Strength of Materials
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Lecture - 23
Bending 1 Euler-Bernoulli Hypothesis

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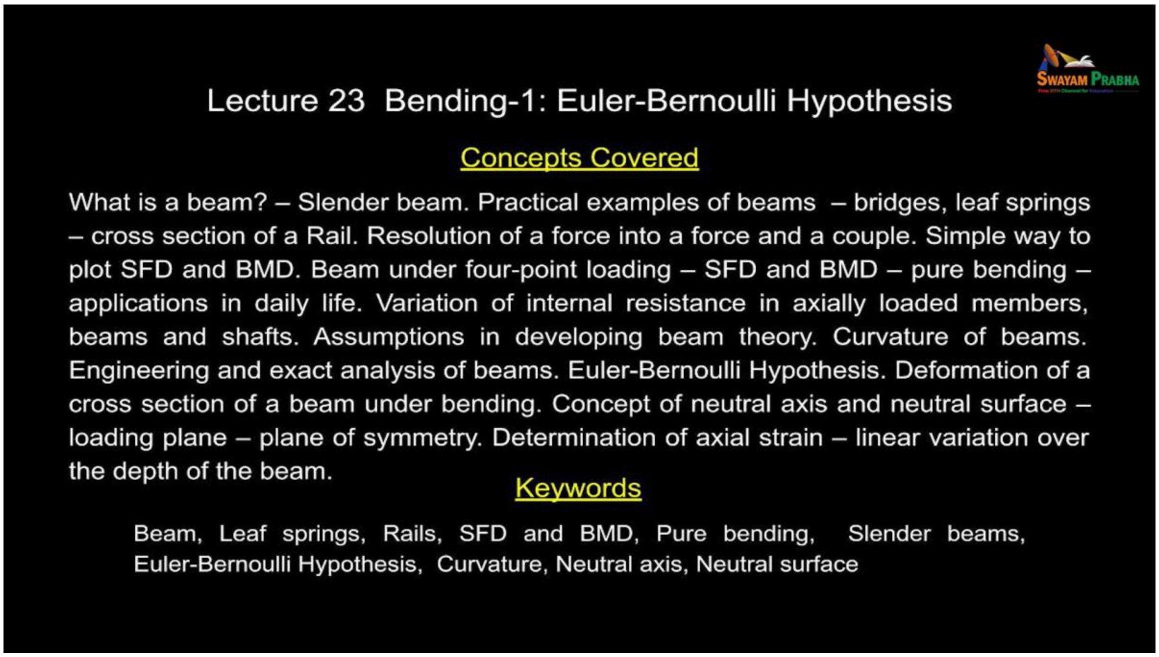

See, let us move on to the next chapter on Bending of Beams. In fact, we are going to threadbare study, how the bending stresses are developed, followed by how the shear stresses are developed. And in fact, in your earlier course on rigid body mechanics, you have been exposed to beams more than the shafts. You have developed shear stress and bending moment diagrams.

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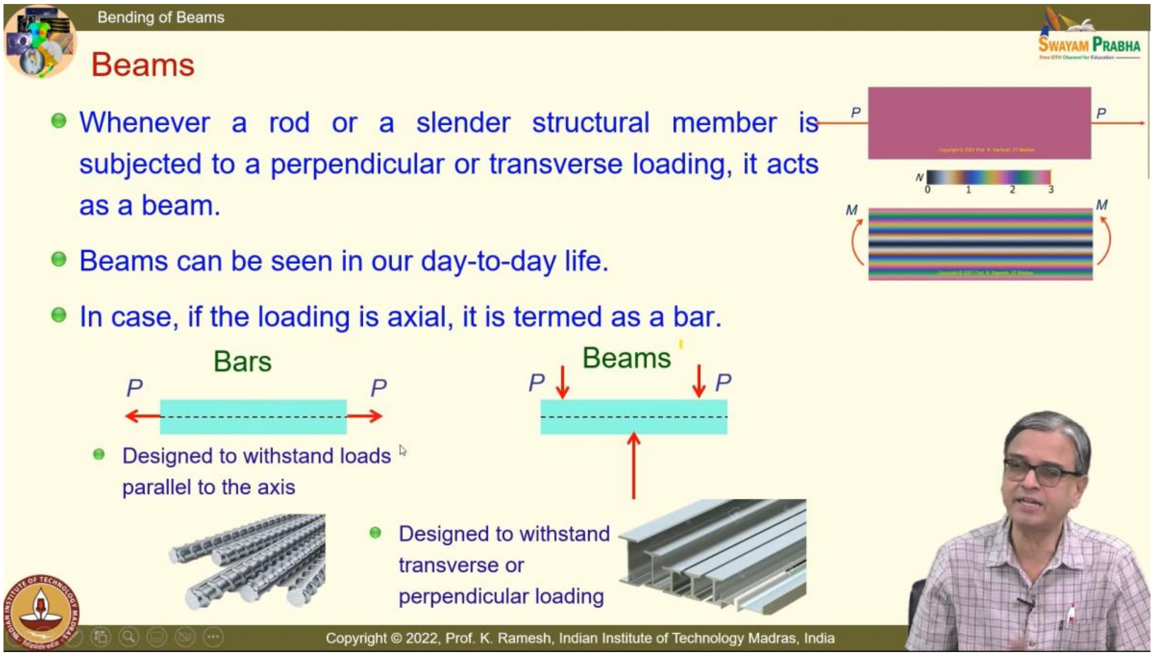

And let us see, what way we get into the understanding of a beam. You know very well that when I have a load transverse to the axis, it acts like a beam. The main point is, if I have this as the member, the loads are perpendicular to that. And here again, we will focus only on slender beams. Never forget that. That is a very important idealization that we invoke in this course.

The important aspect is transverse loading to the axis. And you can also have the rods. When you have the rods, you apply it along the axis. The moment you say beams, you come across the use of such cross-sections very often. You have the I beam, because it is very effective and utilizes less amount of material.

So, these are designed to withstand transverse or perpendicular loading. So, you have to recognize. I can have a beam which is vertical, fine. You do not think that beams are always horizontal. Suppose, I have an antenna protruding from your car, that is what is there in most of the cars these days, you know.

It is subjected to wind effects when you go. So, it is actually bending. So, you have to qualify whether the member is subjected to transverse loading, then you call it as a beam, not by its position. I can have a column which is horizontal, a beam that is vertical, fine. If the loading is axially disturbed as a bar, that is what I have shown it here.

And you know, when we were looking at axial load, we never worried about the cross-section. Did we ever worry about the cross-section? I simply said, I have a force which is acting like this and let us define force divided by area as stress. That came from plotting the graph for different cross-section of the material. We found that is a better way to plot so that I can have just one graph for a given material. We never worried about the cross-section.

On the other hand, when we stepped on to torsion, we were focusing on essentially circular cross-section. We have not gone anything beyond that. Either it is a circular cross-section or a hollow cross-section, that is all we have done.

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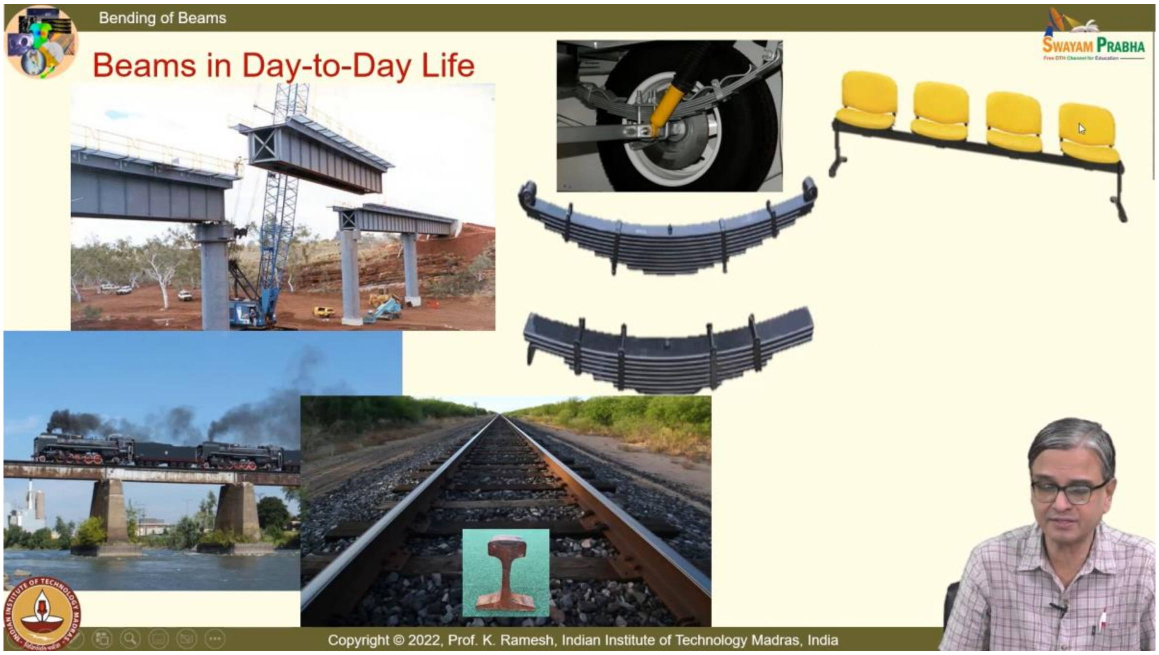

And you know, you have beams in every place, you have these bridges, you have a railroad and you can also have a spring. It is providing a cushioning effect. These are called leaf springs, mainly because these are termed as leaf. Multiple leaves are put. They are essentially subjected to bending and the cross-section need not remain constant. It will vary. And you also have the other important example.

