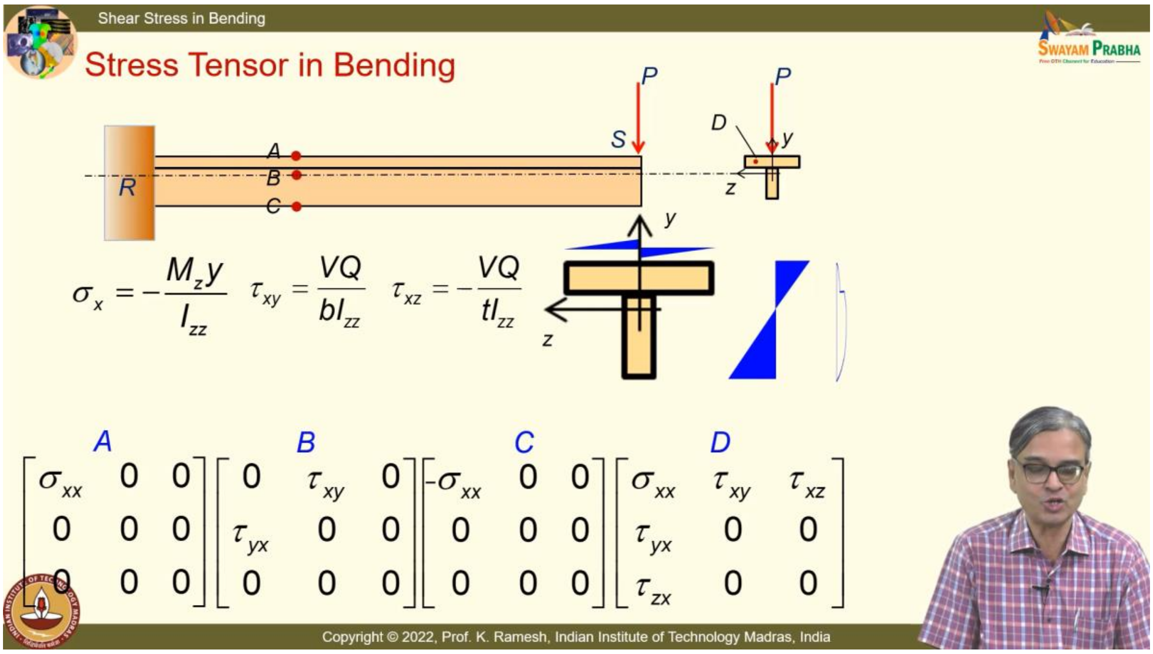

Now, I take a special point *D*. You have to tell me, what all stresses can exist at point *D*? Very good.

So, I will have in the case of *D*, I also recognize that the shear stress varies linearly, τ_{xz}. So, this is a very special point. This is the most populated stress tensor. Do you have an expression for finding out the τ_{xz}? $-VQ/tI_{zz}$.

So, in this class, we have looked at certain further discussions on shear stress in bending. We have also been able to derive the basic expression by integrating the differential equations. By looking at the equilibrium conditions for a rectangular cross-section, we have been able to find out the shear stress distribution.

Then we moved on to what kind of shear stress is possible in an open section like *I*-cross-section. We have seen that there could be a τ_{xz} component which varies linearly and τ_{xy} varies parabolically. And we have also looked at certain inconsistencies on the shear stress equations. And we said that you have to live with it.

And whatever the limitations in flexure formula, it equally applies to shear stress. You have to have slender beams; it is very important. And we have taken a cross section of *T* and we have found out, for selected points, how do you have the stress tensor? It is very very important. You only have the expressions to calculate the components. The components look like scalar. Only when you put it in a matrix form, you can recognize that as a tensor. Thank you.