Strength of Materials
Prof. K. Ramesh
Department of Applied Mechanics
Indian Institute of Technology, Madras

Lecture - 19
Thermal Strain

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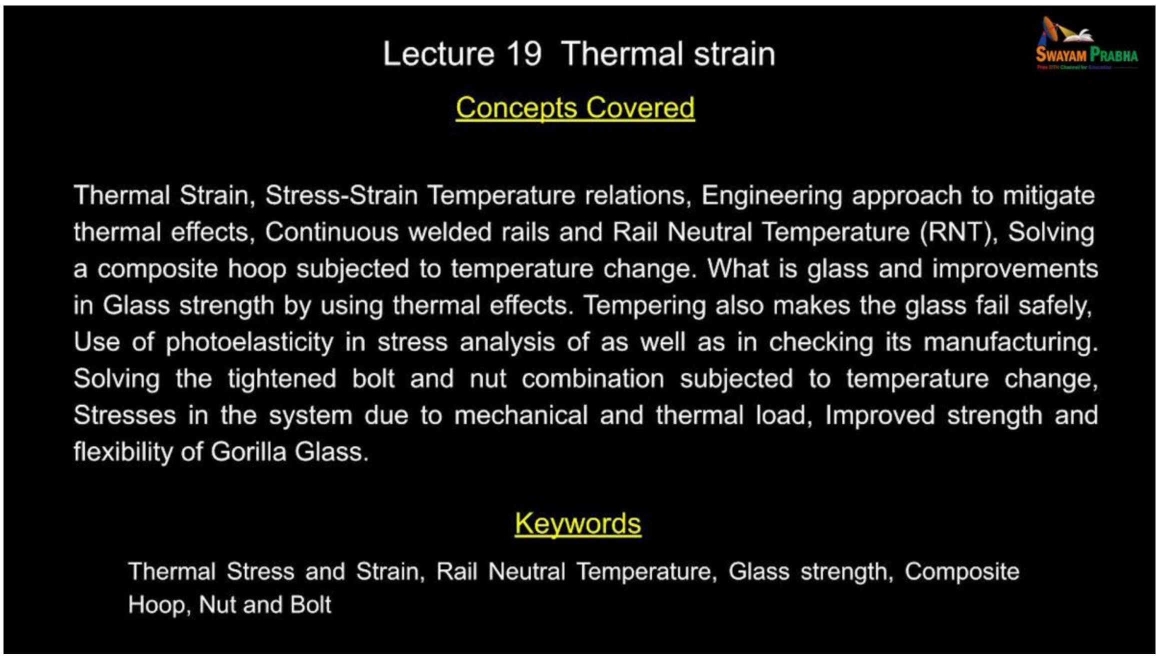

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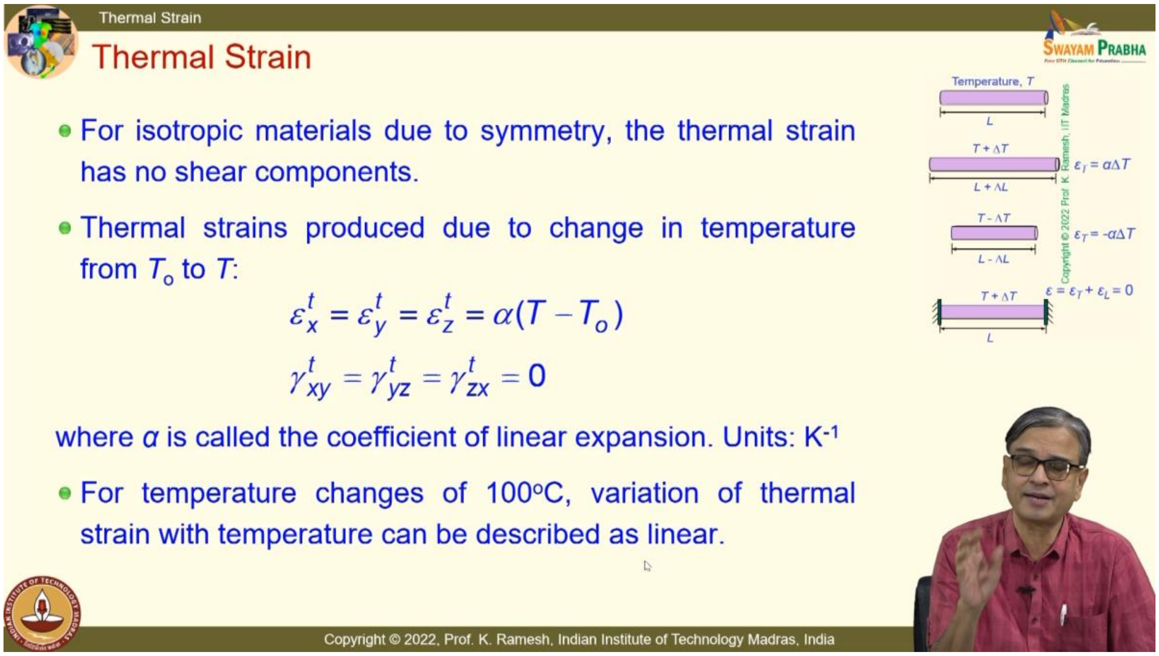

See, while you learn stress analysis, thermal effects are very very significant and we have to see what happens when the structure is exposed to small temperature changes. On the one extreme, you have very high temperatures like what you have in gas turbines and on the other extreme, you have cryogenic engines which operate at very low temperature. Those are extreme environments. We will not get into extreme environment, but we will at least have an appreciation what happens when there is a normal temperature change of the order of 50 to 100 degree centigrade that happens, what way the structure responds, fine. And here, what you will have to appreciate is, we live in a comfort of isotropic materials. So, what is the greatest advantage of isotropic materials? Because of symmetry, every direction is identical from an elastic perspective.

The thermal strain has no shear components. It is a great advantage. So, when I have a temperature change, I would only have normal strains ε_{xx}, ε_{yy}, ε_{zz} to show that this is the thermal effect, either you use T or T_h, whichever way you would like to label it. And that is related to α times the temperature difference.

So, I could have an elongation, I could have a contraction and the problem comes only when I prevent the thermal strain, I get stresses developed. So, you have thermally introduced stress levels. So, the reason why we work on isotropic materials should now be very clear. We need only two elastic constants to characterize. And when we move and look at what happens in the presence of a temperature change, the behavior is also very simple for you to comprehend and analyze.

And α you know, you all have studied in your physics course, it is a coefficient of linear expansion. And this is also very important for temperature changes of the order of about $100^{\circ}C$, variation of thermal strain with temperature can be described as linear. See, we would like to work in linearized elasticity, because that helps you to simplify the problem, split a complex problem into assembly of small problems that can be individually solved and the stress tensor can be added, provided you have the same reference axis, ok.

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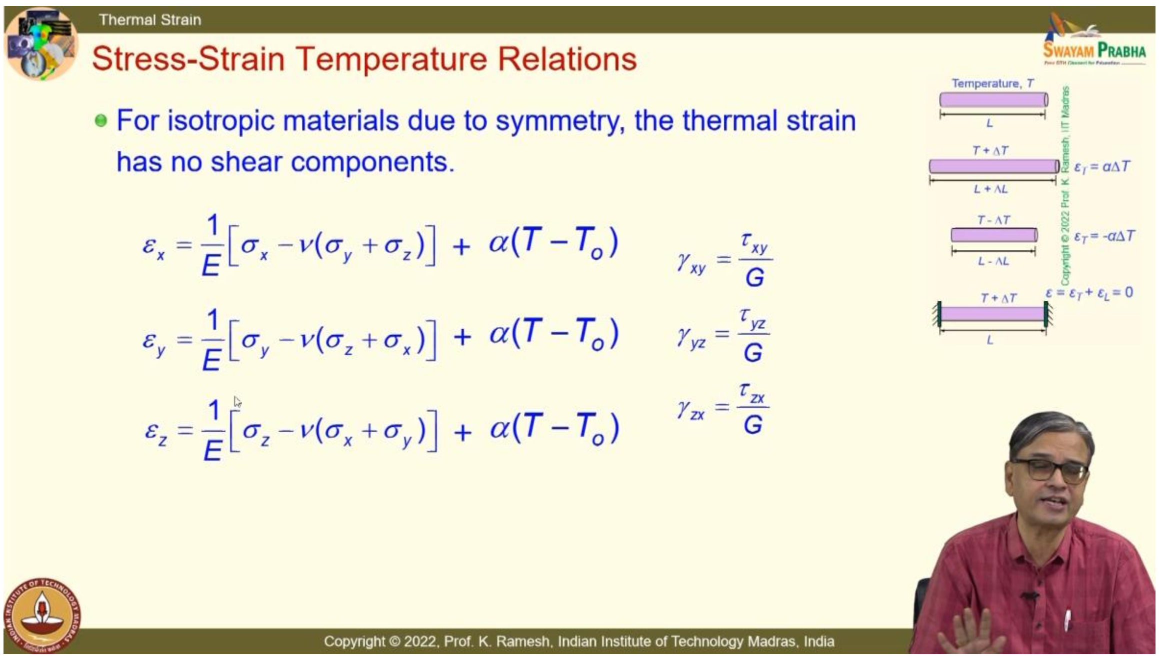

And what happens when you look at the stress strain relation? You have to add one extra term, α times delta t for your ε_x strain component, ε_y strain component and ε_z strain component. And by now, I have sensitized you when you write ε_x, please bring in all the three stress components when you write the expression. For a given problem, identify whichever of the stress components are not existing, knock them off. Do not simply write from your tension test, strain is simply $\frac{\sigma_y}{E}$, that is the mistake that is applicable and valid only for a tension test. It is valid only for a tension test. And fortunately, your shear strain, there is no influence of temperature on it. So, it becomes lot more clearer.