The complete railroad essentially supports a bending load. And we have also said, even before we got into analysis of beams, we have looked at the cross-section. I have removed lot of material from the sides. That is mainly because by inference, we have seen that the central core is not transmitting the load. And even when you go and see, this is kind of seating arrangement is there in many places. So, if you look at the supporting structure, this acts like a beam.

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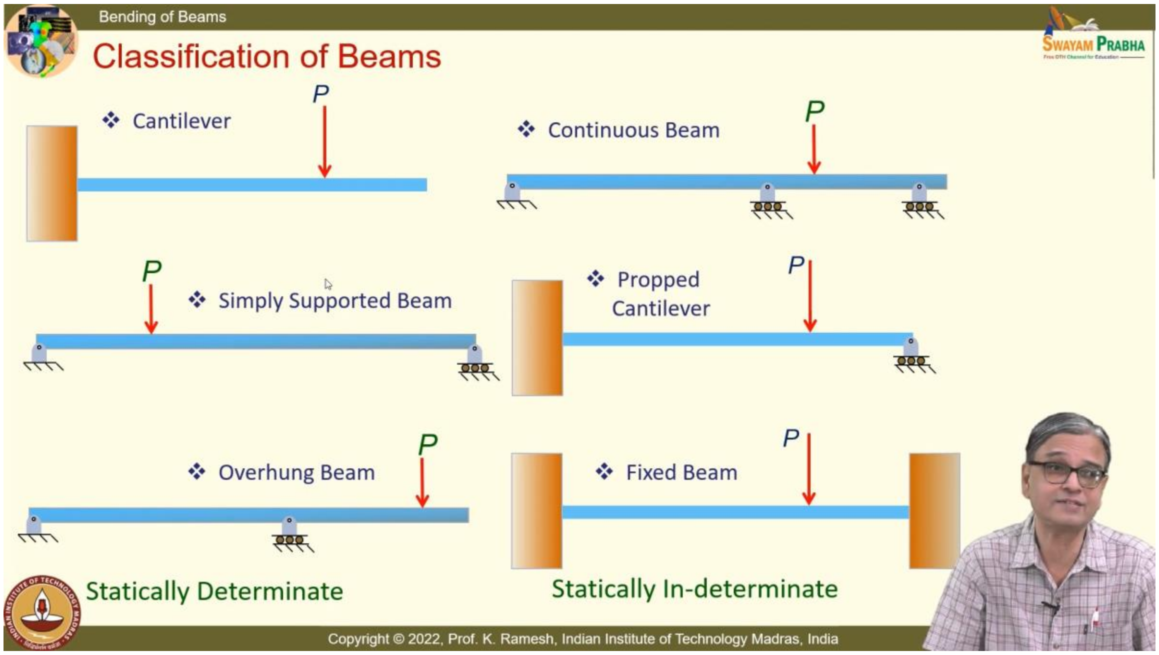

And you know, you have also done in your rigid body mechanics, cantilever, simply supported, overhang beam. All of these you are able to do the shear force and bending on diagram repeatedly, fine. And we have also looked at why do you have a roller support, not just to simplify your mathematics. It is also useful.

You cannot avoid temperature changes. When there is a temperature change, one way of mitigating that effect is, if it is possible for you to allow the expansion in your design, you have to allow that. Do not generate stresses due to thermal effects. So, that is one simplest way of handling thermal effects.

And there are also other class of beams. If you take a railroad, this is continuously supported on the entire length. So, it is little more complicated to analyze. And you also come across propped cantilever, and you also have a fixed beam. See, these are not possible for you to find out the forces by using the equations of statics. So, you call these class of beams as statically indeterminate.

So, you classified in that course, that does not mean that you will not attempt to find a solution. You have to bring in deformation, I get additional equations. With that, I go and evaluate what is shear force, bending moment in any one of these beams, fine. And just because you have been exposed to beams, see whatever the solution that I am going to develop today, you cannot simply jump on to conclusion that this is applicable to all these beams in an exact manner. Understand the word "exact manner".

We may still use the same equations. So, you should carefully observe what is the kind of idealizations we make when we want to find out the stresses in a beam. Watch that very carefully!

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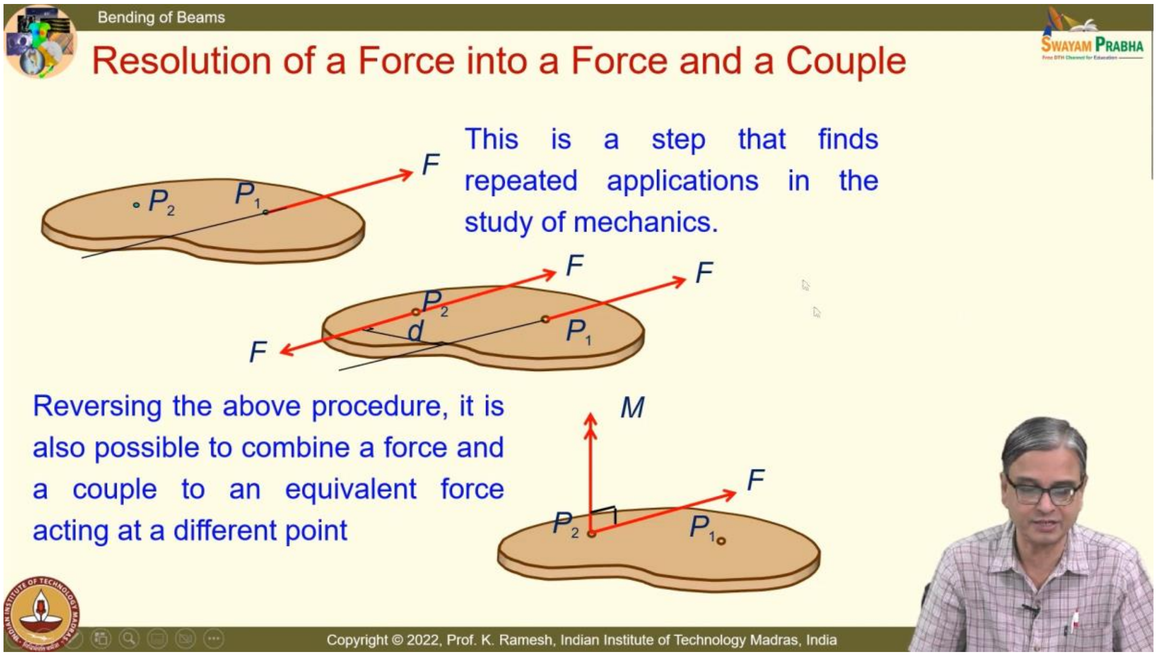

And you know, once you come to beam, you have to learn how to draw the shear force and bending moment diagram quickly. See, in your earlier course, what you have done is, you have taken a free body diagram. Each section you find out what is the shear force transmitted and the bending moment transmitted.

But in a course like this, you should be able to quickly write the shear force and bending moment diagram. And one of the important aspect is, I have a, I want you to make a sketch because this is one of the difficulties I found that you are facing. When I asked in the case of a spring problem, when it is subjected to tension, what is the force transmitted? I did not get the answer with the speed with which I would have expected it from you. That is mainly because you do not know how to move a force from one point to another point.

See, in statics, you learn principle of transmissibility where you can move the force along the axis, along the line of action at any point and the external effect is not modified.

Internal effects can be different, that is a different issue. However, if I have to move the force from point P_1 to P_2, you have to do that very systematically. You should understand how to do it and if you master this, you would have solved the spring problem in no time. You had difficulty, I thought it is very relevant even though it is part of your previous course to continue our discussions further. It is better that I clarify this and then proceed.

And for at the point P_2, what I do is, I add two forces. I want to transfer it to this point P_2. What I would visualize is, if I add a force on either direction, I have not done anything to the point P_2, fine. The net effect is zero. However, when I add a force system like this, I can visualize this force and this force forming a very interesting aspect. What is it forming? It is forming a couple, ok.

And you have the perpendicular distance is given as d and this is shown as yellow lines because we are not going to consider these forces anymore. So, if I have to find out what is the external effect of a force acting at point P_1 to point P_2; has two components. One tends to rotate, and you also have the other force. Is the idea clear?

