Strength of Materials
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Lecture - 24
Bending 2 - Flexure Formula

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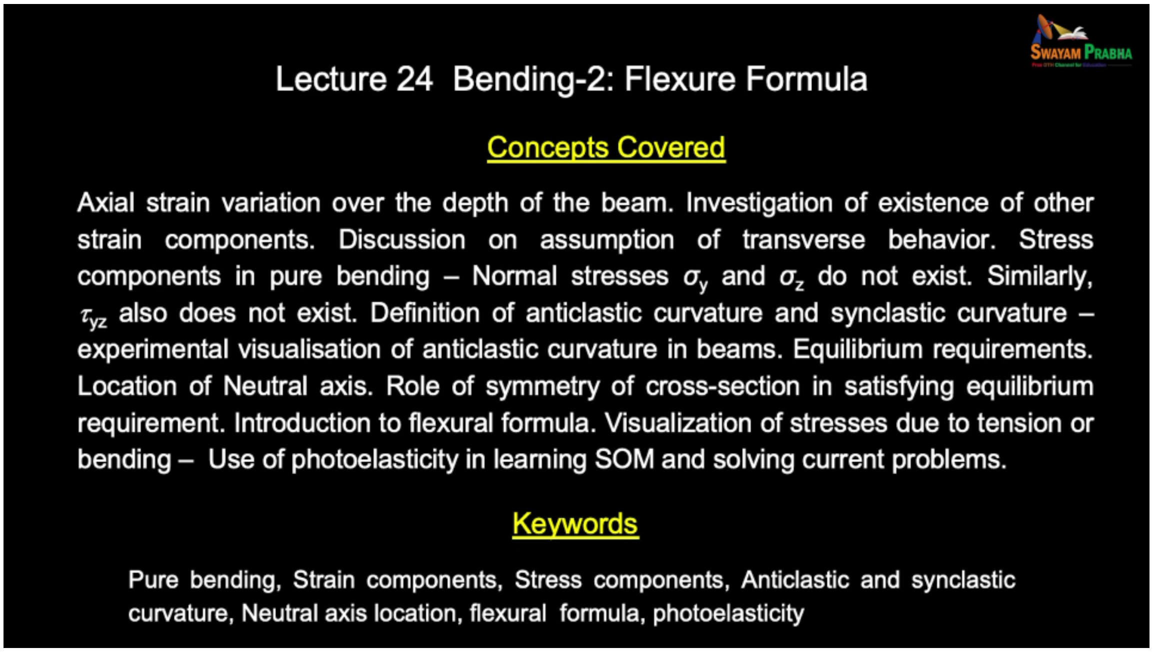

Let us continue our discussion on bending of beams. You know in the last class, we have developed the variation of axial strain. We found that the strain component is related to the distance y. So, it is linearly related.

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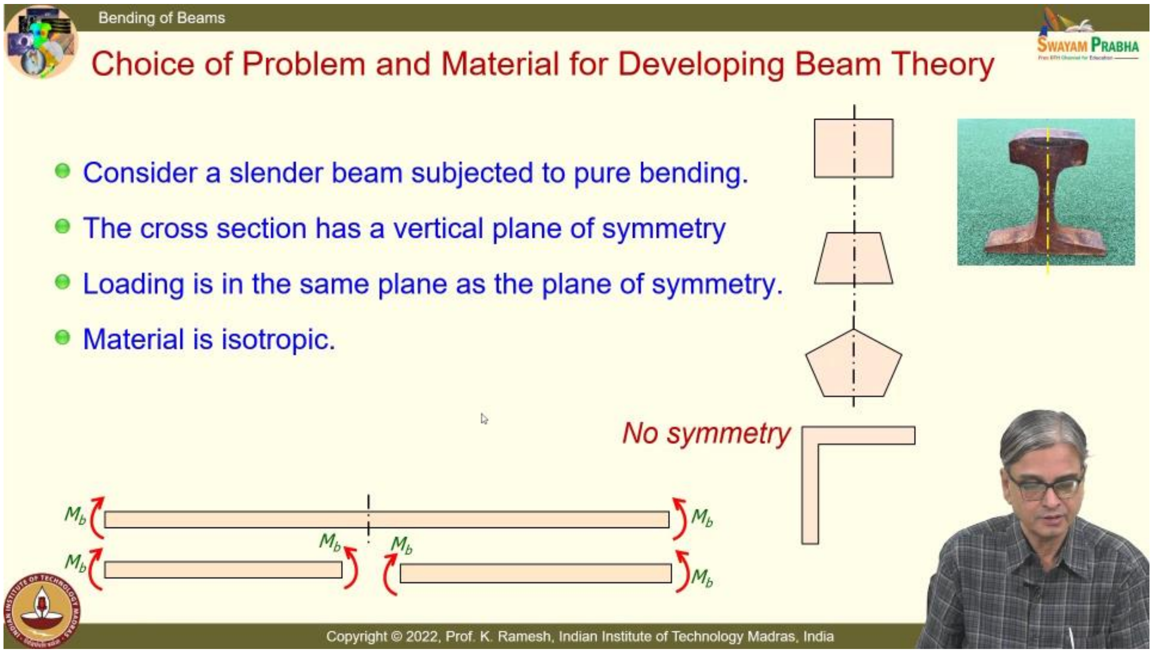

And before we continue today, we again review and emphasize some of the basic idealizations. You should always keep track that, we are looking at only a slender beam and the beam is subjected to pure bending.

So, every cross section transmits only the bending moment, there is no other force. Only for that, we have a hypothesis on how the deformation is. From the deformation picture, we try to write the strain relations. Then we use stress strain relations to find out the stresses and so on.

And the cross-section has a vertical plane of symmetry. I have already said the plane of loading and plane of symmetry should be identical. I could also do it in a horizontal plane. The idea is we would confine our attention to sections that have a symmetry about this axis. This also has a symmetry.

So, in a sense, it is much more relaxed than what we had looked at for the case of a torsion. On the other hand, when I apply the load along the centroidal axis to a section like this, which has no symmetry, you would find when I apply the vertical load, it would bend as well as twist. We will see that later. So, loading is in the same plane as the plane of symmetry and the very important idealization is material is isotropic.

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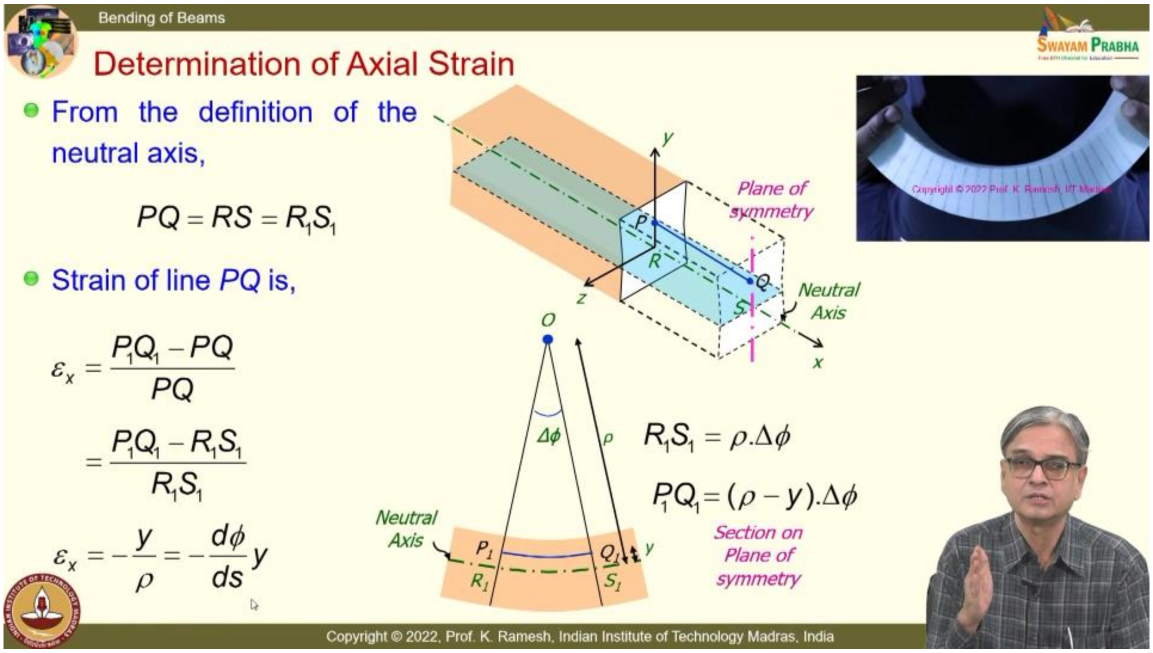

And you know, we have also looked at when you put the radial line like this; they meet at a point. And you know, the same beam continues here. In order I put certain lines here in order to make them clear, I have removed the color. Do not think that beam has suddenly become a wire frame. That is just to bring in clarity on our subject development. And we have defined what is a neutral axis.

Even in the deformed configuration, the length R_1S_1 is same as R and S in the undeformed configuration. So, with that we are in a position to write the expression for strain as change in length divided by original length. And you should understand that this is the undeformed configuration, and this is the deformed configuration. And you know, we have a line PQ in the undeformed configuration. It is at distance y from the line the line RS.

Even in the deformed configuration, we still take the line at a height y from the neutral axis. You will not appreciate it now, because I am showing only reasonably small deformation. Only when we look at very large deformation, you will find this distance need not remain same. We have already said in this course, we would solve everything in undeformed configuration. That advantage we take, fine.

These are all subtle concepts that many people ignore after they got the flexure formula. Unless you remember some of these, you will not know what are the limitations. You get the expression, but the expressions also have some limitations. And you have correctly pointed out in the last class what is expression for P_1Q_1. And since we have said that we would work with neutral axis, we were able to replace PQ as R_1 and S_1. And it is easy to see that R_1S_1 is $\rho.\Delta\phi$ and P_1Q_1 is $(\rho - y)\cdot\Delta\phi$.

And this gives me an expression for strain, which is a function of y. And this is also related to the angle $-\frac{d\phi}{ds}y$. In fact, $\frac{d\phi}{ds}$, we would relate it to the deflection, we will do a separate chapter on it, ok.

