Strength of Materials
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Lecture - 22
Torsion 3 - Problem solving, Hollow shaft

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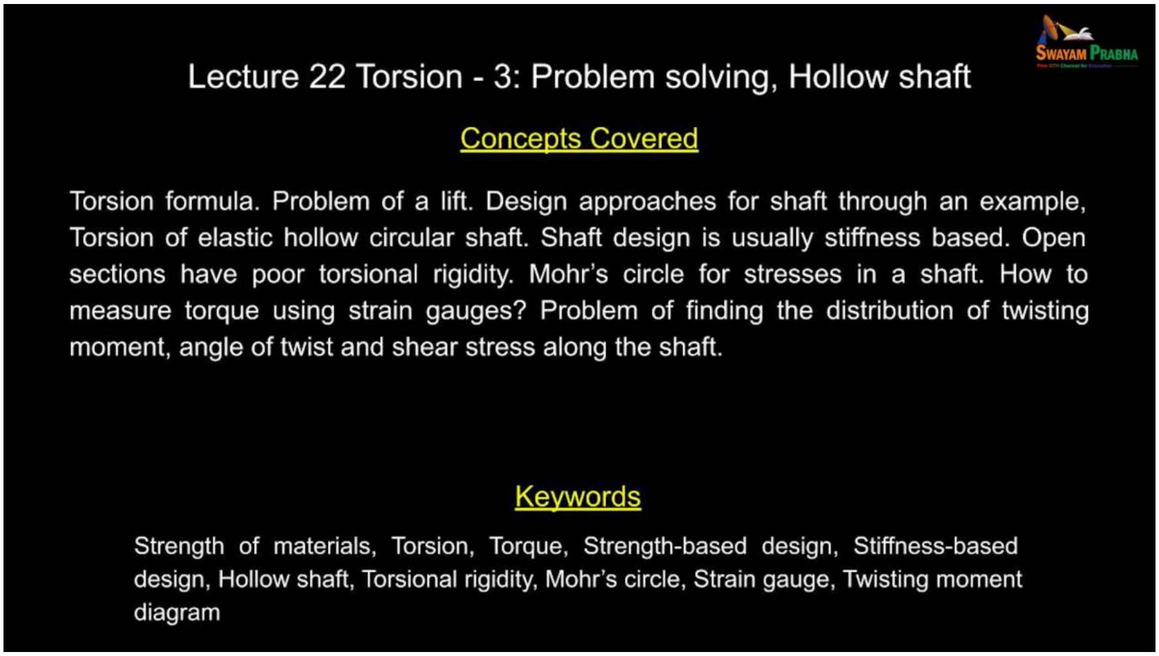

Let us continue our discussion on torsion. And we shall solve some interesting problems today to get the idea of how to use the equations that are developed, what are their limitations, fine?

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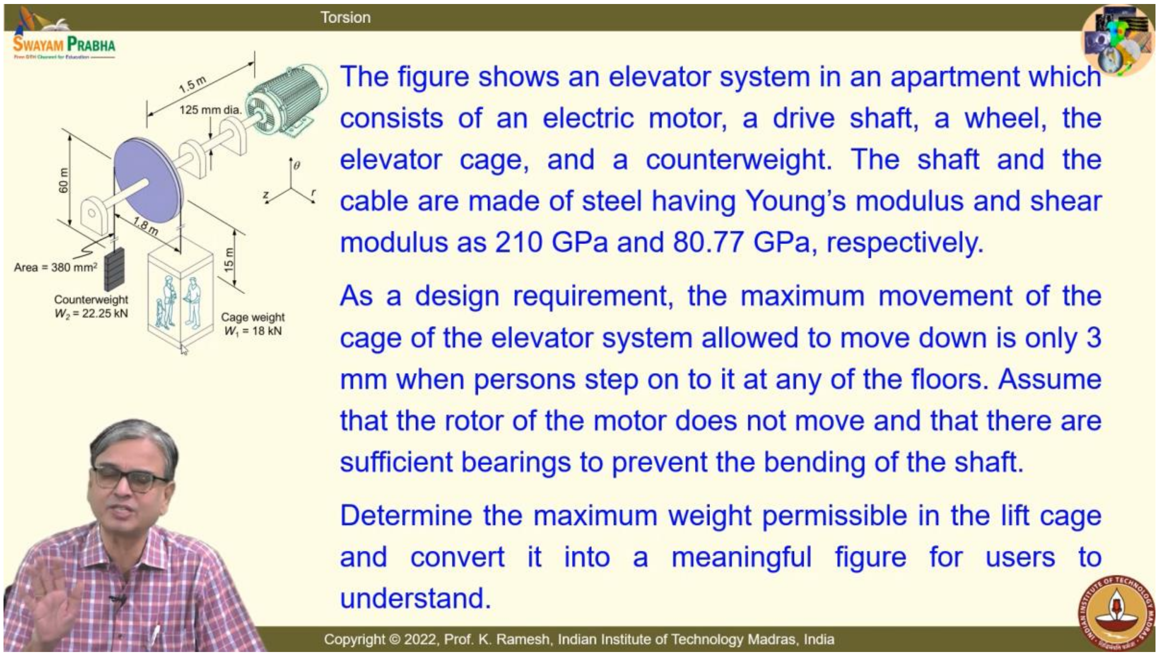

Now, this is a very interesting practical problem. You have a lift system, you have a lift, there is also a counterweight. You know, periodically they will come and maintain the lift. At that time, it is possible for you to see what is the counterweight in the lift system. And you know, this is in an apartment, those lifts are much smaller than what you have in either in our institute or when you go to hospitals, they have a huge lift.

See the question here is, you know if you get on to any one of these lifts, there will be a board that says how many persons can enter into the lift. And you have a system, electrical system, if the persons are more, it gives a buzzer and one person has to get out, only then the lift moves. So, at the design office the person has to decide how the system stiffness is so that it meets all the requirements. Here the requirement is, when the lift is standing in one floor, that means everything is static.

You can permit only a downward movement of 3 mm. So, at the design office you have to find out what is the stiffness, is there anything that you have to do with shaft design; all this you have to take care. And based on this, the designer also has to give, how many people can enter into the lift. So, you have to have a pragmatic approach and then bring in some kind of factor of safety, ok? And you have to do that.

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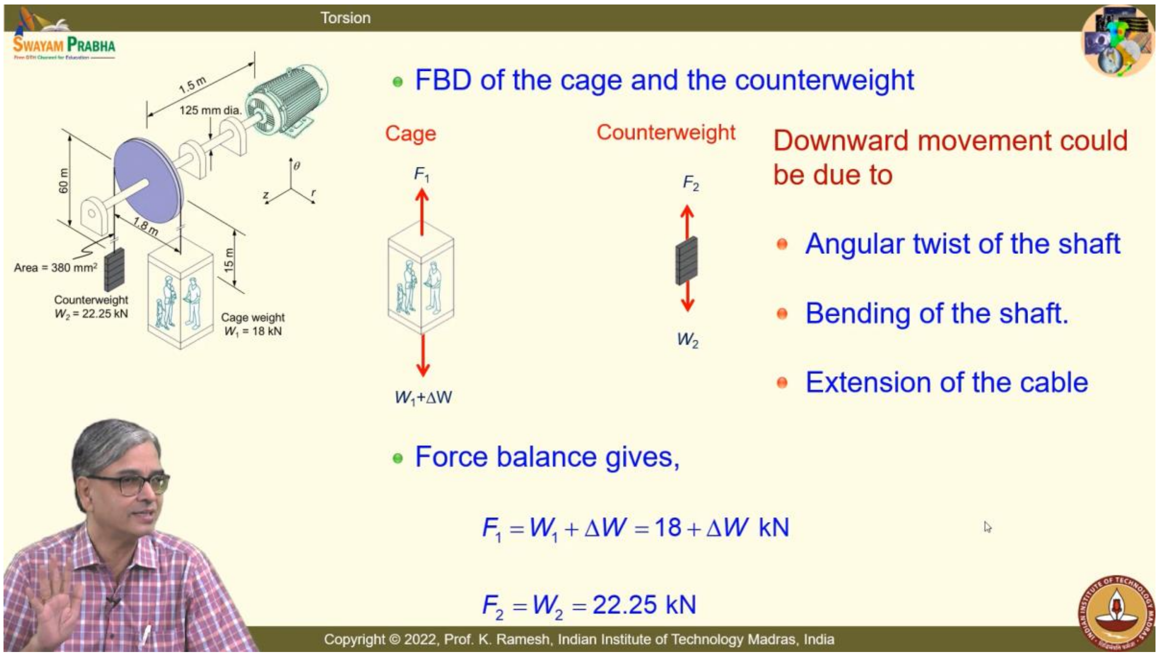

And one of the simple things that you can do first is, look at the free body diagram of the cage. And you can also have the free body diagram of the counterweight. So, what you have is, you have a dead weight of the cage that is put as W_1. And let us imagine that the lift was empty to start with. And some people have entered into this lift that is ΔW or in other words, how many people it can accommodate is the question. So, you have to put a label inside the lift; so many people can enter into the lift.

And this free body is simple and you get the force balance as $F_1 = W_1 + \Delta W$. You are given the cage weight is 18 kN and you have the counterweight as 22.25 kN. Now, the question is, what all aspects of the system will contribute to a downward movement of the cage? You must understand that the motor stopped, there is no rotation from the motor. So, it is standing at a floor and people step into it.

And you know very well that we are discussing torsion, so something with torsion is going to happen. So, that you will recognize. Because many times, you know, when you solve a problem, if it is given in a particular chapter, you take advantage of that. In reality, if you are given a physical problem, you should know how to dissect and find out what contributes to it. Is the idea clear? So, what is the contribution from torque? Can you imagine? I have a shaft; the shaft is supported at two bearings on this side and one bearing on the other side and you have a big pulley which is mounted on the shaft. You have a counter weight and you have the lift gauge. So, you can imagine you are also given the length; this is about 1.5 m. So, you are going to have this shaft. It is not a rigid body. It is deformable so it develops a twist. Because of the cage weight, there is a twist already and because of some people entering into the lift, there is a change in weight, change in torque that contributes to angle of twist and which will get magnified by this big wheel and you will see this as a vertical downward movement of the lift. So, that is very clear. Angular twist of the shaft is very clear that you can employ the equations and then find out what it is. What are the other aspects that can contribute to downward movement? Can you guess what are the other movement that can contribute? See, I am holding it on one side.

What you find is, it is sagging. So, I put a support. I put one support; the sagging is very much limited. And what is given in the problem? The problem says that I have three bearings; 1, 2, and 3. They are sufficiently close enough so that any deflection due to bending, you can ignore it.

That is given in the problem. There are two ways of looking at it. You do not have a background on bending. So, you cannot calculate what is the deflection due to bending. When I put a support, the shaft remains horizontal; that is fine.

But if I have additional load which is very heavy, this is going to cause a deflection. You do not have a background. So, for the time being, let us ignore as it is also given in the problem statement, the bending of the shaft though it can contribute to vertical deflection, let us ignore it for the purpose of solving this problem. Is there anything else that can contribute to downward movement? Stretching of the wire; that is very good. I have a cable on this.

So, I have a stretch of this. That you may miss it, fine? So, you also have extension of the cable. So, we have to account in this problem, only two aspects; angular twist of the shaft and extension of the cable. Once you have understood what is to be done, then rest of the problem is application of what you have learnt earlier; simple and straightforward. Is the idea clear? Say I would like you to use your calculators and then get me the numbers.

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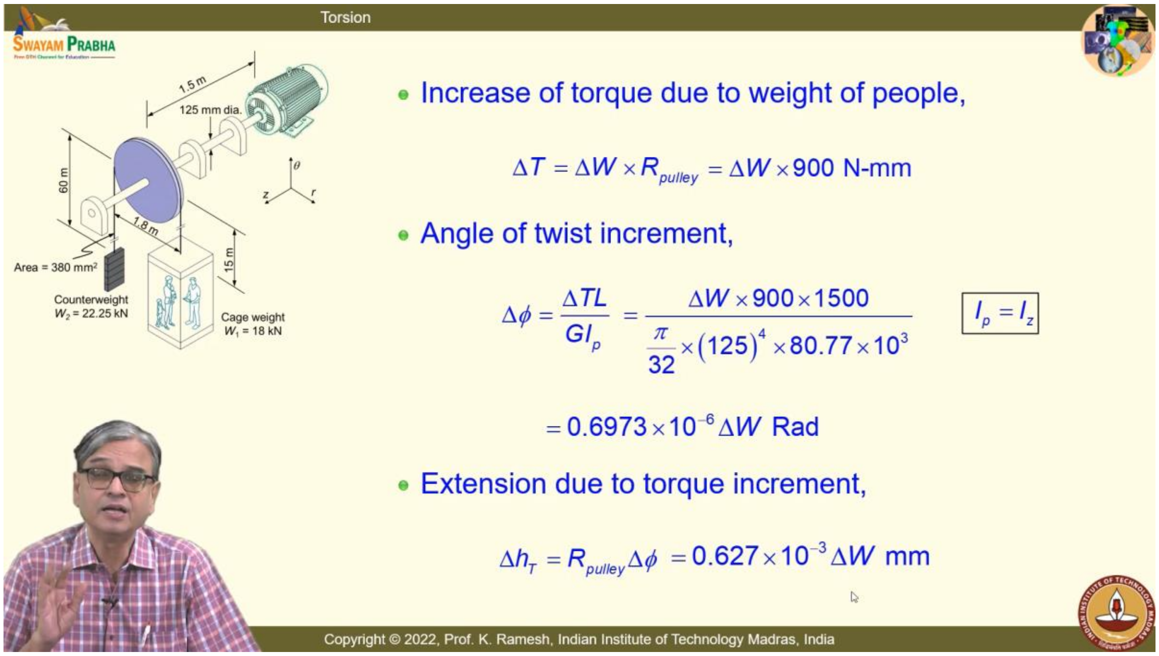

So, increase of torque due to weight of people is ΔW multiplied by radius of the pulley and that is also given; you are given diameter as 1.8 m. So, I get this number here. And what is the increment in the angle of twist? So, I have $\Delta\phi = \dfrac{\Delta TL}{GI_P}$ and you know what is the I_p for a circular shaft.