See, this is a very, very important step. You have to master it. You know, if you master it, you will not have any difficulty, but you think that you have mastered it. This is repeatedly used in strength of materials as well as in your design of machine elements. Though it is taught in rigid body mechanics, it is very, very subtle. So, when I want to move a force from its point of action to any other point in the body, you must find out what is the kind of rotating effect this causes at any other point.

So, if you incorporate that and also move the force, only then it is complete. You cannot simply say this force, simply move it to this point. You cannot do that. It also introduces a rotatory effect. And let us see how we use this for drawing the shear force and bending moment diagram. See, when you give me a problem like this, I will not cut the section and then try to do it.

When you see it, you should also develop that ability because you have done one full course on rigid body mechanics. In the second level course, you should draw the shear force just below the beam with minimal calculations. Is the idea clear? And this also says that by reversing this procedure, if I have a couple and a force, I can find out an equivalent force. So, it is to be understood very clearly and learn how to use it in many of the problems.

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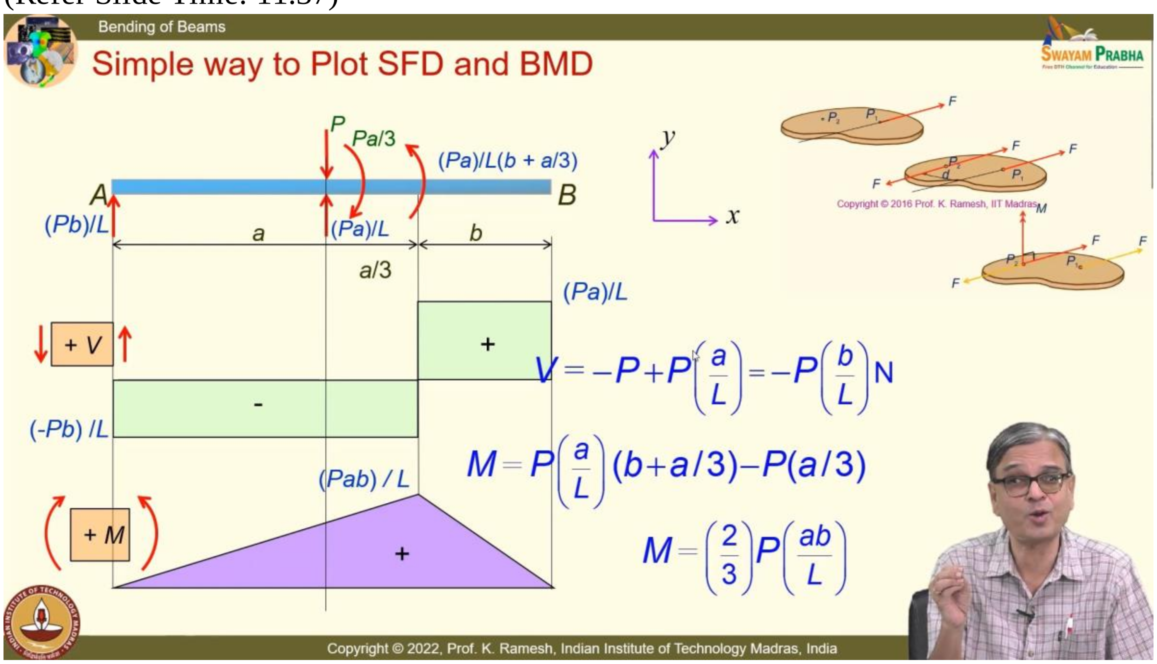

I have a problem like this. I have a three-point bending. See, once you come to bending, you have certain colloquial type of representation. If I have three loads, we call this three-point bending. If I have four loads, I call it as four-point bending. You should also understand what is the language that people use.

And I want to draw the shear force and bending moment diagram. Normally, what you would do? You would find out the reaction. The reactions are already given here. You would cut a section, put the free body diagram and find out what is the force acting. And let me just draw the shear force diagram and it explains you by taking a generic section, how I can find out the shear force and bending moment at that point with minimal calculation.

You agree with me that the shear force diagram looks like this? Fine. And we are using the sign convention on the positive face, positive shear is positive, ok. And you can also draw the bending moment diagram like this and it is labeled, fine. Now, what I am going to do is, I am going to take a section in this portion of the beam, at a distance $a/3$ from the load. And what you find here, I have a value of shear force, I also have a value of bending moment, fine. Which I can quickly calculate.

See, when I write the shear force and bending moment diagram, I am actually saying what is the force acting on the member, fine. That is where the sign convention of Crandall and Dahl also is very, very convenient. And you would also notice, I have drawn the shear force from right to left. There is a purpose behind it. It is not that I have just taken an arbitrary animation scheme.

Suppose I want to find out what is the shear force acting on this. This is nothing but moving this force to this point and moving this force to this point. Is the idea clear? So, when I move this force to that point, I will have a force as well as a couple. So, whatever the couple component will help you to find out what is the bending moment acting at that point, whatever the force component will give you the shear force. Is the idea clear?

So, what you learn here, moving the force from one point to another point is what you are actually doing it. And when you draw the shear force and bending moment diagram, I can find out at important locations, what is the shear force and bending moment diagram and join them, if you know what is the nature. So, I can do very simple calculations and do that. And that is illustrated in the animation here. So, I am actually moving this force. So, when I move this force, I will have a force as well as a couple. Please understand that.

Similarly, when I move this force to this point, I will have a force and I will also have a couple. So, you can do that mentally. Suppose I have this a, b and L are given as numbers, you can do the mental calculation, which symbols it may be difficult. With numbers, it is much easier to do. And if I find out what is the shear force, I have this as, $-P+P\left(\frac{a}{L}\right)$ that is what is we have moved from this point. And this gives you $-P\left(\frac{b}{L}\right)$ N, that is what you have it here. And when you want to get the bending moment, you sum them up and finally, you get this as $\left(\frac{2}{3}\right)P\left(\frac{ab}{L}\right)$. Because you know, you can understand this as a triangle and we are talking about *a*/3. So, this is two-third. So, I have this two-third coming into the picture.

So, with very simple appreciation of the logic, you can quickly draw the shear force and bending moment diagram. This you have to master it. Because in this chapter and in the subsequent chapter where you want to find out the deflections, you should be very comfortable with shear force and bending moment diagram. And you repeatedly use this resolution of a force into a force and a couple. You should understand that if you have, if you would have understood this, the spring problem would have given it in no time.

You had difficulty in that. I noticed the difficulty; I thought it is relevant that we go back to this discussion and then proceed with the course.

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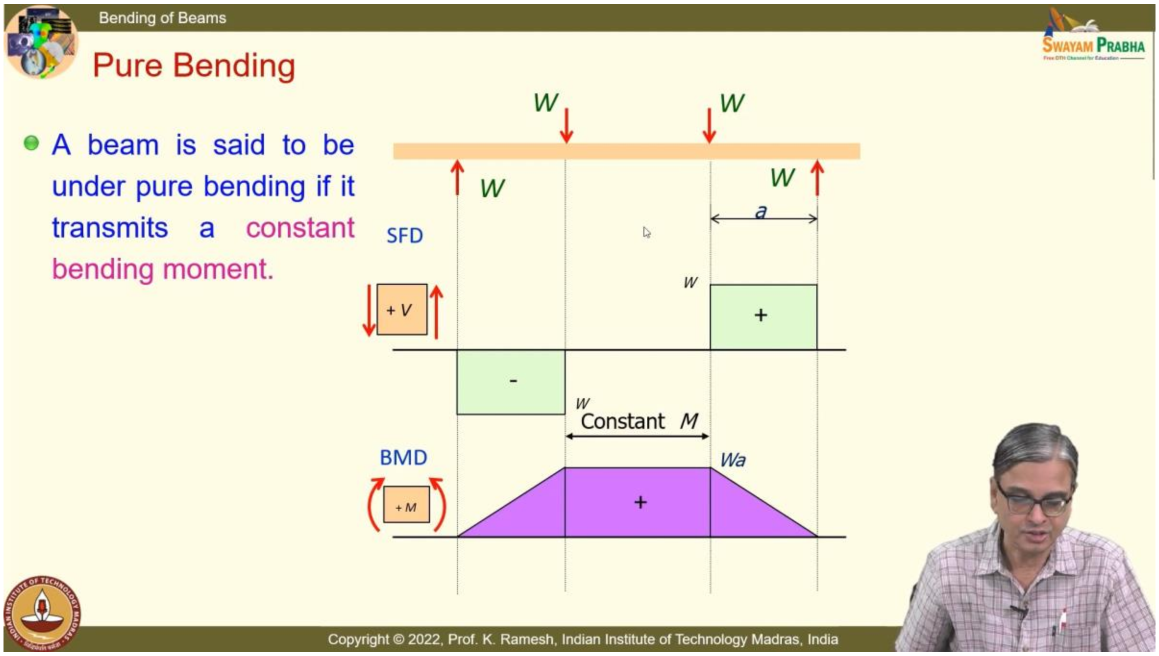

Now, I have a four-point bending, it is also called as pure bending. Why I call this as pure bending? Please draw the shear force and bending moment diagram quickly! I have given you this. Draw the shear force and bending moment diagram in the way that we have discussed. First thing you have to, you need to know is the reaction. Reaction is also very simple to get.