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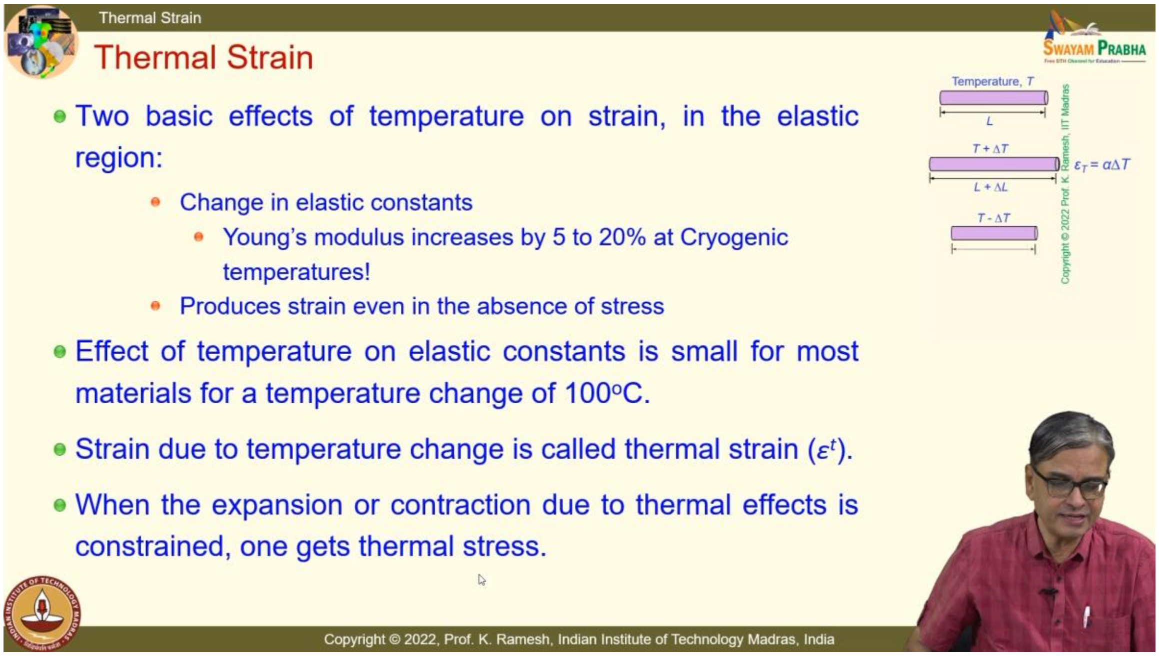

So, if you solve a problem dealing with thermal strain in one stroke, you know how to handle these expressions as well as accommodate the temperature variation, that is what we are going to do today. And you will also have to look at what are the basic effects of temperature on strain. There could be change in elastic constants and you know you have Young's modulus increases by 5 to 20% at cryogenic temperatures. I have told you earlier, it is very difficult to change Young's modulus. Your heat treatment does not modify and your alloying does not modify, but a temperature change can modify.

It happens at cryogenic temperatures and cryogenic temperatures are becoming important. When you have a rocket that you want to send it to moon, you would like to pump in lot of energy and cryogenic engines are very powerful. So, right now we deal with cryogenic temperatures also. So, what you will have to look at is when you measure the strain, you will have to recognize the Young's modulus is higher. So, the stress levels introduced are much higher than what you normally anticipate.

So, this you have to consider when you go to extreme environments. And the other important aspect that we have seen is it produces strain even in the absence of stress and that is used to mitigate when you design a structure, when you recognize that thermal strains are introduced, allow it to expand, do not constrain it. That is one of the ways to mitigate the effect of temperature change that is very well exploited in engineering. And this is also stated when you are talking about changes of $100^\circ\mathrm{C}$, do not worry about change in elastic constants. And as I have mentioned earlier anything related to temperature, you call that as a thermal strain.

When the expansion or contraction due to thermal effects is constrained, one gets in addition to thermal strain, you will also have thermal stress. You have the strain relation, the strain relation takes care of the it has stress components as well as a term related to temperature. So, if we use the expressions properly, you can find out the thermal stresses and thermal strain comfortably.

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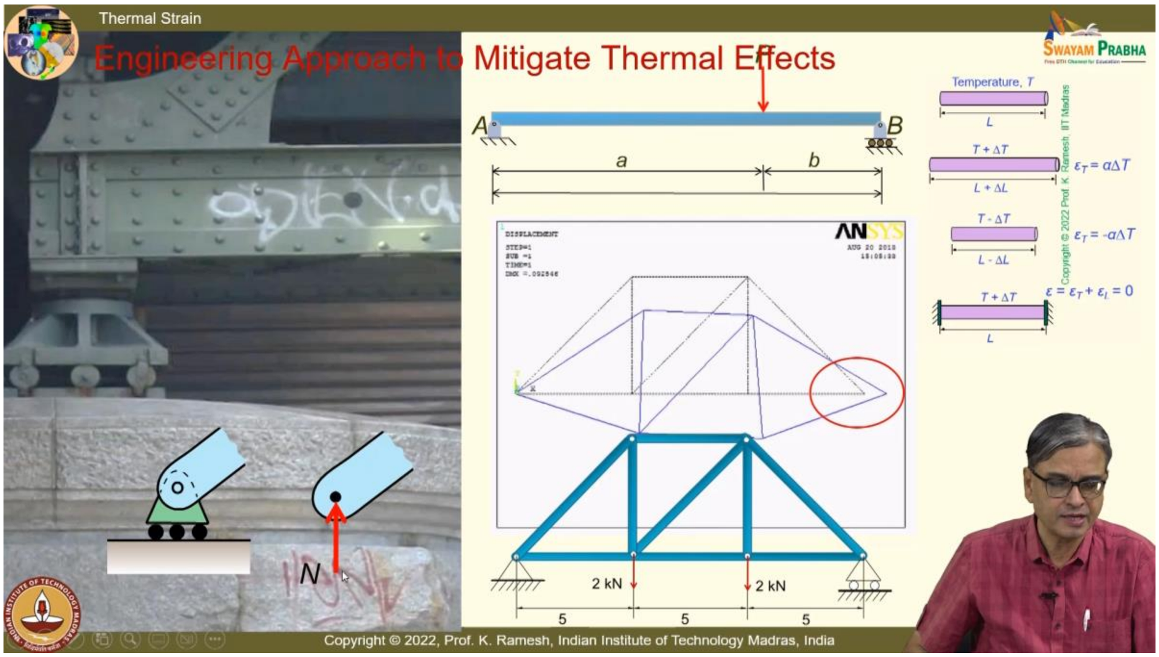

Now, what we will see is what are the ways people have mitigated the thermal effects. You know you may wonder when you are riding a beam, you take one side is simply supported beam, one side is hinged, other side you put a roller.

And if you go to actual bridges, you have such big rollers put, you may not have the opportunity to go and see how they are supported. So, this is practically implemented. And you know I have also shown in the previous class that you have analyzed the truss, when it is subjected to loads, it is an exaggerated deformation. And look at this edge, this edge is moving this way and you have provided a roller. So, you allow the expansion to take place. As long as you allow the expansion to take place, no harm is done in your structure, no additional stresses are introduced. So, it is not only for simplifying your analysis that you have a simply supported beam, it is statically determinate and solve the problem. Even in a practical structure, you can mitigate the influence of thermal effects to an extent possible, fine.

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And there are also other applications. You know before we go into that from your engineering mechanics, you can replace this support as a simple normal reaction. There are also other applications. See this is in Rameshwaram, you have a rail bridge on the sea, also have a road bridge. And I was surprised when I visited that place, the road bridge has this kind of features. So, you have interlocking steel plates that come in and go that accommodate the temperature change. It is very interesting, people have accommodated the thermal effects in the design so that you minimize the additional load due to thermal stress.

And when you go to the rails, conventionally what they were doing is you have rail of particular length and then you have gap in between them and then you have a bolted connection. This was the old type of handling the thermal stresses in the case of rail road. So, this is on a actual concrete bridge like this. So, this is really done and you go to talk to a civil engineer who is constructing, he will know more of it. When you go and visit any place of historical interest, look at the engineering aspects also.

Only when you look at practical structures and ask a question, why they are constructed in a particular manner, what is the advantage? Then you learn some aspects of engineering.