So, I get this as $\dfrac{\pi}{32}\times125^4$ mm^4 and your G is given in the problem statement as 80.77×10^3 MPa. And I have $\Delta W\times900$ Nmm is the torque additionally developed. And the length, it is also expressed in terms of millimeters. So, I am in a position to get what is $\Delta\phi$ when I see from this.

This is a fixed end and then it comes to this portion, fine? So, I have extension due to torque increment. So, I will have to multiply this angle of twist by the radius of the pulley. So, it gets magnified because of the diameter of the pulley. You have taken the radius; from the diameter get the radius and you get the extension or downward movement due to torque is given as $0.627\times10^{-3}\Delta W$ mm.

We still do not know what is ΔW. See, a problem becomes a design problem when limits are given to you. This is a design problem because limits are given, you are given the maximum downward movement permitted is only 3 mm. How you arrive at it? That is a big question, fine? It is not like somebody has a dream and then says 3 mm. There are physical considerations. So, you have to look at. That is a difficult aspect of it. How to set limits for you to do the design?

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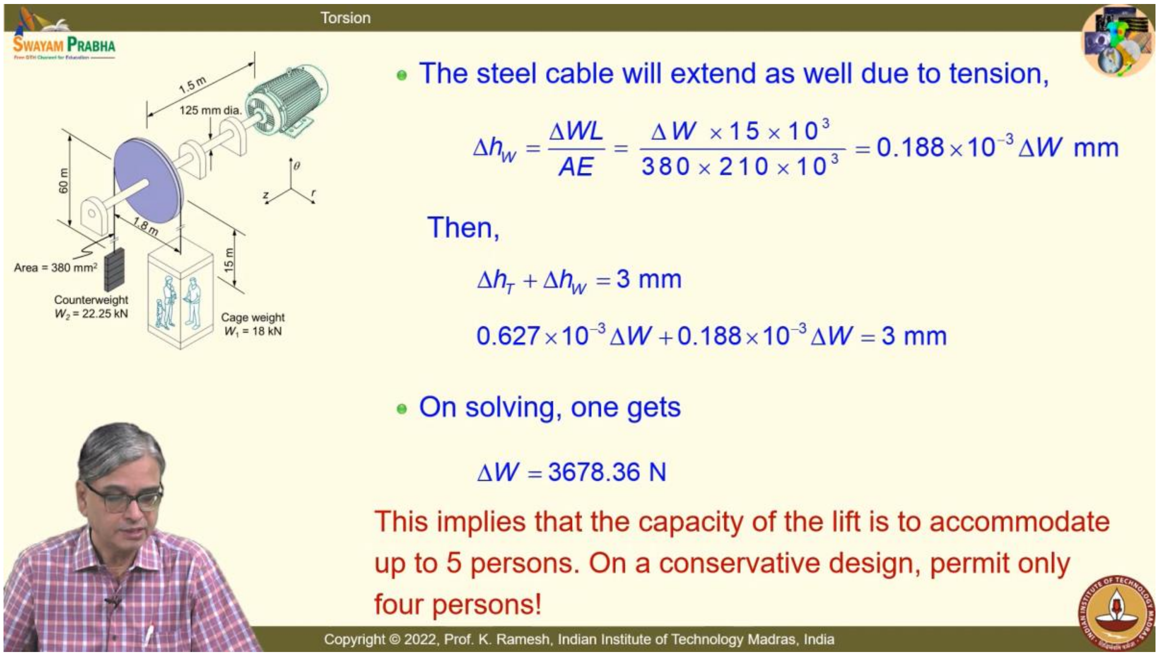

And steel cable will extend as well due to tension. And you know, for axial load, a very famous expression we have got. Do you remember, in terms of applied load P? The extension is given as; these are all very famous expression you should remember, PL/AE; do not forget that. Now replace what is P, what is L, what is area of cross-section and what is Young's modulus, ok? We are talking about ΔW as the additional load because of number of people who step into the lift. So, this is P. So, PL/AE. So, I have this numbers substituted. So, I have this area of cross-section. The cable the dimensions are given. So, you have to look at that, fine? Area is given. Area is given as 380 mm^2. You do not have to do the calculation yourself. It is given in the problem statement and Young's modulus is given. So, this gives me, when I substitute the numbers, I get this as $0.188 \times 10^{-3} \Delta W$ mm.

And it is given in the problem statement that this extension should be limited to 3 mm. So, you can easily find out what is the ΔW acceptable, fine? It is child's play; just substitute these numbers and equate it. And when you get the ΔW, it comes to be something like 3678.36 N. See, engineers you know, look at things from a physical perspective, fine? So, this amounts to how many people? Suppose you have 72 kg ~ 75 kg is the weight of the person you can imagine. So, it can accommodate about 5 people. And you are a designer, you know, you want to have some kind of factor of safety inbuilt.

So, even though I can allow 5 people to step on it, on a conservative design permit only 4 persons. So, put a label, this lift beyond 4 persons, do not get on to it, fine? This is one way of learning; while you design, you take care of anything that you have not calculated or included, any manufacturing defect, any problem in the cable, all that you inbuild by bringing some kind of factor of safety. See, the cable what you have here is, it is not simple rod of wire; you will have multiple strands twisted and factor of safety for a cable design is altogether different. You will have factor of safety of the order of 10 because any small problem there, it can be disastrous. That is what is needed when you have a cable car not in the lift, but in the cable car you are exposed to natural elements and also high wind.

You have, you must have seen in the papers that people got stuck in a cable car and the army helicopter goes and then rescues, all that happens, fine? So, factor of safety is a very very important concept; you have to apply depending on the context. See, for a car design it was initially 4, now they have brought it down to very close to 2 or less than 2, but when you go for a cable design the factor of safety is very high; a factor of 10. When you are designing a nuclear power plant, factor of safety will be very high. You do not want to take chances because nuclear radiation deteriorates the material. So, there are many considerations that go into it.

So, a problem becomes a problem of design when limits are set. How to select the limits? That is a very big job to be done, fine? Once you set the limits, you would know how to use the equations and get the calculations.

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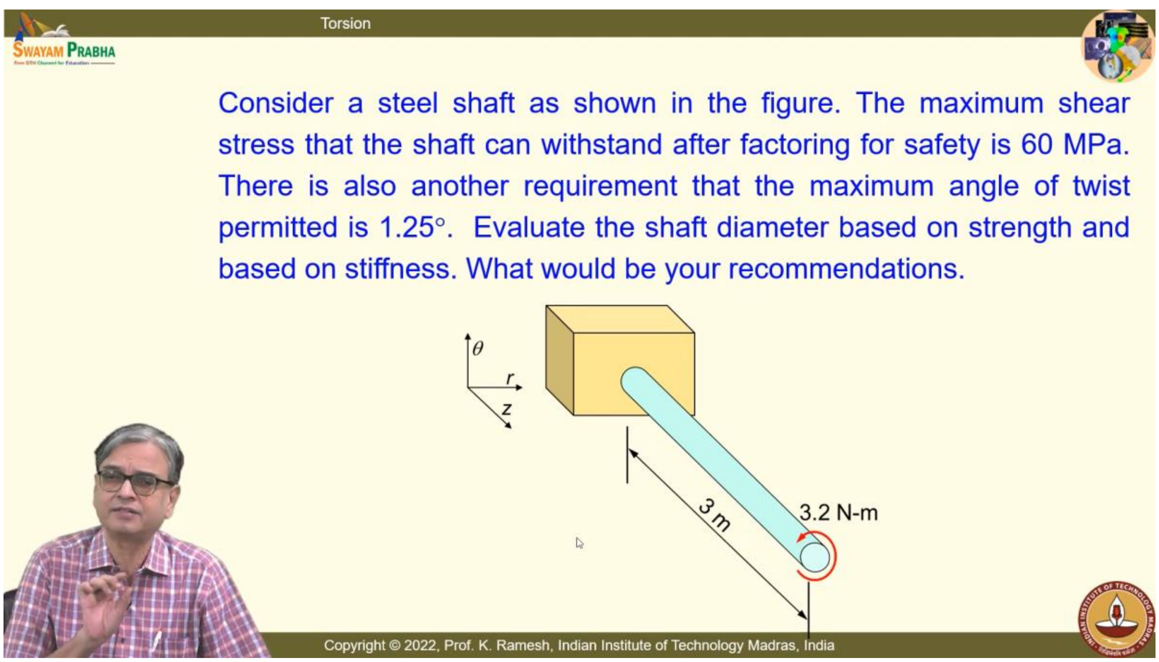

Now we move on to the next simple but interesting problem. Consider a steel shaft. I have deliberately taken simple problems. The focus is to learn the use of equations and also appreciate what happens when I do a design from one perspective. Because here what it is done is the shaft can withstand a maximum stress of 60 MPa. There is also another requirement that the maximum angular twist permitted is only 1.25°. And the question is, evaluate the shaft diameter based on strength and based on stiffness.

See, what I want to do is, while you solve this problem, let me also circulate this so that you can have a feel of what happens in the case of a shaft.

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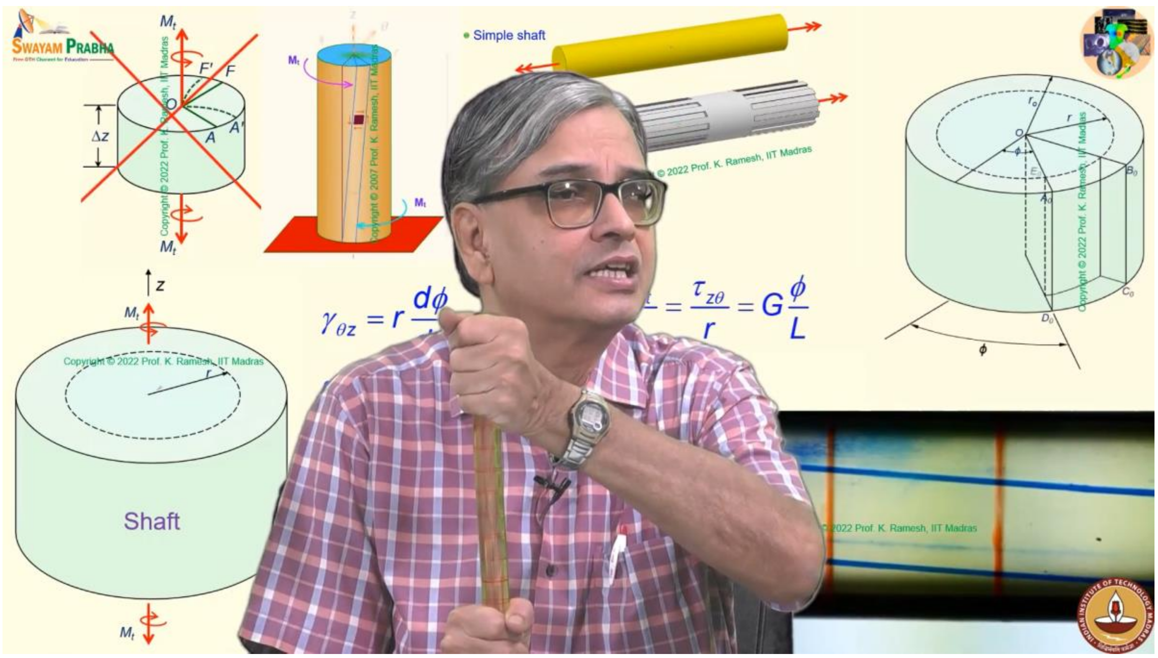

You twist it yourself and then see what happens. Can you collect it and then let this be circulated? So, the problem is simple, straightforward; there are no steps in the shaft. It is a simple cylindrical member and I have the twisting moment is given, ok? And you should also understand.