I have the support, the reaction will simply be W. Can you draw the shear force and bending moment diagram? You do not have to take a section, write the free body and do it. It is not necessary, because you have to graduate. In the initial learning, you have to do all that, so that you get a grip of how to draw the free body diagram. In the second level course, once the loading diagram is given, just below that you should be able to draw the shear force and below that you draw the bending moment.

I always recommend when you have the loading diagram, below that you should draw the shear force diagram, below that you should draw the bending moment diagram, so that even if you have made any calculation mistake, that diagram would prompt you to look at.

See, as engineers, you should come out with correct calculation. It is not speed alone, speed is also important, correctness is equally very important. Have you been able to get the bending moment and shear force diagram? Ok. I have drawn the bending moment first, because the interest is in this section of the beam, that is the central portion, the beam is transmitting only bending moment. Is the idea clear? Which you can confirm when you draw the shear force diagram.

When I have a shear force diagram, I have like this. Shear force is completely zero in this section and in this section, it transmits only the bending moment. See, we are going to develop our logic to find out the stresses only for a beam which is experiencing constant bending moment. If it is anything other than constant, you have to see how the results are applicable, fine. That is very, very important. So, I have constant bending moment.

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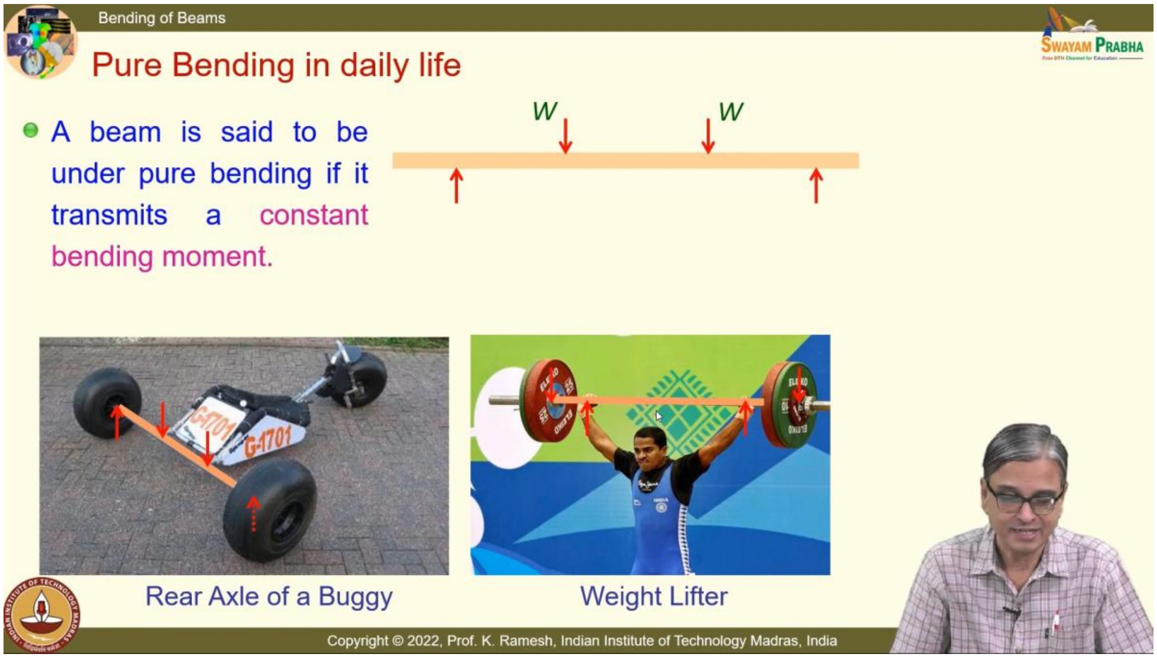

And you know, this you find in many, many applications. You have a buggy and then this has forces in two places. So, the central portion is subjected to pure bending.

Similarly, a weightlifter is a very nice example. This portion of the beam is subjected to pure bending. And we are going to develop the stresses strain only for a member subjected to pure bending, that is slender member, ok.

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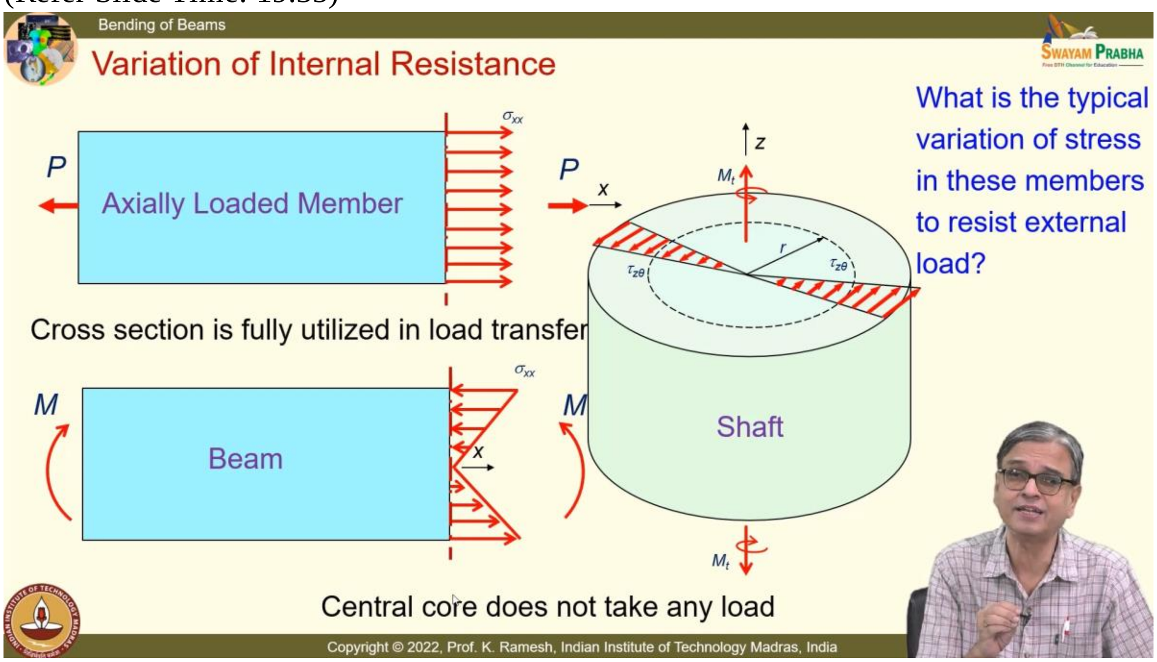

This follows the same logic. You know, when we have looked at the axially loaded member, we had only axial load, fine. We had only axial load along the length of the member, no other load was acting.
And we moved on to torsion. We took a circular cross section. We applied only twisting moment and I have discussed at length, when I twist the member like this, every cross section transmits only the twisting moment, constant value of twisting moment. And we found that the shear stress is varying linearly, fine. And when we come to bending also, I take only a situation where I have a constant bending moment. There is no other force, no shear force.

Only for that, we are going to develop the solution. And in all this, what we were actually doing? We were conceptualizing what could be the deformation. We have developed an hypothesis, investigated the hypothesis, satisfied ourselves with the experiments. Then we used the displacement or deformation to write our strain definition, then stress strain relation. Is that what we have done? The same thing we are going to also proceed.

And you know, I have put the force which is tensile positive here. I have also taken an anti-clockwise twisting moment as positive. In similar lines, you know, whenever I show the beam, I was always bending it like this because it is so easy and natural for me to bend rather than bending it like this, fine. Because we want to have the equations with proper sign convention, I apply a positive bending moment. Positive bending moment means it is anti-clockwise positive.

So, that means, I should have shown you all along only like this, fine. But it is much convenient for me to show that it bends. So, that is the reason why we have changed this. And this is the result that we are going to get, fine. What do you get from statics? What do you get from the strength of materials? When you have a beam, just now we have seen four-point bending. Your bending moment diagram gives you what is the bending moment at that point in the beam.

So, your statics can give you what is the resultant of the force system at that particular cross section. It is a very subtle point. I am discussing a subtle point. Please understand that! How the bending moment is affected through a distribution, your rigid body mechanics cannot give you. Only when you come to deformable solids, you will know how the distribution is.

Suppose I have this as a rectangular cross section. Can you tell me what is the net force acting on this cross-section? Net force? That is zero. You are able to see that. But if you find out what is the moment, you will find and we will also equate it in the later development that this moment will be same as the moment applied at that point.