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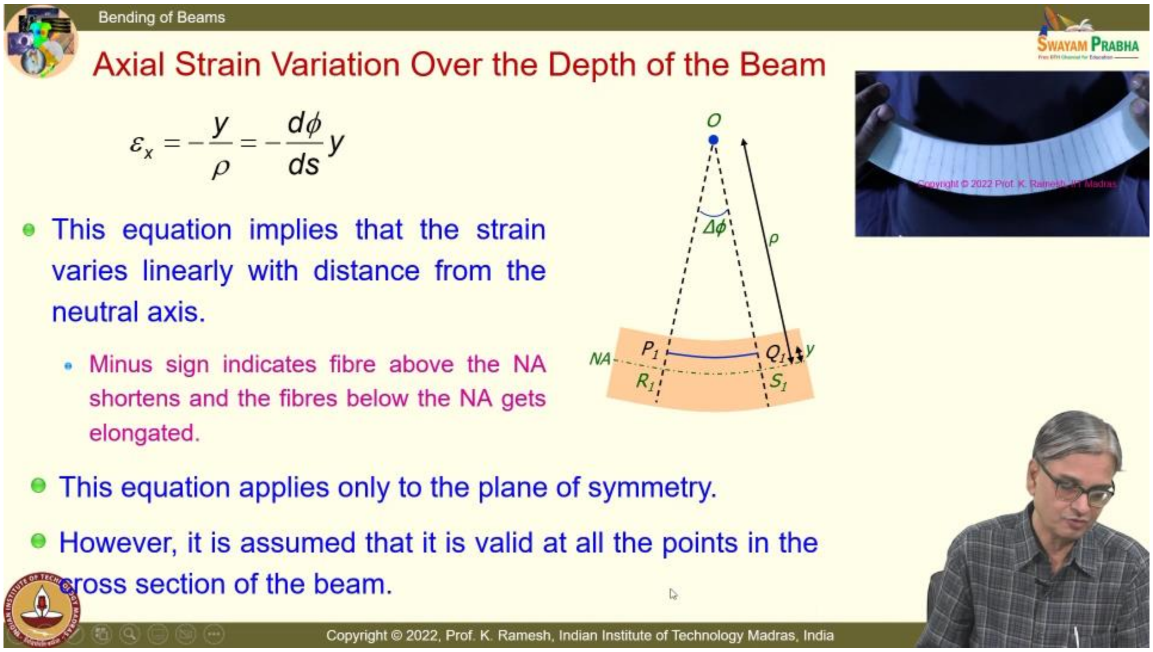

So, this is the starting point for today's discussion basically. Since I have this as a function of y over the depth of the beam, the variation of the axial strain is clearly linear. So, the reference is neutral axis. I have the neutral axis from which I measure the distance y. If the neutral axis is not coinciding with the centroidal plane, I have to determine the neutral axis and then find out what is the distance y. And this also implies if I have y farther away, that is $y_{\max}$; your strain values also will become larger and larger. The maximum is reached when you have the $y_{\max}$.

And you know we have taken a sign convention when you take anti-clockwise moment is positive, that produces compression on the fibers above the neutral axis and produces tension below the neutral axis. That is the reason why you have the minus sign. If you look at some other books, they may not have followed the sign convention. You will find that simply they will say $\varepsilon_x = -\frac{y}{\rho}$.

So, you will have to be very careful about the sign convention and that is very important. And from all the discussion we have done so far, this equation applies only to the plane of symmetry. We have never recognized whether the specimen has any thickness, fine. We have still not brought that aspect at all. Even though it is valid only for the plane of symmetry, it is assumed that it is valid at all the points in the cross-section of the beam. So, this is the happy approximation.

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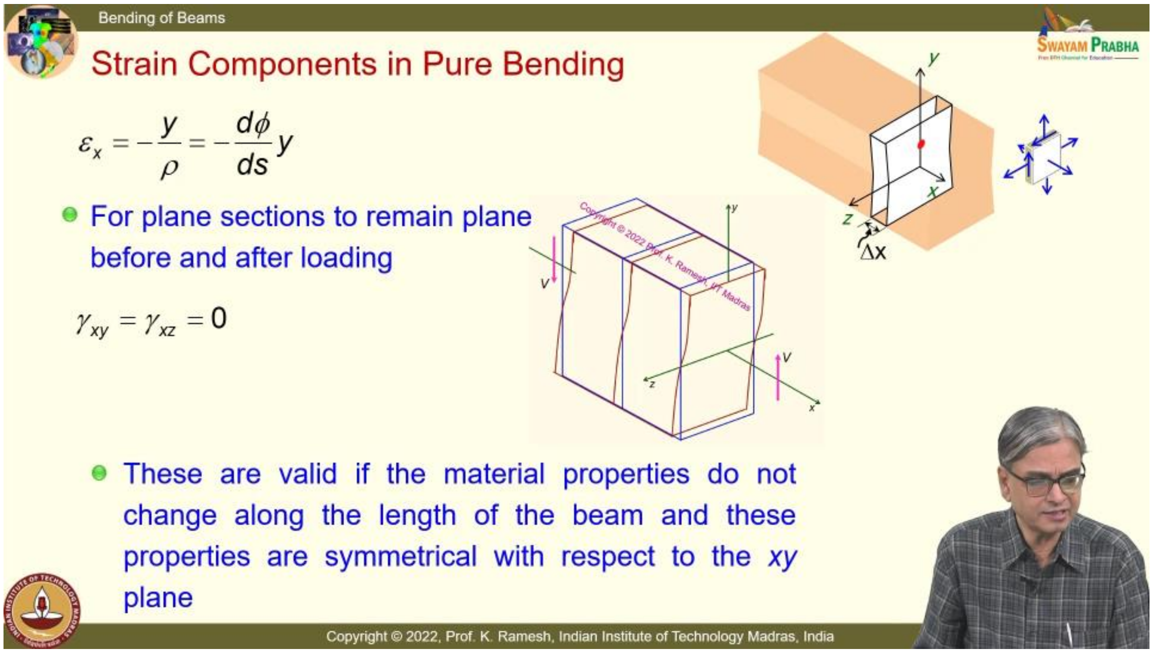

And you know we will also investigate other strain components. What are the other strain components that can remain? We have brought in symmetry. We had a hypothesis; straight lines remain straight lines after deformation. So, anything that distorts the straight line, we will say that strain component is zero, fine.

Right at the moment it is difficult for you to see, I will have to show you an animation which we will discuss it once we look at the shear stresses. But nevertheless, for this discussion, I will show you that. So, I have the same beam, I am considering a small element, I have taken a point of interest here and I assume that it has the stress components indicated like this. We will have to find out which are the stress components are zero, which we can assume them to be zero, fine. And you have important symmetry condition, plane sections remain plane before and after loading. That implies $\gamma_{xy} = \gamma_{xz} = 0$, which you can appreciate only when you see. Suppose I have a shear stress, when shear is varying, you will find the straight line get deformed this way.

And we have already noted, when we take a beam and then apply only pure bending, the straight line still remain straight. So, in this problem, the experiment also shows straight line remains straight, that is possible only when γ_{xy} is zero. The similar argument if I have γ_{xz}, I will have warping in the horizontal direction, ok. So, based on that futuristic consideration, you take this as zero.

See, the stress analysis subject matter itself is the concepts are interrelated. You cannot have top to bottom approach in a uniform fashion, you may have to go to the bottom, bring some of those ideas on the top. So, the sequence cannot be in one direction, you have to know certain facts from other experiments, other understanding, use that to say that this is zero, ok. And you have this plane section get deformed in the presence of shear stress, but in the development of the beam theory, we are only looking at constant bending moment. So, when I have this shear force is zero, we will have γ_{xy} is zero. Because I have shear force in the z direction, I would have γ_{xz}, because that shear force is also zero, γ_{xz} is also zero.

And till this discussion, we were worried only about that the cross-section has xy as a plane of symmetry. We are not demanded anything more, it has to be symmetric about the xy plane, but still we have not used that symmetry in the mathematical sense, we will see where it affects in the mathematical sense.

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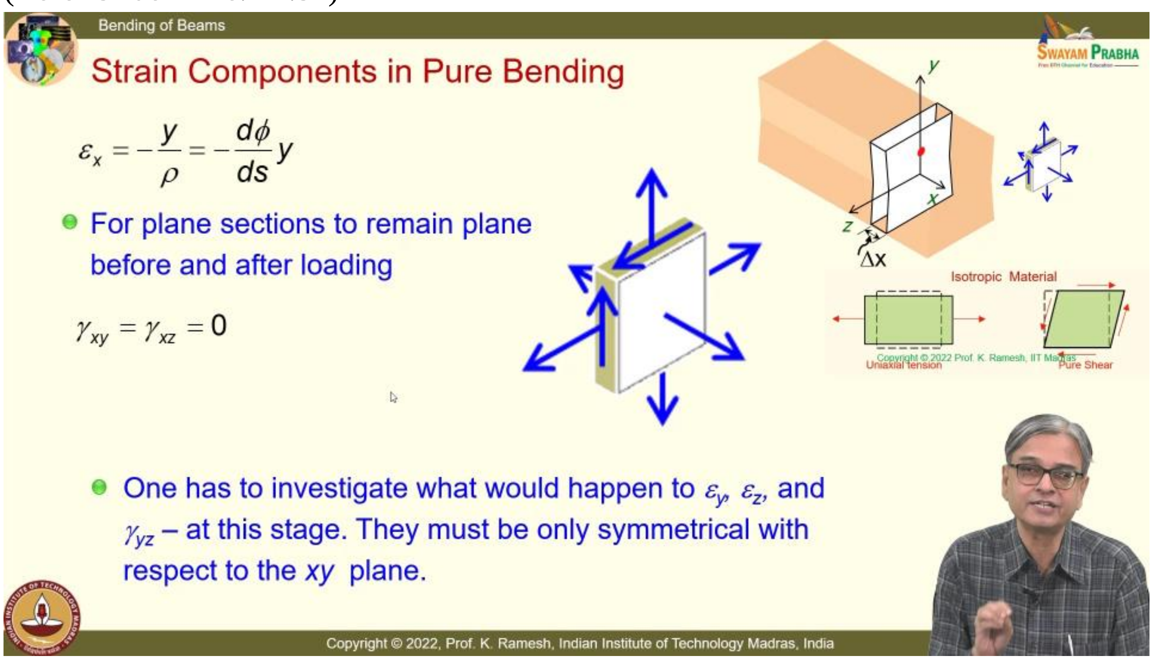

So, I have these quantities and you know we are dealing with isotropic materials, where normal stress produces normal strain, and you have shear stresses produces shear strain. So, I will have to have τ_{xy} and τ_{xz} should go to zero.