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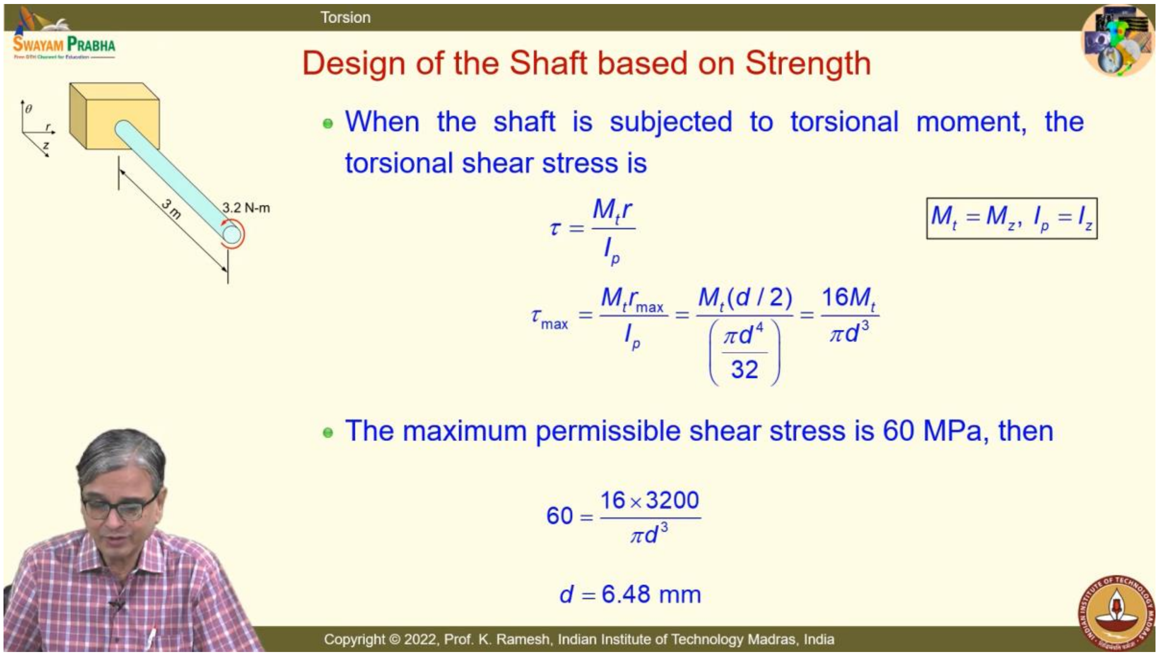

See, we have taken the axis of the shaft as z-direction because it is circular shaft. We have taken polar coordinates to define the circular boundary. And you should appreciate that the twisting moment is nothing but M_z and then the moment of inertia that we want to calculate is actually the polar moment of inertia. Because when you go to bending, there you will again see I_z, but that will be different moment of inertia. In order to avoid confusion, I thought that we would adopt this symbolism. So, I have this maximum shear stress introduced from our torsion formula.

You can have this as $\frac{M_t r}{I_p}$ and the stresses are maximum at the outer diameter. So, r_{max}, that is the $d/2$, and then I_p is $\frac{\pi d^4}{32}$. So, the maximum shear stress in a cylindrical shaft is $\frac{16M_t}{\pi d^3}$. In fact, this expression also is worth to remember. When you go for design calculation, design of a circular shaft, because when you have a shaft, it is subjected to torsion, it is subjected to bending, it can also have axial force.

So, once you learn bending, you must also get the stresses because of bending. And when you look at that, the shear stress you can easily calculate as $\tau_{max} = \frac{16M_t}{\pi d^3}$.

Like I said PL/AE for extension of a member subjected to tension, the maximum shear stress developed in a cylindrical shaft as $\frac{16M_t}{\pi d^3}$, is a very famous expression, fine? Certain expressions you need to remember; nothing wrong in it. It is not mugging it up. That means you should have solved at least 20 problems.

Then you will automatically know that this is an important expression. And it is given, the maximum shear stress is 60 MPa. I would like you to appreciate. See, you have looked at mild steel, the yield strength was something like 240 MPa. And I said that you will never go to yielding in real life structure.

You will work much below that. So, I work half of it, that is 120 MPa. Then I said that you have repeated loading. See, in the case of a problem which I have shown, there is no torsional oscillation. On the other hand, if you go to an IC engine you also have torsional oscillation. So, there, the torsion is going to change as a function of time.

Suppose I want to bring in, when the shaft is rotating, because of rotation, the bending stresses will oscillate. Suppose from that consideration, I would like to have the maximum stress limited as 60 MPa, that is one-fourth of the yield strength of even the mild steel. Mild steel has very low yield strength, alloy steel is about 550 MPa or so. So, even the cheapest material, we do not want to load it. And when I do the calculation, what is the diameter that you get? Is it a shaft or a rod? It is no longer behaves like a shaft; like a very thin rod.

And naturally, you would expect that this is going to have a very large value of angle of twist. Is the idea clear? See, I have taken to start with one-fourth of the yield strength. I have not applied very close to the yield strength, very very small stress I have taken. Even when I take a small stress like this, I get a number which is very funny; d is 6.48 mm. I want you to calculate the angle of twist for this, do the calculation.

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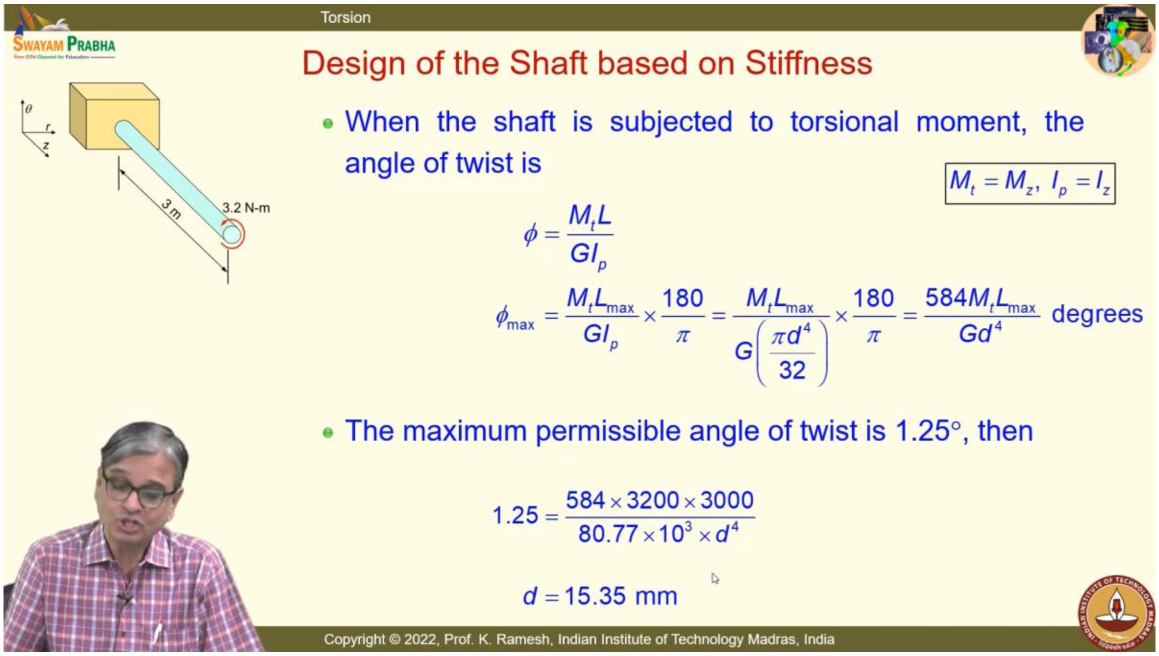

And in the meanwhile, let me also go and proceed. We have also said the angle of twist should be limited to 1.25°. Isn't it? That is given in the problem statement. And I have the expression for this is $\frac{M_t L}{GI_p}$, and I have already said GI_p is the torsional rigidity. So, when you solve problems in future, calculate this GI_p and keep it at one place so that you can repeatedly use it for various purposes. And that angle of twist will be maximum when L is maximum, that is at the free end. So, I get this as, when I substitute these numbers, I get this as $\frac{584 M_t L_{max}}{Gd^4}$ degrees.

I have already told you, in the development of the equations, ϕ was in radians; never forget that. It was in radians; what you get from this basic expression is in radians. I convert the radians into degrees using this; $180/\pi$, I do that. So, I get this is in degrees.

So, when I limit the angle of twist to 1.25°, I get a reasonable size of the shaft which is 15.35 mm. It is something better than 6 mm, fine? So, now we have to do two calculations. One calculation is when I take a diameter of 15.35 mm, what is the maximum stress generated? Even before calculation, can you tell me which will be more or less? When I have a shaft, whose diameter is increased, we have earlier calculated the diameter is only 6 mm if the limiting stress is 60 MPa.

Now, I have increased the diameter, almost double the size. It was about 6 mm; this is about 15 mm; slightly more than doubled. Do you expect the stress developed to be more or less? Why do you say it is less? That is good. He has coupled, see, when the radius is more and more, stress is high, but inertia also changes. So, he has coupled and then said its proportional to d^3, fine? So, you will anticipate the stress levels will be very much smaller when I go to this, fine?

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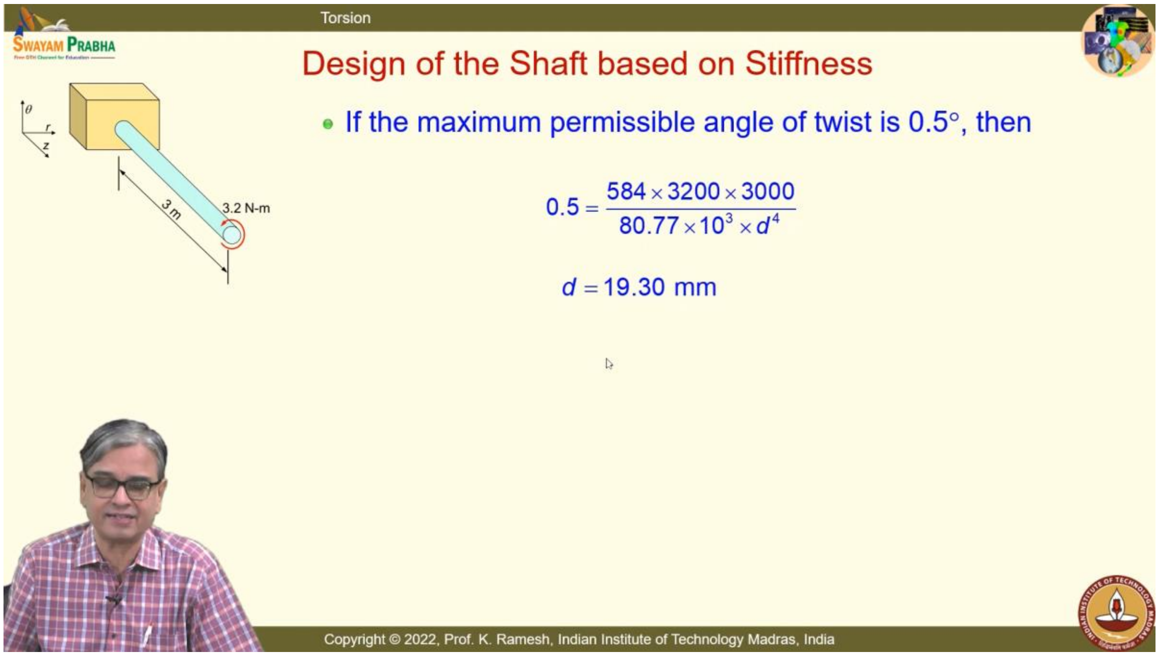

Suppose I also put another restriction, even 1.25° is not acceptable. See, I want to bring home a point.