See the idea what you will have to appreciate is, we are graduating from rigid body mechanics to deformable solids. Rigid body mechanics gives me the resultant of what is the force acting at a particular cross section; It does not give you the distribution. In the second level course, I get the distribution. Is the idea clear? There is no contradiction between the two. We will verify whether our solutions are satisfying the equilibrium conditions.

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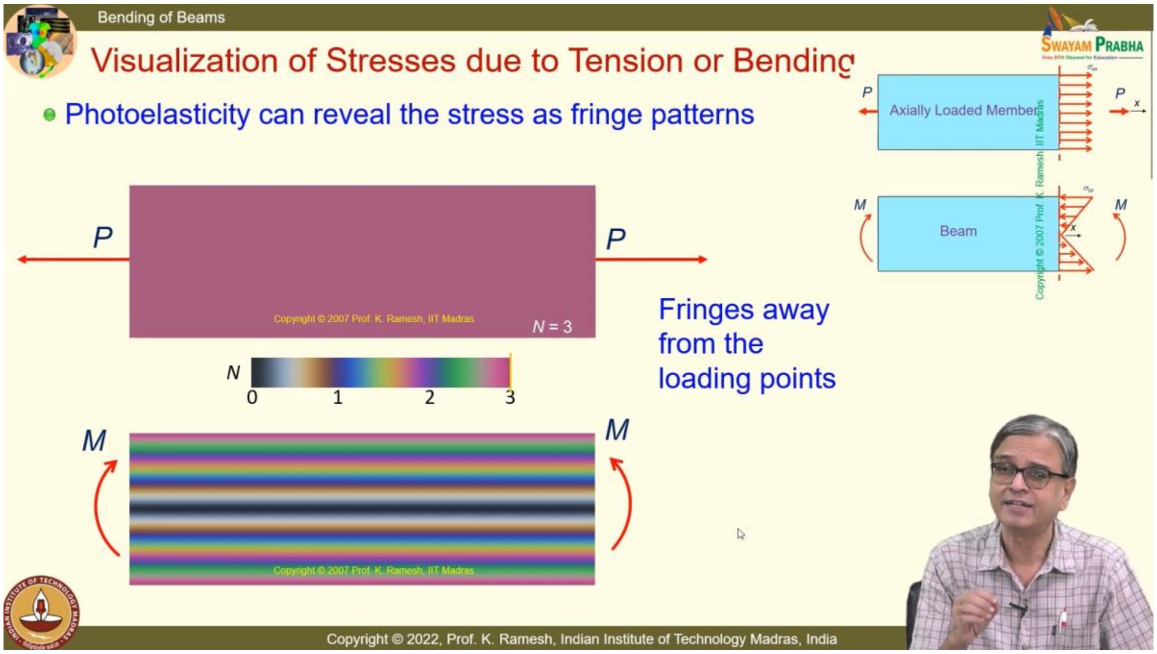

And we have shown this by an inference. I have taken a photoelastic material, apply the axially loaded this one, ok. And you found that this is of uniform color for any increment of load. You have multiple fringes get developed. And when you finally look at the distance between the fringes was remaining constant, suggesting that the variation is linear. We have also seen the counter example. When I showed you the thick cylinder, the distance between the fringes were not uniform; It was varying and it shows a non-linear variation, ok.

So, you can get rich information from photoelasticity in a jiffy. What I need to do is, take the model, stretch it and put it in a polariscope. So, even before we solve the problem of beam under bending, we know what is the nature of stress that it is transmitting.

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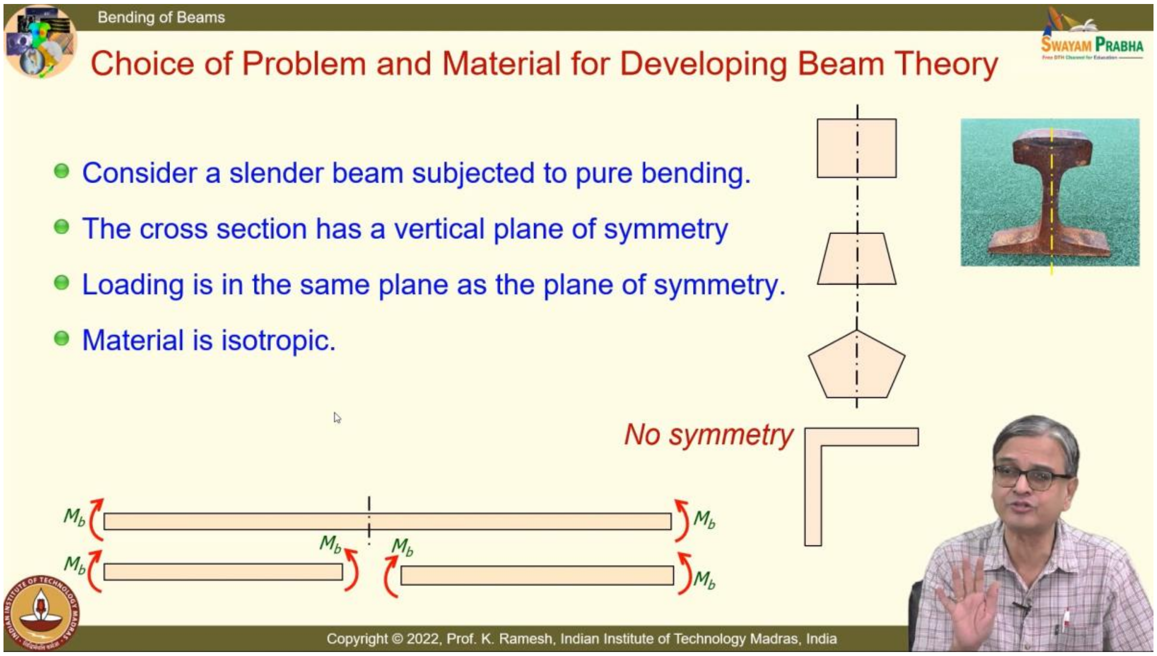

And you have to appreciate what we are doing. See, I am emphasizing it again and again. Because I have done it while developing torsion also, we are considering a slender beam. For the purpose of illustration, I may blow it up. So, always keep in mind the cross-sectional dimensions are much smaller than the length. So, slender beam we are taking. And we also restrict what is the force it should transmit. We want the beam to transmit only a constant bending moment. Please write it in bold letters.

If anything, other than a constant bending moment, the solution you get is not exact for that problem. So, if I take a cross-section and then find out what is the force balance and find out what is acting, it will have a same bending moment transmitted. That is what is illustrated. Like what we have illustrated in the case of a shaft. And another very, very important aspect which many students ignore is that it is applicable for cross-section that has a vertical plane of symmetry or a plane of symmetry.

I can have the loading applied in that plane. Usually, we have transverse load is shown vertical. So, it should have a vertical plane of symmetry. That means, this cross-section whatever the solution I get, whether it is valid for this cross-section or only for this plane, that also you have to keep in mind. While developing the theory, we will confine our attention only to the central plane.

In the case of a shaft, we looked at a square shaft as well as a circular shaft. We said our strength of material solution is valid. We have developed only for a circular cross-section. Here you have little more relaxation. Whatever we discuss as a beam theory, it is applicable for a variety of cross-sections, but they should have a plane of symmetry.

You also find L section is used as a beam in many applications. Does it have a plane of symmetry? It is not having a plane of symmetry. And you will find it is very, very interesting. When I apply the bending load, it will also twist. We will see that later. And if I have to find out how to find out the stresses due to bending, I have to do little more work.

I cannot get it easily. So, I should have a plane of symmetry. So, in a sense, it is slightly more generic than what you have learnt in the case of torsion. Loading is a same plane as a plane of symmetry. So, that is what I said. I can also have the beam rotated horizontal and apply the load in the horizontal plane.

The loading plane and symmetry plane should be identical. That is the meaning here. And that is a very subtle point. Please understand that! It is not emphasized normally, but that clarity should have. And finally, our very important idealization material is isotropic. Only for an isotropic material, we are going to develop. So, we are going to have a hypothesis and see what is the deformation. Once you have taken care that the deformation is correctly understood, rest of it is straight forward.