So, that is what is reminded here, when you apply a normal force, it produces only normal strain, when you apply a shear force, it produces a shear strain, you take that as an advantage and then you can comment about what happens to these shear stresses τ_{xy} and τ_{xz}.

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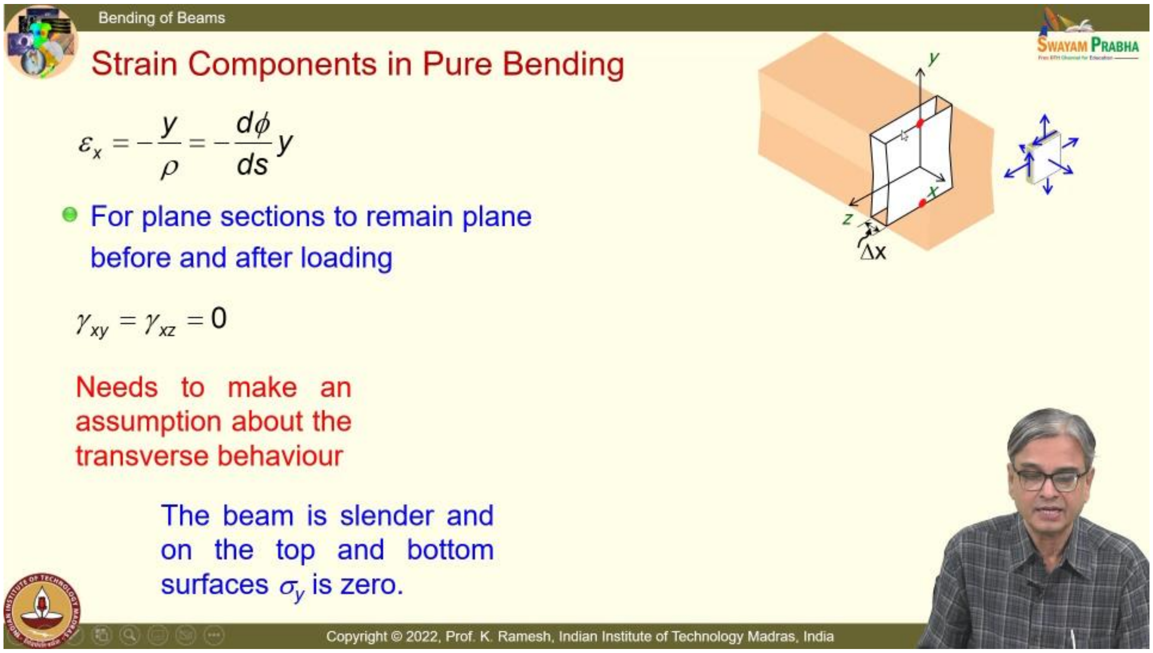

And you will also have to look at what happens in the transverse direction, ok. And you will have to appreciate that the beam is slender. And we have also noted when I take the beam and then apply the load from your concept of free surface, the complete beam where I have not applied the load is free, fine. That means, when you look at the traction, the traction will be zero.

So, from that argument I can say that because this is a free surface, I do not have anything in the *y* direction, so σ_y should be zero. And generally, what they bring in is, you know I have the beam is slender and the depth is very, very small; depth is very small. So, if it is zero on the top, zero on the bottom, it should be zero in between also. That is one line of argument. Other line of argument is, I have not applied anything other than the bending moment. So, there is no possibility for σ_y to exist. On the similar vein, we can also say, I have removed the σ_y here. On the similar lines, I can also say σ_y cannot exist, σ_z also cannot exist and τ_{yz} also cannot exist.

See, all of them are zero on the top surface. So, one argument is bring that as a slender beam. So, over the depth, because the depth is very small, top surface is zero, bottom surface is zero, everything in between also should be zero. You know that same logic may not apply when I want to find out the variation of shear stress when I apply shear. Through the depth, you will find a shear variation. So, a better way of looking at it is, I have applied only the bending moment, because I have applied the bending moment, the forces required to cause any one of these stress components are not applied. So, they go to zero is another way of looking at it, fine. Whichever is convenient to you, have a look at it.

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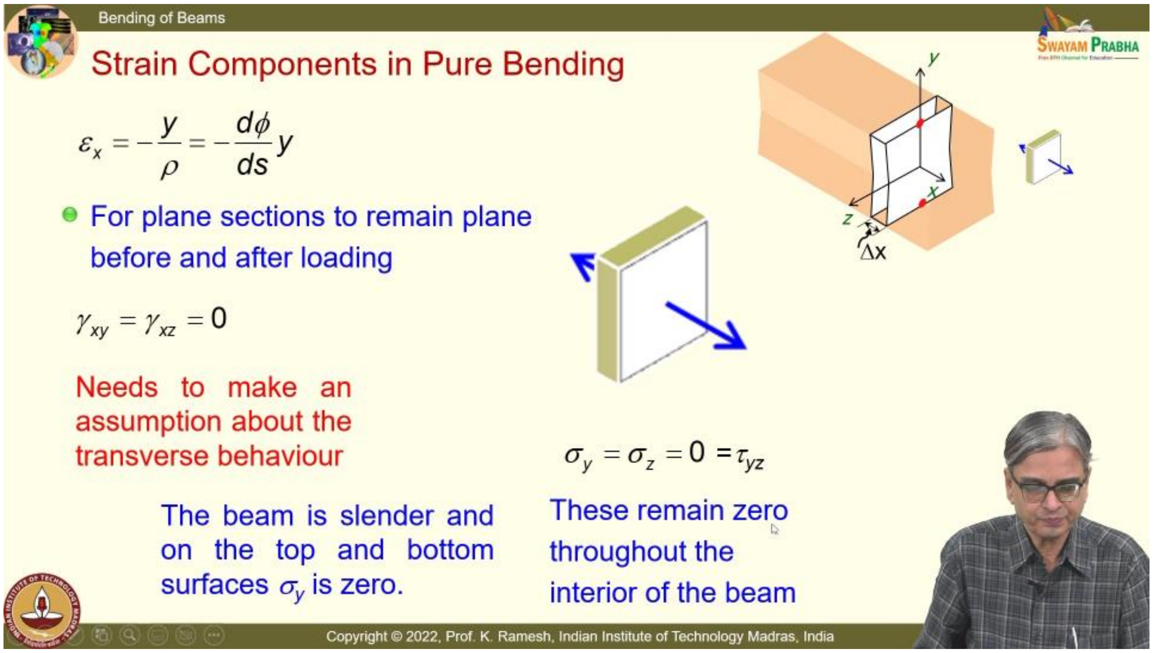

So, I progressively remove these and finally, end up that for any point inside the cross-section, I will have only the axial stress. There is no other stress is existing partly by symmetry, partly by you can also say assumption, but all these are verified with the exact solution from theory of elasticity. See, theory of elasticity is the basis. Whatever the assumption that I have made in the development, if they agree, if they do not violate what is done in theory of elasticity, then our discussion is on the right track and we can proceed with it.So, this is the logic that we do. These remain zero throughout the interior of the beam. So, here we bring in the thickness, fine.

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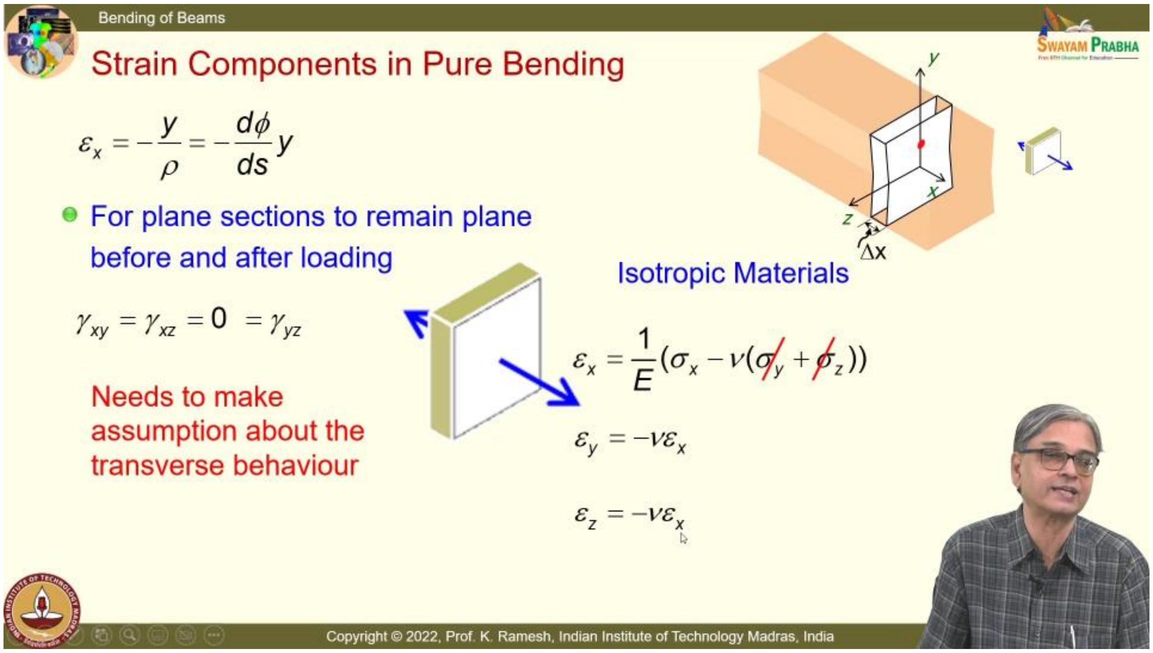

And you know, you have taken a generic stress state at any point inside. Now, we have idealized how it should be. We have removed all of these stress components.