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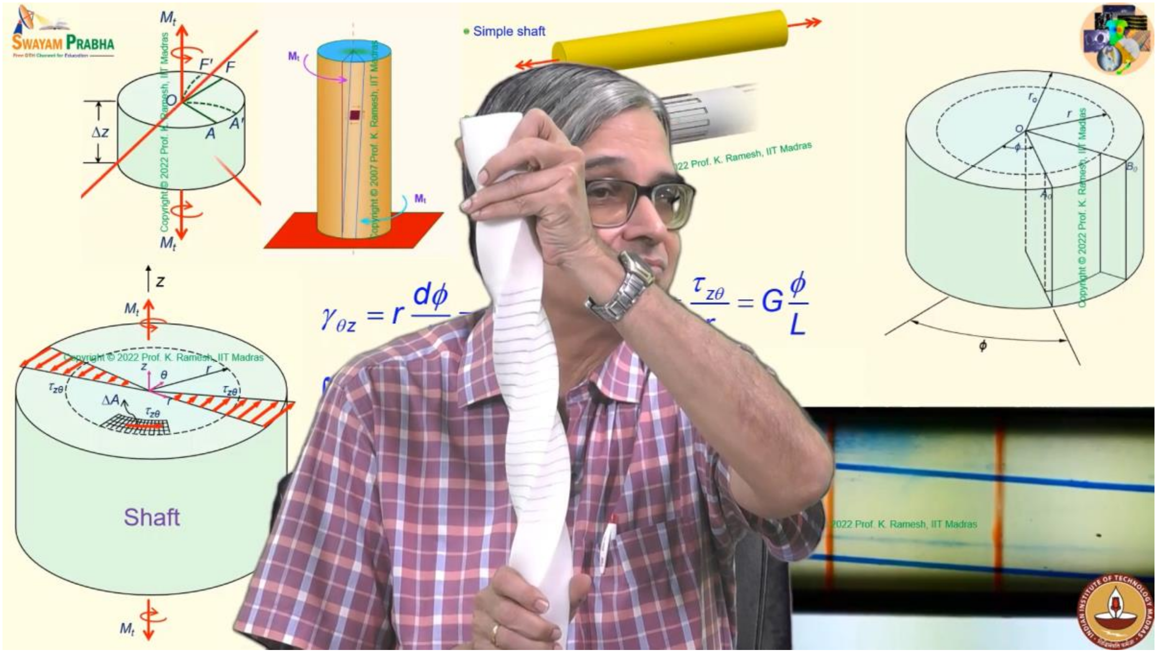

See, whenever we want to understand anything, we want to have very large deformation, ok? I can do this only if I have a very large deformation. Almost like close to one circle I am doing it. That is what 360°, I have given. Only then, I am in a position to show that these horizontal lines have a warping.

For you to visualize, you need large deformation. But mathematics will not work, it is not applicable when I have large deformation. Just to drive home this point, I have said the permissible twist is only 0.5°. In reality, you will have only such small angle of twist, fine? In all our discussions, in order to drive home what kind of deformations take place in torsion, I deliberately show a large deformation for you to visualize the interrelationship and what happens. But in reality, these angles are very very small and our equations are applicable. Our equations are applicable only when you have small deformation; never forget that. And I find the diameter has slightly increased, 19.3 mm, fine?

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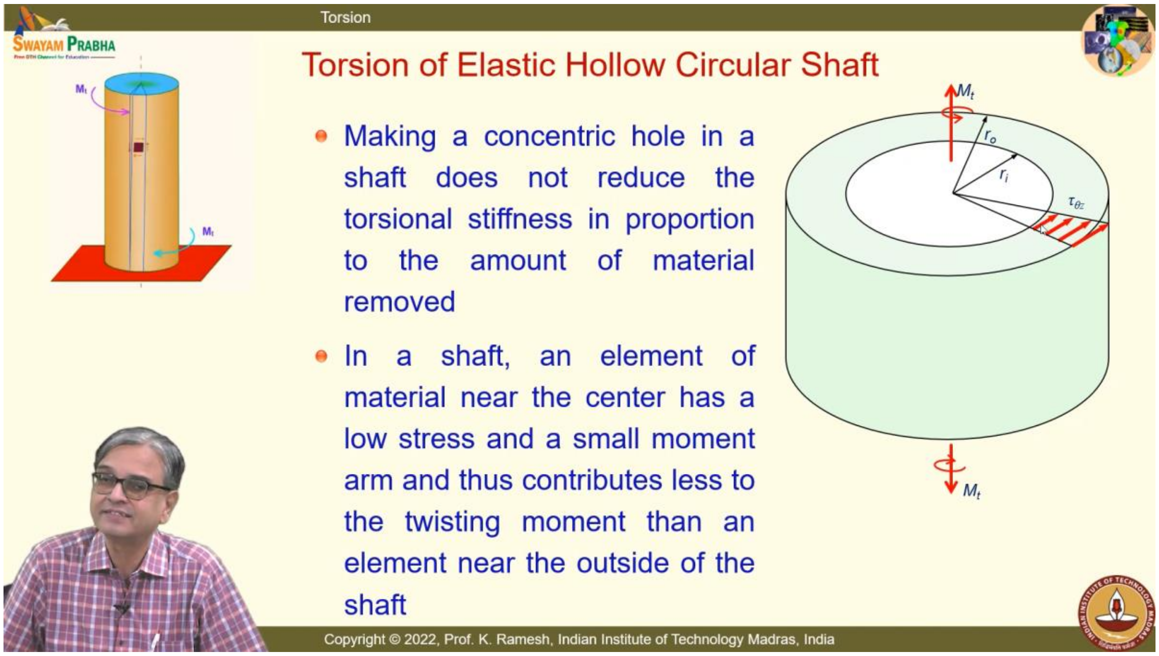

And before we discuss on the problem further, you know I want to also discuss what happens when I have a hollow circular shaft. It is very very important. The same equations are applicable, so you make a concentric hole and I have only a shaft which is like this. I have also brought this. So, you have a tube, ok? You have a particular thickness for this, you can see this.

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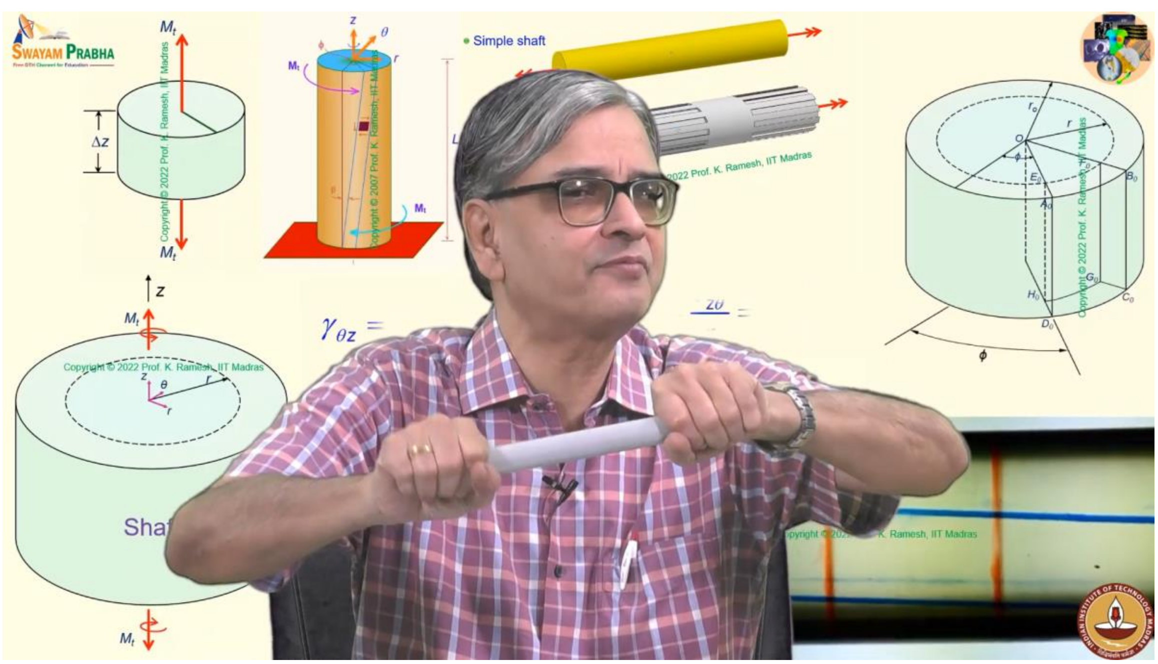

And it is also very strong, when I see, when I twist it, I am finding it very difficult to twist. It is made of plastic only.

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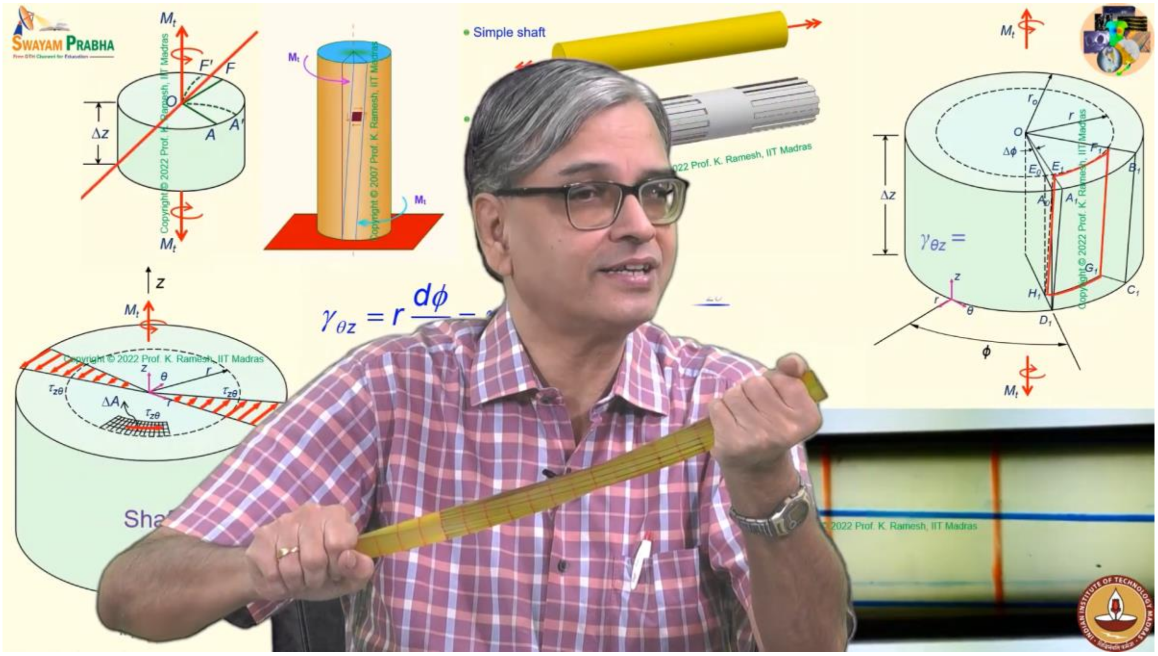

Whereas, the other one which I have circulated is made of soft material. This is also plastic, ok? But when I have it in the form of a tube, it gives an impression that it is very strong, don't you feel so? Let us see what the mathematics says, ok? So, this is also applicable, instead of this going to zero, it stops at whatever the shear stress that this radius can develop.

And the difference is coming in the moment of inertia. See, I told you, among the various mathematical quantities, which one recognize the cross-section. You must appreciate when you change the cross-section, moment of inertia changes.

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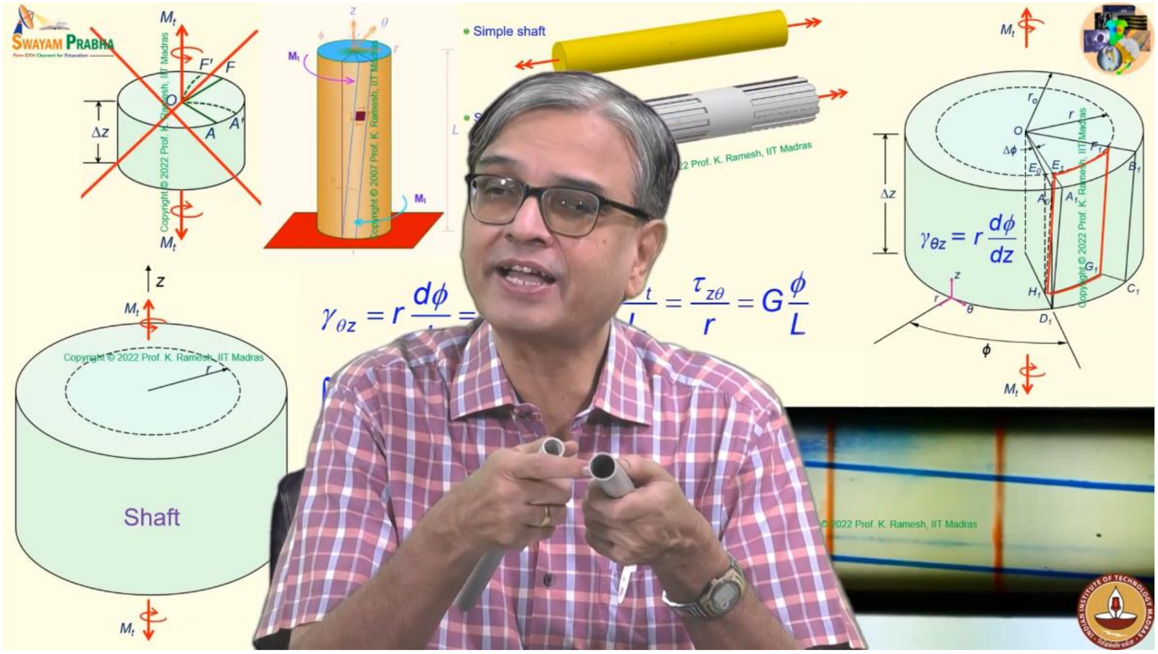

And you all know when I put, if I have the axis like this, suppose I have a smaller tube like this, I have a bigger tube like this, this moment of inertia is going to be higher, because the material is put away from the center. As the material is farther away from the center axis, you get higher and higher values of moment of inertia.

