Strength of Materials
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Lecture - 27
Bending 5 - Composite Beams

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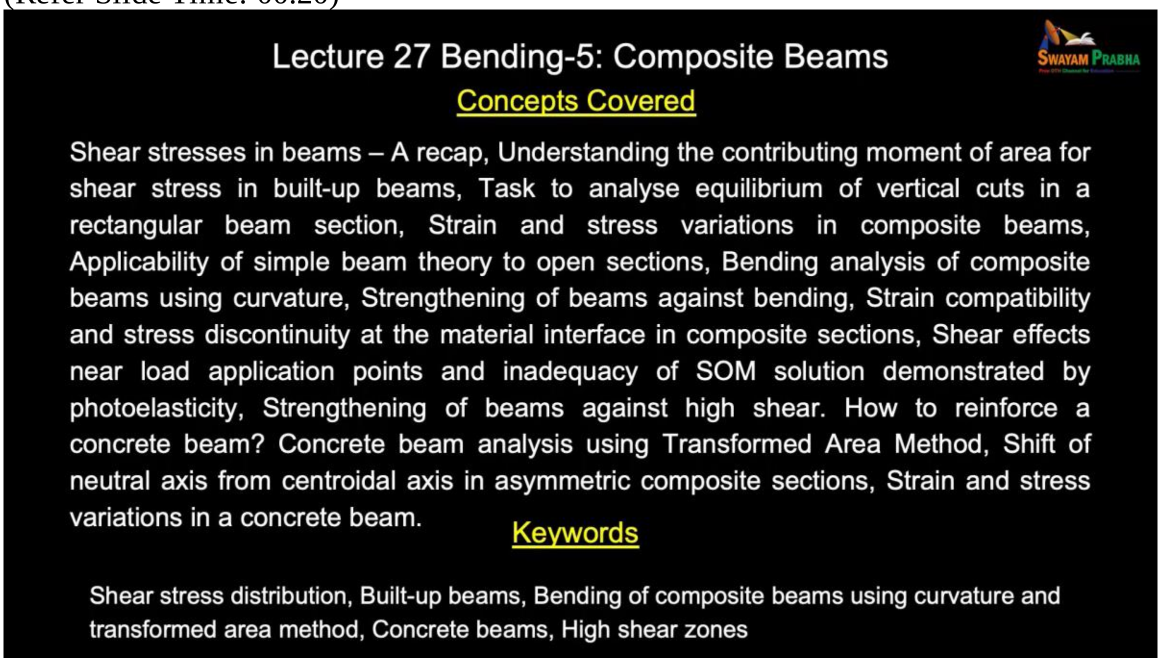

See, we have learnt in the previous classes, how to find out the bending stress and also the shear stress induced in bending. When you have shear force transmitted by the beam, you have shear stress introduced. When it transmits only bending moment, that happens when I have only four-point bending, the transmitting only constant bending moment, you get bending stress. And when I have a combination of shear force and bending moment, we said look at the bending moment diagram, pick out the bending moment and the shear force, use these equations to calculate the bending stress as well as the shear stress. Now, we shall solve a few problems in this class.

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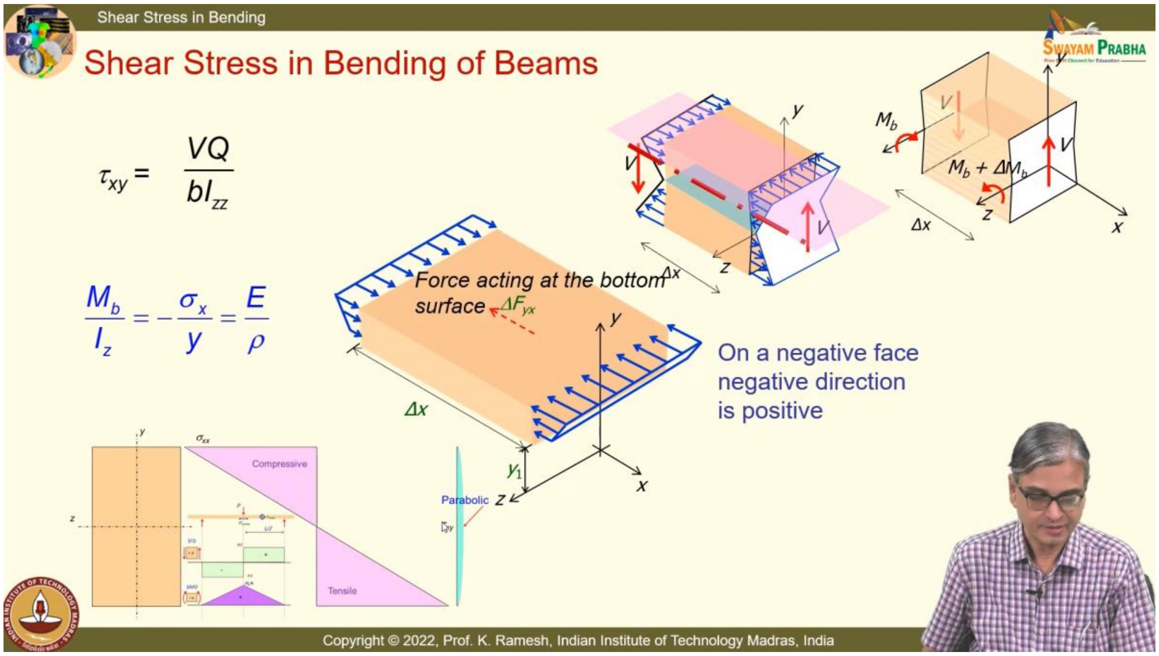

See, the whole idea is, you know, you will have to appreciate, what is the imbalance comes because of variation of bending moment along the length of the beam. And that is what is shown here. If you do not account for the shear stress developed, you will have the element moving like this. And to get this, what we have done was, we have sliced the beam horizontally. In a new situation, you should know how to look at, how does this imbalance causes relative motion because you may want to solder it in a particular way or weld it in a particular way or you may want to nail it in a particular way when you want to construct anything.

So, the basic idea here is, because of variation of bending moment along the length of the beam, when you take an element by slicing it horizontally, you find this is not in equilibrium unless you consider there is shearing action at the bottom of the surface. So, that is the conceptual discussion and you should be able to visualize it and exploit it depending on the problem context. And once you have this, you know, you have a basic expression. So, you pick out the shear force from the shear force diagram, calculate the shear stress, pick out bending moment from the bending moment diagram, calculate the stress that varies linearly over the depth, we call that as a bending stress. And I have also emphasized, if you take a rectangular cross section and if you consider a simple problem of three point bending, if you plot the normal stress and the shear stress to scale, you find the bending stress is of very high magnitude and the shear stress is of very, very small magnitude.

Many books emphasize the variation; the bending stress is varying linearly, whereas shear stress varies parabolically, that is also an important knowledge. It is also pertinent that we realize that the shear stress magnitudes are very small and for this case, we have discussed if I have my length, it is 10 times the *h*, then I will have this as 20 times smaller. So you have to recognize, occasionally we will also say that shear stress are negligible, ignore it. There are instances where shear stress effects are very significant. You will have to be sensitive to that; where to use it, where to neglect it.

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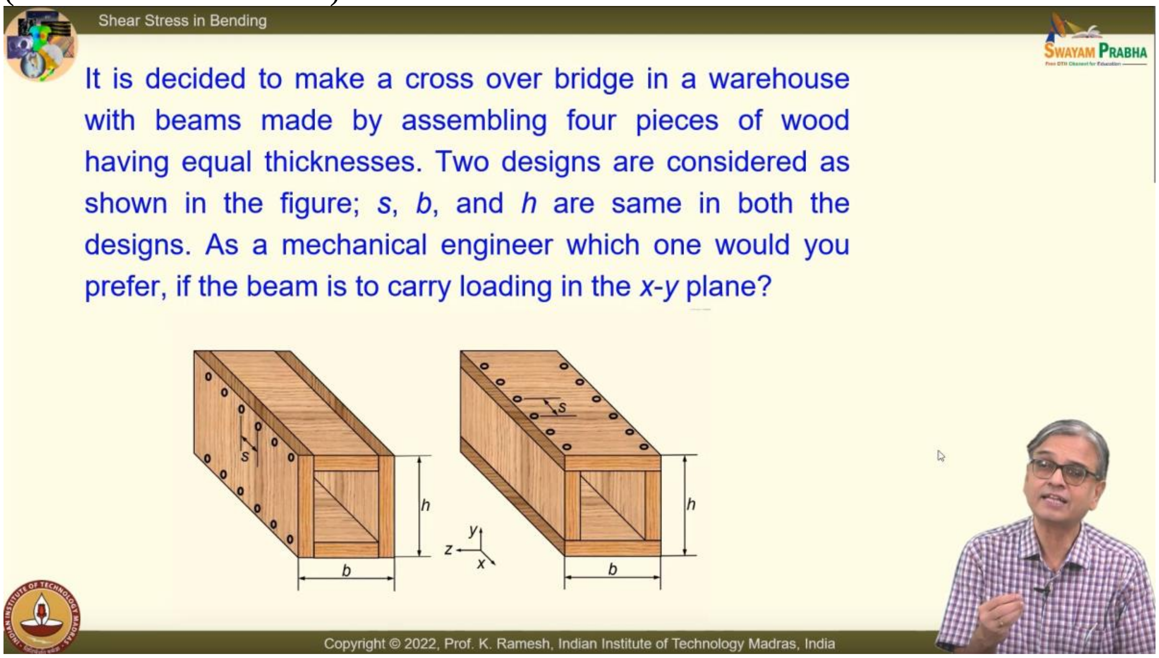

And you know, you have an interesting problem. In fact, this problem sheet was circulated, the idea is once you read what the problem is and ponder, then you get an idea how to approach the problem. And whatever the discussion that we are going to have, you are in a better position to follow because you have already looked at the problem that we are going to discuss today. See, it is a very simple problem, what you have is you have planks and then you want to make a box. I can make a box like this, I can also make a box like this.

See, this is just to give you some food for thought to realize how the imbalance because of the variation of bending moment is going to cause the shear stress from an academic point of view. Is the idea clear?