We have said it has only the axial stress. And we should look at that we have this as a isotropic material. And you should tell me the complete expression for strain in terms of stresses. When I write ε_x in terms of stresses, what all I will have in the right-hand side? ε_x can be contributed by σ_x, σ_y as well as σ_z, never forget that! When I write the generic expression for an isotropic material, I can write ε_x only in terms of

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu(\sigma_y + \sigma_z))$$

And in this simplified loading, I have applied only bending moment and we are handling small deformation and we are also handling slender members; σ_y and σ_z go to zero. And can you write anything about ε_y? Does that exist or not?

See, you have studied strength of material so far. When I apply bending moment, we have seen that axial strain varies linearly. So, you can imagine that the loading has caused some kind of a strain in one direction. When there is strain in one direction, what happens to other two directions? You will also have strain because of Poisson's ratio; you cannot ignore that! So, I will have

$$\varepsilon_y = -\nu\varepsilon_x$$

So, it indicates when there is an elongation, there will be a contraction and vice versa. When there is contraction, it will bulge out. I will also have ε_y, I will also have ε_z.

$$\varepsilon_z = -\nu\varepsilon_x$$

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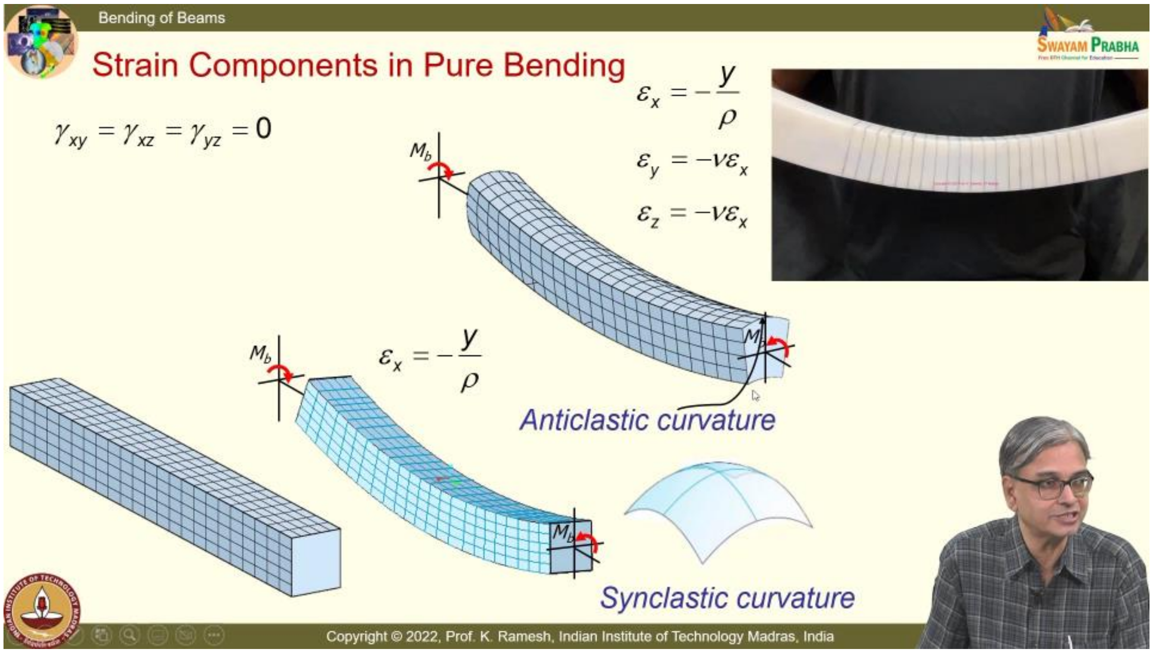

Now, let us understand, you know I have drawn a sketch, this is the beam which is reasonably slender. And if you consider only $\varepsilon_x = -\frac{y}{\rho}$ - the beam has bent, fine. And this is what we have considered because you know till now, we have not really recognized that the beam has a thickness.

When I recognized that the beam has a thickness, I have to recognize I have ε_x, I have ε_y as well as ε_z. And when you plot the lines in a diagrammatic sketch, the beam would be bent like this. Now, what you see is you have a hump, it has moved up and this has become like a cup. So, it has one curvature on this direction, there is another curvature and in the mathematics, they call this as anticlastic curvature because the curvatures are opposite in direction.

See this is only a diagram, fine. Once I take the model and I bend it, how it comes that we will have to look at. And to appreciate what is anticlastic curvature, you have synclastic curvature in which I have curvature on both the directions in the same way. It is increasing and decreasing, it is increasing and decreasing. Whereas this is cupping like this, and the curvature is opposite in this, fine. So, when they are opposite, you call this anticlastic curvature, when they are identical you call this as synclastic curvature.

And in the case of beam bending, you have this kind of phenomena. And when I bend this beam, I want you to look at very carefully, you will see this hump, fine. You will also see this hump, but you have to look at it very carefully and you will also see that this has bulged out very well. And you will also be able to see the bottom curvature, fine.

So have a close look at the beam, fine. And you will see, you will see, you will see now; you will see when I rotate it. You will see, do you see that there is a hump? You are able to see this bottom curvature very well and you see the projection out. But the hump is very difficult to see! And I have done literally a very, very large deformation, fine. So you know, we can have an argument that yes, I have all these happens and you have a finite thickness, I do have anticlastic curvature, but it happens at a very, very large deformation. So, for the small deformation, I do not have to worry about it in my calculation. Is the idea clear? You have comfort from that.

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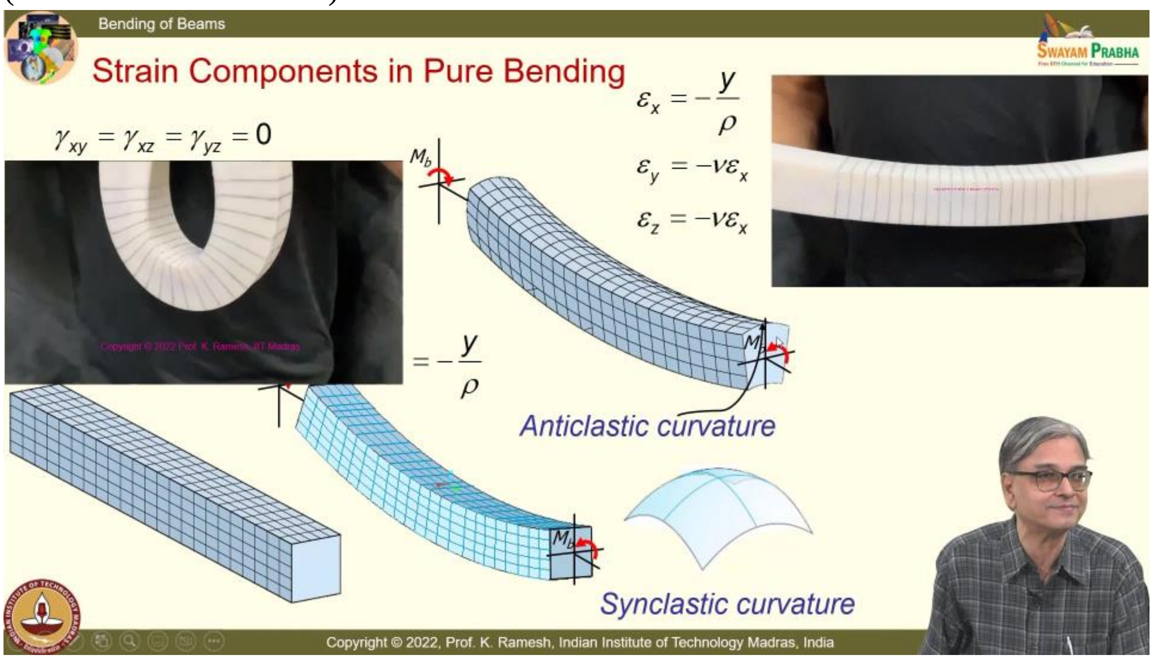

For structural analysis, I am going to show again the bending. You can see the hump, ok. And you can also see that is coming out because of compression, it is bulging out, because of tension it is compressed, you can see the bulge. See, I see only a small section of the beam is having this bulge, whereas when I do the diagram, I can draw it nicely. These are all exaggerations.

See when you are developing a concept, you should understand it threadbare. You should ask a counter question and see what happens. And if you say that it definitely happens, but it happens at a very large deformation, then you have the comfort that I can comfortably use these equations for general stress analysis. And I was also very surprised, you know when I was looking at anticlastic curvature; see the human body is very, very complex. I was very surprised the biomechanics people when they want to analyze the swallowing action, they consider tongue as a beam under bending. It is initially curved, and it is so soft, and they find that it also has anticlastic curvature.

You do not have any of the structure as soft like your tongue. So, as long as you are not dealing with such soft material, even though mathematically you can argue that there is a curvature like this, these are all second order effects. See in engineering, you should appreciate how the equations are developed, how to take a middle path. You should also have the courage to ignore it and carry on with your design, bring in appropriate factor of safety. This is how engineers operate, ok.