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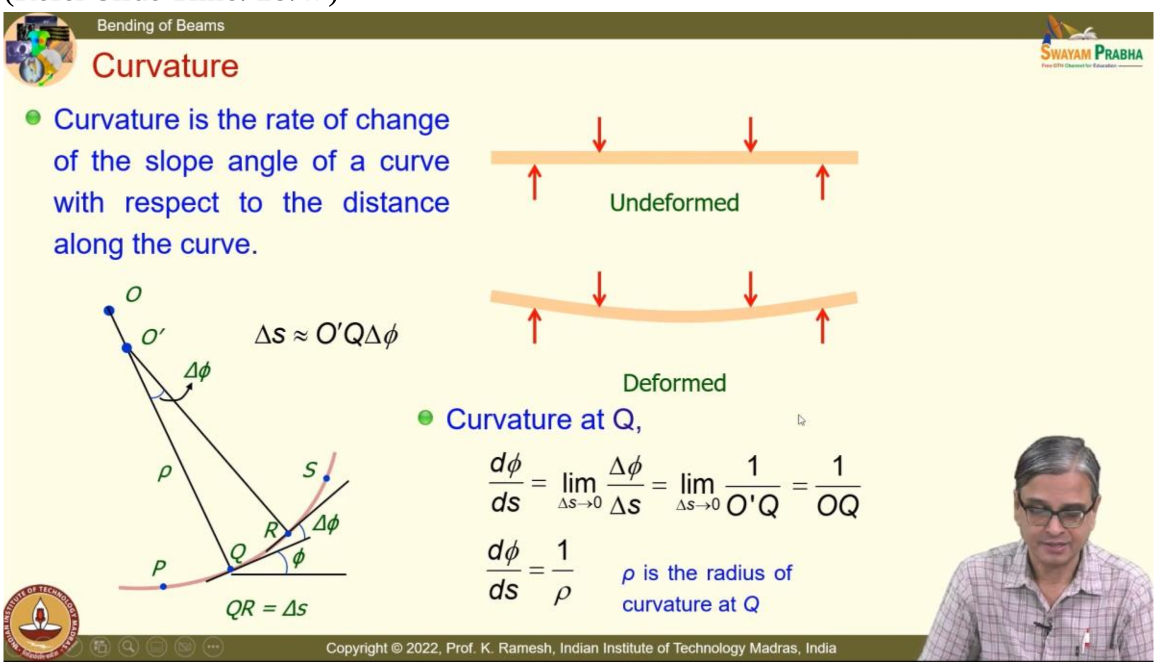

And once you come to the beam, you know, when I am going to bend it like this, you will also have to bring in your knowledge of curvature, which you have studied in your analytical geometry. But nevertheless, we will get into that understanding. So, I take a curve. I have marked two points *P* and *S*. And curvature is the rate of change of the slope angle of a curve with respect to the distance along the curve, ok. So, if you put it in pictorial representation, so this you have this as a radius of curvature ρ, ok.

And I have the point Q. So, I write the slope. This is at angle ϕ. I take a neighboring point. Please draw the sketch along with me. Draw the neighboring point, which is at a very short distance from this. And I have labeled this as R.

So, when I draw the slope, I have this angle as taken as $\Delta\phi$. So, this will also be $\Delta\phi$. And we want to link this curvature to the bending of the beam, fine. And you also have QR, you take the distance as $\Delta\phi$. This is a small distance.

These are all blown up pictures. So, you should understand that these are all blown up pictures. And what is the definition of curvature at Q? We want to find out what is curvature at Q. It is defined mathematically like this.

$$\lim_{\Delta s \to 0} \frac{\Delta\phi}{\Delta s},$$

I can call it as $\frac{d\phi}{ds}$. And you have $\Delta s \approx O'Q\Delta\phi$,

$$\frac{d\phi}{ds} = \lim_{\Delta s \to 0} \frac{\Delta\phi}{\Delta s} = \lim_{\Delta s \to 0} \frac{1}{O'Q} = \frac{1}{OQ}$$

$$\frac{d\phi}{ds} = \frac{1}{\rho}$$

So, that is the definition of curvature. And ρ is defined as the radius of curvature at Q. And the curvature can change from point to point. Suppose I have a uniform bending moment, we have an advantage.

See, we would like to have a very simple deformation picture. In order to achieve a very simple deformation picture, we restrict what is the cross section, what is the material and what is the loading.

You should appreciate why we do all that, fine. So, I take a beam like this, which is subjected to four-point bending. I am repeating it again and again. See, when we started the lecture, I showed different beams. We may use the result obtained from analysis of a pure bending to those that would be termed only as an engineering analysis. It is not an exact analysis! Exact analysis we develop only for a beam subjected to pure bending. So, it goes into a curved position when it is the bending moments are applied, ok. And you can also visualize it in a different perspective.

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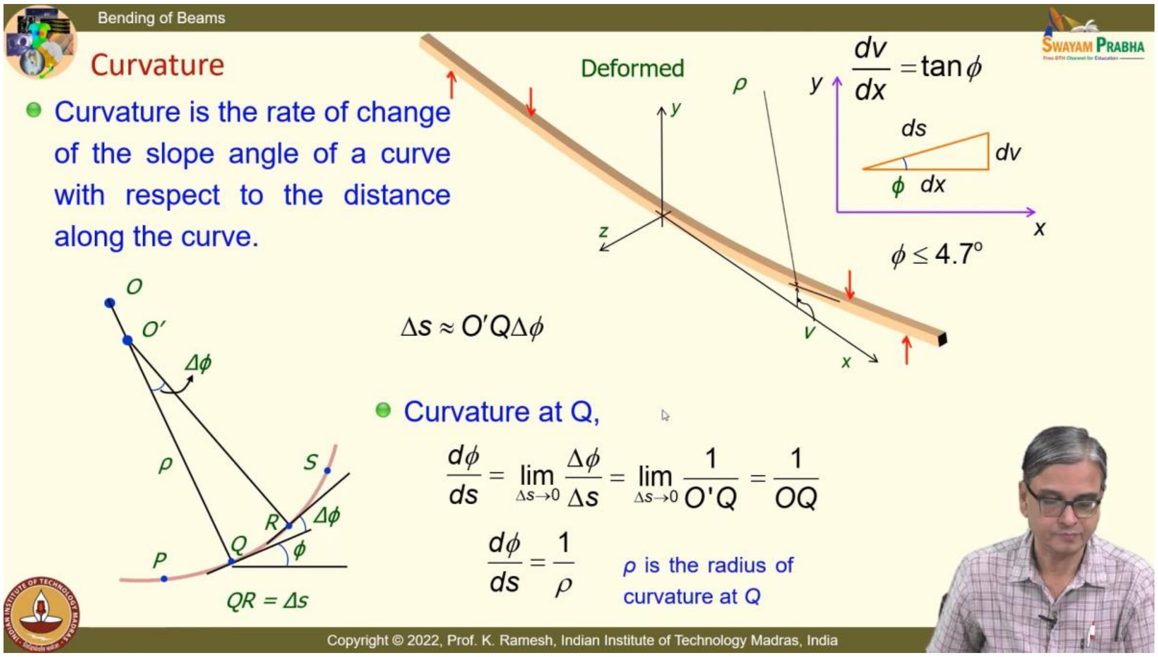

So, wherever possible where I can bring in the slenderness, I am bringing it. Please make a sketch. This would also help you to appreciate the deflection, what we are going to develop a few classes later. I have the reference axis. Along the beam axis, I am going to take the direction as x and you have z and y direction shown like this. And I would have my radius of curvature like this. I would define this displacement as v; that is the deflection.

And you can also write from your analytical geometry, I have the distance as ds and I have dx, ds and dy. So, I will have this as $d\phi$. And you know we always live on small deformation. Small deformation you should never forget. For illustration, whatever I show in the class is very large deformation.

Unless you see large deformation, you cannot conjecture what is the nature of deformation. Our idea is to capture all aspects of the nature of deformation. So, the ϕ should be within 4.7 degrees, fine.

And whatever you have this, I do not have to repeat it. You have seen it in the earlier one. So, the same is repeated for appreciation what is the definition. So, I have this as ρ and we have looked at this ρ in a four-point bending also bringing in the deflection. We will again use this picture later for developing the deflection. And you have an hypothesis even before I go to an hypothesis, I have a very nice animation. What do you see? I have parallel lines. You see parallel lines? You see parallel lines here. How do they deform? Do they remain plane? Why do you say it does not remain plane? It remains plane! The straight line still remains straight. It is getting rotated, fine.

See, you have an advantage of a soft material and you are able to flex it like this and you get it in no time.