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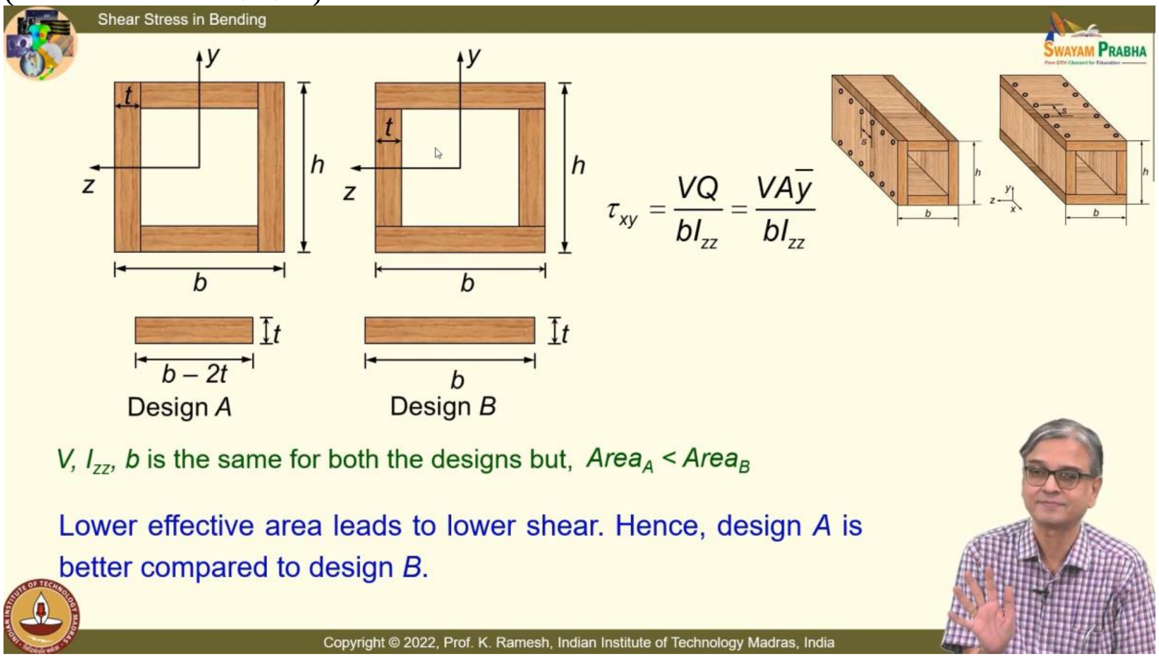

I have two cross sections like this. We have already discussed what is the expression for shear stress. We know that this is nothing but

$$\tau_{xy} = \frac{VQ}{bI_{zz}}$$

And then you know Q is nothing but first moment of area of the cross section. So, what happens in this situation and what happens in this situation? Because I am nailing it horizontally here, so this is the one which would try to have a relative motion which needs to be arrested by the nails. You have to be firmly connected, you cannot simply insert it and then say make a beam out of it, or even if you apply the glue which is very weak, it is also not going to work because we have already seen in the glass plate problem where you want to support the books. If you put an extra glass plate without any connection between the earlier glass plate, its strength was very very small because we have done the experiment when we want to understand how shear stresses are developed in the beams. So, the nailing is very important and you have to recognize that this is the area which is going to contribute to the relative motion. So, whatever the shear force generated because of this.

And the idea is, it is an academic exercise because you know when you construct the beam in one way, it is much smoother. When you construct the beam in another way, it is not smooth. And you find area A is smaller than area B, so obviously I am going to get smaller value of shear stress. Is the idea clear? Because I am going to get smaller value of shear stress from your calculation point of view, this is the better design. But if you look at you know workmanship, whether the carpenter does this workmanship, whether he smoothens the surface or not, whether it looks elegant for you to look from the top, you would go for this kind of a design.

But the variation is very, very small. You can use either of the two from practical point of view. From your calculations point of view, this will have a lower shear stress which gives you some food for thought to visualize the shear effects. Is the idea clear? Here you are going to experience shear effect on this surface, vertical surface, on this vertical surface; whereas here, you will experience it on this horizontal surface. So, there is a difference from what you have learnt earlier. And particularly when you have to handle the shear effects, you should understand the concept and invoke your usage of those concepts appropriately. Now let me give you a homework. You know you have taken the slice horizontally.

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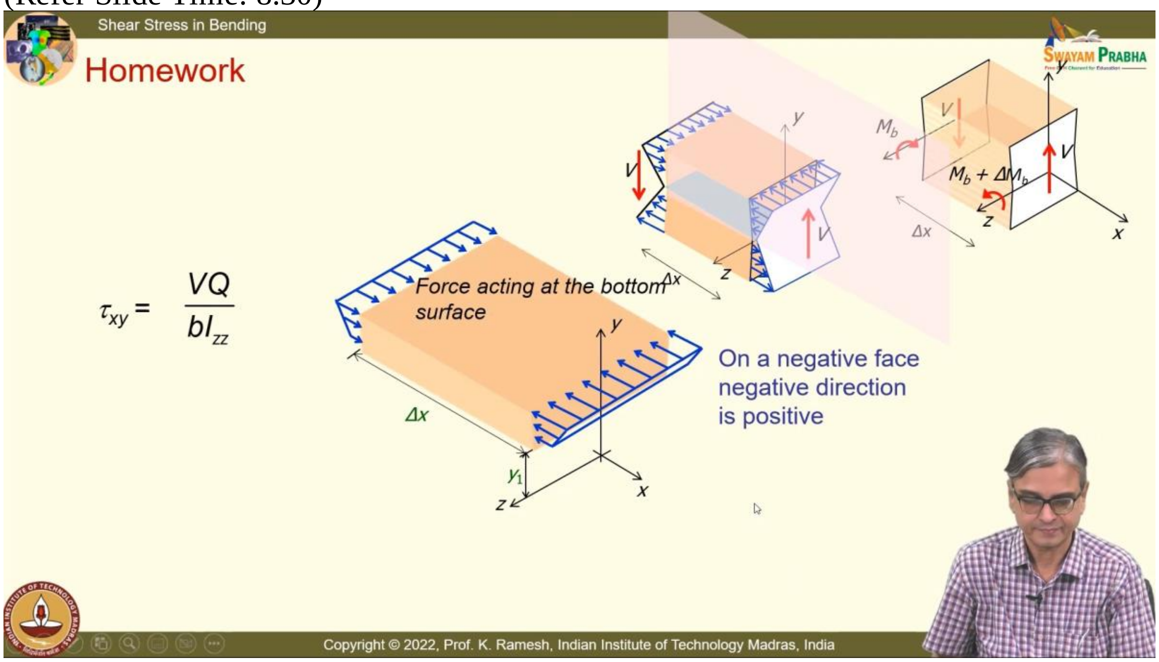

Now what I want you to do is, I want you to slice it vertically and go and investigate whether the sliced portion is in equilibrium or do you need any shear stress developed at appropriate places. Is the idea clear? Instead of slicing it horizontally, slice it vertically at any section: one-third, one-fourth, any section that does not matter. It should be sliced vertically. Do that as a homework.

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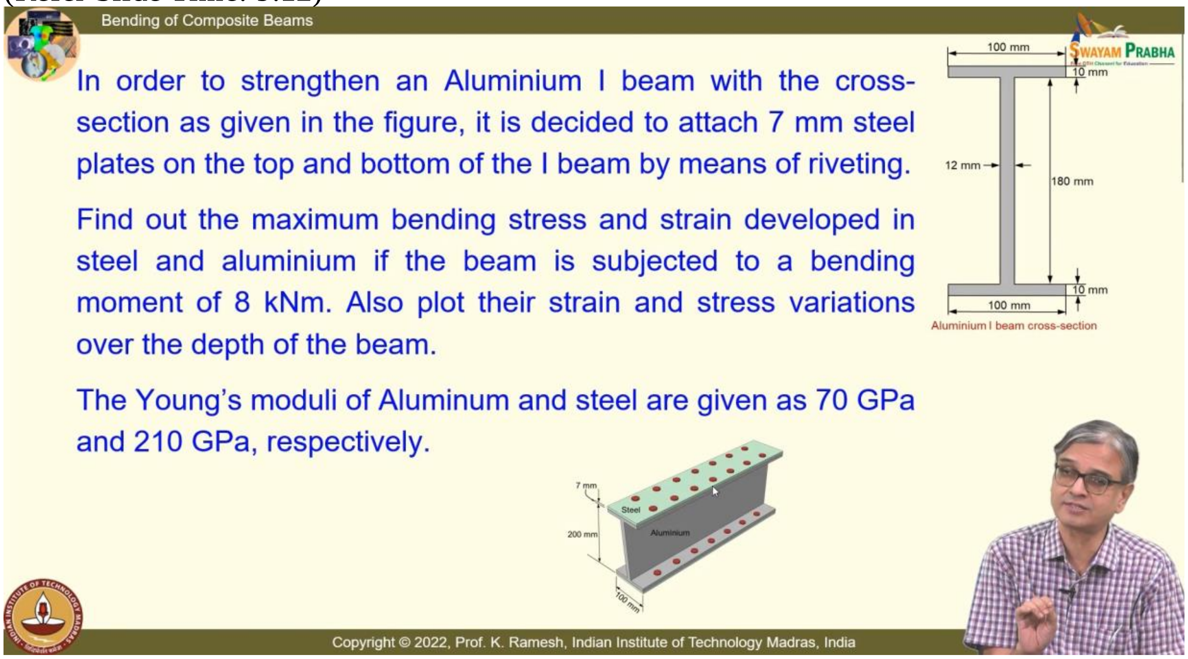

Then we move on to a very interesting problem. You know I have to strengthen my I-beam and one way of strengthening the I-beam was simply to put a steel plate on top of it and bottom of it and firmly connect it with rivets. That is a very important statement.

It is not just placed on the original aluminum I-beam. The steel extra sheet what is put, it is put on the top as well as put at the bottom symmetrically and this is also made integral with the I-beam. That is a very, very important statement, fine. Now this has to support a bending moment of 8 kNm and you are given the properties of aluminum and steel and your geometric details are given. You have to find out what are the strains developed and what are the stresses developed.

Let me ask a question because before we solve the problem, we also anticipate I am going to have a beam made of two different materials. We have to look at the strain, we will also have to look at the stresses. We have learnt in our basic development of the theory, the strain variation is linear and we have also learnt stress variation is linear. How do you expect when I have the aluminum and steel which is bimaterial here, how do you anticipate the strain to vary? How do you anticipate the stress to vary? Is the question clear? The concept that you have to learn as part of this problem is also that. We have learnt from using a single material, we have learnt that strain varies linearly and stress varies linearly.

But now I have a different beam wherein I have put steel plate on top, steel plate at the bottom, I have not disturbed the symmetry. When I have not disturbed the symmetry, first question is where do you expect your neutral surface? Will it coincide with centroidal surface of the complete cross section? It will coincide with the centroidal surface of the complete cross section because it is symmetrical. Suppose I put my steel plate only on one side of the aluminum beam because of accessibility. I do not have access to the other side and this side is weaker, I put a plate. Then you will have to find out where the neutral surface is, symmetry is disturbed.

So, here we are taking a simpler problem where symmetry is not disturbed, but I have a beam made of two different material. And also mind you, you are also having an open section. See while discussing torsion, we have never looked at open sections. I only said open sections have very poor torsional rigidity. I said that usually the evaluation of shear stresses in open section is relegated to the second level course.

So, whatever we have learnt in simple beam theory is quite applicable for a variety of cross sections. We have only put a restriction that it should have an axis of symmetry and the loading should be in the axis of symmetry.