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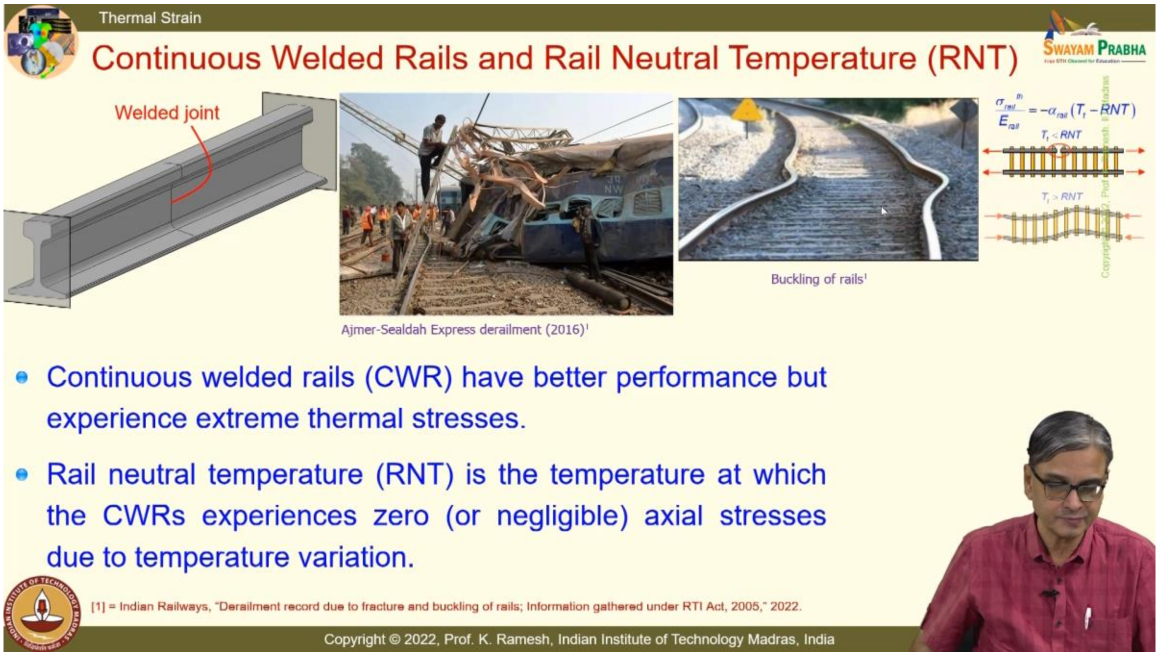

See, the modern rail network is different, you do not have a gap, you have that as welded. The reason is it improves your riding comfort and we have also seen that people are going to have forged wheels and even the life of the wheels can be improved if I do not have gaps there. But when you put a welded joint, these are called continuous welded rails. Structurally they have a better performance, but experience extreme thermal stresses.

You have something called a rail neutral temperature (RNT) that has to be determined, that is a very complicated job to do it, at which the continuous welded rails experience negligible stresses due to thermal effects. See, you often hear there has been a derailment and you wonder whether it is due to a sabotage. Many times people conclude that it is a sabotage. If you really look at from structural point of view, when you have the temperature stresses due to improper management of RNT, it can fracture if I have tensile stress, it can buckle, you have this buckling recorded across railways in the world. Because of these reasons you can have derailment. So, if you have to have a riding comfort, you need to address the effects due to temperature.

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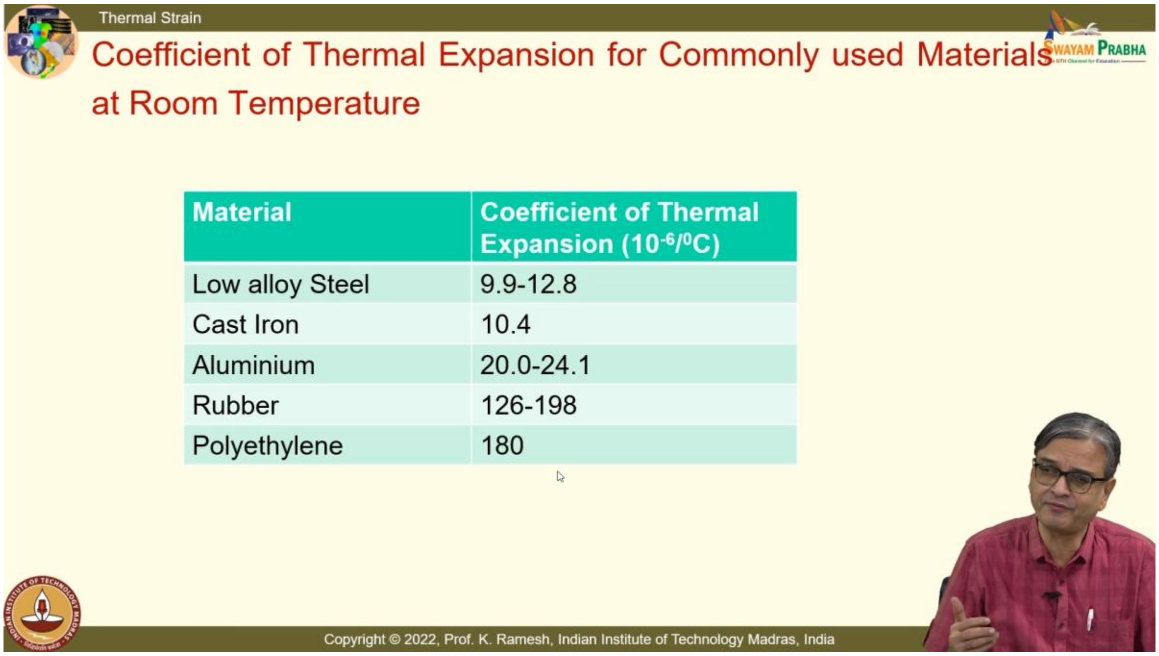

And you have a set of coefficient of thermal expansion, you can take a photograph, you know this is a data, it can help you to solve some problems if data is not given.

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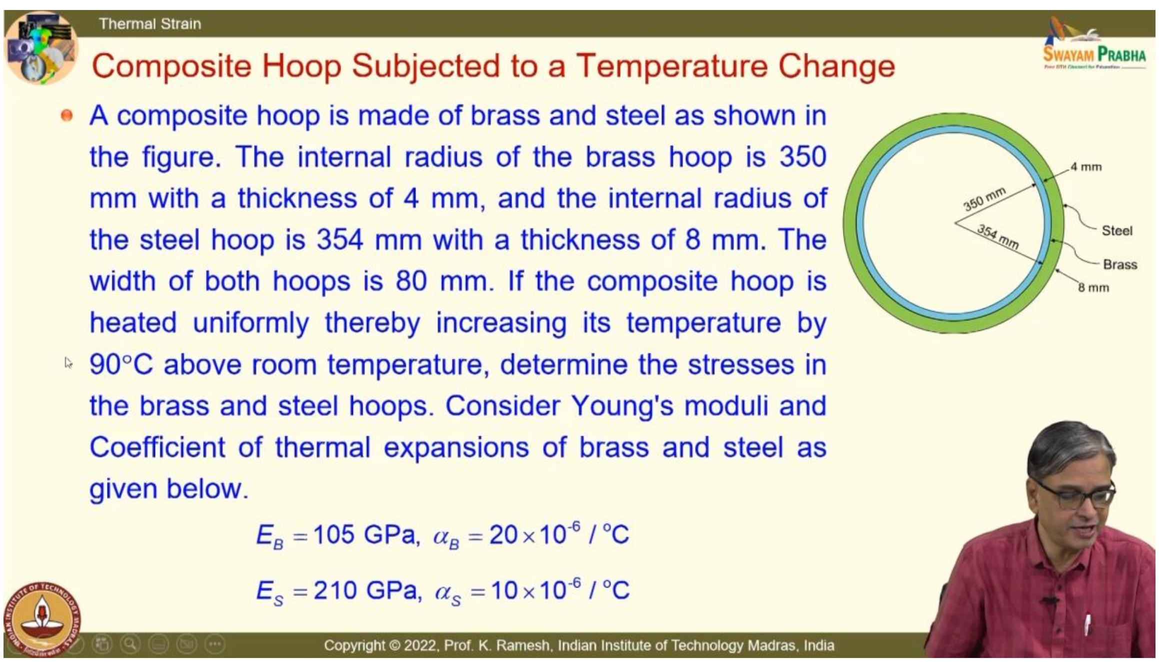

And let us solve one simple problem, where I will only worry about the thermal effects that is easy to start with. What I have here is a composite hoop, this is made of one material, softer material. See, this may be precipitated by a requirement of what fluid that you send through the tube, what fluid you send it through the tube. If it is corrosive, you need to have appropriate material. Depending on the pressure levels that may not be sufficient to contain the pressures, so you need to have additional strength. So, that is what you gain by putting an another tube that is made of steel in this case. We have also seen one such problem, you had a tube made of wooden staffs that is tied by a steel ring, they are all identical, they are all again composite. The idea here is there is a temperature raise of $90^{\circ}C$ and you will have to look at how do you go and solve the problem, what is the way that you should approach the problem.

When there is a temperature change, you find the inner one is made of brass, so that has a higher coefficient of expansion and your steel has a smaller coefficient of expansion. So, the brass will try to expand and steel will prevent it from expansion. Is the idea clear? You should first understand the physics, only when you understand the physics you can translate your understanding into equations and you have to bring in a geometric compatibility, fine. How do you bring in the geometric compatibility?