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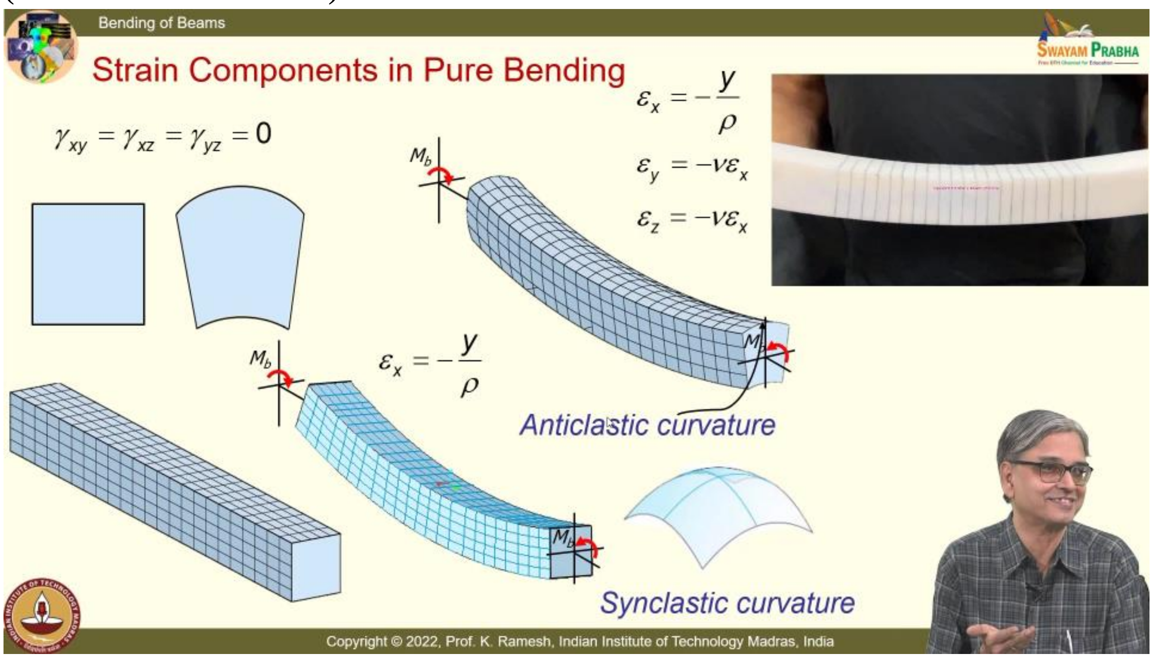

And what you have here is a rectangular cross section is shown, when you have anticlastic curvature, it has a hump, and it also has a cup like thing at the bottom and your neutral surface also will have a curvature. And now you can appreciate, when I said while developing the strain relation that this is related to $-\frac{y}{\rho}$; y is in the undeformed configuration, y is the same height in the deformed configuration. If I take this as the actual bend of the beam because of small deflection from the neutral axis, the y in undeformed and deformed will be more or less identical. Suppose I look at very large deformation and I bring in this, the height need not be same, it is moved up. So, you should also understand where do I use the idealization, small deformation and I live in the comfort of undeformed configuration. This is very important! These ideas are embedded in the books, but they do not emphasize it. When you start thinking on your own, you get stuck or when you want to do research and you want to capture vital phenomena from your experiments, you should have mental clarity. For that all these discussions will help. And this anticlastic or synclastic is a name comes from your mathematics.

When you have curvature, people have different types of curvature and they also have monoclastic curvature. When they say that I have only the bent beam like this, there is no another curvature in the second direction. So, you call this as monoclastic, you call this as synclastic, you call this as anticlastic. So, it is very interesting your tongue has anticlastic curvature. So it is very interesting.

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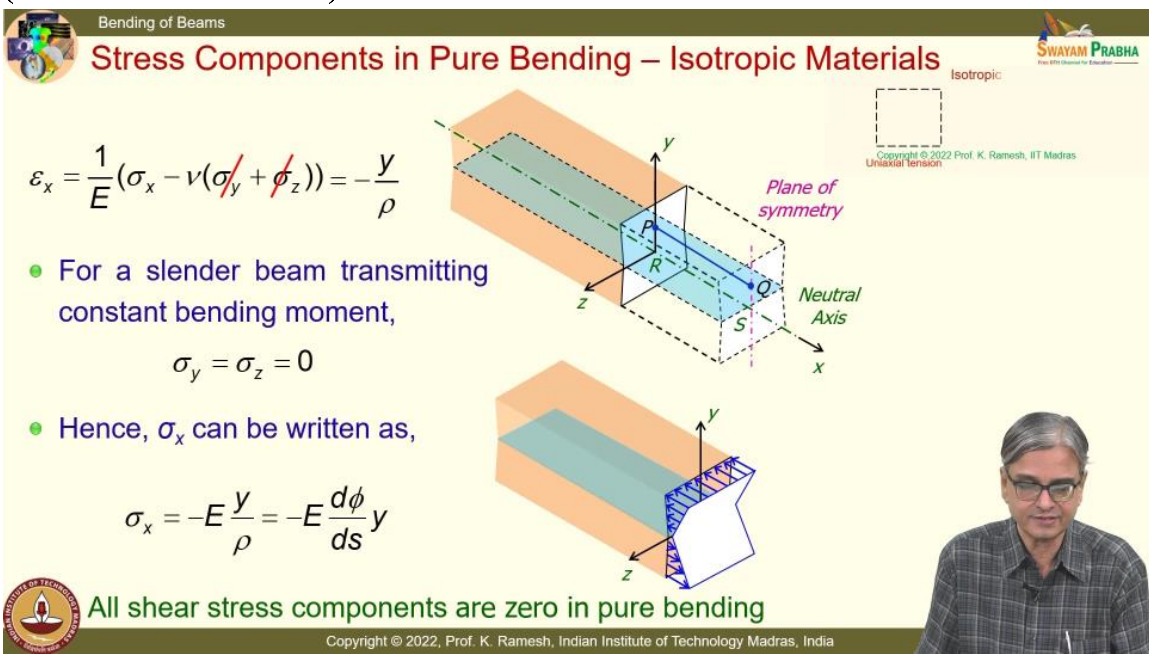

And let us look at stress components and pure bending and you should appreciate that we are living in the comfort of isotropic materials. And we have seen that,

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu(\sigma_y + \sigma_z))$$

Based on our assumption, we have removed this σ_y and σ_z. You have to appreciate that slender beam transmitting constant bending moment. When you say slender beam transmitting constant bending moment, what do we do? I apply bending moment somewhere here, but I look at far away from the loading. You all know Saint-Venant's principle. Saint-Venant's principle is a very nice principle that we would invoke. If I go close to the loading conditions, none of these simplifications you can invoke. It is very complicated and when you work very far away from the actual loading, I can have the assumption $\sigma_y = \sigma_z = 0$. And,

$$\sigma_x = -E\frac{y}{\rho}$$

So like strain varies linearly over the depth of the beam, stress also varies linearly over the depth of the beam! Like strain reaches maximum at y_{max}, stress also reaches y_{max} -

maximum at $y_{\max}$. And what you find is in one case it is compression; in another case it is tension. But algebraically you can say one is maximum, another is minimum. But if you look at only the magnitude, magnitude wise for a symmetric cross section, tensile stress and compressive stress will be identical, because $y_{\max}$ will be same for compression as well as tension in a symmetric cross-section. If I do not have a symmetric cross section, if whichever one where you have $y_{\max}$ is larger from the neutral axis, whichever end you will also have problems. Suppose I have a T section, I would have one side is closer to the neutral axis, another side is far away. So $y_{\max}$ decides the maximum values of the stress or strain. So, I have this

$$\sigma_x = -E\frac{y}{\rho} = -E\frac{d\phi}{ds}y$$

So, it varies linearly like this, make a sketch out of it. It varies linearly and I have shown this as a three-dimensional picture-isometric picture. When you look from the side view, it varies linearly. So magnitude wise, when I have a symmetric cross-section, where the both the sides are at equal distance from the neutral surface, the numerical value of compressive stress and numerical value of tensile stress would be identical. One is having a positive sign; another is having a negative sign.

And all shear stress components are zero in pure bending. What we saw in the case of torsion formula? When you have a torsion transmitted by a circular shaft, all normal stresses are zero. See, we have carefully selected problems for discussion in this course. We had only axial stress in the case of a simple tension member, when I pull it. We had only torsion in the case of a torsion and generated only shear stress which varies linearly. And when I do the bending, I have only bending on transmitted and it has only normal stress. So, we have taken problem consciously, so that we have very simplified results.

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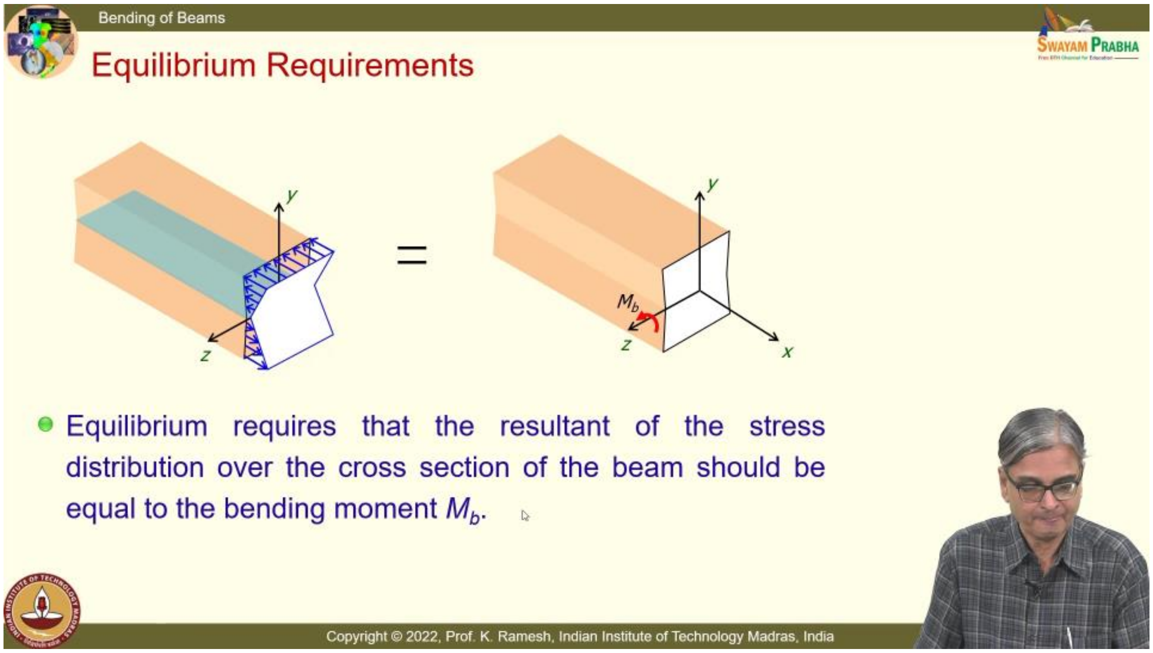

And what are the equilibrium requirements? We have already said even before we started looking at it, we have already said that normal force is zero. We are going to write it in mathematics. So here we are definitely going to look at the cross-section. We are going to recognize that it is not the discussion confined to the symmetry plane. We also recognize the complete cross-section of the beam and then ensure that these are satisfied.