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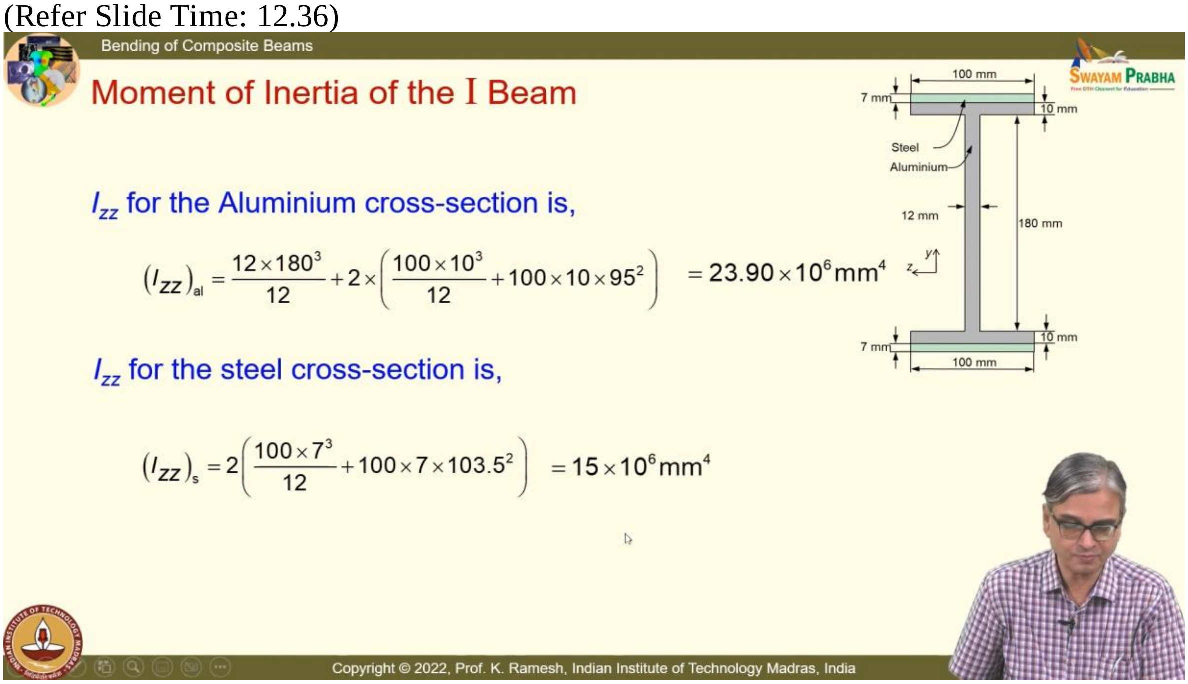

So, first thing you have to calculate is moment of inertia of the beam as a whole. So, you develop it for aluminum. I want you to do the calculations. See you will also have to pick up speed and you will also have to remember all those parallel axis theorem. You have studied all that, isn't it? All those aspects you have to remember. And I can imagine I section as one rectangular cross section like this and two rectangular cross section kept away. So, I can find out the moment of inertia about that axis and use the parallel axis theorem and I want to find out from the centroidal axis. So, when I go to aluminum, I have

$$(I_{zz})_{al} = \frac{12\times180^3}{12} + 2\times\left(\frac{100\times10^3}{12} + 100\times10\times95^2\right)$$

So, this you can calculate what is the value of this. And this turns out to be $23.90\times10^6\,\text{mm}^4$. Please do the calculations.

On similar lines, I can also find out the I_{zz} for the steel cross section and you are given the dimensions. So, I have

$$(I_{zz})_s = 2\left(\frac{100\times7^3}{12} + 100\times7\times103.5^2\right)$$

So, now we will have to look at how do you find out the curvature? Because that is needed for us to calculate the stresses, fine. Instead of one material, I have two different materials. So, we I have also got this as number $15\times10^6\,\text{mm}^4$.

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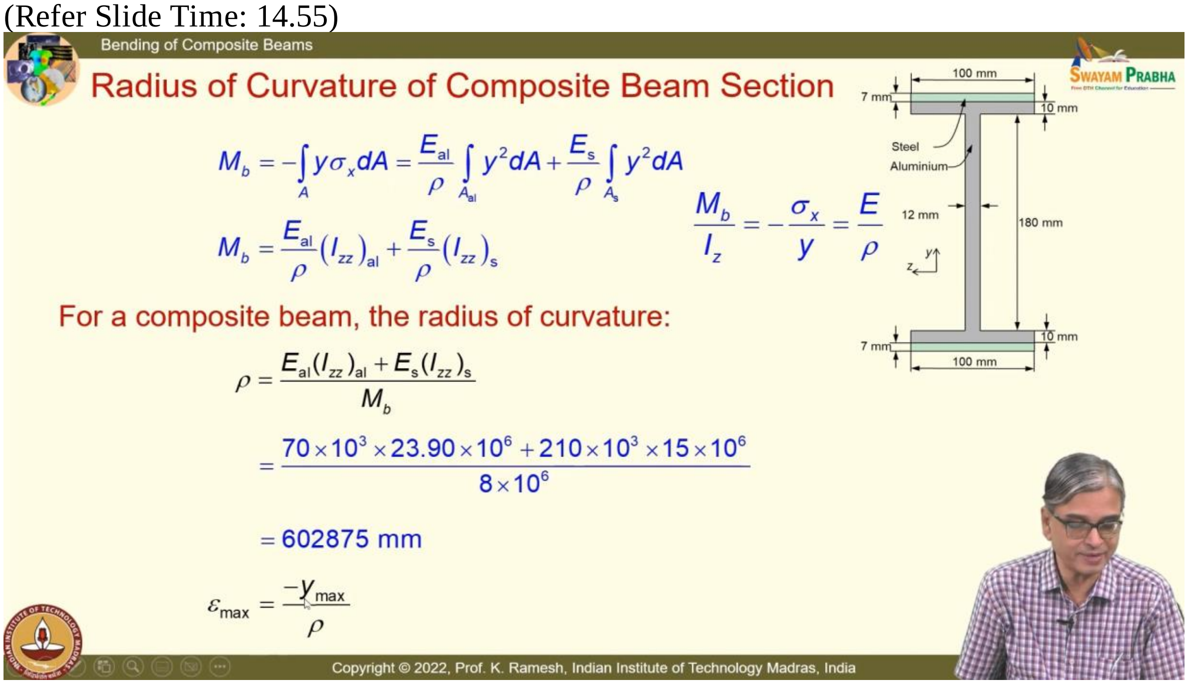

And basically, you look at what is the bending moment, that is related to

$$M_b = -\int_A y\sigma_x dA$$

and we have to write in long hand what is σ_x. Because now I have two different materials, isn't it? Earlier we had E was constant. Now, E changes. When you change a material is different means I have one Young's modulus for aluminum, another Young's modulus for steel. And let me look at the flexure formula.

$$\frac{M_b}{I_z} = -\frac{\sigma_x}{y} = \frac{E}{\rho}$$

So, I can express σ_x in terms of E, ρ and y.

I can substitute it here. So, I will have one for aluminum, another for steel. Is the idea clear? So, what I do is I basically rewrite this expression from my basic flexure formula in terms of the Young's modulus. So, now I have a relationship between the bending moment and the radius of curvature. I have bonded the steel plate firmly. It is integral with it. It is done by riveted joint. If they are separate, I cannot write like this. Because they are integral; I can write like this. I can simply write this in a known expression which we have already calculated.

$$M_b = \frac{E_{al}}{\rho}\left(I_{zz}\right)_{al} + \frac{E_s}{\rho}\left(I_{zz}\right)_s$$

And we want to find out what is radius of curvature. That is simply *EI* of material 1 and *EI* for material 2 divided by bending moment. So, very very important relationship. This is mainly because it is bonded firmly. I have not just put, see in this diagram, we want to calculate only the moment of inertia. We have not shown the riveted connection. I should have shown some dotted lines that the rivets are somewhere at the behind which is not shown. But you should not consider that the steel plate is just put on top of it. So, once you substitute these expressions, I also get this radius of curvature as a number 602875 mm.

So, once I have this calculation of strain is straight forward, calculation of stress is also straight forward. Now, I want you to think and then tell me how do you anticipate the strain variation? How do you anticipate the stress variation? Both have to be linear.

You have to visualize what happens at this joint and at this joint. What do you anticipate for strain and what do you anticipate for stress? Do you expect both of them to be identical? Please be louder! Strains are equal. Strains are equal. What happens to stress? Discontinuity. Discontinuity, I am very happy! That is the purpose of this problem, fine. You have got the idea clearly. So, I have

$$\varepsilon_{max} = \frac{-y_{max}}{\rho}$$

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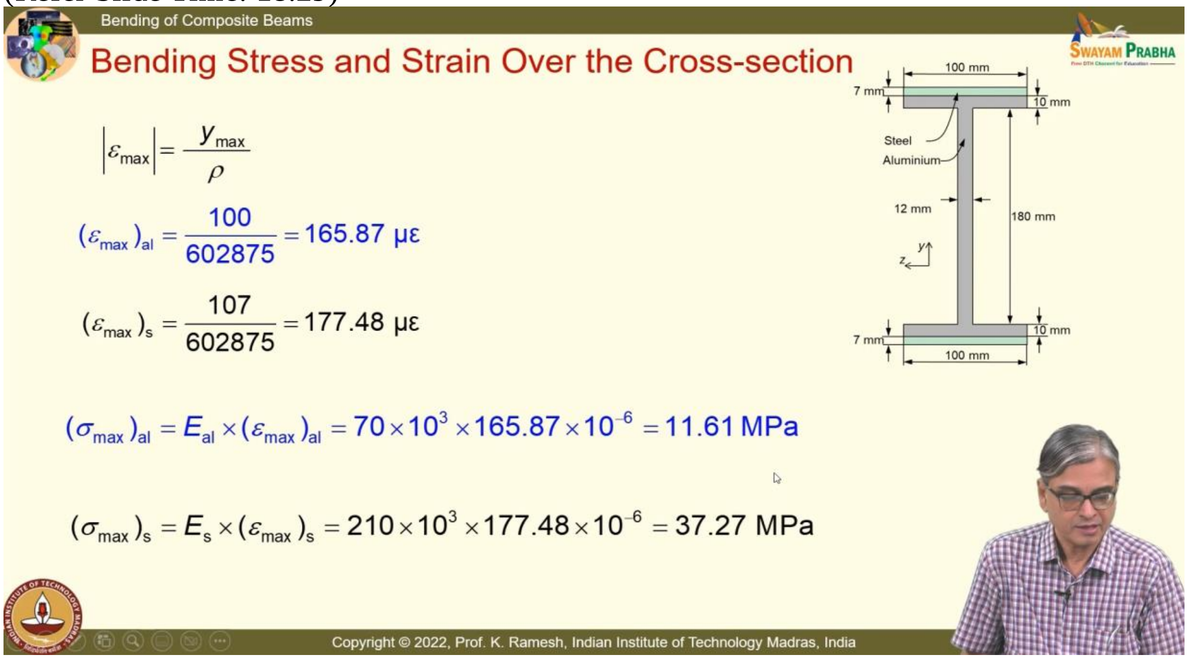

And you know, as you grow older and then solve more and more problems, the moment you come to bending, even though my basic equation contains the symbolism sign convention, which says that if I have a positive bending moment, the top surface is compressive. So, I have negative sign coming and bottom surface is tensile. Once you are accustomed to solving problem, we normally look at

$$\left|\varepsilon_{max}\right| = \frac{y_{max}}{\rho}$$

fine. I will not worry about the sign. I will just get the numbers. Then from the physical appreciation of the bending of the beam, I will say whether it is compressive or tensile, which is practiced normally done once you go for design of machine elements. It is better to learn that here. We have learnt it very systematically with proper sign convention and when you solve problems after problem, you want to find out what is the maximum strain. You want to know what is the distance from the neutral axis to the fiber, which is farthest away. Here, I have a symmetrical beam. If I have a T section, one side will be nearer, another side will be farther. So, the farther stress can be compressive or tensile in section like a T section, but here it is symmetric.

So, my focus is on the number. Why I focus only on the number? It can have both tensile and compressive depending on the beam is bending. In a symmetrical section, I have equal values of tensile and equal values of compressive strain.