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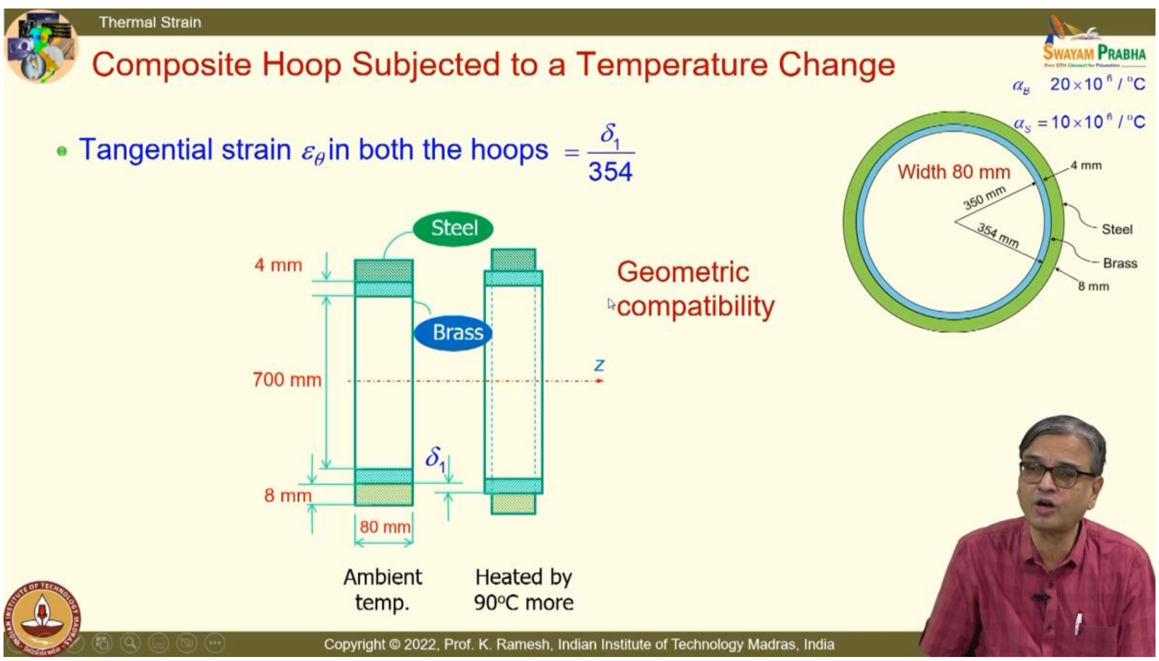

You have the basic dimensions given and when this is heated, what do you anticipate? I anticipate the diameter to change and you know you also imagine that these surfaces are ultra smooth, so that there is no friction and it allows free expansion of brass and you have steel that has not expanded that much because of the lower value of coefficient of expansion. On the radial direction it has expanded, when you have a circle when there is a change in the radius, it also has a change on the circumference, fine.

And because you have pressure developed, the composite hoop still remains a composite hoop, it is not that they come out of it, fine. Because I have this expanding and it can expand more than if there is no steel ring above it, so it will get constrained. Is the idea clear? Suppose I have a material which has a lower coefficient of thermal expansion, what will happen is the outer one will expand, the smaller one will come out of it. We are not looking at a situation like that, we are having a situation the inner one has a larger coefficient of thermal expansion. And what is the geometric compatibility that you can think of? This circumference remains same, ok.

When it is expanded by the inner, by the inner hoop. The other one also expands, but it does not expand to the same level. So, the inner hoop is subjected to compression, outer hoop is subjected to tension. So, in both the cases whatever is the tangential strain that should be identical. Is the idea clear? Before we look at the tangential strain, we will look at what is the way that we can put the, I have put the tangential strain itself.

So, I have this, if I have this as δ_1, we have also taken that interface radius as 354 mm, for that I am directly computing. See when you deal with thin rings, the question always comes should I take the center of the ring or should I take interface radius, because the thicknesses are very very small, the error committed would be very very small, fine. And it is prudent to take, because you have the interface still maintained and it is better to take what happens on that radius. So, this tells you how to proceed and solve the problem. See we have already noted, when you have a pressure applied on a hoop, what is the stresses developed? You have the expression, do you remember that expression? These are all very famous expressions, it is a very famous expression. We have seen, we have also seen that when you analyze it for a coca-cola can, you have in addition to the hoop stress, you have a longitudinal stress, fine. And there is a standard expression for that, please try to recall that before I come to the next slide.

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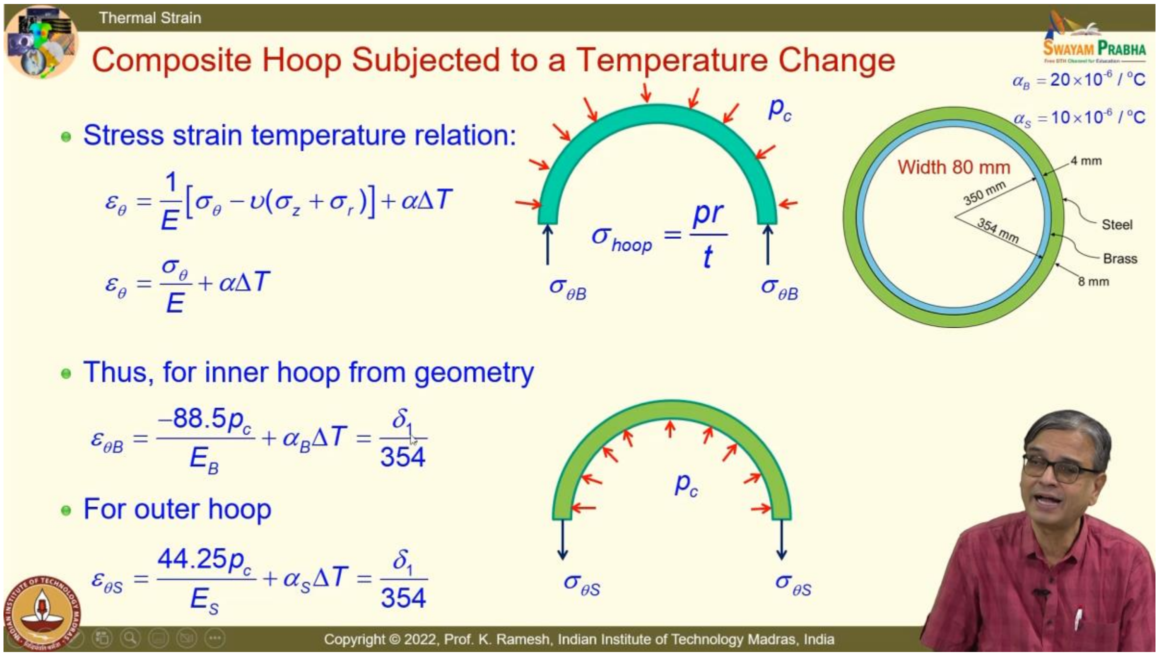

So, I have the stress strain temperature relation, I write it faithfully. So, it has a component from stresses and also a component because of the temperature change. And in this case, I have only $\sigma_{\theta\theta}$, because I have a thin hoop, I do not have σ_z and σ_r.

And we have already derived hoop stress is nothing but $\frac{p\mathrm{r}}{t}$. We do not have to redo the same calculation again, you can take this as a starting point and proceed further. And let us look at what happens to the inner hoop. So, I can substitute what happens to this value of σ_θ. And we have already understood that inner hoop is constrained by the steel hoop, so it develops compressive stresses.

So, I have this as when I put $\frac{p\mathrm{r}}{t}$ and that pressure is taken as what is the interfacial pressure, I have a call this as p_c, this is a composite hoop. So, this is because of the construction, you get this pressure. When I have the temperature change, this pressure gets developed. And you have the term related to $\alpha_B\Delta T$, this is for the inner hoop and the force balance is also properly put the way $\sigma_{\theta B}$ is shown. And what happens for the outer hoop? Outer hoop expands, so I have this as positive, I have $\alpha_S\Delta T$ that also goes to $\delta_1 / 354$.

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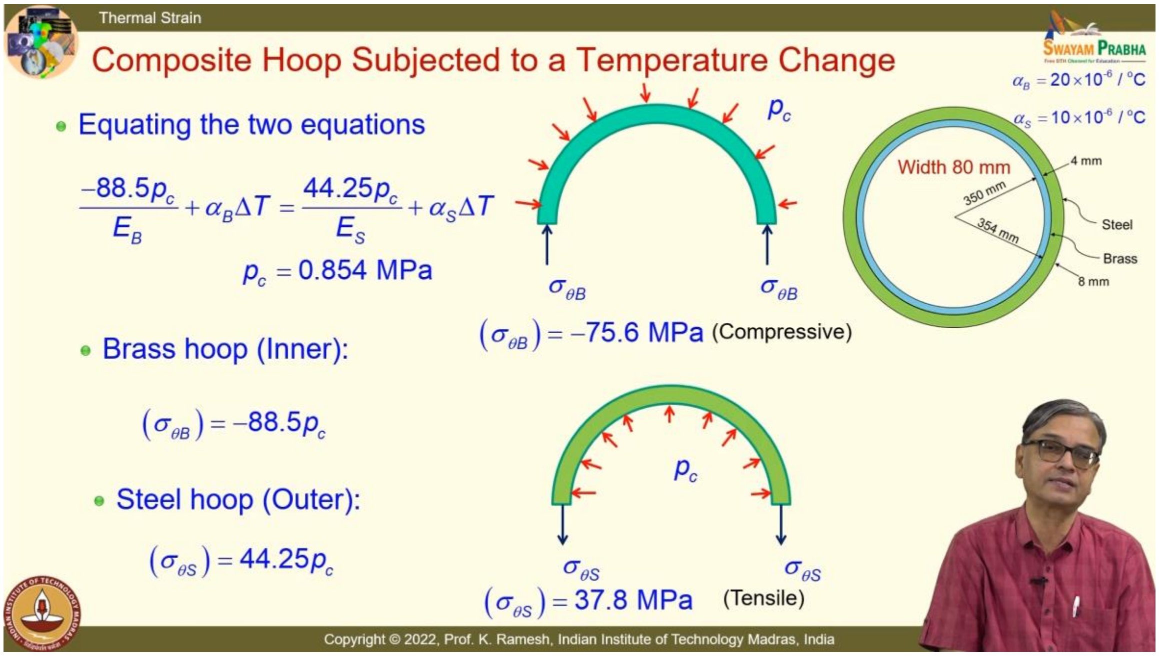

From geometric compatibility, these two should be identical, you have that expression and you get the interfacial pressure as 0.854 MPa. So, if there is no temperature change, p_c would have been 0, because there is no internal pressure acting on the composite cylinder. I can also think of a problem, where a fluid is going at a particular pressure and it is also subjected to a temperature change. And you all know we live in linearized elasticity, solve this as two problem and sum them up, fine.