So, we have applied only bending moment, so the resultant should result only in the bending moment M_b, nothing else should remain. We will see what are the kind of simplifications we have started with, where do we use them.

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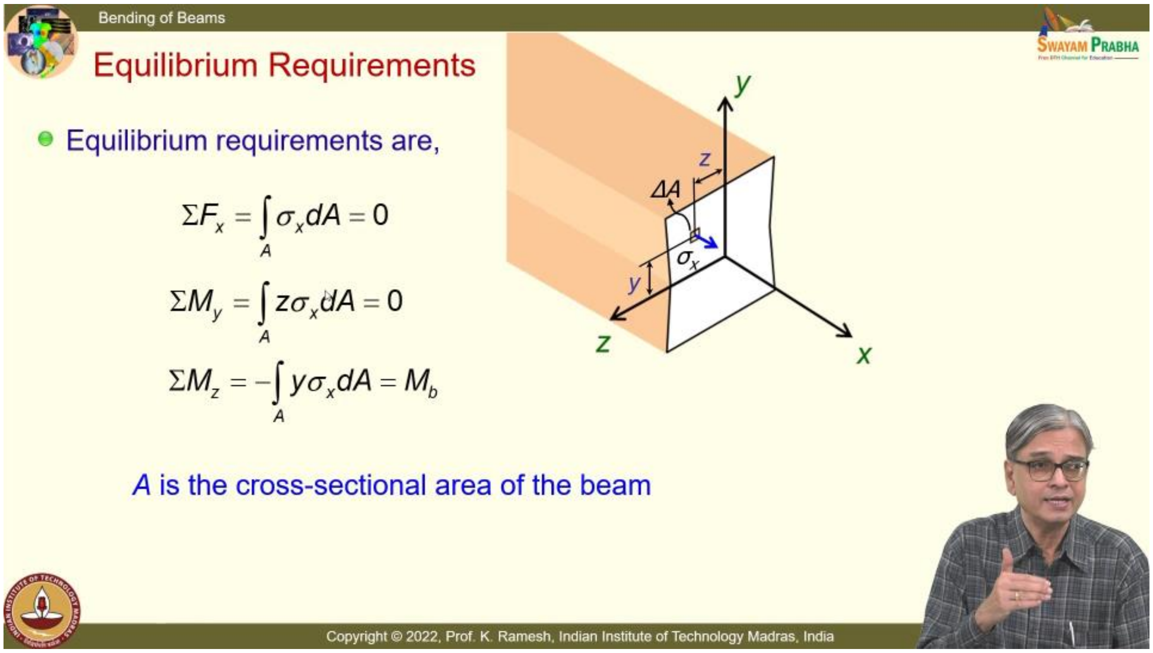

So, I take a small area, I know what is the stress acting on this. This is located at a distance *y* and *z* from the axis, ok. So, when I have stress, when I have the area, I can find out the force and *A* is the total cross-sectional area of this cross-section. So, I have the first requirement, summation of force in the *x* direction should go to zero. This is to be integrated over the area *A*, so

$$\Sigma F_x = \int_A \sigma_x dA = 0$$

And you can go back and write what is the expression for sigma x; we have already determined that.

And the second condition is,

$$\Sigma M_y = \int_A z\sigma_x dA = 0$$

And the third condition is, I find out the moment about the *z* axis, so

$$\Sigma M_z = -\int_A y\sigma_x dA = M_b$$

We will do that one by one, we will substitute the expression for σ_x and we will get a very important learning from this equation. You know the development is also very important when you want to solve problems that are not discussed in the class, fine. Because many modifications can be done in problem formulation.

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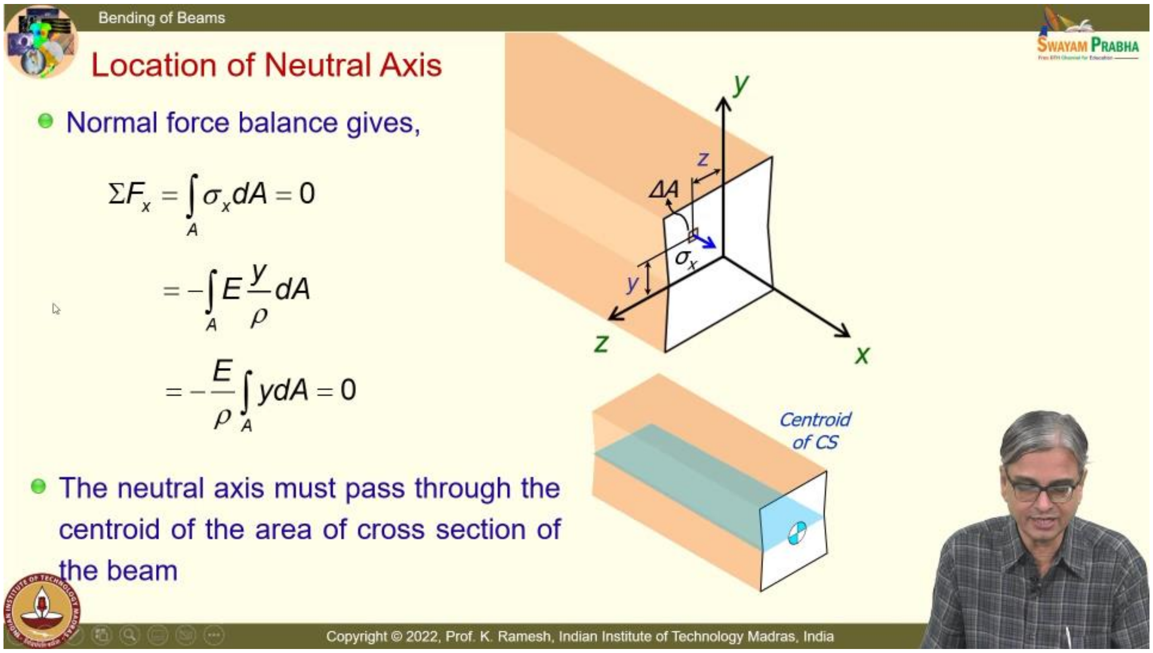

So, what does the normal force balance gives when I have the expression, I replace σ_x in terms of $E\dfrac{y}{\rho}$, I have an relationship here because we have said, this is made of one material. See, $-\int_A E\dfrac{y}{\rho}dA$ is a very important step. Suppose I have this made of multiple materials, we have seen composite hoops, I can also have composite beams, then I cannot take *E* out of the integral, I have to accommodate that. Suppose I have a material which is non-linear, when the material is linear I have only one *E*, when material is non-linear *E* keeps changing. So, what is fundamental here is this equation is more fundamental for you to extend whatever the understanding that you have developed to other problems than isotropic material of constant material across the complete beam.

Suppose I have a concrete, if you go to concrete you have reinforced bars which is steel rod is embedded in the concrete. Suppose I want to extend my knowledge of beam theory to concrete, then I have to hang on to this expression, but the second expression when I take it out $-\dfrac{E}{\rho}\int_A ydA$. What is $\int_A ydA$? First moment of area. What is the definition of your centroidal axis? You define centroidal axis based on that, isn't it? So, that is the reason why we have said when the cross-section is symmetric and is made of one material on the strength of this expression, the neutral axis must pass through the centroid of the area of cross-section.

This is a specific situation when I have a constant material and linear. If the material is non-linear or if I have a concrete where I have steel rods embedded in concrete which is made of aggregates and cement; I have to go back to this and this is also a very important expression for you to find out where the neutral axis is located. So, you will use this expression to find out the location of the neutral axis. Once you determine the neutral axis, only from there you find out what is the distance *y*; you do not do it from the centroidal axis. There is a special case when I have beam made of one material and it is a linear elastic. So, this is what it says that it coincides with the centroid, that is the simplicity that we have taken.

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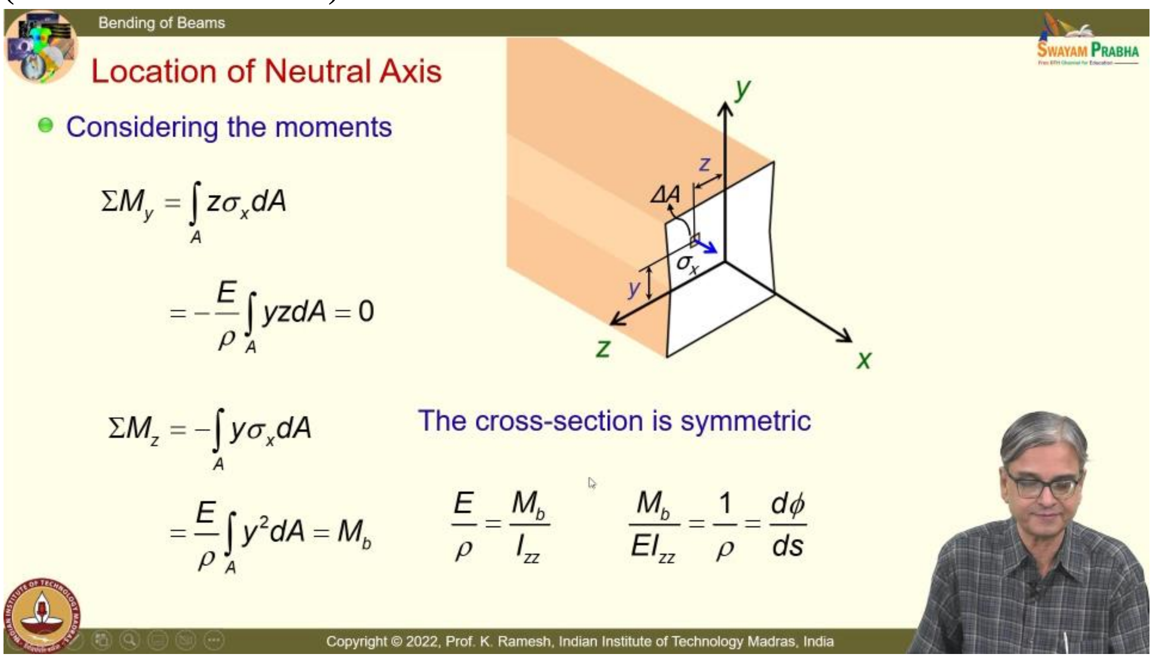

Let us look at the moments. See, we have always been saying that cross-section is symmetric about *xy* plane. Have you ever used it till now? I have expression,

$$\Sigma M_y = \int_A z\sigma_x dA$$

and substitute σ_x.

$$= -\frac{E}{\rho}\int_A yzdA = 0$$

Is it satisfied or not? Is it satisfied on the strength of what idealization that we have started with? When I say that this is the plane of symmetry because of that, this quantity goes to zero.