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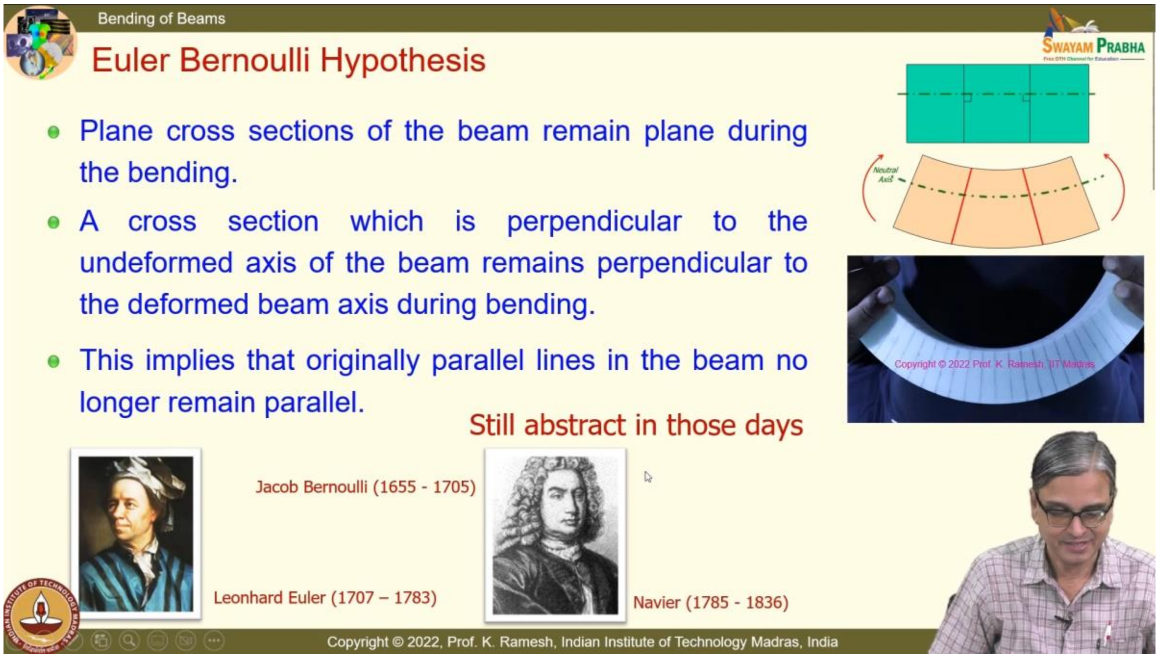

Euler and Bernoulli, you have this picture of Euler here and Bernoulli. This hypothesis is credited to Euler and Bernoulli. First statement they make is plane cross sections of the beam remain plane during the bending. A cross section which is perpendicular to the undeformed axis of the beam remains perpendicular to the deformed beam axis during bending. See, this hypothesis took how many years? 400 years! Fine.

So, when this hypothesis was stated, the hypothesis was correct. People did not understand what this hypothesis is all about. It was quite abstract at that time. When I illustrate this, I have blown this up. I have a line here and this is perpendicular to that; I have parallel lines. These parallel lines have got rotated; They remain plane, ok.

The plane sections remain plane before and after loading is very well seen. Mind you, we are discussing for a beam under pure bending. Never forget that! Suppose I draw a tangent to this curve and also find out what is the angle between this line and the tangent, it would be 90 degrees. It would be 90 degrees. That is the meaning of this statement. Remains perpendicular to the deformed beam axis during bending. This implies that originally parallel lines in the beam no longer remain parallel and when you extend them, they all meet at a point. You get the idea? That is what you see here. We will use this for us to capture the deformation.

See, in the case of torsion while we developed, we took a wrong hypothesis. We said radial lines deform into curve. Then we investigated, did the thought experiment and said it cannot be like this. Radial lines will remain straight. In the case of beam, we start with the right hypothesis and people at that time did not understand what it is because they did not have access to soft material where they could see the deformation very clearly. Now, I have a soft material.

So, I can easily bend it and then show and draw the lines and then see because this is magnified many times. If I have a deformation like this, none of my mathematics is correct. Please understand that! I understand this deformation and then factor it in my theoretical development but apply it for very small deformation. You should understand the difference. So, it is, this hypothesis was very abstract in those days. People did not understand. I have a beam like this. The only restriction is it can have any cross-section.

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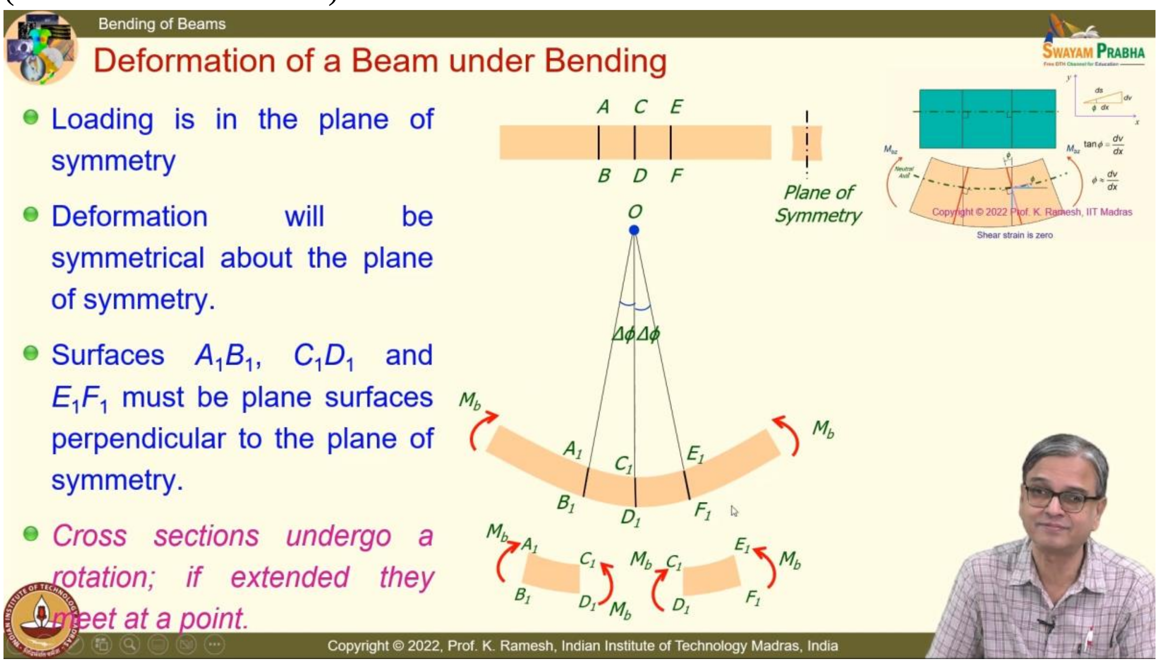

I have taken a cross-section which is having a plane of symmetry because this solution is applicable for cross-section that have a plane of symmetry and my loading is also in the same plane of symmetry.

The deformation will be symmetrical about the plane of symmetry. So, I take parallel lines *AB*, *CD*, *EF*. I have taken three parallel lines. We have already seen how the beam is deformed, fine. And these parallel lines get rotated and these rotated lines are shown. Please make a sketch and then draw this. Allow sufficient space because these lines have to meet at a point. That is very important.

So, do not draw immediately below this figure. Leave a gap like what I have given, fine. So, I will have the line A_1B_1 rotated like this, C_1D_1 like this and E_1F_1 like this.

So, you can see here when it is rotated, what I find is if I draw a tangent to this, this angle is 90 degrees. So, plane sections remain plane without distortion. It is very well validated. So, when I extend these lines because in this complete cross-section, I have constant bending moment. The lines if you extend, they will all meet at a point. This again emphasizes that I am having only a constant bending moment transmitted and if extended, they all meet at a point. I have this angle as $\Delta\phi$, this angle as $\Delta\phi$. Is the idea clear? And we are developing this for which portion of the beam? Have you ever recognized that the beam has any thickness? It is a subtle point! We are always talking about plane of symmetry.

So, we are looking what happens at the plane of symmetry. We have not bothered about the thickness. The moment I bring in the thickness, there is something else which is going to affect. That we will postpone. See, we get the central core of the idea, then bring in modifications. Then your conceptualization becomes very simple, your equation becomes very simple, and you can also appreciate what happens and what could happen if you consider the thickness.

What aspect of the material is going to affect when I take the thickness? Can you visualize? Can you guess? I have elastic constants. Which elastic constant is going to get bothered by the thickness? Tell me, you are right! Tell me louder. Poisson's ratio! Poisson's ratio, fine. We will postpone that later. When I bring in the thickness, I will have to look at what way the Poisson's ratio plays havoc. Right now, we take a beam which is slender. It has a plane of symmetry, and the loading is on the plane of symmetry and we analyze only that plane. We are not analyzing anything else. That should be very clear.