So, once you know the interfacial pressure, the pressure that happens on the interface of the brass and steel, I can compute the stress on the inner hoop, I can also compute the stress on the steel hoop, please check the computation and then tell me that my numbers are alright. I get $\sigma_{\theta B}$ that is the brass hoop has -75.6 MPa. And when I have a steel hoop, I have the pressure as 37.8 MPa. See, it is a very simple problem that illustrates, how do you handle the temperature change? We started with writing the strain stress relation and also the temperature effect completely and knocked off the term, that is the right procedure, do not jump steps, ok. Now, I am also going to ask you one more thing, see you when you go to a mall these days, you see glass everywhere, am I right? Where you were having steel railings, you have all of that barricaded by glass. Now, I want to ask you, see you have these two benches, in these two benches I put a glass plate, fine. And then in the other set of benches I put an aluminum plate and I want some brave hearts to come and stand on the glass and stand on the steel or aluminum. Everyone will go and stand on the aluminum, you know very well that is nothing is going to happen, fine.

But even if you fall it is only one or two feet, why do you fear that you will not go near glass? You have inherent feeling glass is brittle, it cannot withstand, is it not? That is what you have a feeling.

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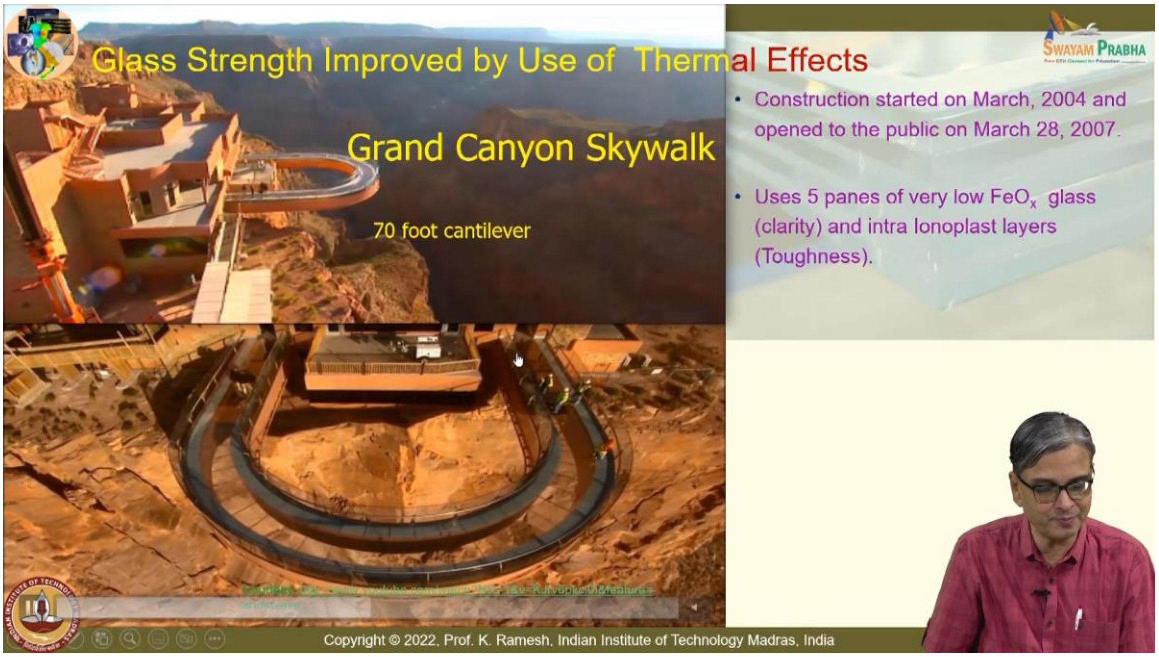

Let me show a counter example, in the Grand Canyon you have a 70 foot cantilever made of glass, you can walk on it, you can see it is not one feet or two feet like what you have your bench and people can comfortably go walk on that and see what happens may be 300, 400 feet deep below. Do not you think that people have tamed glass? See when you say strength of materials, it is not that you look only at failure, you also look at how to utilize the material, why glass is important? Glass is a green material. See, you are in a generation where environment friendly construction processes and practices have to be implemented. When I talked about cork, I said it is naturally available material, it is also green, it is replenishable and glass is also replenishable.

If there is a concrete structure, if you would demolish it, you do not know what to do with the concrete debris, you can only use it as a landfill or something like that. Whereas, glass if it is broken, you can melt it and reuse it.

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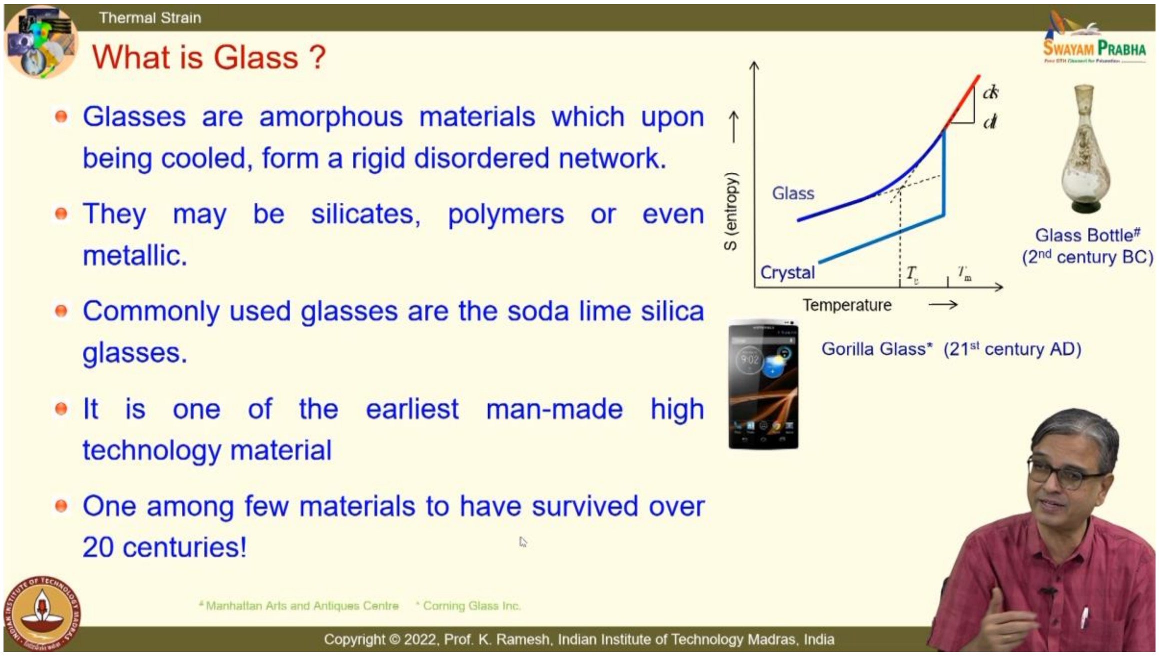

So, people have tamed glass and what you have is you have five different layers are put and in between you have a plastic layer that absorbs shocks and things like that, people also have improved the friction and it is in operation. So, that means, people have understood glass what is glass Please make a sketch out of it. See there is a difference between if you take aluminum and then make it as a take it to high temperature and then cool it, it will cool at one temperature.

On the other hand, when you have a glass, it will cool over a range of temperatures and it becomes an amorphous material. And there is also a very interesting temperature, this is called glass transition temperature put as T_g. And you can have glasses out of silicates, it can be out of polymers or even metallic glasses you have. Commonly used glasses are soda lime silica glasses. And surprisingly this is one of the earliest manmade high technology material. You have a glass bottle which is traced to second century BC and all of you flank your beautiful cell phone and drop it 100 times, nothing happens to it, fine. And why it is because of gorilla glass that we have, it is by Corning and it is one of the few materials to have survived over 20 centuries. And glass is also used in bioreconstruction, fine. If you have any fracture or something like that, they have some kind of scaffolding made out of glass.