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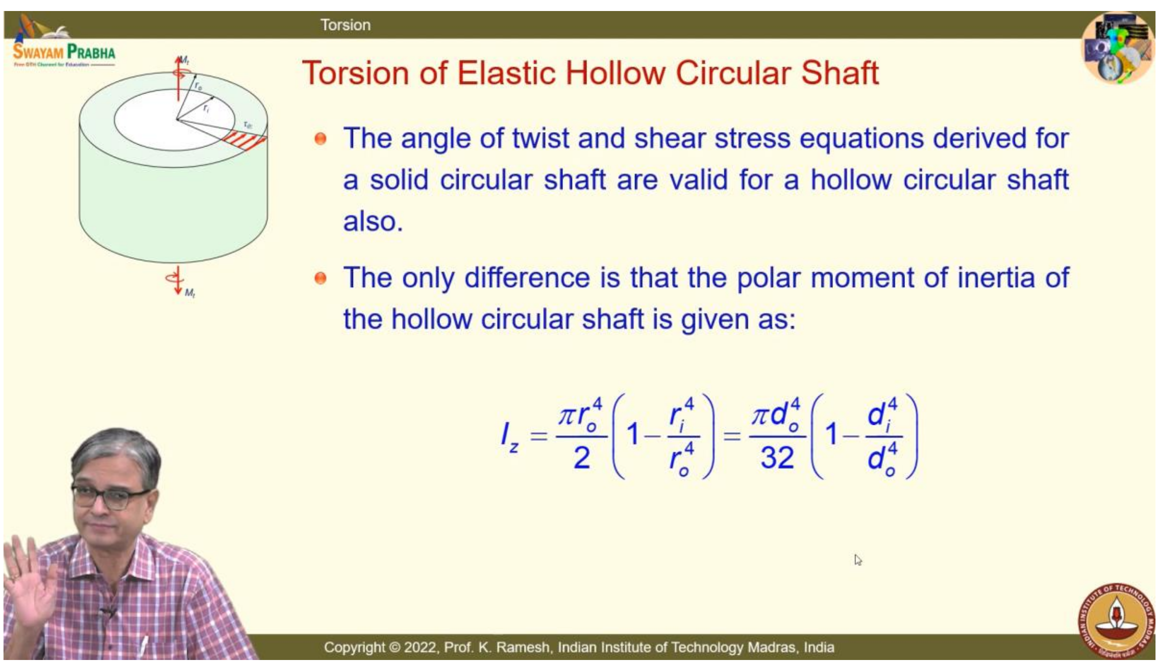

So, the moment of inertia calculation will be different, ok? The equations that you have derived are valid. And I_z or I_p is now $\frac{\pi d_o^4}{32}\left(1-\frac{d_i^4}{d_o^4}\right)$. And also in your design, people may specify what should be the ratio of d_i/d_o, that makes your calculation simple. And when you take a hollow shaft, you try to make the diameter little more so that you use less material and higher inertia, because you have to satisfy the stress levels as well as the twisting requirement; both are necessary. So, now we will go back to the same problem and calculate what should be the dimensions of a hollow shaft to support the same load. That will give you a clarity whether solid shaft is better or a hollow shaft is better. Is the idea clear?

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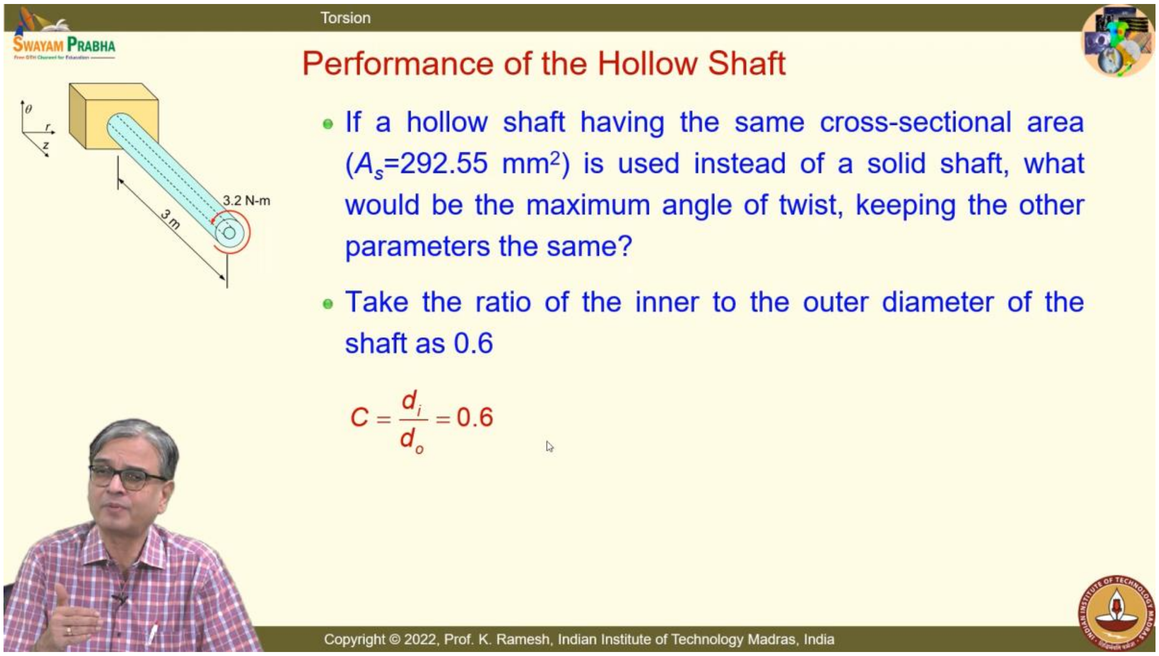

See, the area of cross-section is given right here; we will also calculate. And you have to remember that we have limited the angle of twist as 0.5° and we got the diameter as 19.3 mm, fine? And you are also given another simplification. Take the value of d_i/d_o as 0.6; that is given in the problem statement. Because I can construct multiple hollow shafts even if I want to map the same area of cross-section.

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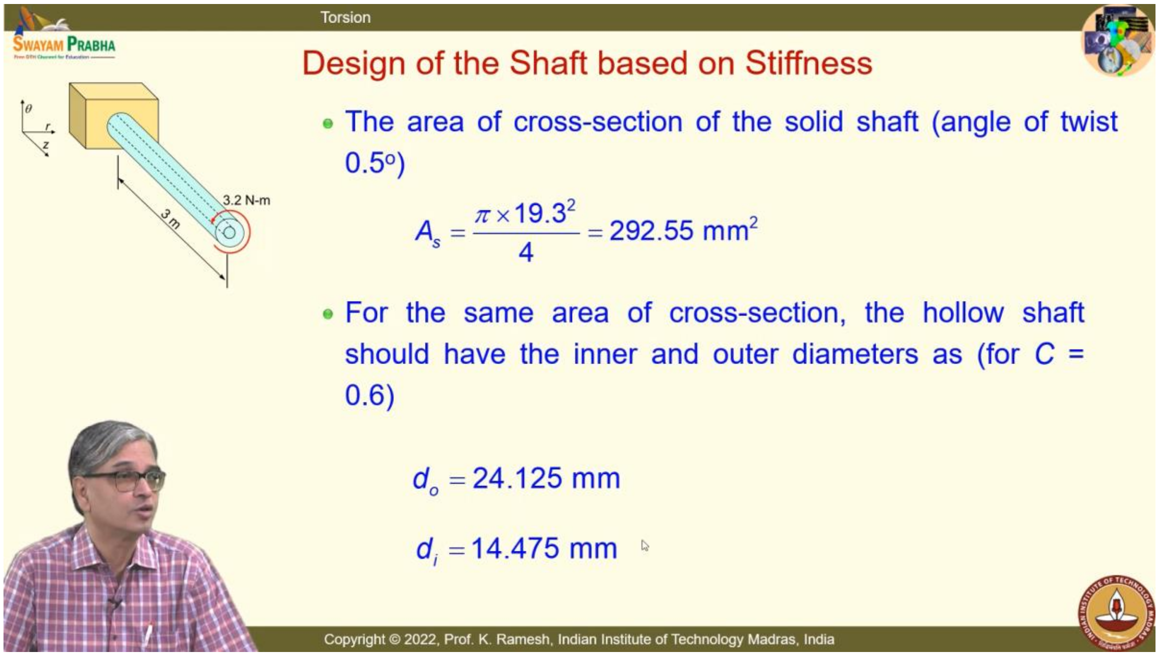

So, what you have is the area of cross-section of the solid shaft; I have again calculated it so that you really associate the results to this problem. So, I have limited the angle of twist as only 0.5° and I have got the result as 19.3 mm as the diameter. So, I calculate the area of cross-section. See, the comparison would be valid if I use the same material to construct a hollow shaft, fine? So, when I do that, when C is also given, you know if you do the calculations, it is possible for you to find out that d_o should be 24.125 mm and inner diameter is 14. 475 mm. So, the wall thickness is something like 10 mm. You know why I mentioned the wall thickness? Because you want to have higher and higher value of moment of inertia, increase the outer diameter and keep reducing your wall thickness. You can also do that, isn't it? Why I am talking about wall thickness? Can you guess where do you anticipate a problem? Where do you anticipate the problem? I am going to twist it; I am going to take the shaft and twist it. When I twist, what is the kind of state of stress there? Shear can be viewed as what? At 45°, what do you see a shear as? Tension and compression. So, compression is going to give me a problem. When you have a thinner and thinner section, when I have a compressive load, it can buckle, I can have torsion buckling.

So, you need to have a trade-off. If I increase the outer diameter, my moment of inertia is going to increase. But if I keep reducing the thickness, I may also reach the other extreme. When the twisting moment is transmitted, the shaft may develop buckling because of twisting. See, people normally forget at the end of the course because we develop the critical load when you have compression on the columns.

Then people think only columns buckle. I have discussed it even in my initial lectures. Buckling can happen whenever you have a compressive load. Torsion also develops compressive load at some other planes. You should never forget that, ok?

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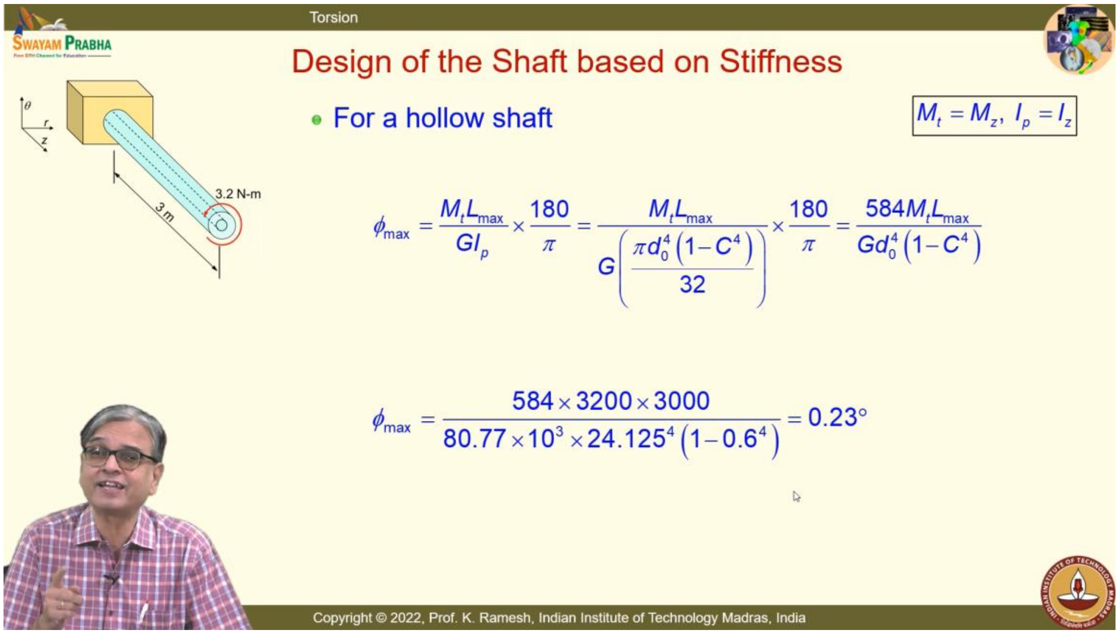

So, I go and calculate what is the angle of twist for this shaft and when I substitute all these quantities, only the inertia changes here, ok? So, you can take the same expression and change inertia and substitute these values. You find for the same cross-section of the solid shaft, if I use a hollow shaft with C = 0.6, that is specified in your problem, the angle of twist is just half of it. That means, solid shaft is weaker in comparison to a hollow shaft of the same cross-section. So, what does this imply? I can also satisfy my design requirement by having smaller material than a solid shaft so that I can save material. Is the idea clear? Ok.