So, the focus is more on getting the numbers. So, I am just putting this as modulus. The same discussion is continued. I have not put the modulus separately here. I am only looking at the numbers. So, I have similarly strain on this steel because it is away by 7 mm extra and it is 177.48 $\mu\epsilon$. I can also calculate the stresses. How do I calculate the stresses? You have the fracture formula simply multiplied by the Young's modulus. Obviously, the Young's modulus of steel and aluminum are quite different. So, you will definitely have a discontinuity at the interface from stress point of view, which is acceptable. You cannot have a discontinuity from strain point of view. If the beam as a whole has to bend as one unit, the strain variation has to be identical in both the materials. And when you do the calculation for stress in aluminum, this is coming out to be 11.61 MPa. And when you find out for steel, see this is the maximum stress 37.27 MPa. I must also calculate what happens at the interface.

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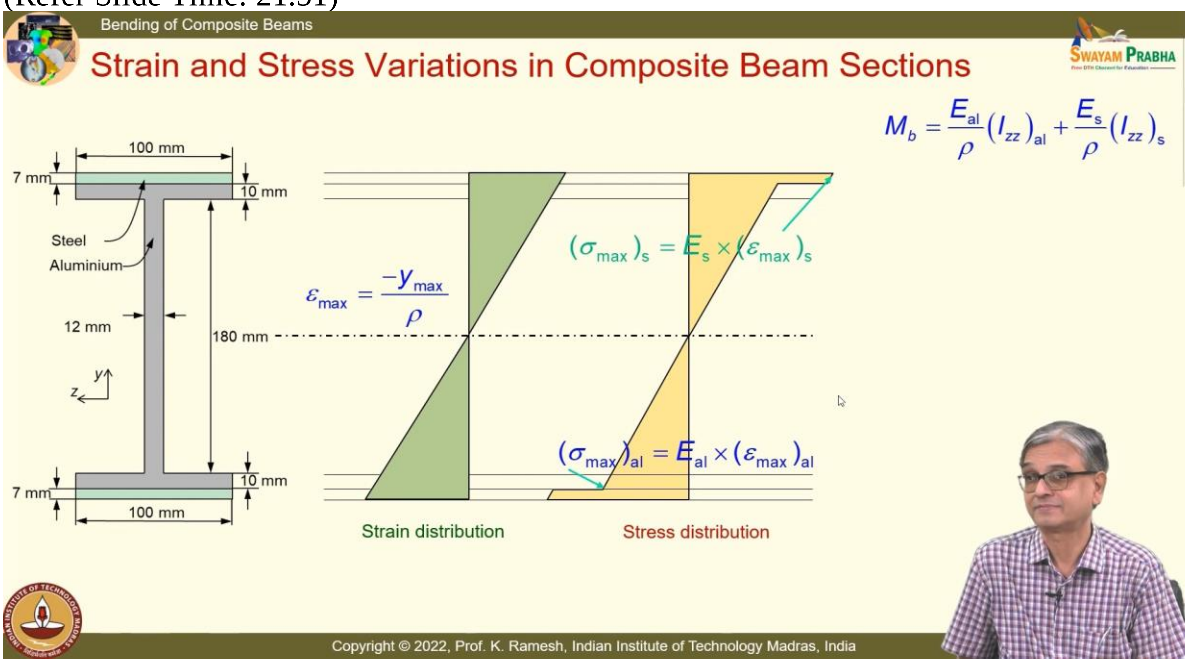

That is shown in a diagram. So, I have the expression for strain is put with the sign convention and when I plot the strain distribution, strain distribution is linear. See, I have been drawing this triangles in various forms. You know, I can label this as depending on the problem, depending on the sign of the bending moment, I can say whether it is tensile or compressive. I think in this problem, just the number is given, nothing else is given, fine.

We have just put without any sign here. I have also shown this triangle this way. I have also shown it with arrows. There are multiple ways it is put in books. So, you should also be comfortable with any one of them.

So, strain distribution, there is no problem. Even when I have a change of material, the strain has to be continuous. You know, we have also solved long back the problem of a composite cylinder. We said that strain at the interface $\varepsilon_{\theta\theta}$ should be identical, but we have not plotted the strain variation or stress variation, fine. There is one subtle difference that is also a thin hoop, fine. And here also I have, if you look at this thickness is only 10 mm and this is only 7 mm, isn't it? Again it is very thin, but the moment I put the stress distribution from my basic expression, they have to be linear-they are linear.

But what happens is at this interface because of Young's modulus difference, stress value is higher. Even though I have a very small thickness of 7 mm, you find this is varying linearly inside the material. That is the importance of what happens in a bending situation. In the case of a composite hoop, we had similar thicknesses of the hoops. They are not very thin compared to this, but the way hoop is constructed, the way hoop is resisting the load, it is going to have a constant stress in the outer hoop and constant stress in the inner hoop.

Whereas here, even though the thickness is only 7 mm, because this is resisting bending from our basic expression, this stress will vary linearly over the small depth. So, you have to recognize these subtleties. Is the idea clear? So, the focus of this problem is to appreciate strain variation will be linear, whereas stress variation can have a jump. That is natural.

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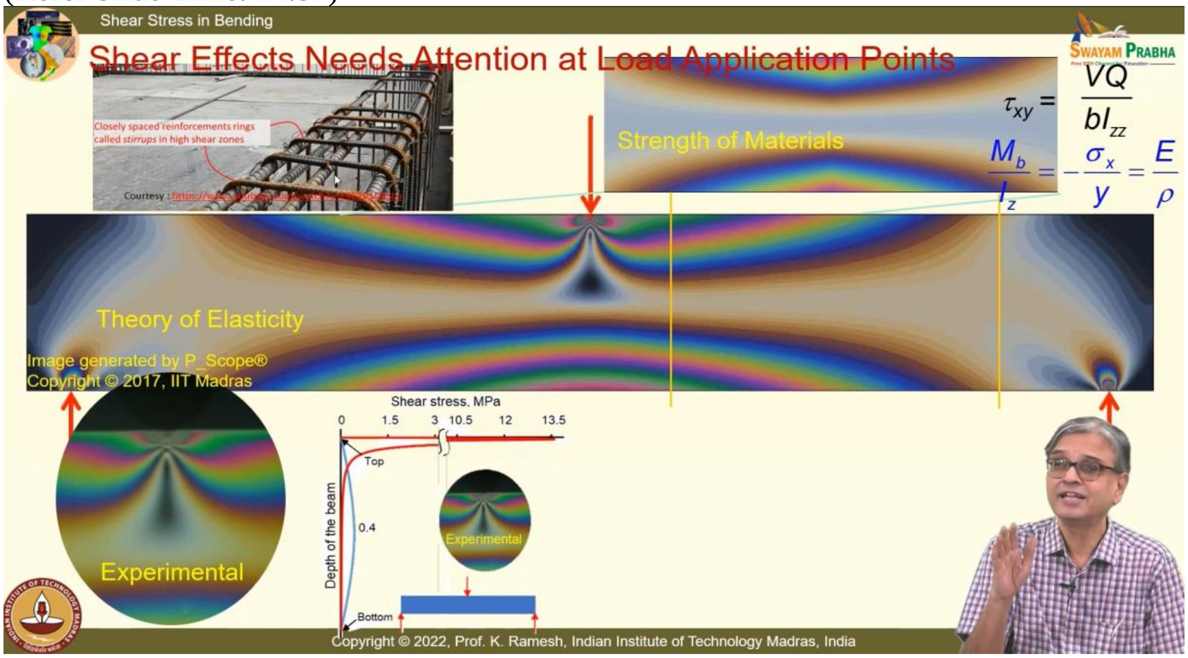

See, I said the shear stresses are very very small. In many situations, we ignore the shear effects. In fact, when we want to develop deflection of beams, many problems we would solve without even worrying about what is the influence of shear stress. May be in one problem finally, we will find out how to include the shear effects and find out what is the small change it introduces on the deflection. But there are problems where shear effects need attention; you cannot ignore. See, if you are smart, you would say that I would use

Saint-Venant's principle, I will not go near the load application points; I will apply my strength of material only in a zone far away from it.

But what happens is, you know, we have always said the Saint-Venant's principle, the load is applied at the end. When I have a three-point bending, load is also coming in between the beam. Many times, while applying, you know, people simply say I have got the shear stress, let me use it for every cross section along the beam. And this is done by theory of elasticity. And in theory of elasticity, if I have a concentrated load, it is very difficult to solve.

I need to have Fourier components, and with 200 Fourier components, this image is generated by the virtual polariscope which we have seen. And let us look at what I get from strength of materials. That means, you have learnt shear stress,

$$\tau_{xy} = \frac{VQ}{bI_{zz}}$$

and your normal stress is,

$$\frac{M_b}{I_z} = -\frac{\sigma_x}{y} = \frac{E}{\rho}$$

From this, you can calculate the normal stress. If I use this for the entire length of the beam in this zone, I find a marked difference between what I see in strength of materials, what I see in theory of elasticity.

Let us look at what happens in experiment. This is done by an experiment. Which solution matches with the experiment better? Theory of elasticity. See, we have also taken up a problem where there is a UDL. We got a very surprising result from theory of elasticity. Even the bending stress variation is not strictly linear; there was a correction form term and this was non-linear. And we also found that there is a small variation of sigma away over the depth of the beam. And we also discussed that these are second-order effects. We do not have to worry about it, but recognize that what you have learnt in strength of materials is good for engineering analysis.

Whereas, when you come to shear effects near the load application points, even though strength of material does not give me good result, you will have to look at what is the actual value and the stresses are comparable to the bending stresses. And you find this is at least 13 to 14 times larger than the maximum parabolic variation near the load application points.

So, your strength of material solution probably you can apply between these two lines which is far away from the load application point, far away from this load application point. In this segment, it is still parabolic reaching a maximum at the neutral axis, ok. Whereas, when I go close to the load application point, shear is still zero at the top point as per your requirement of free surface. Just below the top point, it reaches a very high value of 13.5; this is 0.4, and this is 13.5. So, you can calculate the ratio how many times it is magnified. So, it is now the magnitude is comparable to the bending stress maximum.

So, you will definitely have failure occurring because of this. And in construction practice, in civil constructions practice, they put what are known as stirrups, they closely spaced extra reinforcement there. And if you are having a metallic beam, you may have to put an extra plate and then rivet it on wherever you have the loads. So, your strength of material knowledge, bending stress varies linearly, shear stress varies parabolically is useful at places where it fails, you will have to accommodate that otherwise your design will be disastrous, fine. So, you should know what are the limitations, fine.