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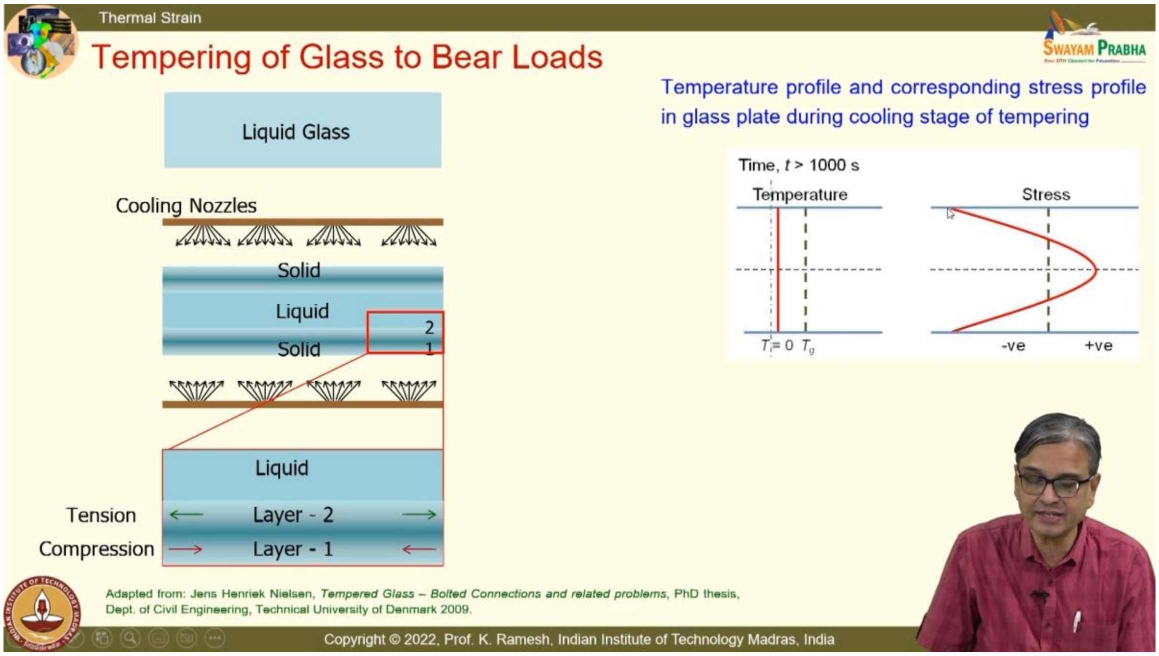

And you temper the glass, you introduce intentional stresses while glass is being fabricated. And what you have is, you have a liquid glass and then you have cooling nozzles. So, I have a liquid, so this gets cooled first and so you have this will prevent the other this one to come back to its original position. So, it will you have a compressions compressive stresses developed on the outer layer and then you have a tensile stress developed. And as a function of time this is what it shows. Finally, you have a stress state after the tempering process parabolic like this, it is highly compressive on the surface and tensile at the center of it.

You intentionally make it, fine, while you process the glass because people have understood how to tame the glass.

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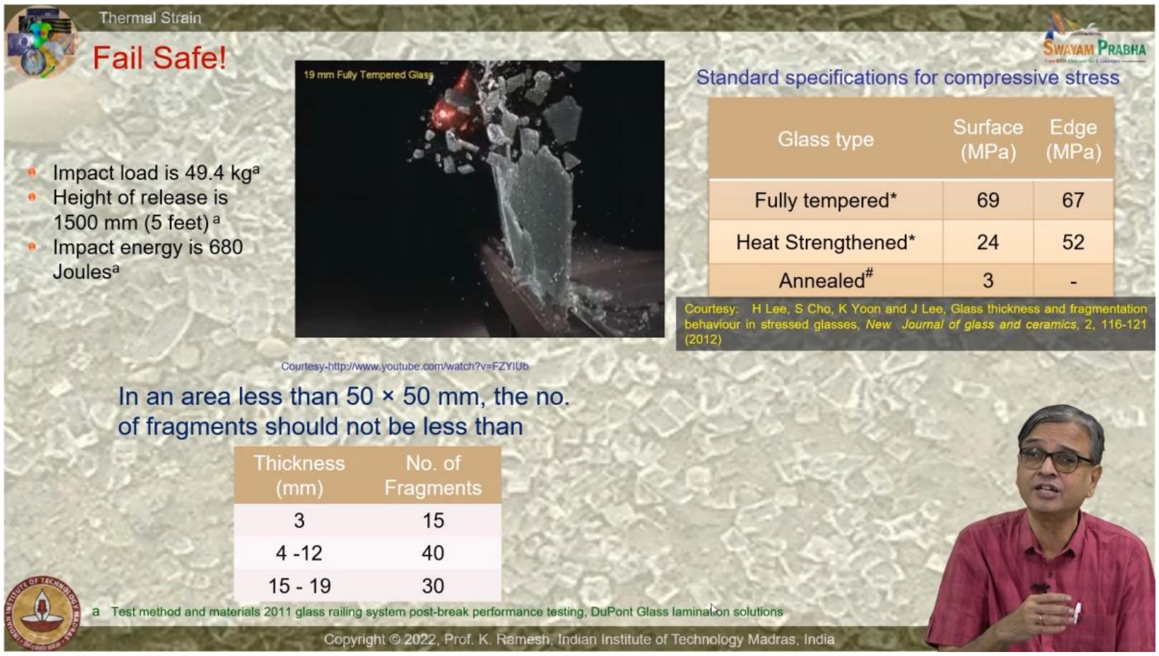

And not only this, you know when you have failure it should safe, fail safely that is also another important design perspective these days. Failures do happen, but let the failure happen safely. So, when you temper the glass like this, there are also standards that say for different thicknesses of glasses, how many fragments it should break into and what should be the level of stresses developed if it is fully tempered, heat strengthened, what should be the level of stresses developed. So, the advantage of tempering a glass is, it improves the functional aspect of the glass, but even if it fails, you get small particles.

You would have seen when there is a road accident you see pulverized glass on the street, you would be wondering from where the glass has come. If you have that windshield whatever that is there, it is all tempered and they make it intentionally to get into fragments, so that it does not harm you. Once you say it has to be tempered, it has to develop a particular level of stress, you must also have a measuring system to measure this. You have to ensure that, is the idea clear?

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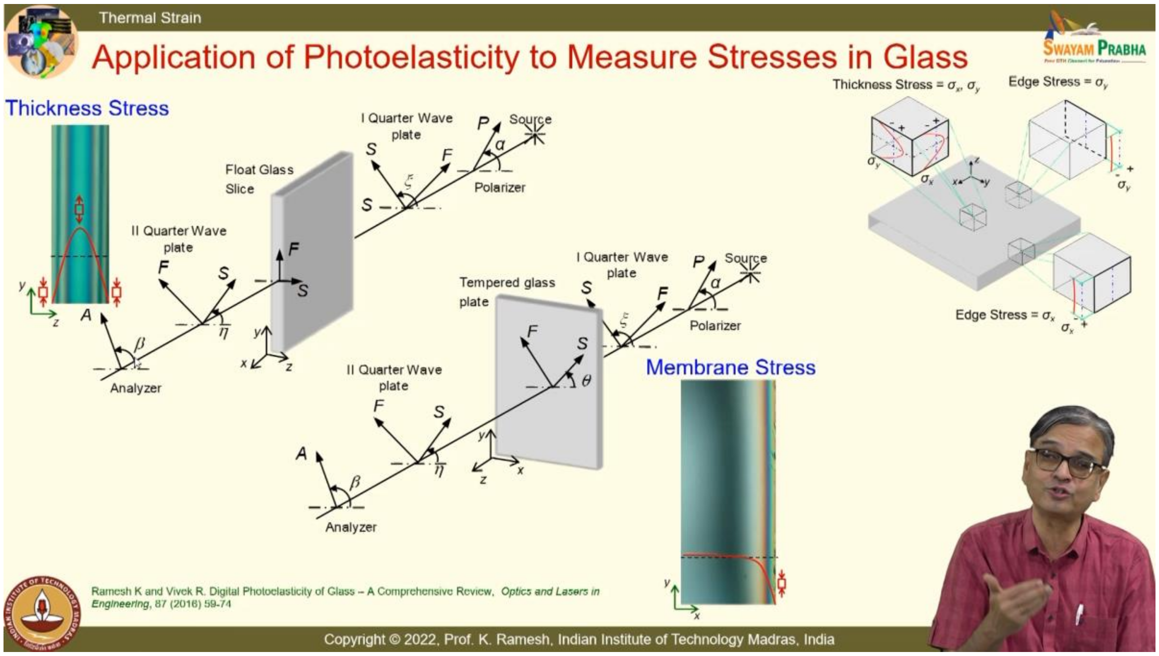

And you find photoelasticity fits in very well because glass is photoelastically sensitive and we have seen that by tempering it has a parabolic variation where you have a compressive stress and then at the core it is tensile, which you can easily measure if you align the glass plate in this manner in the polariscope. If I align the glass plate perpendicular to that, I get what are known as the edge stresses, you have the fringe patterns.

See the photoelasticity what we are looking at, it is helping you to understand solid mechanics and it also meets the current requirements of the modern society because we are going from homogeneous to heterogeneous materials. We have seen that photoelasticity can be applied for analyzing concrete by using a CAT scanner, you can generate a complex three-dimensional model with pores and all the pebbles aggregates put inside and you can analyze what happens. So, you can go to heterogeneity and we have also looked at when you have a soft material like a skin where they want to do the epidural injection. Soft material means you need to go for finite strain, there again photoelasticity is applicable and you find glass is a green material and people want to understand glass and also manufacture it properly. So, I should know how to measure it and I should also monitor it while I manufacture it.