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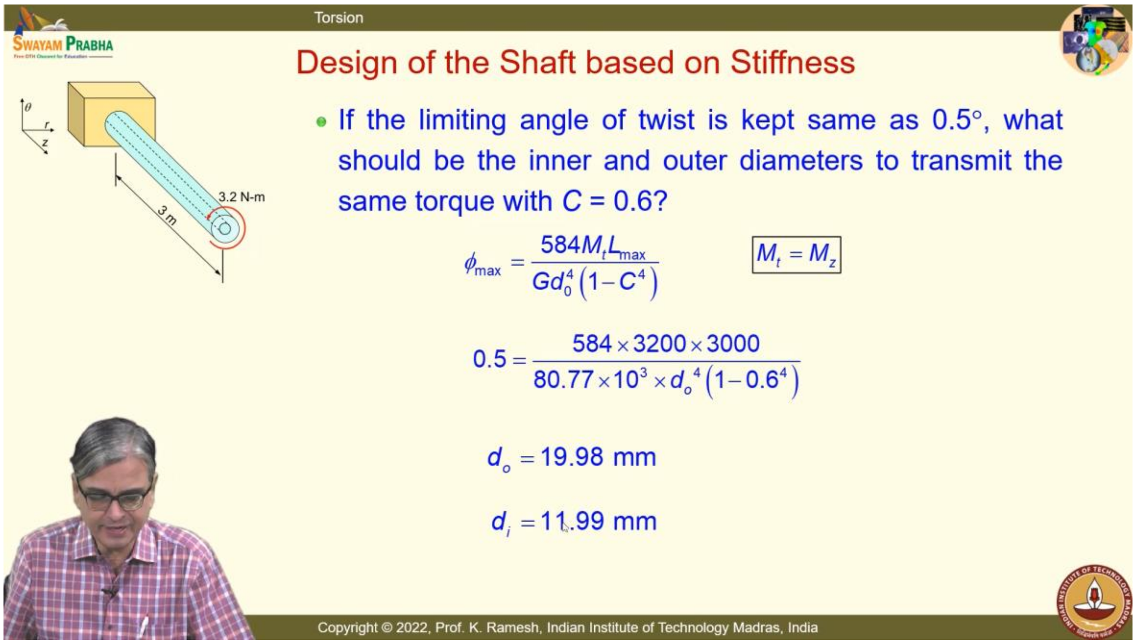

And we will also do the calculation. If the limiting angle of twist is 0.5°, I do not want to change it. Now, I want to find out what should be my outer diameter and inner diameter. The outer diameter and inner diameter are calculated and then you find outer diameter is 19.98 mm, slightly greater than what was there earlier. And inner diameter is 11.99 mm and the difference is about 8 mm, considerably thick, ok? It is not a very thin shaft. So, if I want to maintain my angle of twist as 0.5°, it is enough that I use d_o and d_i like this.

You can also calculate the area. You can find out what is the change in the cross-sectional area. And if you have the length, you can also say what is the change in weight. Suppose, you have a design requirement where the weight is a consideration. See, if you look at LCA aircraft, where weight is a very important consideration. When you have a power drive shaft, it is all hollow in an aircraft because every kilogram you save, you save enormous amount of fuel in its complete life.

But you go to Attachaki, you go and find only a solid shaft. Am I right? Why people use solid shaft when you find that hollow shaft is better? It uses less material. You have to look at holistically. I have to remove material; I have to pay money to get the material removed. You understand? And it should also be concentric. So, if you look at the manufacturing cost, in many of the design application, day to day application, there is no great advantage by using a hollow shaft and also there is no great advantage in going for an alloy steel.

It is enough I work with mild steel. Isn't it? Because when you see everywhere, you do not normally come across the hollow shaft. Am I right? In many of the applications where you have access to, you find everywhere the shaft is solid. Whereas, your mathematics now says a hollow shaft is better.

When it uses less material, it is better. That is one way of interpretation. Other way of interpretation is to get that as hollow; you have to pay a cost. So, you have to have a trade-off and that cost advantage is necessary when I want to go for an aircraft design where it is going to save enormous amount of fuel, understand? So, you are also learning strength of materials and also get insights on aspects of design. That is what you are going to do as a next course.

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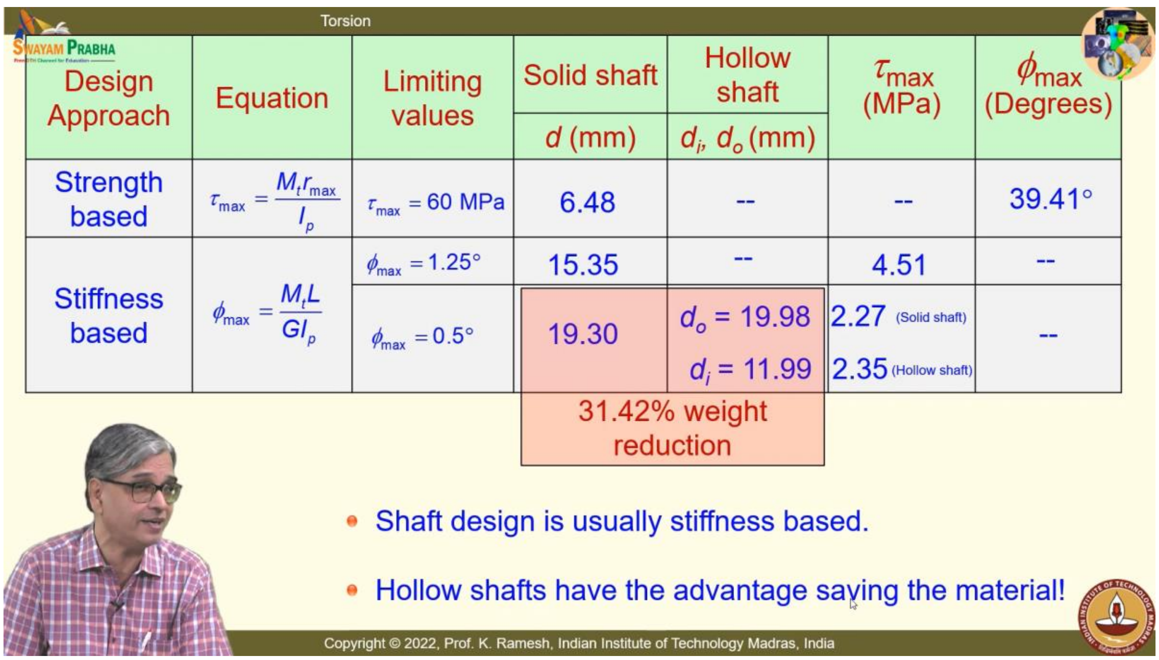

Design Approach	Equation	Limiting values	Solid shaft d (mm)	Hollow shaft d_i, d_o (mm)	τ_{max} (MPa)	ϕ_{max} (Degrees)
Strength based	$\tau_{max} = \frac{M_t r_{max}}{I_p}$	$\tau_{max} = 60$ MPa	6.48	--	--	39.41°
Stiffness based	$\phi_{max} = \frac{M_t L}{G I_p}$	$\phi_{max} = 1.25°$	15.35	--	4.51	--
		$\phi_{max} = 0.5°$	19.30	d_o = 19.98 d_i = 11.99	2.27 (Solid shaft) 2.35 (Hollow shaft)	--

So, I have two design approaches we have looked at. One is a strength-based approach. Another is stiffness-based approach. So, in the strength-based approach, we say what is the shear stress permitted and I said we have not taken the 60 MPa arbitrarily. I said, it is just one-fourth of the yield strength. It is very very small. You are not permitting a very high level of stress. And when I do this, I get the shaft diameter as 6.48 mm which is ridiculous, ok? I have asked you to calculate the angle of twist. Anybody has calculated the angle of twist for this diameter? Please do the calculation and then verify with my table because my table also has what is the final shear stress developed and what is the ϕ_{max}.

I want to do the calculation. Use the calculator and do the calculations. And we have also analyzed the same problem based on the stiffness. And you know the expression for stiffness as $\phi_{\max} = \dfrac{M_t L}{GI_p}$. I said that you have to calculate GI_p and keep it aside for your calculations. It is a torsional rigidity and in this, I have also said an expression for the $\tau_{\max}$, what is the value? We have also, I said it is a very important expression that you have to remember, ok? So, that is also a very important expression for you to remember.

$$\tau_{\max} = \frac{16M_t}{\pi d^3}$$

So, when I limit the angle of twist as 1.25° for a solid shaft, we got the diameter as 15.35 mm. And when we limited it to 0.5°, it slightly increased, 19.3 mm. And when you go to a hollow shaft, it has gone to 19.98 mm and 11.99 mm when I limited the twist. And I have also asked you to calculate what is the weight advantage, fine? If you do the calculation, I get this is about 31.42% weight reduction, about 30% weight reduction. I can easily get by switching over to a hollow shaft.

Has anybody calculated the angle of twist for this shaft of diameter 6.48 mm? Anybody has number? You are not doing the calculation. You should do the calculation and then check because you should also pick up speed. You should pick up speed. It is only one simple expression you have to calculate; only one simple expression. Because you should also know what happens to the angle of twist. Ok, in the meanwhile, when you calculate, let me ask one more question.

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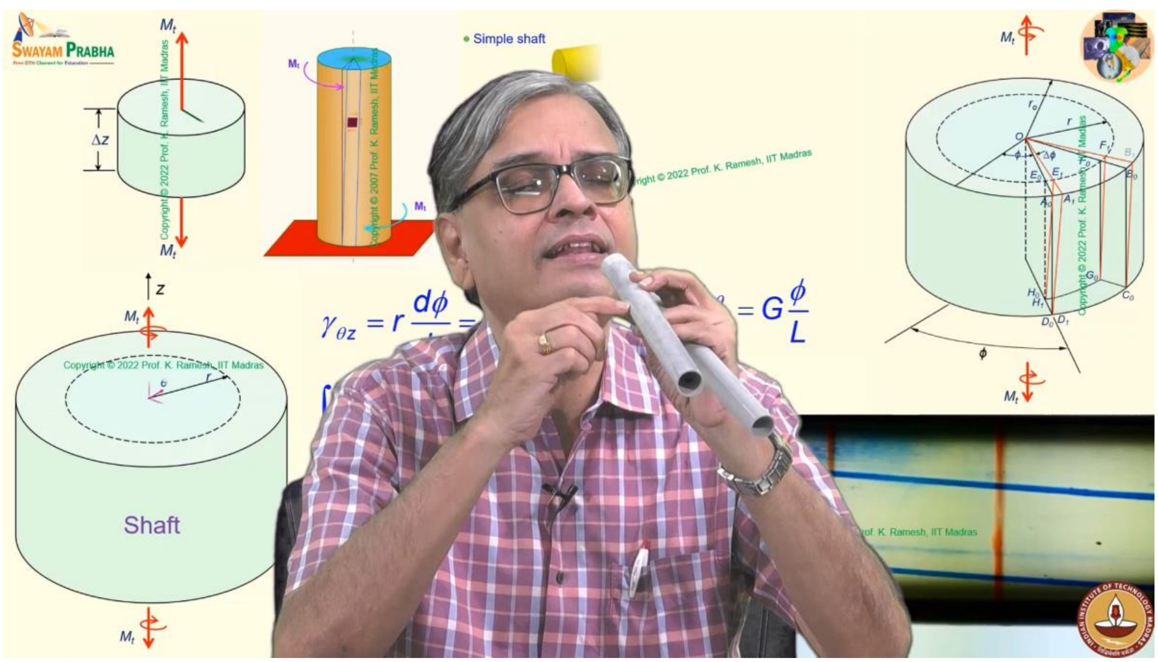

See, I have shown a circular shaft, fine? What I do is, we find that hollow shaft is very very strong. Is the idea clear? Now, what I do is I go and make a thin cut here. I made a small slit here. What do you think it will happen? What will be its torsional rigidity? Because we have seen, compared to a solid shaft, a hollow shaft is very very strong. Isn't it? Now, what I have done is, you know, I have asked him to make thinner than this, but he did not have a cutter. So, he had made this as a slit, fine? What is your take on it? What will happen to the torsional rigidity? It will not change. You know, it is like mathematician.

You have an infinite plate with a small hole was equivalent to a plate without a hole. I am happy. I am happy. You have to give that answer. Let me twist and show you. I am not a very strong man, ok?