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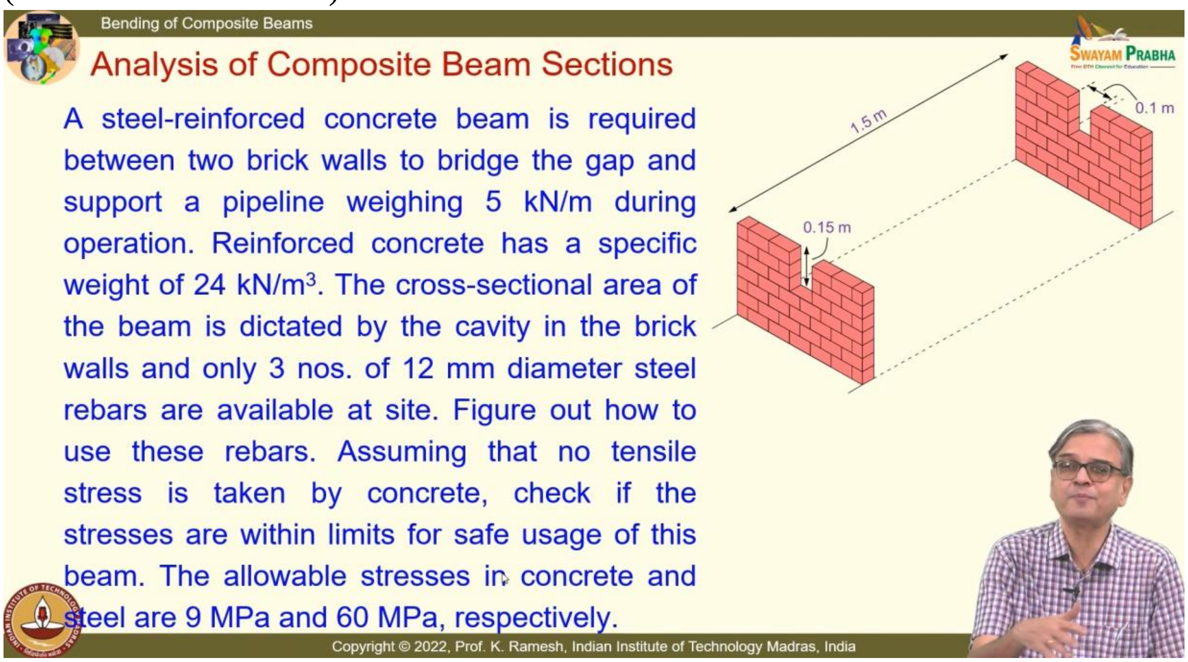

Then we move on to a very interesting problem. This is again composite beam section and you have a pipeline that needs to be supported between the two walls. We want a beam to be made of concrete and then it says what is the weight of pipeline that is 5 kNm. And for the concrete, you are given the specific weight, and you know I can have the beam fitted into this cavity. So, my size of the beam is the same as the cavity. And it also says that three numbers of 12 mm diameter steel rebars, reinforcement bars are available at the site. If at all I have to reinforce the concrete, you should ask a question: Why you have to reinforce the concrete? What sort of a material concrete is? We will see all that. And we will also have to see how do I reinforce. And the question is figure out how to use these rebars? Assuming that no tensile stress is taken by concrete, here itself comes even the question it is embedded.

Check if the stresses are within limits for safe usage of this beam. And you are also given allowable stresses in concrete and steel which are respectively 9 MPa and 60 MPa. And you find you know people have applied their mind to decide why these values are limiting values. That is what you learned in theories of failure, how to fix those limits. Once you are given the limits, your problem becomes a design problem, ok.

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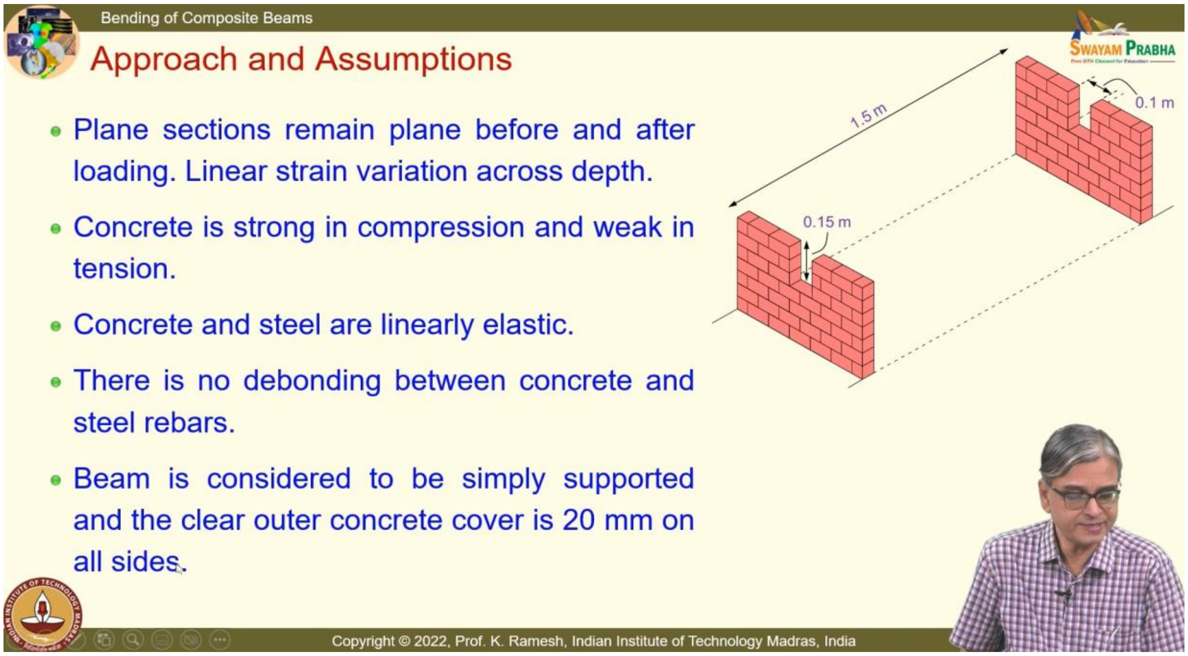

And you know I am obviously going to have a heterogeneous material. We are not studying a heterogeneous material from our analysis. We have just simply said a homogeneous material. So, we will simply close our eyes and say concrete is homogeneous. We will say plane sections remain plane before and after loading, and linear strain variation exists across the depth. This is an assumption reasonably good, but understand that you are doing an engineering analysis. See this is what you have to appreciate. We are not doing an exact mathematical analysis, but an engineering analysis.

Concrete is strong in compression and weak in tension. So, that is why you need reinforcement. And we also assume that concrete and steel are linearly elastic. And a very important statement, there is no de-bonding between concrete and steel rebars. See it may so happen when you construct the beam, it may be like this. But if you have not taken care of water seepage into the system, you may find that you know steel will corrode and it will increase in volume, and you will have de-bonding occurs between the steel and the concrete. But now since we are designing the beam, we had a comfortable situation. We asked the contractor to make a good beam following all the good manufacturing practices. So, that you ensure that the steel rebars are integrated with the concrete. That is a very very important statement. There is no de-bonding.

And even though this looks like some sort of a fixed support for the purpose of simplified analysis, beam is considered to be simply supported and the clear outer concrete cover is 20 mm on all sides. See, I have asked one of my TA's who is exposed to civil engineering to coin this problem.

So, he has put in his experience of some importance on the field. So, you have 20 mm. So, that limits where you can put the steel rebar. You cannot put it you know beyond the 20 mm distance. So, you can put it only 20 mm below that.

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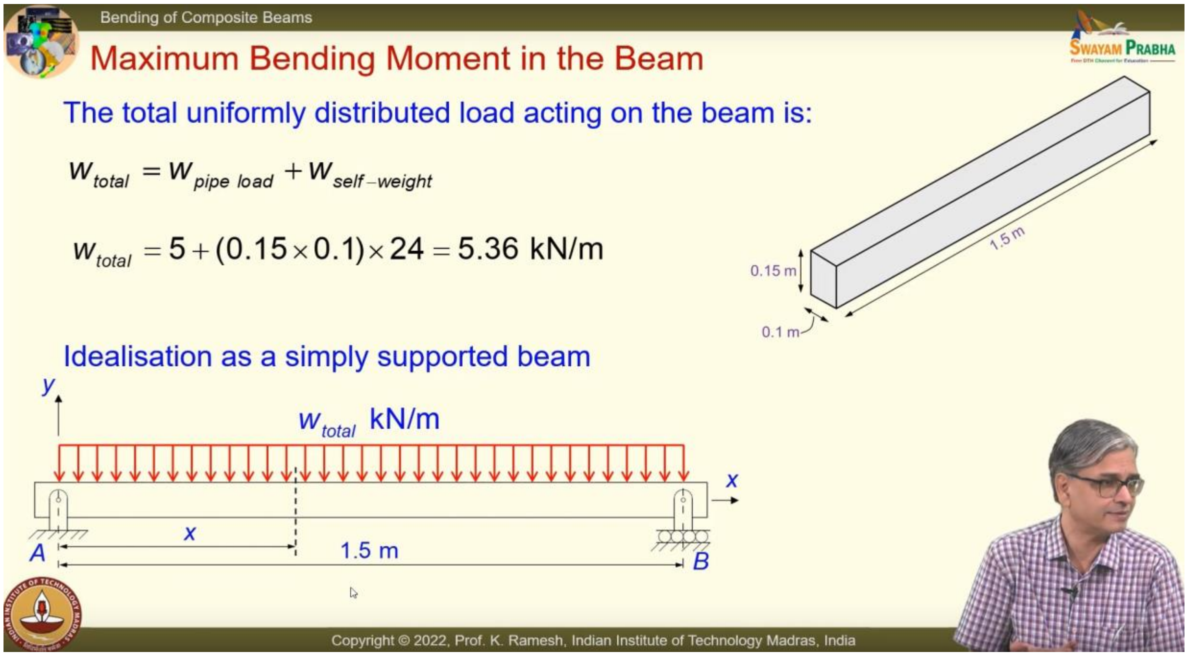

And you all appreciate that the beam is transmitting distributed load. I have an UDL and this uniformly distributed load comes from two sources. One is the weight of the pipe, second is the self-weight of the concrete. See one distinction between mechanical, aerospace and civil is one of the major loads that come in civil construction is self-weight. In the case of mechanical and aerospace, compared to the loads the member is experiencing the self-weight is very, very small. You can consider that as a second order effect and omit it for your engineering analysis, ok.

So, here I have to take care of pipe load as well as self-weight and you are given all these calculations. These calculations are very very simple and its straight forward and you know I get the W_{total} as 5.36 kN/m. All these information is given in the problem. So, straight application of simple equations is what you will have to do. And we have already said and it is also shown in a picture that I have a roller support on one end and pin support on one another end. And the idea is what? When we have to do the design, I have to find out what is the maximum bending moment. Is the idea clear? I have to know what is the shear force and what is the maximum bending moment. And I do not think we have going to worry about the shear force now.