So, whatever the development of beam theory, we have developed it for a cross-section which has a plane of symmetry. That is very important, that is why this is identified. And you have the other expressions,

$$\Sigma M_z = -\int_A y\sigma_x dA$$

$$= \frac{E}{\rho}\int_A y^2 dA = M_b$$

This is also familiar to you. $\int_A y^2 dA$ is called as what? Louder! Second moment of inertia. And that is what you determine from about this axis. So, I can put this as I_{zz}. So, I get a very famous expression,

$$\frac{E}{\rho} = \frac{M_b}{I_{zz}}$$

See now we have used all the idealizations. We have recognized the beam has a finite thickness even though we discussed about influence of Poisson's ratio because our deformations are very small. We can ignore that as a second order effect. We can still work on undeformed configuration to locate the points. Since we have taken that this has a plane of symmetry, we are now able to get only M_z exists; M_z is nothing but M_b. You do not have M_y because of symmetry of cross-section. So, I get

$$\frac{E}{\rho} = \frac{M_b}{I_{zz}}$$

And if you recast this, I can also recast EI_{zz}. Have you seen something similar to this in torsion? We have called it by a special name. In torsion we have GI_p we call that as torsional rigidity. That means to what extent the shaft can withstand the torsion. On similar lines, I can also coin the product EI_{zz} as bending rigidity.

See, people do not want to forget that they have graduated from rigid body mechanics to deformable solids. So, even the deformable solid, whether it has resistance towards bending, resistance towards torsion; they qualify it by their respective rigidities. If those rigidities are high, it will not twist when I have high torsional rigidity. If I have high bending rigidity, it will not bend that easily; that is what you want. If you imagine that the roof beam sags like what I show in the class, I will be the first one to go out whether you go out or not, ok. So, we live in the comfort of very small deformation. It is not an idealization; it is reality! Ok. Not just an idealization.

And you also have the complete expression,

$$\frac{M_b}{EI_{zz}} = \frac{1}{\rho} = \frac{d\phi}{ds}$$

Then we would exploit this when we want to determine the deflection, one full chapter will devote on this.

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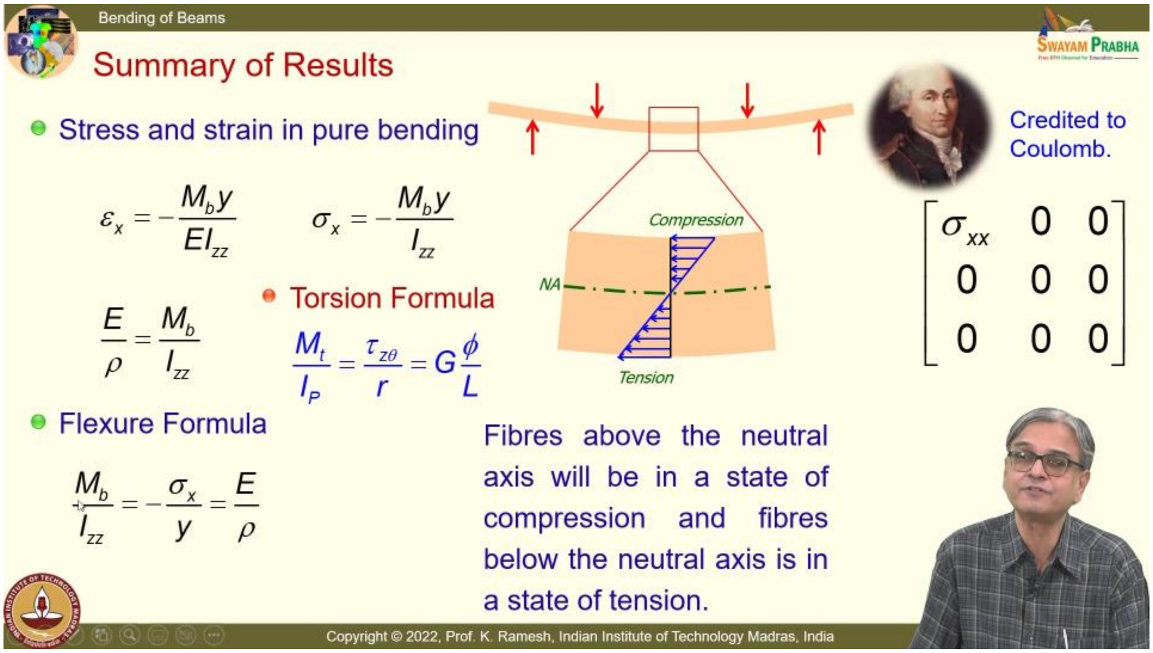

So, we have a summary of results here. I show a bent beam; you see the deformation. I have been saying repeatedly in this course the moment you see deformation, it is large deformation. But for the purpose of appreciation of relative movements, we need to show large deformation, because these deformations are very small. So, I have this,

$$\varepsilon_x = -\frac{M_b y}{EI_{zz}}$$

and,

$$\sigma_x = -\frac{M_b y}{I_{zz}}$$

And I have always said these are all components of stress. I would always appreciate that you visualize this as a stress tensor. When you put it in the matrix, I have shown it as a 3×3 matrix; you can also have it as a 2×2 matrix. Then you appreciate that this varies at every plane and you have to find out the principle stresses all the other aspects of it, ok. When you look at this, this looks like a scalar. Always visualize this as a tensorial quantity, and this understanding is credited to Coulomb.

And you know I also bring in consciously, you have a Saint-Venant's principle. That means away from the point of loading which affected this pure bending, the solution is valid. We know what is the strain and we know what is the stress, which is varying linearly. Please make a neat sketch! You should have this sketch, and our expression also embeds the sign. So, fibers above the neutral axis when I apply positive bending moment is subjected to compression; below the neutral axis they are subjected to tension. And from the equilibrium consideration, we have also related the curvature to the bending moment.

So, I can write the flexure formula,

$$\frac{M_b}{I_{zz}} = -\frac{\sigma_x}{y} = \frac{E}{\rho}$$

You know you can complete your second course on machine design just with this formula. Because there you directly take it as slender members, even if it is not slender you force it as slender, you get the first level of results and you may not also have multiple material and all that the entire course you can live with this. Do you have an equivalent in torsion? You have a torsion formula that gives,

$$\frac{M_t}{I_P} = \frac{\tau_{z\theta}}{r} = G\frac{\phi}{L}$$

So, I have this flexure formula and only in our development you will find this as minus sign because we have followed systematically the sign convention. And you have got this minus sign because it also tells you that when I have a positive bending moment, I will have compressive stress developed.

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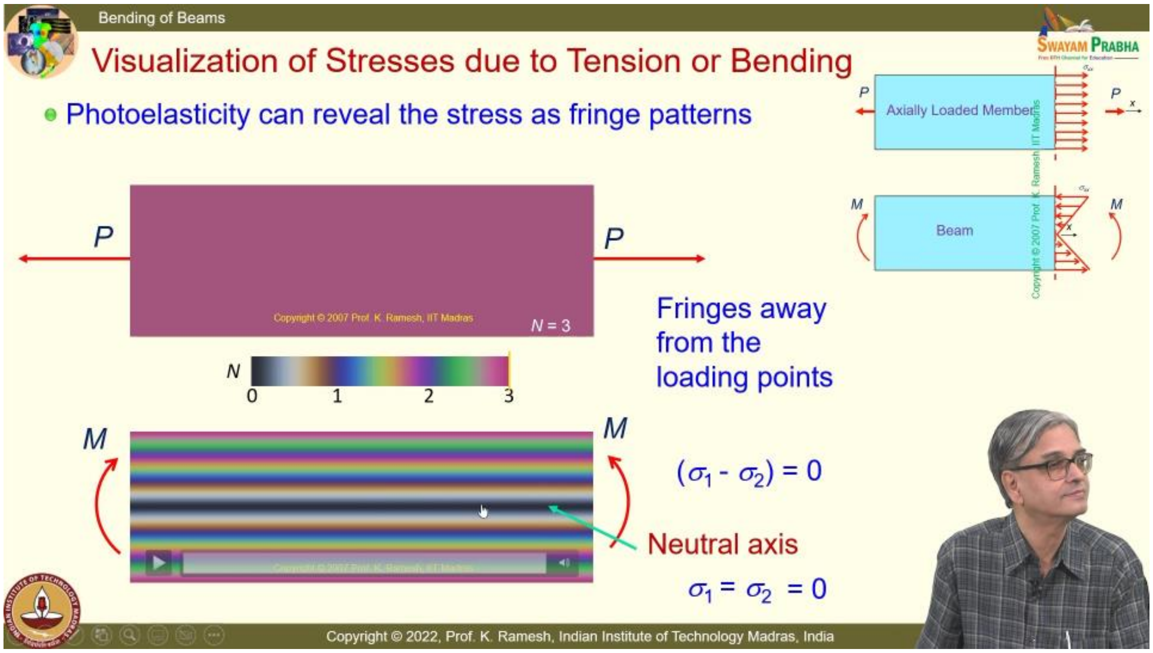

You know when we started this course, we started with photoelastic fringe patterns where I have this as constant color for a axial load; for bending you have this as densely packed lines. And we found that the distance between the fringes remain constant for a particular load and now we also go back we have learnt what is known as a neutral axis. Can you identify neutral axis in photoelastic fringe pattern? What is a neutral axis? Neutral axis is the line which does not get deformed, ok.