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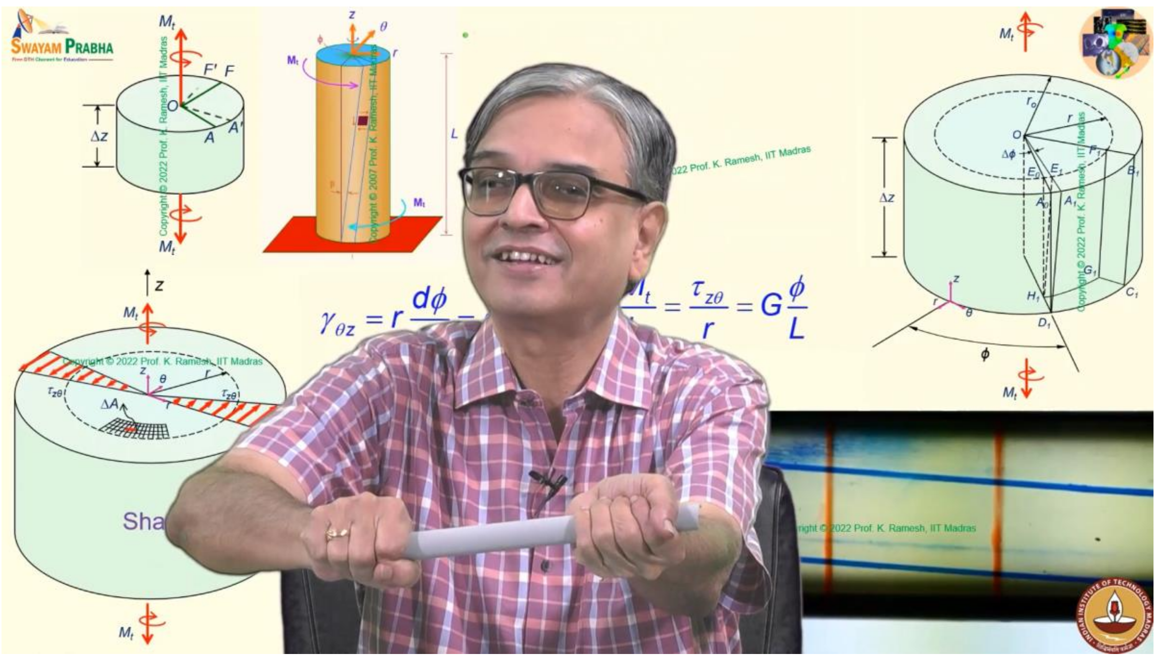

Let me twist and show you. What is happening? What is that you are seeing in reality? Drastically reduced. Do you get the feel? It is drastically reduced.

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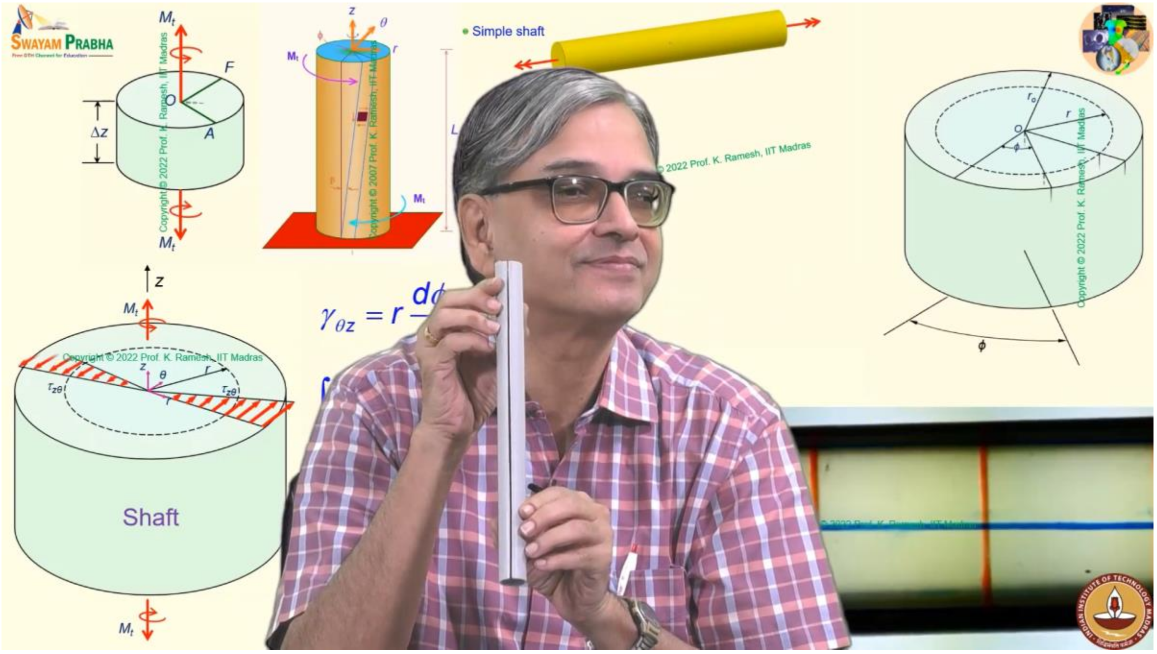

So, technically, these are called as open sections. So, write down. Open sections have poor torsional rigidity. So, I am also going to circulate this so that you have a feel of it. I am circulating a hollow shaft as well as a shaft where I have removed this material, fine?

So, you have hollow shafts. One has slit. Twist both of it and feel it. See, feeling is very important. If I have an open section, even if I have a small slit, even slit smaller than this, the width is smaller than this also will behave in the same way. So, open sections have very poor torsional rigidity. Can I get the number for this table?

I have discussed all this, in the meanwhile, somebody will calculate. They are not doing it, ok. The value comes to be 39.41°. Very very high angle of twist, unacceptable. You cannot have a shaft supported on bearings when you have such large twist. In the problem of the lift also, it will not stop at 3 mm. It will go to the next floor, ok? And when I go to a hollow shaft, look at what are the stresses developed. You guessed correctly that it is proportional to d^3. So, you have said that it is going to be smaller. So, it is just 4.51 MPa. And when I go to 0.5°, when I increase the diameter, for a solid shaft, it goes to 2.27 MPa. For a hollow shaft, it goes to 2.35 MPa. Let me ask a question. Suppose you have to design a shaft, out of these two approaches, which approach would you adopt? Stiffness approach, that is very very important. That is a learning in this class.

So, when you want to design a shaft, shaft designs are usually stiffness based. It is never strength-based. On the other hand, if I go and design a spring, it is always strength based. You use high alloy steels for this one. And you also have an understanding that hollow shafts have the advantage of saving the material, fine? And you found, if I have a small slit, you make the hollow shaft as open, it loses its torsional rigidity. You have methods to calculate for the open sections. Normally, it is taken up in the second level course. So, I am not going to discuss that.

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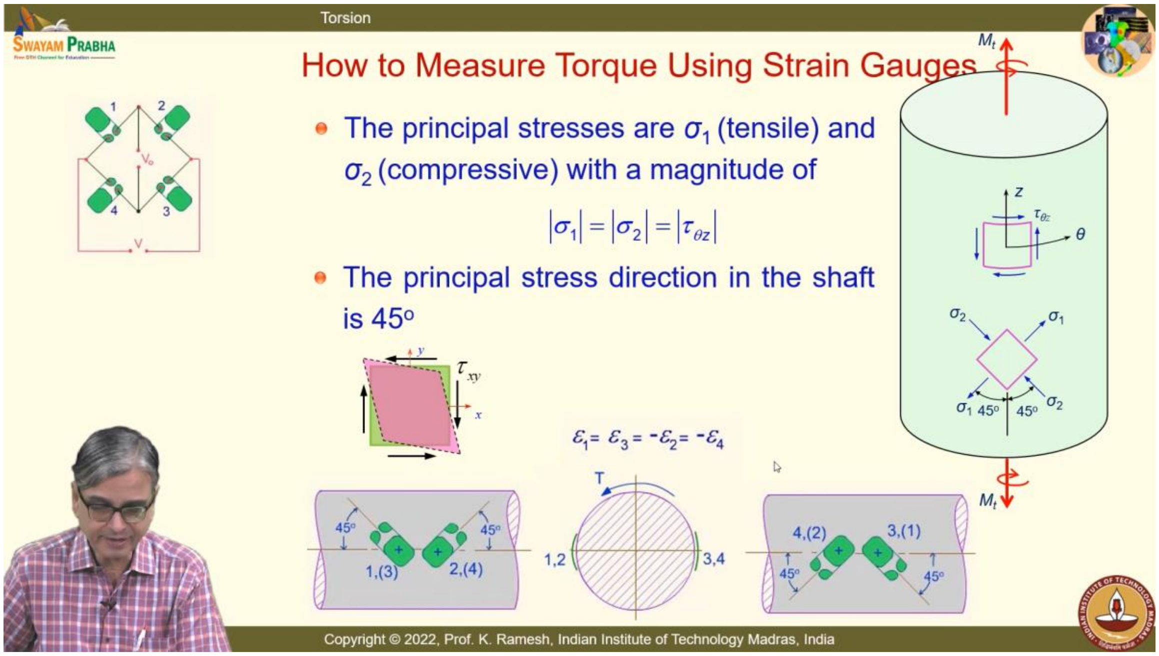

And we have already seen Mohr's circle several times for the pure shear stress state. And you should recognize that torsion is a situation where I have a pure shear stress state. And I have the principal stress direction, the shaft is at 45°.

So, this is shown in a different purpose, ok? It is kept in a vertical direction and this is drawn. Can you draw the element at 45° and show tension and compression just below, and check with my diagram? See, you should also apply your mind. While learning, keep applying your mind and then find out what is happening. So, I have the shear stress indicated like this.

Tell me where I will have tension, where I will have compression. Just below this, draw the element which is at 45°. Tell me which is the direction where you have tension, which is the direction where you have compression. When you draw the Mohr's circle, you can easily locate. But even without the Mohr's circle, the moment you see the stresses here, you should be able to do that. These are all training, fine? You understand the concepts better and then you find out how to get this.

So, I have this as tension and this as compression, which you can also verify from the

Mohr's circle, fine? You can verify this from the Mohr's circle. And we have also seen how to paste strain gauges. Am I right? How to paste strain gauges? Let me go back to that because you know these are the things that engineers have to do repeatedly. You also have an experiment in your strength of materials lab where you find out the torque using strain gauges.

So, you have the stress state drawn for you to visualize very clearly. And this is what we had seen it when we developed strain gauges for measurement of strain. We have looked at the problem like this. It is twisted anticlockwise. And then, when you draw the side view, the strain gauges are pasted like this, that is at 45°.

Why 45°? You have the reason. Strain gauge can measure only the axial strain. It cannot measure shear strain. So, you measure the tension and compression and you should also recognize what is the way the; for this I am drawing, for this picture I am drawing, ok? Do not confuse this with this. This is drawn vertical, this is drawn horizontal, for this, I am drawing the shear. And you also know that ε_1 and ε_3 are positive, ε_2 and ε_4 are negative.

You should also connect them appropriately in the Wheatstone bridge, fine? So, they are connected like this. So, I have 1, 2, 3, 4. So, when the adjacent arms because of opposite signs they add up; opposite arms, like strains, they add up. So, I get 4 times the signal. So, when you want to use strain gauge as a torque sensor, you want to maximize the signal. So, you use Wheatstone bridge all the 4 arms of the bridge and you use the knowledge of what is the stress state introduced in torsion in making the measurement also.

Is the idea clear? So, this is also a review for your strain gauges. You should take it. See, I consciously repeat concepts. The same concept you hear in more than one lecture so that I expect that your assimilation should be better.

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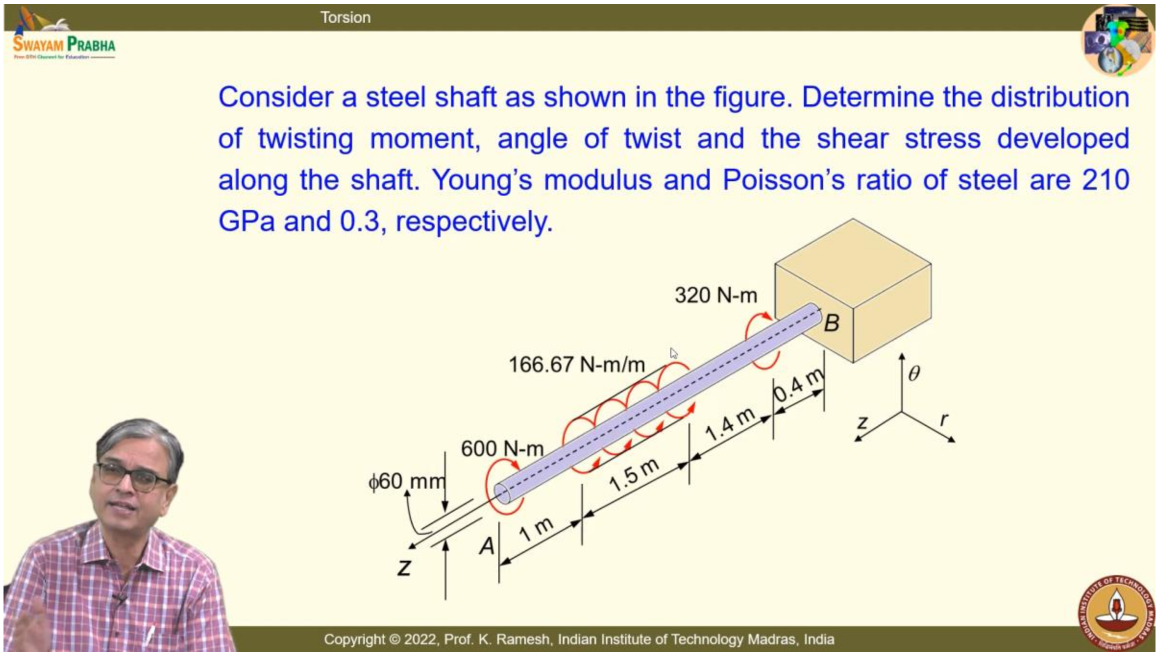

Now, we will also look at the problem that we have taken up earlier. We have already determined what is the variation of twisting moment. For the same problem, calculate the angle of twist as well as the shear stress. We will see some of it in the class. Rest of it, I want you to solve it as part of your home assignment.