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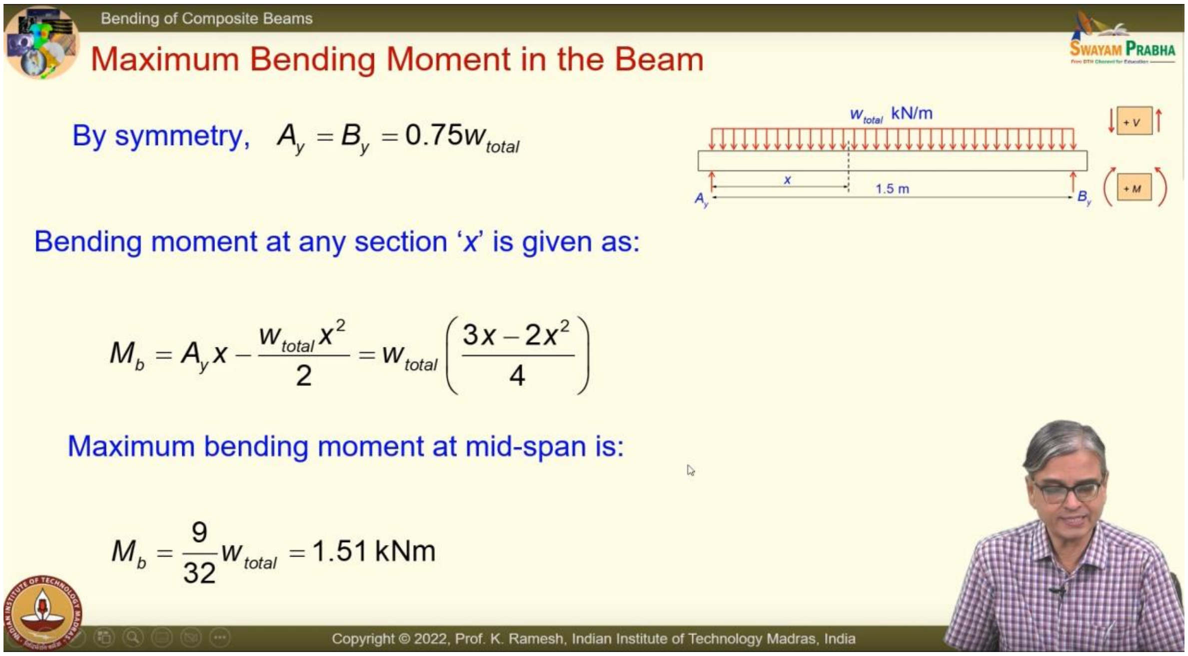

So, you have from rigid body mechanics, you have to calculate the reactions because the problem is simple straight it comes to 0.75 W_{total}. And bending moment at any section, you can take a section and then find out the basic expression that can be written as W_{total} which is N/mm or kN/m. This consisting of pipe weight as well as the self-weight of the beam that comes out to be

$$w_{total}\left(\frac{3x-2x^2}{4}\right)$$

Where do you expect the maximum bending moment at the beam? Middle point. It is a middle point, whenever shear crosses zero, you will have extremum values of bending moment. And shear would cross zero at the middle point. So, I can also find out the maximum bending moment at mid-span. I can get that as 9/ 32 W_{total}. So, this comes out to be 1.51 kNm. So, we have got the maximum bending moment.

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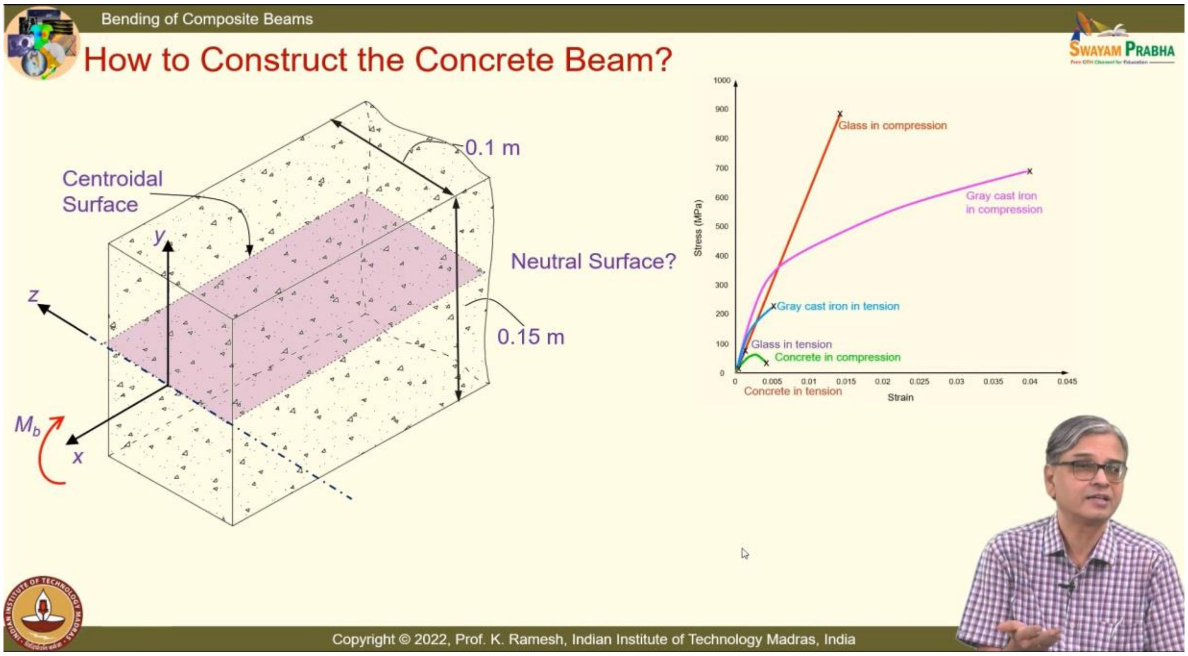

And now we will have to figure out how do I use my steel? How do I construct the concrete beam? See if we go to machine drawing, engineering drawing there is a way that you have to depict the concrete. So, this is the way you will depict the concrete. But you know later I would remove all these triangles so that I have a clear picture for you to see other details, fine.

Because the idea is to learn the subject, not get enamored with complicated figures. So, I have this centroidal surface. If I have only a concrete beam, the neutral surface and centroidal surface will coincide. And if you look at the stress strain graph of concrete, you will find the values of tensile and compressive stresses are quite different and it hardly has any strength for supporting tension, ok. So, what you find here is your concrete in compression is somewhere here, your tension is almost close to zero.

So, you need to support. Suppose I imagine that I mean in the from the problem statement itself, we know that the bending moment is acting like this. And from the physics of the problem which side is supporting compressive load and which side is supporting tensile load? The top half is compressive and bottom half is tensile. And once you know from the stress strain graph, it cannot support any tension. I need to do something. I need to use the steel rebars so that I strengthen the bottom portion, fine.

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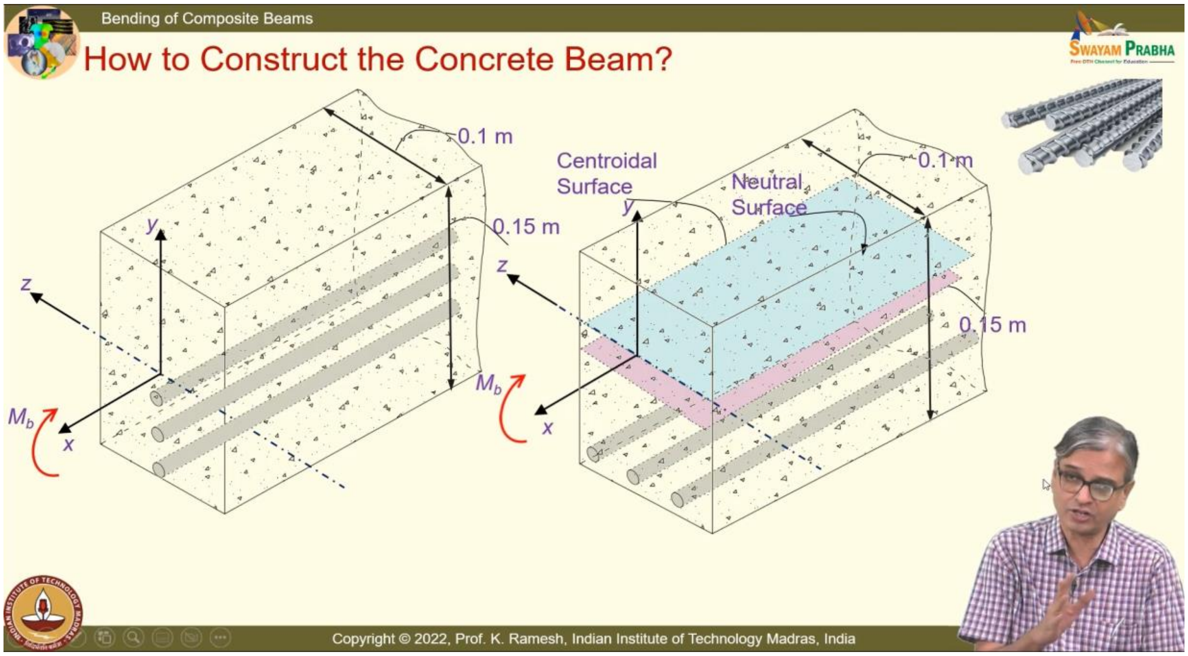

So, we need to do some reinforcement. I have three steel bars and I propose to start with, because we know we need to support the tension side. I propose, I have a steel bar put like this and then reinforce the beam. Is it a good idea to do that? Because we have discussed the top portion is compressive, bottom portion is tension.

And to safeguard from failure, I want to put three steel bars like this. You know in isometric, you know it is not very easy to show and I agree with that. He says that it should not touch the bottom. It should be slightly inside. That also will take care while constructing.

We will take care of that while giving the release of engineering drawing. We will take care of that. Is it a good way to do it? I get one nod; It is not the right way to do it because you know you have to use the material properly. Is the idea clear? We have already learnt when you want to have bending strength, I should have higher bending rigidity. Higher bending rigidity means product of E and I should be as high as possible. And I will have a better moment of inertia when my material is far away from the centroidal axis.

And I am given only three bars. So, I must use the three bars very effectively. So, a better way to do this is, do not put it in the vertical axis. Put them in the horizontal axis far away from the centroidal plane, ok. This way I would effectively use my rebars, ok. For the purpose of drawing, I have just shown this as cylindrical. We will also see how they are in actual practice. And what this will influence on the behavioral of the beam. I will have a centroidal surface like this. Because I have introduced the steel bars, they are integrated while construction itself I insert it, fine.

And it is firmly bonded to this concrete. Because of this, I have a cylindrical material variation. So, I will naturally anticipate my neutral surface would be shifted. I have to find out the location of the neutral surface. That is part of the problem now. And you will also have to recognize that all our expressions are referred with respect to neutral surface. It is a very important step.

My reference axis also shifted to the neutral surface. Only from there I measure what is the distance of the outer fiber, fine. So, I have to find out where the neutral surface is. And to ensure that I have good bonding, you know you will have the rods, they are not perfectly cylindrical. They have this projections like this so that it cannot come out easily. And people also put some kind of a polymer coating so that it does not corrode. The challenge is polymer should bond with steel as well as it should bond with concrete. All these are field challenges, ok.