And from your stress variation stress is zero. So, what happens to $\sigma_1 - \sigma_2$? That is zero. When it is zero, we have in the color code this is black. So, a black fringe indicate that you have a neutral axis, and in this case $\sigma_1 = \sigma_2 = 0$ because it is a triangular variation. So, I have this as a neutral axis, fine.

And we have also looked at certain modifications in our earlier development. I put a hole in the tension plate, you have beautiful play of colors and we also said that is stress concentration. And the stresses go in a finite I mean, when I have a infinite plate with a very small hole it goes to three times, but for a finite plate it goes higher than that which you can determine only from experiment or from a numerical study. And we have also looked at a stepped beam. See, this is again subjected to pure bending. When it is subjected to pure bending, do I see black color here? I see some disturbance here. Is the idea clear?

But we are clever! We know Saint-Venant's principle. So, when you exclude this zone, we have essentially captured in our strength of materials development, what happens in a beam over the depth of the cross-section. That is a success. This zone requires special attention, fine. Suppose we wanted to frame the problem how to analyze a beam like this and if you are concentrating on how to accommodate this in your development you would not have got any answer. So, the success is we have understood that there is linear variation of stress over the depth of the beam, and you also have neutral axis seen, fine.

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And you know I find this knowledge whatever we have looked at, is it used in general practice? Definitely! Because we know the core is not transmitting load; we have a rail network instead of using a cross-section like this which is filled with material, we have removed material you have I cross-section.

So, significant weight reduction has been achieved. So, the idea what you will have to look at is you have to extract the essence that is beautifully done by taking a very simple problem.

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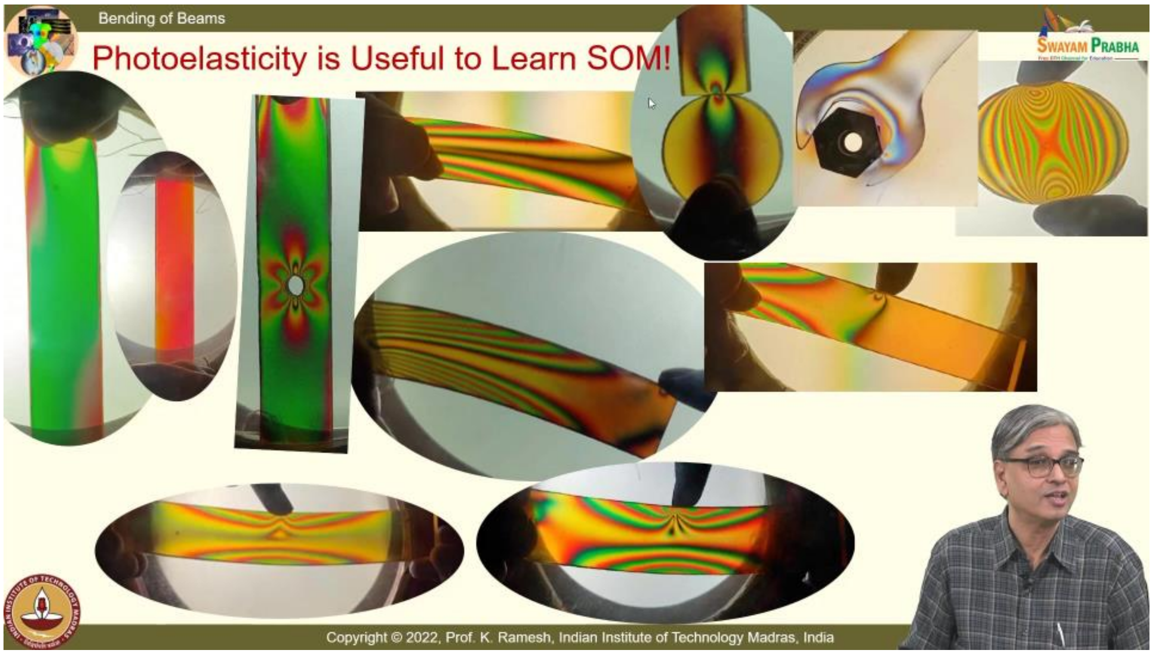

And you know, I also want to compliment that you have done a good job in your laboratory. I have just given you a model like this and in your exposure to photoelasticity lab, people have provided this beautiful fringe pattern. These are all recorded by you in the laboratory class. I picked some of them. So, you have a grip. You know I have tension applied and compare these two; this is done better. You find there is uniform color. So now you agree, when I apply an axial force, I get only uniform color. When I show in the slide, you see it is Greek and Latin. But when you go to the lab and then pull your own specimen you find happens, actually. It gives lot of confidence in taking a call on this, fine.

And the model is same, just a simple flexible member like this. You have got the cantilever, another cantilever, this is also very nice. So, this is loaded in between you have taken it in a bright field; that is why you see this as bright. If you have taken it in a dark field, this would be completely dark, which we have seen in one of the problems.

And I was also happy that some of you have tried to flex it as a three-point bending. Please note down this fringe pattern, it is very, very important. We are going to discuss it. It is not trivial; we will have to see what is the result that we have got and how our strength of materials compares with the theory of elasticity. There are deviations. You should also know the deviations. Is the idea clear?

And you know by putting a hole, you also found there is stress concentration. And you also had the pleasure of tightening a bolt and saw what happens in the spanner. And some of you have also tried contact stress. I am happy! When models are given and you are given time to explore in the polariscope, your mind also becomes fresh and you want to experiment more and more aspect of it.

So, this definitely says and brings out photoelasticity is useful to learn strength of materials.

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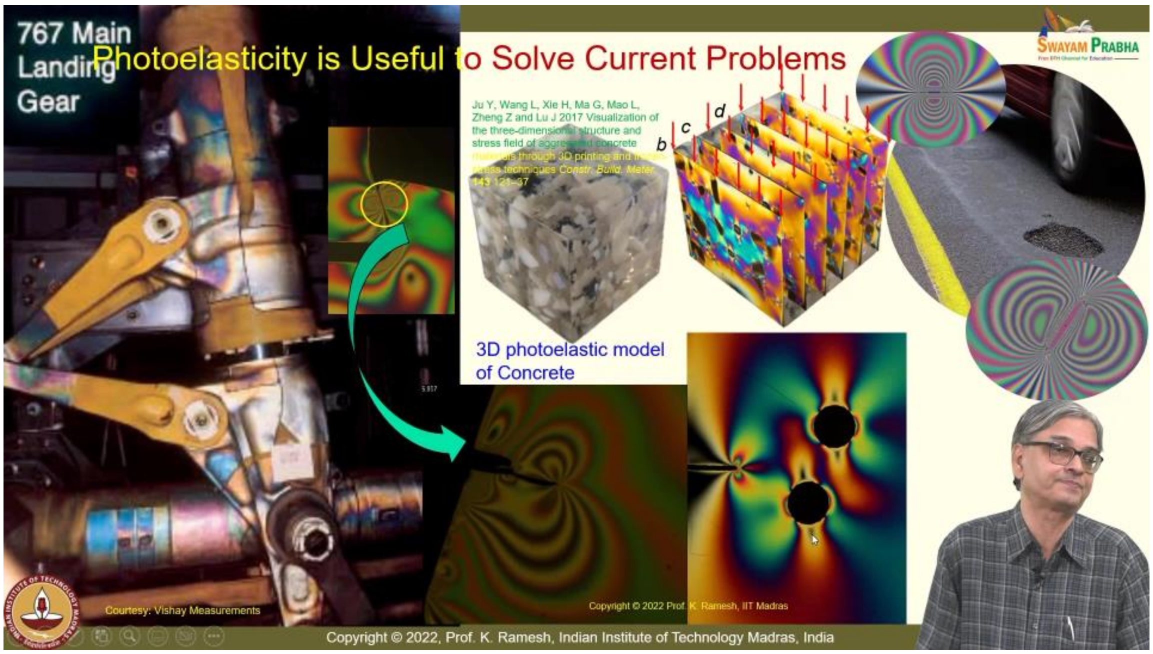

Do we stop there? It is also very useful to solve current problems. See, look at this is the picture I got very recently for 767; Boeing 767 main landing gear. This is the fringe patterns that people have got. These are all photoelastic coatings. And you know some of these are all confidential results. And you have now 777 and 787, isn't it. So, its being used currently.

And I have also said, you have potholes in roads. Recently you must have listened to that they transported the statue of Netaji from Telangana to Delhi, and they had to put several wheels to the truck. See, there is a design strength, you have to transmit only this much amount of load to the road. So, in order to support that load they need to put several axles. If you overload your tracks because of contact stress, this is what is going to happen that is one of the problems. The potholes is a very big nuisance.

Then you know I said that we are going to deal with heterogeneous material, and you have a beautiful rapid prototyping which you can make this kind of complicated material and analyze complicated heterogeneous material with pores and aggregates.

And you also have a very interesting problem of a turbine blade root. And you have a crack generating, you have very interesting fringe pattern; these were developed in our laboratory. And I have also said there is also a current problem on interaction of cracks with macroporosity.

So, what you will have to look at is, in strength of materials we have taken graded problems. By analyzing a beam with pure bending, we have been able to capture the essence of linear variation of stress and linear variation of strain, when I have beam made of one material.

We have captured the essence of it which is also used in many engineering applications. And in the laboratory, you have been able to experiment with simple photoelastic models which are very flexible; you do not have to spend time on designing a loading rig or anything like that, simple hand is sufficient to make this that has given you a comfort that what you are learning in the course is true in reality. You have seen it in your own hands. And photoelasticity is shown to be useful for learning the strength of materials; not only that it is also useful for solving current as well as future problems. Thank you.