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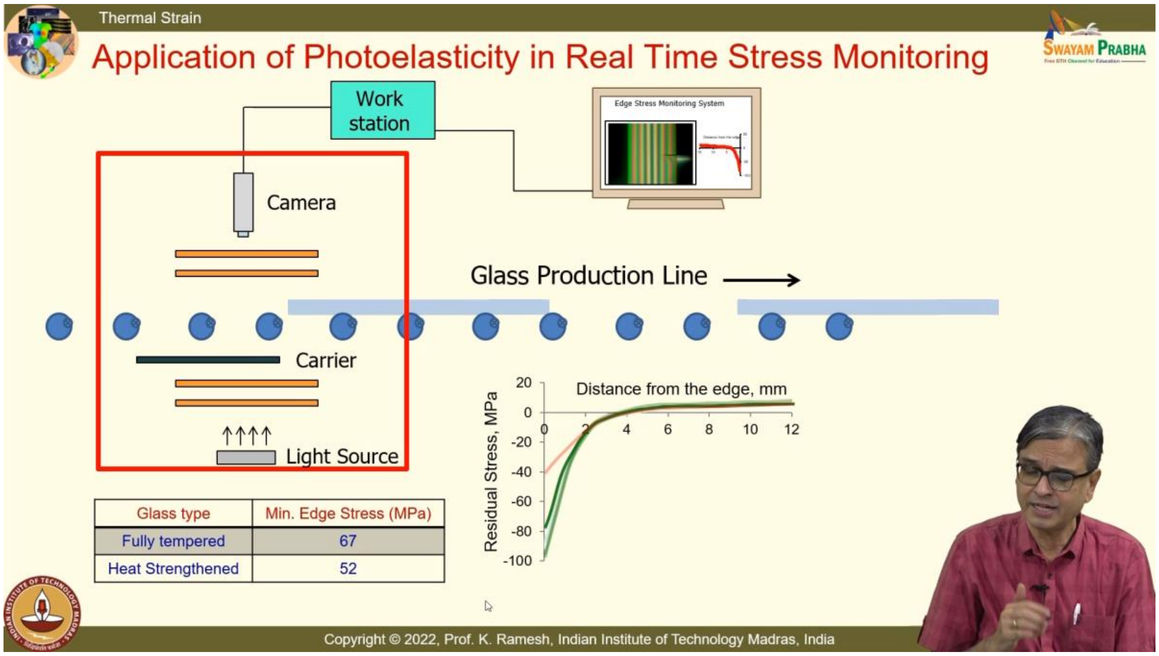

So, I can have an online system where I would look at the digital image and have a software to evaluate the edge stress developed. So, I can accept whether the process is going on properly, whether I can accept that batch of glass or not. So, right from measurement and also in manufacturing photoelasticity helps, fine. Then we will also solve one interesting problem.

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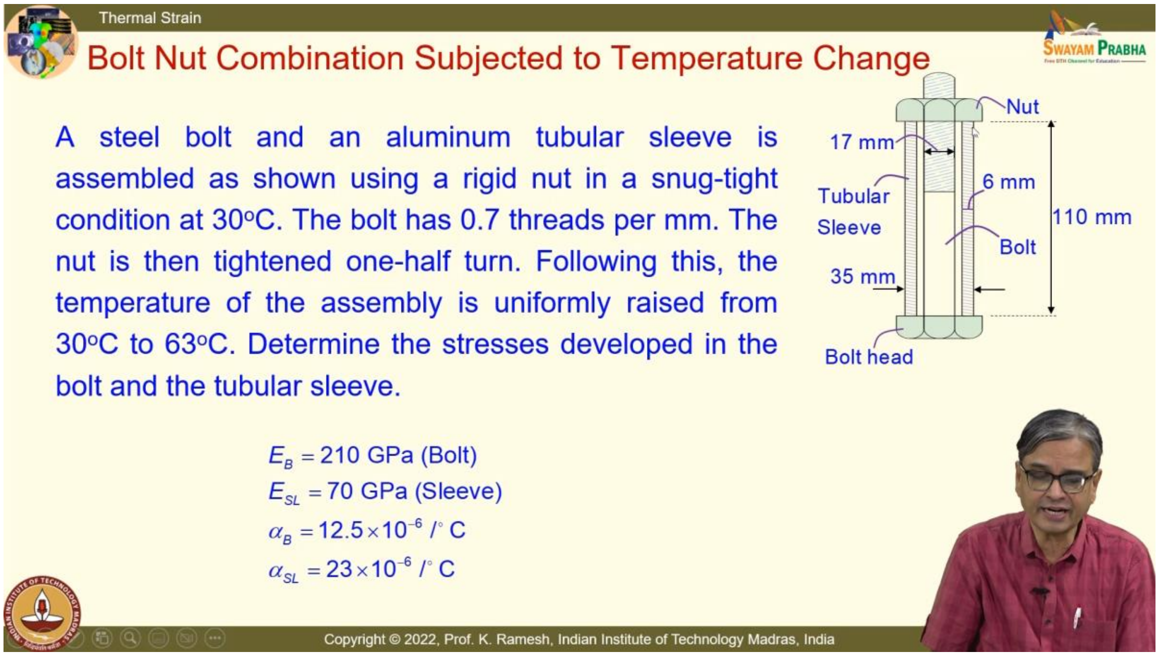

I have a bolt nut combination subjected to temperature change. You know I have a bolt and I have a sleeve, you have the geometry of the system is shown here. And you know as engineers by now you should have understood why I have threads. When I have threads I can have pitch, I can have multiple threads, I can have lead, all that physics you should know. That is if I rotate in one turn, how much it will move in the horizontal direction, ok.

Linearly how much it moves that you can find out from this data. And what is seen here is initially the bolt nut is tightened by a half a turn by snug tight, the nut is then tighten one half turn. So, that is also given, you are also given the information to calculate what distance it can travel. And this is very common, see if you go to north India, the temperature changes from morning to afternoon quite a bit in winters, it can have 10 to 15° is very common. So, you have several systems even in your automobiles there are your engine is fitted into your chassis with some bolt nut connection. So, you have this kind of a practical situation, a bolt nut is connected and then they are tightened and they are exposed to a temperature change.

So, it is a very common occurrence that needs to be handled. And you are given the elastic properties of the bolt as well as the thermal coefficient of expansion. How do we go and solve the problem? In this case I have mechanical stresses introduced as well as temperature effects have to be considered. And I need to know what is the final stresses developed and we live in the world of linear elasticity. So, solve this as two problems, one is a mechanical problem, another is a thermal problem and finally, add up the stresses, is the idea clear? Ok. And we have discussed about this problem long time back when we discussed force deformation.

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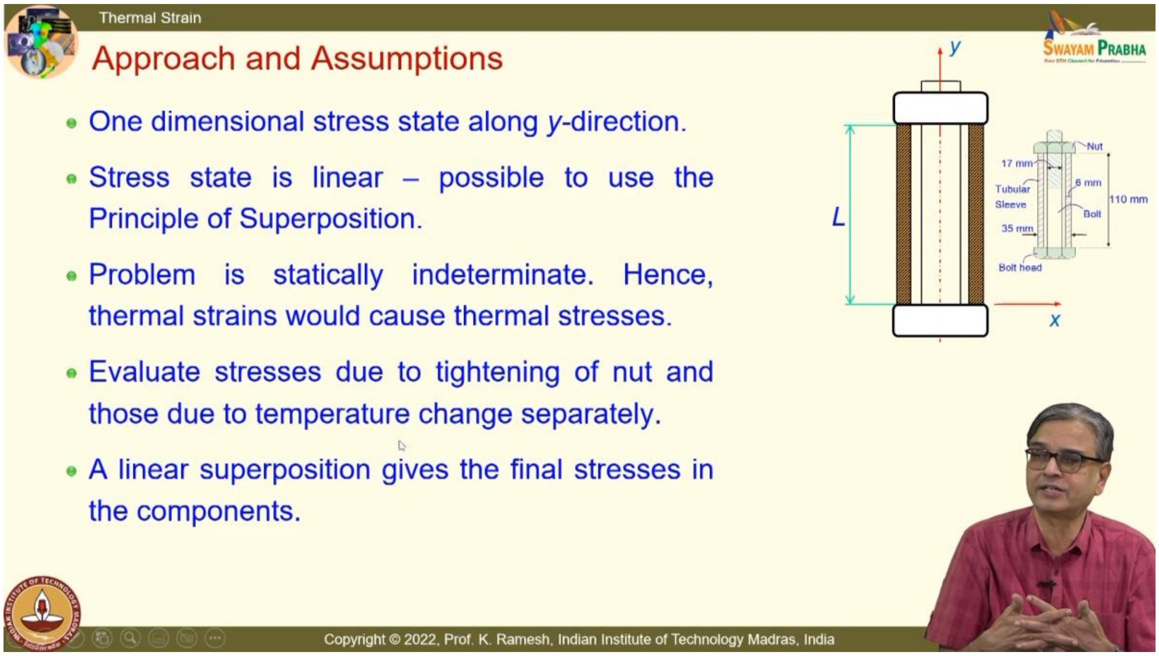

And when you have a nut tightened what happens? How do you generate the geometry compatibility? See whatever I have said is summarized in this slide, I have the linear elasticity. So, I can apply principle of superposition. You will have thermal stresses developed because there are constraints, it is a statically indeterminate problem. So, you have constraints because of that thermal stresses are introduced. So, evaluate stresses due to tightening of nut separately and those due to temperature separately and add them together, you can do the principle of superposition.