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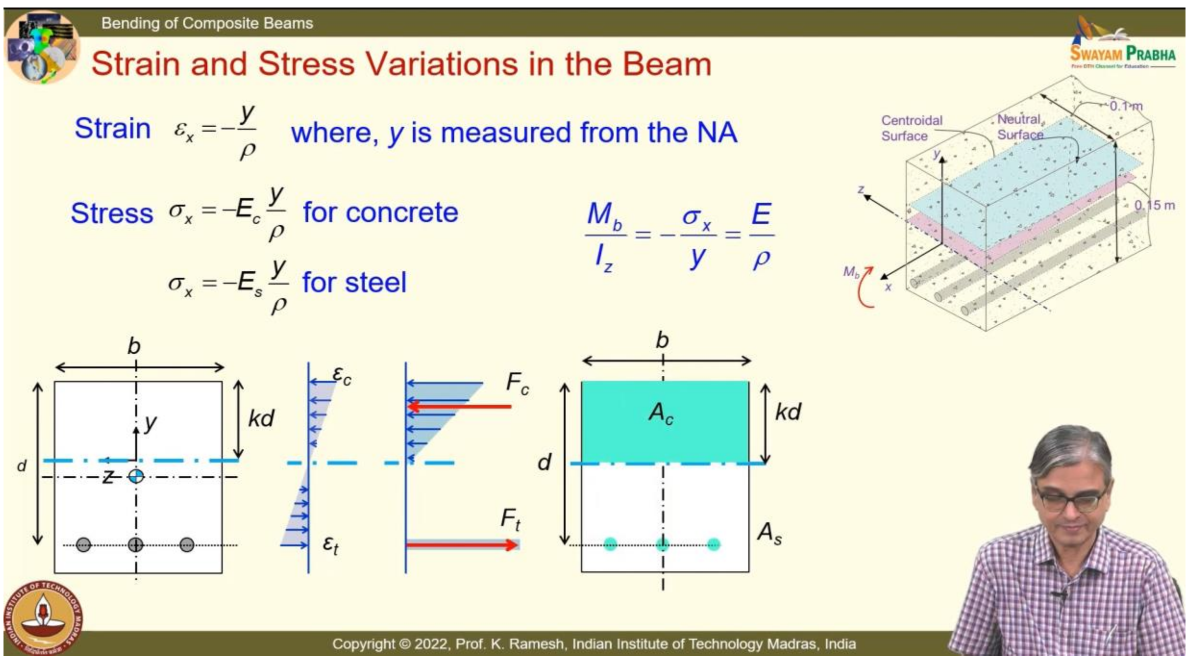

Now, how do I visualize what is happening in the beam? ok. Now, I would have my beam like this, imagine like this. I have this made of one material. See, I have stopped my distance *d* not till the end of the beam where I have to insert the steel rebar. Because the tension side of the concrete we say it is not existing at all. It is a very very soft material, ok. So, I have my neutral axis is shifted to slightly above. I have to find out, I have to look at the mathematics.

To simplify the mathematics that this distance is taken as *kd*, k multiplied by *d*. And I have the reference axis put with the neutral axis. And we know that it varies linearly from the previous problem we have seen no matter what we do with the stress, strain has to be linear.

Strain variation is linear, there is no problem at all. But the moment I come to stresses, I need to find out how do I go about to solve the problem, ok.

I have the strain variation like this. And if I plot the stress variation in the case of concrete side, no problem that is going to be linear. The compressive side it is going to be linear. We will have that as linear, there is no problem. In tension side, we are treating concrete as such a soft material. It is not capable of transmitting anything. The entire load is supported by the steel bars, ok. So, I will have only steel bars transmitting the load. And whatever the force transmitted by steel bar will be the force transmitted by the compressive side of it, ok. That is how we are going to do the force balance. And that will also help us to calculate the position of the neutral axis, fine.

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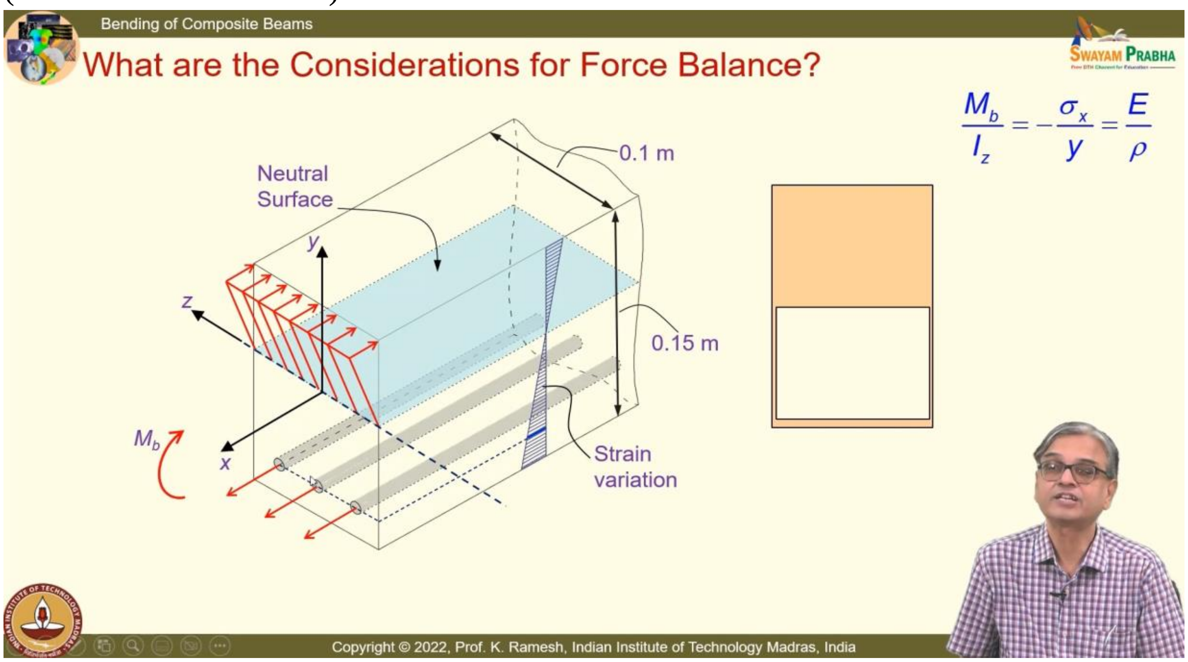

And we will also visualize it in a different manner, ok. I have this as strain variation like this. So, in a sense what we are trying to do is, you know you have this varying linearly that is not a problem at all. To find out what happens at the bottom, we are picturizing the beam like this.

It is hollow in between a very thin, this one is connecting and I have something which is happening at the rebars, fine. This is how we are visualizing it. So, it is a combination of a complete material in this segment and no material here and only this, ok. This is an analogy, the same analogy we will also extend it. And I have the strain and I would have the stresses transmitted by the steel rebars, and this should go to the force balance. Is the idea clear? Ok.

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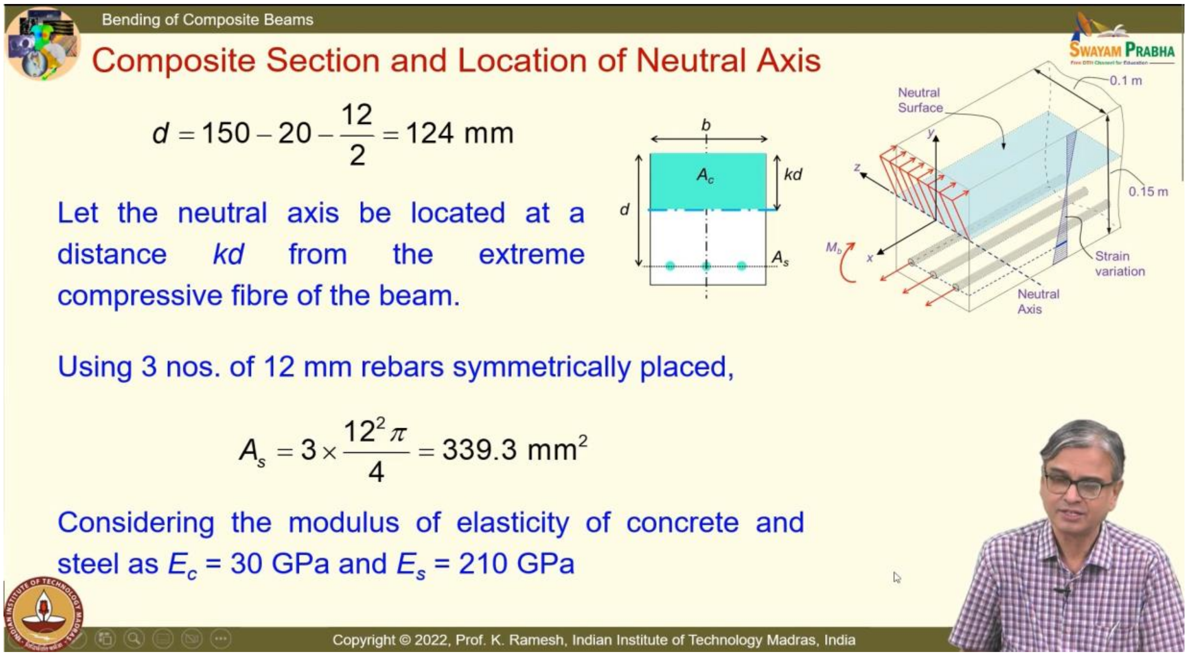

And you have the minor details, ok. So, I have what is *d* because I have to count for what is the depth and what is the size of the steel bar and so on and so forth. I get this as 124 mm. And we have said that the neutral axis is, we have measured it from the top compressive side. So, this distance is *kd* and three rebars are symmetrically placed and your area of cross section is also given.

So, with this I am in a position to find out how to locate the neutral surface, ok. The neutral surface I have to locate it. See, there are multiple definitions for neutral axis. While developing the strain, we consider this as a neutral axis. You can also consider that when I want to measure from this, we are doing all the calculation only on the symmetry plane. I can also find the neutral point and then go up. Some books also say that you can also call this as neutral axis, whatever the line that is representing the neutral surface this way. So, it is a very small nomenclature variation.

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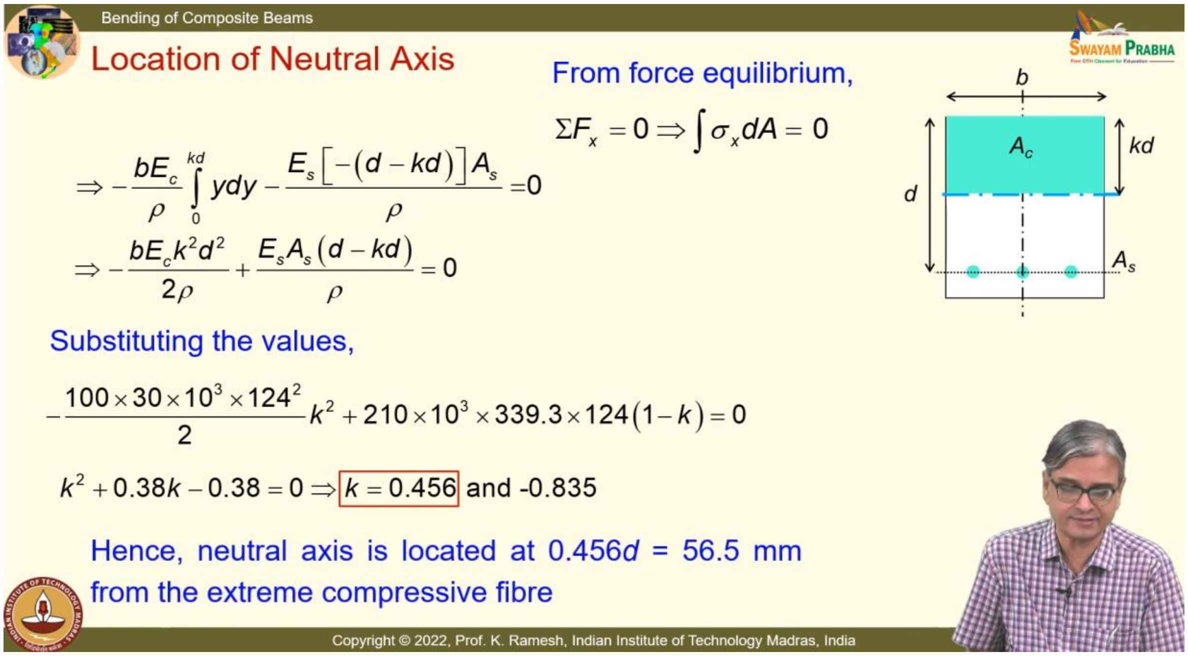

So, from force equilibrium, I get this

$$\int \sigma_x dA = 0$$

Now, I have to bring in the expression properly because I have two different materials now, ok. So, I will, I can write for the concrete side easily. From my basic expression, I can replace it like

$$\Rightarrow -\frac{bE_c}{\rho}\int_0^{kd} y\,dy - \frac{E_s\left[-(d-kd)\right]A_s}{\rho} = 0$$

So, when I substitute this in terms of the distance *k*, the factor that we have taken, I get a quadratic expression involving *k*. So, I get this *k* as 0.456.