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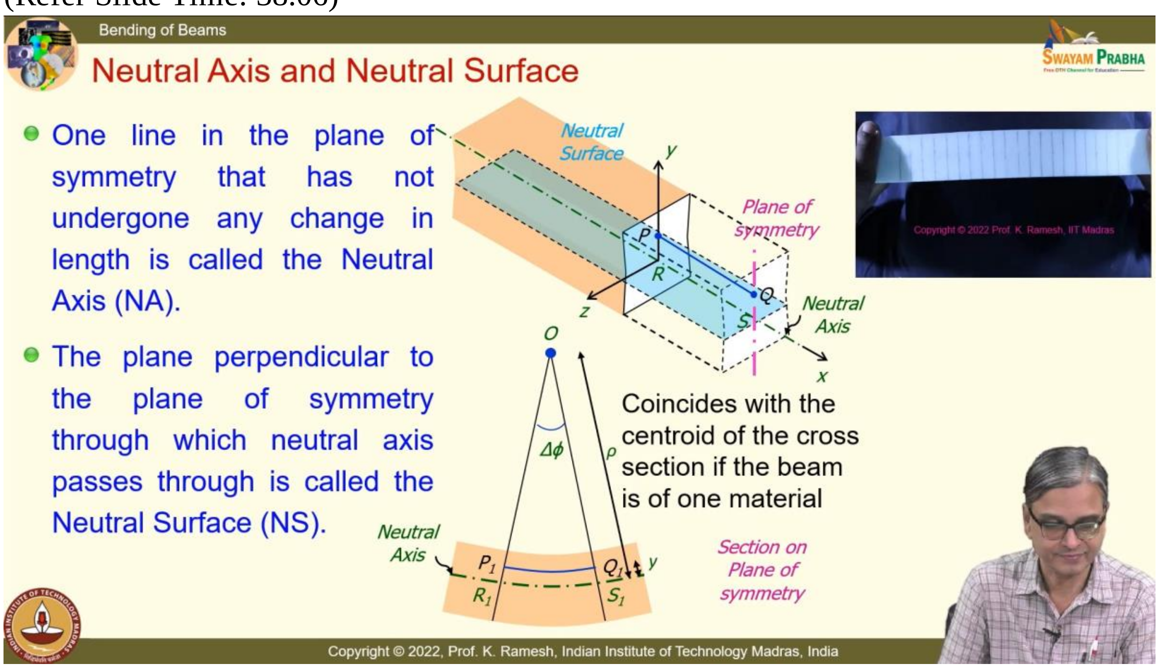

And that is what is shown here again. I have the lines going like this and they meet at a point and you extend they meet at a point. And you know, you also bring in certain other observations because these observations help us to write mathematics clearly. When the axis is rotated, when it is meeting at a point, I can also have a value at a particular location. Even after bending, the length remains constant. That is possible because it has only rotated, is not it? So, one line in the plane of symmetry that has not undergone any change in length is called the neutral axis, fine. This is called the neutral axis. It will help us to write the definition of strain. Because strain means we want to find out change in original length divided by the original length. That is what we have defined as strain. So, I should know how to calculate the change in lengths, how to get the original length.

So, in order to get that, I define a neutral axis. So, I have this as R_1 S_1 and I call this as neutral axis. And you may also have a question, where will the neutral axis be located? Suppose I have a beam made of the same material, the neutral axis will coincide with the centroidal axis. If I have a beam made of multiple materials, it is not going to coincide with the center of centroidal axis.

So, I have the plane of symmetry and then I have drawn the axis. I am showing it slowly so that you can make the sketch. Please make the sketch. Because from neutral axis, we are bringing in the thickness just to satisfy ourselves. Once in a while, we see the thickness, what happens and come back. But all our analysis is restricted only to the plane of symmetry. So, now I will have one axis which will remain of same length even when I bend the beam, coincides with the centroid or cross-section with the beam is of one material, which I have already said, ok.

The plane perpendicular to the plane of symmetry through which the neutral axis passes, that is shown here is called the neutral surface. So, wherever we want to recognize the thickness, we will recognize the thickness. Otherwise, the discussion confines only to the plane of symmetry, never lose track of it. Now you know, we will use this definition because our interest is I want to take a general line PQ.

We want to see how it gets distorted. So, I should find out what is the change in length, what is the original length. The only parameter I get from the diagram is the radius of curvature. So, I will try to link it to that, that is my interest. So, I have a deformed line P_1 Q_1 and this is an undeformed beam, fine.

In undeformed beam, what is RS and what is PQ? They are of identical lengths. In a deformed beam, they are different. I have R_1S_1. You could quickly see R_1S_1 should be same as what? RS. Because that is what we defined as neutral axis. And we have taken the line P_1Q_1, which is at a distance y from the neutral axis. And then this neutral axis is located at radius of curvature ρ.So, with this, you are in a position to find out what is P_1Q_1, what is original length and what is the expression for axial strain.

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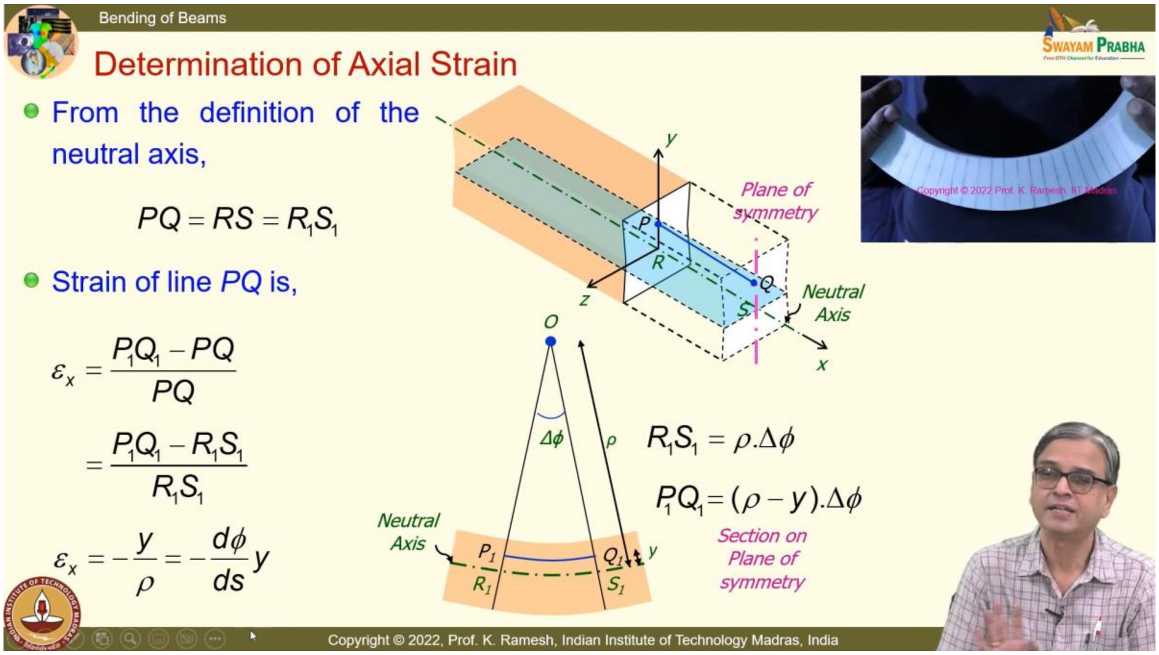

So, determination of axial strain is our interest. We have discussed the basic requirements. I am redrawing the same diagram, so that you have time to fill in the blanks if you have missed some of it.

And we have already discussed what is the original length, ok. So, from the definition of neutral axis, originally the line *PQ* was having the same distance as *RS*. In the deformed configuration, this is nothing but R_1S_1. Is the idea clear? Is there any difficulty? No difficulty at all. Then what we have to write the strain of line *PQ*? We have to write change in length.

The deformed configuration has a length P_1Q_1, original configuration had *PQ*. So, this gives a change in length, and this gives the original length. Now, we would like to express P_1Q_1, *PQ* etc. in terms of ρ. and *y*. Let us see how we are in a position to do that. ρ, *y* and $\Delta\phi$, these are all the quantities that we have. I can also write this only restricting ourselves to deformed configuration. Instead of *PQ*, fine. I can replace it by R_1S_1, so that I use only this diagram for me to fill in all the blanks. Is the idea clear? No difficulty at all. Now, we have to write what is R_1S_1 and what is P_1Q_1. Can you write what is R_1S_1 ? $\rho\Delta\phi$. $\rho\Delta\phi$, very good. R_1S_1 is $\rho\Delta\phi$. What is P_1Q_1? $(\rho - y)\cdot\Delta\phi$. That is all! You have got it!

And when you substitute this P_1Q_1 is $(\rho - y)\cdot\Delta\phi$. So, I get this strain quantity. We have taken the axis as *x*. So, this is ε_x and you should also put the sign properly. So, I get this as $-y/\rho$. And if you write it in terms of the $\dfrac{d\phi}{ds}$, it is $y\dfrac{d\phi}{ds}$; $-y\dfrac{d\phi}{ds}$.

So, you recognize why the variation over the cross section is linear. I have the factor y coming in, fine. And in the case of torsion, we had that as r; r is changing from 0 to r maximum. So, this brings in that there is a linear variation, fine. So, in this class, we have looked at what happens in the case of a beam. Though we have looked at many different beams in your rigid body mechanics, while we want to develop the expression for stresses, we take a beam subjected to only pure bending.

And we have said it should have a plane of symmetry and the loading should also be remain in the plane of symmetry. And our mathematical development really confines only to the surface, the central surface through which it is symmetric. With that, we are in a position to write the axial strain because we have taken the axis as x axis. We have got the expression for ε_{xx}. It is a linear function of the distance y. Thank you.