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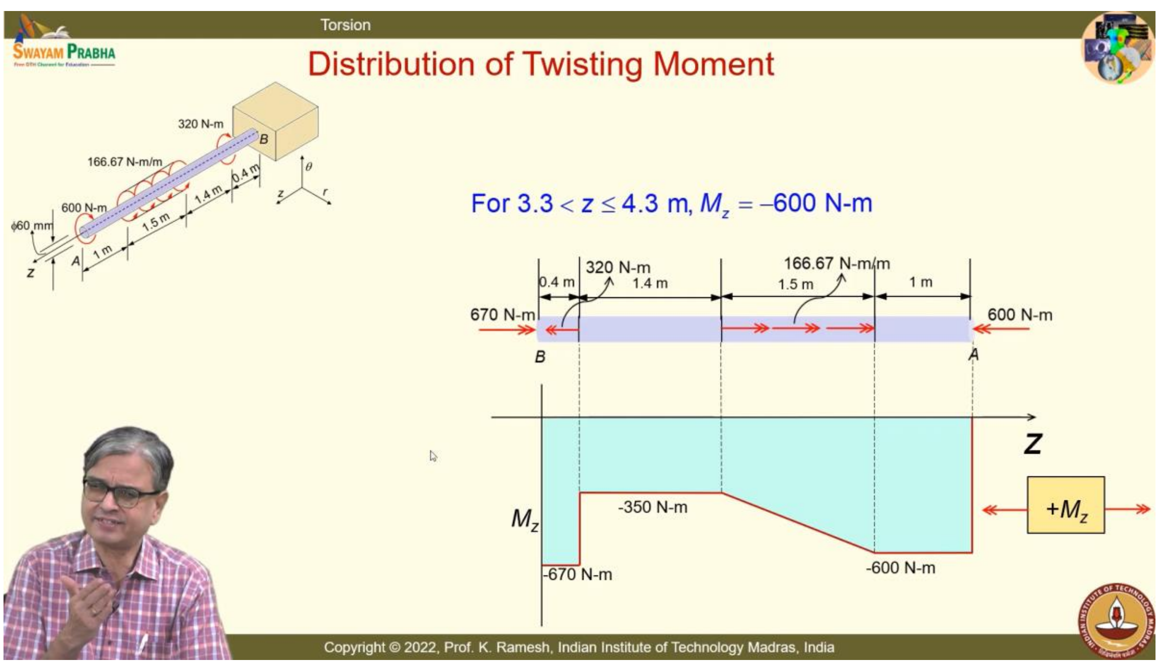

And we have already looked at the twisting moment. I am just going to show this so that we need these numbers for us to calculate. If I want to calculate the angle of twist, I need to know what is the twisting moment at that cross-section.

So, I am going to show one after another what is happening. This, you already have in your notes. For the purpose of solving this problem, when you want to calculate the angle of twist, I need these numbers, ok? Between the two, angle of twist and shear stress, shear stress calculations are very simple and straightforward. Angle of twist, you have to keep track what is happening along the length. So, we will take up angle of twist to start with first.

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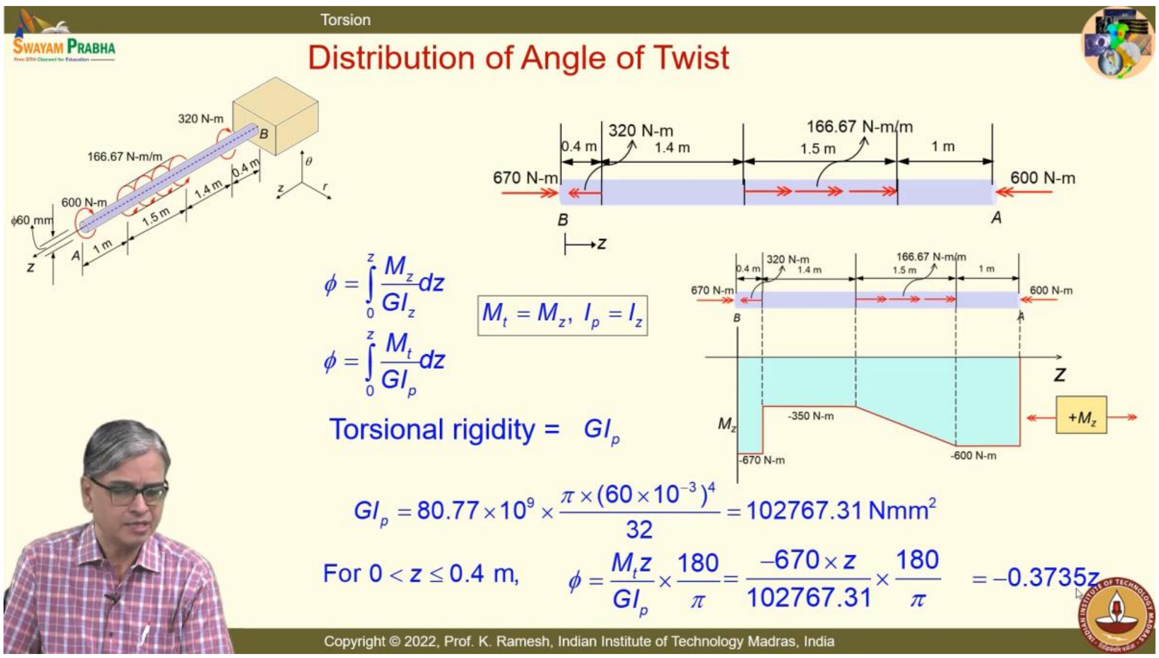

So, I have this shaft shown and we have the expression. See, at some section of the shaft, we need to use the integral appropriately because I have a situation where the torque is varying as a function of length.

The problems we have solved, we have taken a constant cross-section and constant twisting moment. I may need to use this if I have a tapered cross-section where I_z will be changing. I can also have a shaft where I have one step, ok? I can do that independently, but near the step, our calculations will not be correct. So, we hang on to Saint Venant's principle.

Away from the step, I know what are the values on either side, whether it is a smaller shaft or a bigger shaft. All that idea you have to make it clear. And I can also have a shaft constructed out of two materials, like we have seen a composite cylinder. And we equate the strain. Strain will be equated, but stress will be different. All these concepts are identical. So, you can construct variety of problems. So, I said that, you know while writing this expression, we have got it like this with the reference axes. And I said, it is better to put M_t=M_z and I_p=I_z. So, I will use this for doing the calculation.

And I also said you calculate torsional rigidity and keep it. Because in this problem, the entire cross-section is same. There is no change in the cross-section, only the twisting moment changes. So, once I calculate this, I can use it for all other sections. Ok? So, if I use for the first section, it is 0 to 0.4 m. I have the twisting moment as -670 Nm. So, when I substitute this, I get this simply as -0.3735*z*, which is similar to what you have done. Now, I have to move from this section to the next section. I will just indicate what is the catch here and I will stop at that.

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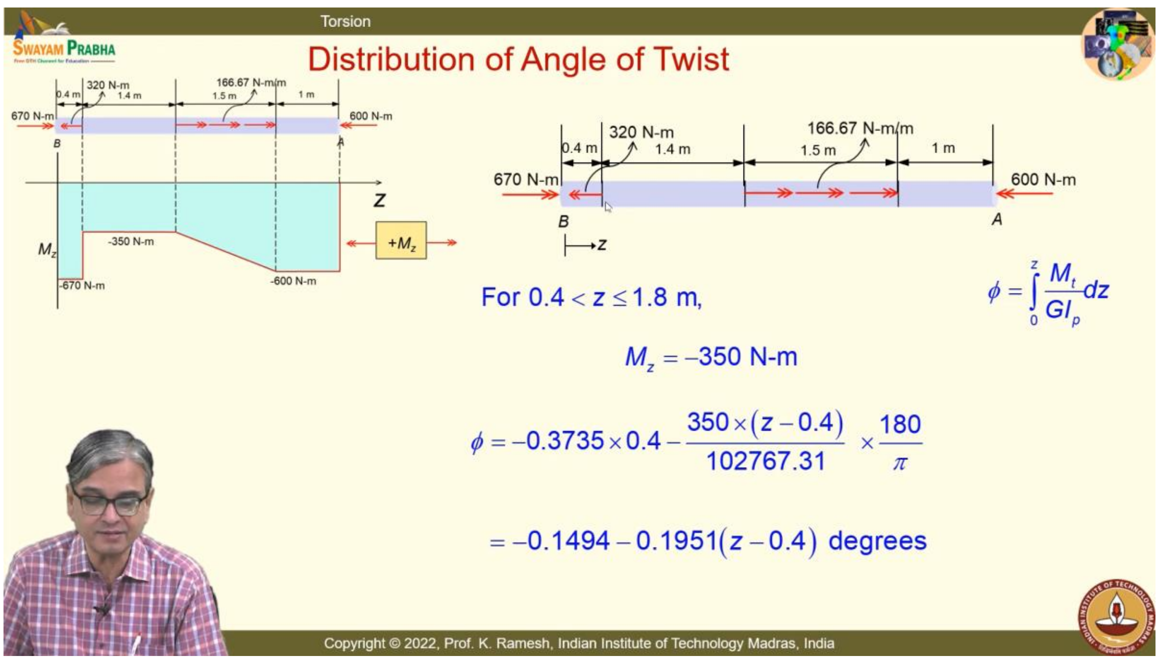

So, here I should have the angle of twist at this end and then bring in what is due to the next one. So, I have *z* is here and then I start from this, that is 0.4 m to 1.8 m. That is why I have a factor (*z* - 0.4) coming in here. Then I calculate this, ok? And when I go to the next one, what I will have to do is, I have to do the integration of this, ok? I will just show that and stop for this class.

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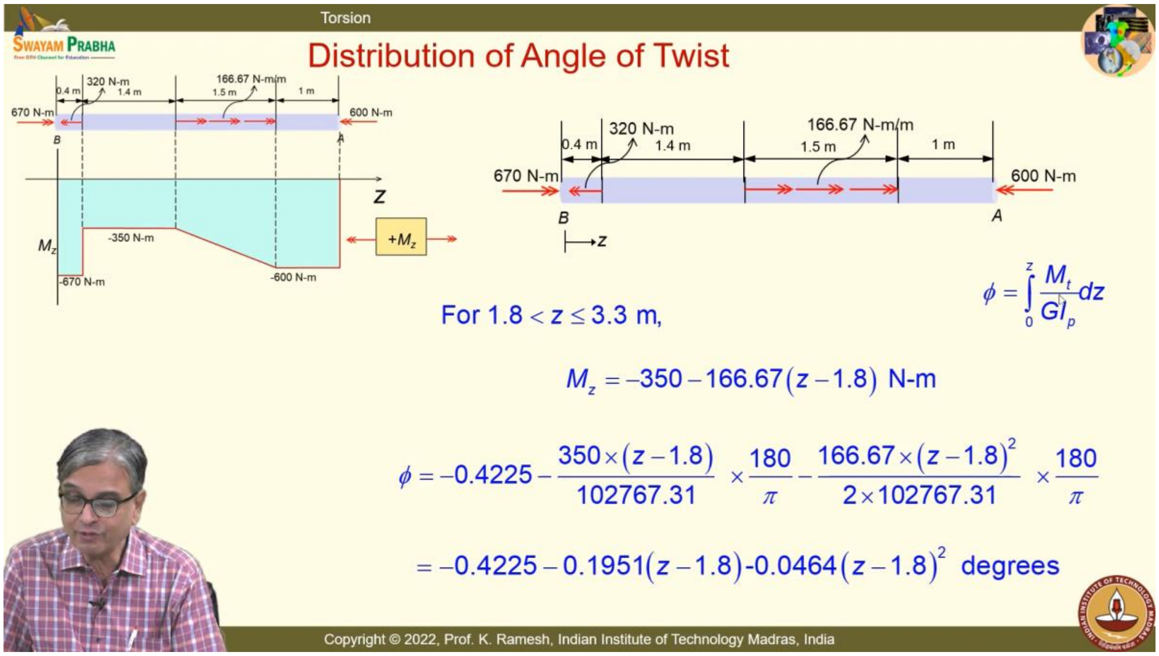

So, you know the expression for M_z. So, I write the initial angle of twist, then this, I have to integrate, I have to do the integration. All along you have not done any integration. Because the moment is varying, you have to bring in integration here.

So, in this class we have solved some interesting problems. We have learnt, if you use the hollow shaft, it is stronger in comparison to a solid shaft. And when you look at any design of a shaft, it is always stiffness based, never strength based.

And we have also discussed, suppose I have a thin slit cut in my hollow shaft, we found the torsional rigidity has reduced drastically. Such sections are known as open sections. So, any open section will have poor torsional rigidity. Thank you.