So, if you have to locate the neutral surface, you go and solve the force equilibrium. You have to satisfy the force equilibrium to get the neutral surface. Once you have calculated that, rest of it is simple calculation, ok. See, from here I can also use curvature and find out what are the stresses.

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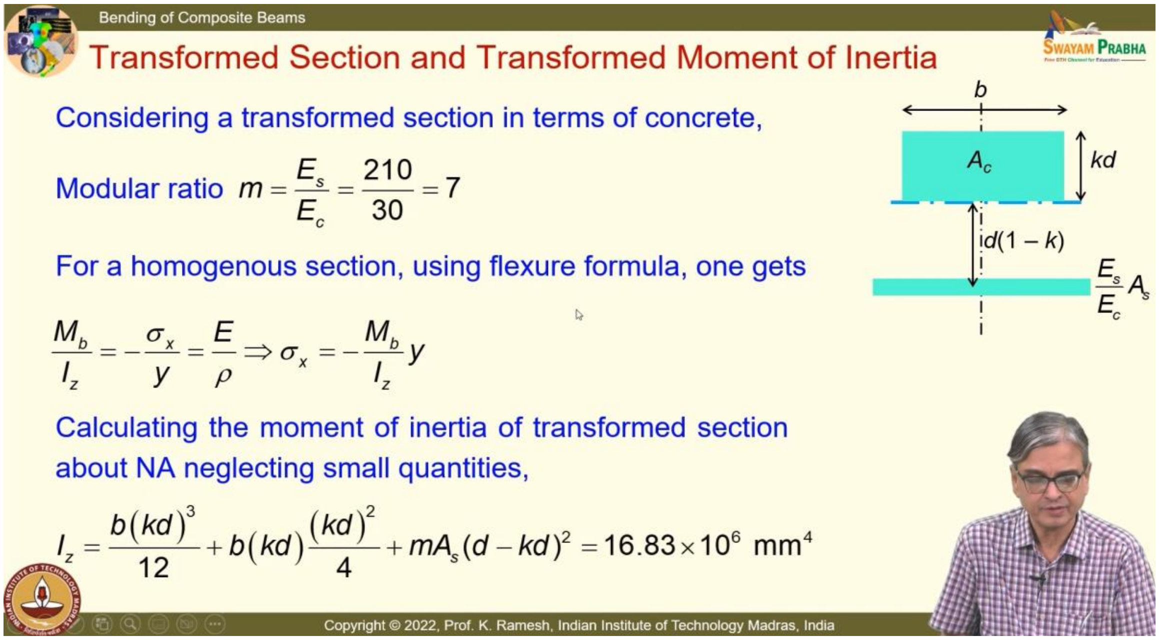

I can also use another approach which is usually done in civil engineering, where I said we will look at the cross section as this is made of steel, this is made of concrete.

I would replace this area equivalent to concrete and then visualize this as a beam made of one material, ok. And this is called a transformed section in terms of concrete. So, that means, I have fully filled section, I have only material away. You can imagine that is a very slender line is connecting these two. That means, this inner core is hollow.

What you will have to recognize is, we still use the strain variation in the beam, directly use it for this section also. Like we have used expressions for the torsion from solid shaft to hollow shaft. On similar lines, you also use it for beam, ok. And it would automatically adjust because we are doing the force balance, it will automatically develop the appropriate stress levels to satisfy, ok. And to simplify the calculation, we have the ratio put as a symbol *m* and you use the flexure formula, find out the stresses and you will get all this in terms of transformed section.

Because now we have replaced this as a larger area because E_s/E_c is 7. So, 7 times the A_s area is what you are going to use. And so you can do this calculation because the concept is very well discussed, ok.

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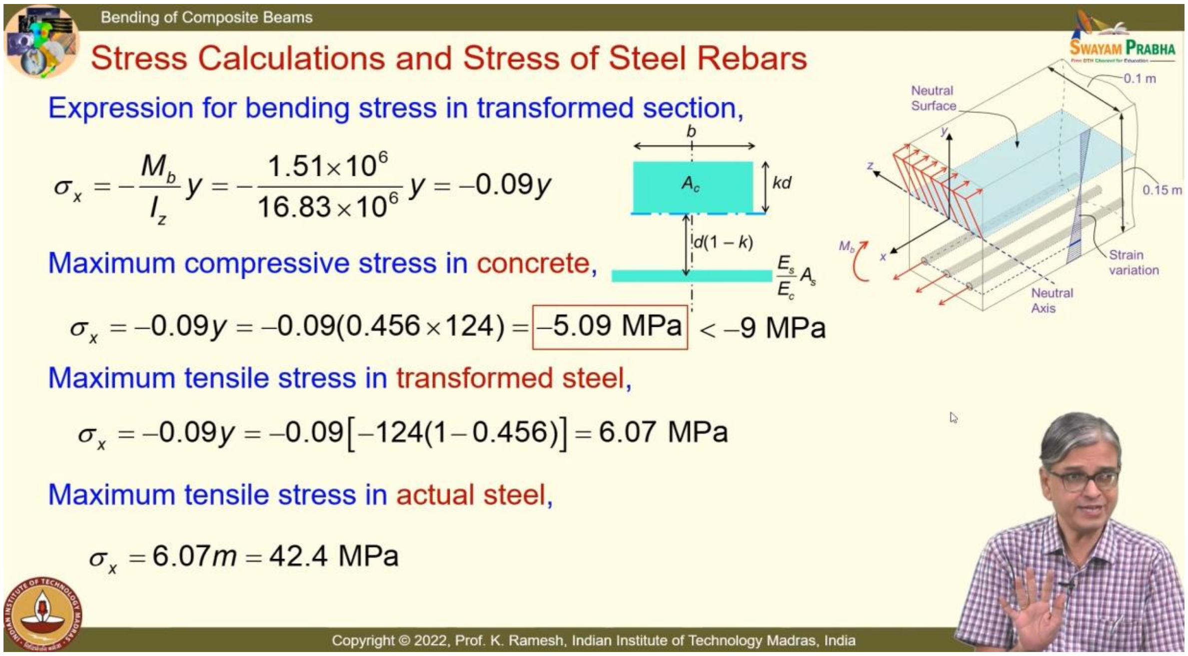

So, you can get the bending stress in transformed section, you can get. I get this as - 0.09y and then maximum compressive stress in concrete. I have to find out what is the *y* maximum. I get this as minus 5.09 MPa and tensile stress in transformed steel, I would get this as 6.07 MPa. This is only for the fictitious one that we have used.

Now, you have to convert back to what is the force in steel. So, when I do that, this will be multiplied by 7 times. So, I that whatever the ratio that we had that was 7, I get this as 42.4 MPa. And if you look at your basic requirement, concrete should withstand only 9 MPa and this about 60 MPa, I have got these numbers very close. And these two figures summarizes what happens in the cross section. It is a very interesting problem, fine. And the key point here is the steel bars are perfectly embedded.

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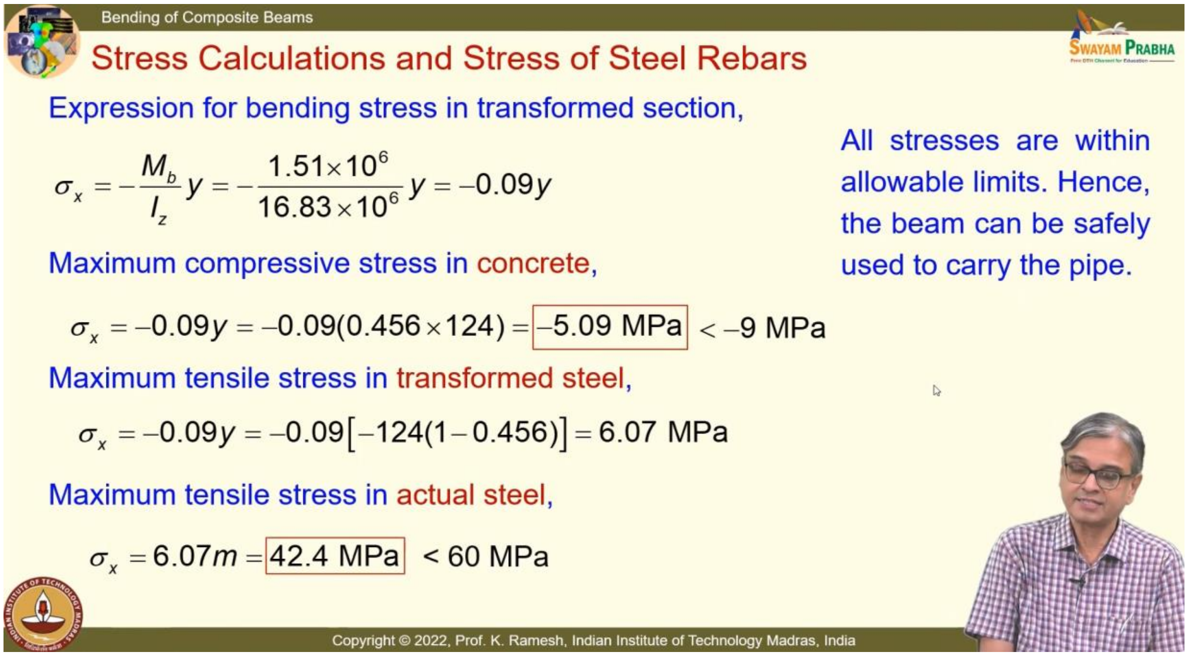

So, all stresses are within the allowable limits. Hence, the beam can be safely used to carry the pipe. So, in this class, we have solved very interesting problems. The focus here is how to visualize the imbalance because of variation in bending moment causes the formation of shear stresses. You have to understand the basic idea, so that problems can be coined in many different ways. Unless you understand the basic idea, it may be very difficult for you to apply what you have learnt to solve a given specific problem. And I have also given one interesting homework.

I want you to find out if I take a vertical section, please do that. Only then I can discuss something very interesting tomorrow. So, please do the homework and then come back. And we have looked at composite sections.

In one case, the symmetry was not affected. So, I do not have to find out the neutral surface. I could easily start from that. The important concept was strain variation is continuous. There can be discontinuities in stress variations. And I also noted, even though I have a thin section here, even within the thin section because it is transmitting bending, the stress varies linearly in contrast to what you saw in hoop. Then we have also looked at how to reinforce a concrete beam.

See, one of the very important application is you should know the bending moment diagram so that you find out which is the tension side, which is the compressive side. You should not put the steel bars in compressive side and say that you have reinforced the beam; the whole building will collapse. All that happens and they have also narrated one instance somebody made the concrete beam without even putting a reinforcement. Now, you have an alibi. How do I reinforce? Suppose somebody has done a mistake and you have to do retrofitting. What can I do? I can bond a steel beam in the tension side firmly. That is one way of alleviating the problem, ok. Thank you.