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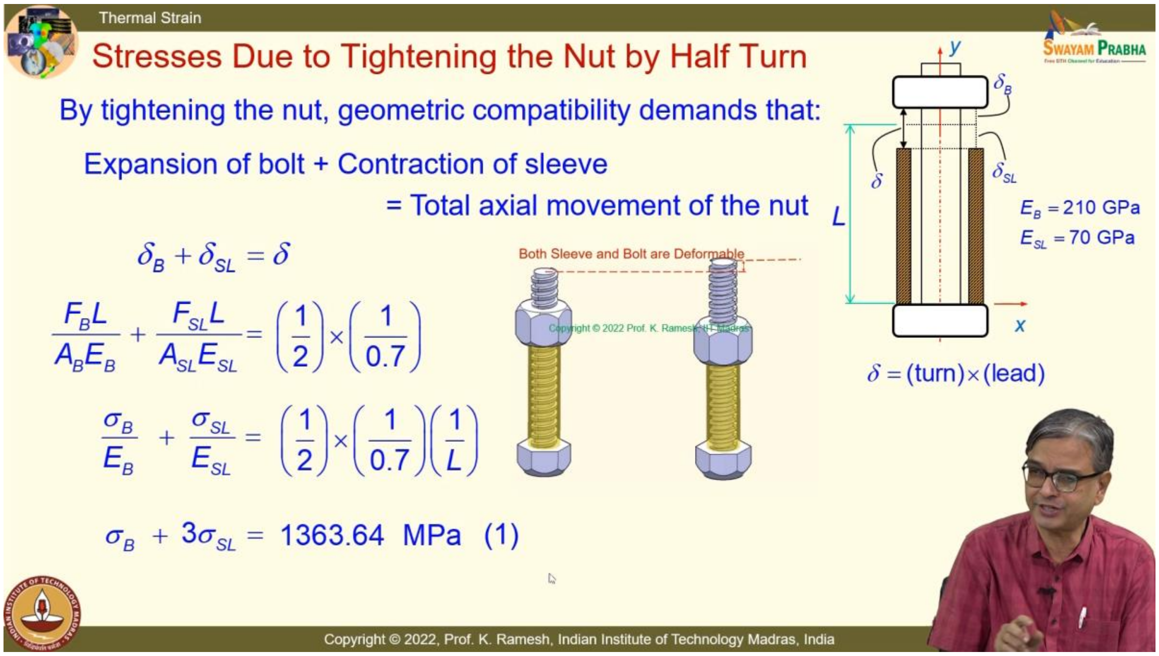

Now, we will first take up the tightening of the nut and what is the geometric compatibility? See, if you remember the discussion, you can very well go ahead. But even if you do not remember, I gave you a methodology how to get the geometric compatibility. You visualize of the two members that are interacting, you make one of them as rigid, another as deformable. Then understand what happens in extreme situations, from that you hypothesize and find out what could happen when both are deformable. Is the idea clear? That we have seen, if it is sleeve is rigid, whatever the moment of the net is determined by the elongation of the bolt.

If the bolt is rigid, whatever the moment is done by the sleeve, when both are deformable, I have elongation of the bolt as well as compression of the sleeve. See, while you write all of this, you must keep track of what way you are putting the signs, ok. We are looking at it physically and then we are looking at this as expansion and this as contraction and we are adding them. When you add them, that is the moment what you could get by tightening the nut, fine.

So, you should understand that and write your equations appropriately. So, I will have A' $\delta_1 + \delta_2 = \delta$. So, when I write this expression, I will have to recognize that I am talking about what is the contraction. So, I have $\delta_B + \delta_{SL} = \delta$ and you know for a axial load, we have learnt this as $\frac{PL}{AE}$, where P is the load and in this case, F_B is the load, L is the length that is the length that is shown here and you have details of what is the cross section that you can easily find out. A bolt is a simple circular cross section, your sleeve is an annular cross section. So, you can easily calculate from your geometry, that is not a big job and you are also given what is the pitch. So, I have 1/0.7, I put it so that I get the axial length it can move and I have rotated by half turn. So, I know what is the value of delta. So, I get one equation from this. It can also be simplified if you substitute the numbers, I can also get a simplified expression and that gives me $\sigma_B + 3\sigma_{SL} = 1363.64$ MPa.

Please substitute the mathematics, substitute this cross sectional details and check whether this equation is alright, please do that verification. If there is any change, please alert me. See, I have two unknowns, I need two equations to solve, isn't it? I have brought in one equation based on the geometric compatibility, other equation comes from simple equilibrium, ok.

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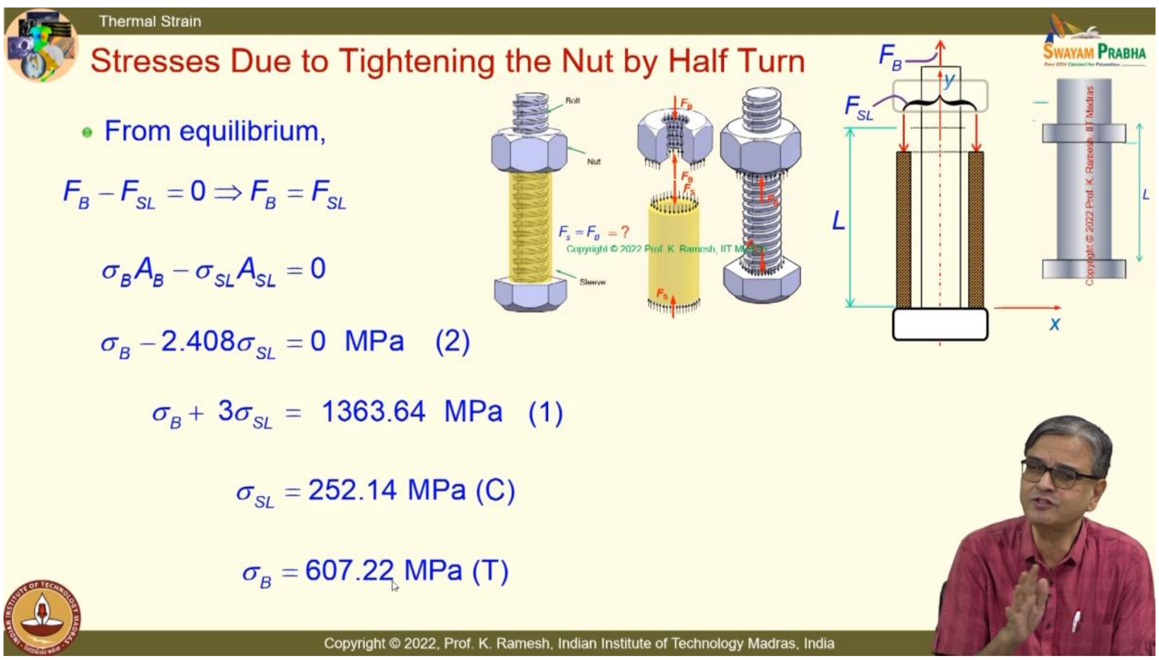

That also we have seen, when I have a bolt and nut tightened, we have said the forces on the sleeve and forces on bolt should be identical, but we do not know how to calculate based on statics. You need to bring in deformation into the picture and in this case, we have first looked at the deformation, wrote the equation and this equation is simple and straight forward. I will have to have F_B equal to F_{SL} and then I can substitute the values in terms of stresses and the geometry of cross section. So, I get the second equation. The second equation reads as $\sigma_B - 2.408\sigma_{SL} = 0$. So, I have two equations, I have two unknowns, it is simple to solve, it is quite simple to solve.

And when you solve this, I get the stress on the sleeve as 252.14 MPa and on the bolt as 607.22 MPa, fairly straight forward. This became fairly straight forward mainly because we have already spent some time on how to look at the geometric compatibility. Since we have looked at what happens when we are tightening the nut, the discussion was simple and straight forward for you. So, you generated an additional equation. So, with two equations, you have been able to solve the two unknowns.

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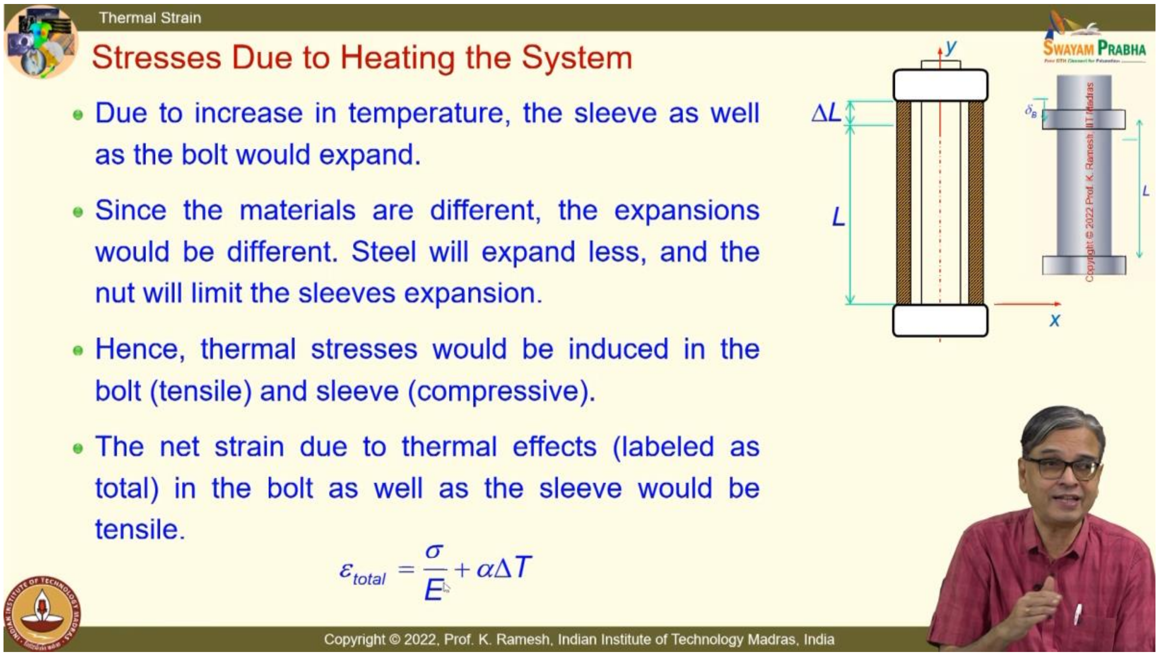

Now, we will go to the thermal part. See, this is a simplified representation. You know, I have not shown the bolt, I have just shown to visualize because of temperature change, what happens when there is an increase in temperature, bolt also will elongate, sleeve also will elongate, fine. So, that is what is shown here. I have a nut which is moving up, but you find the coefficient of expansion of the sleeve and bolt are different. Sleeve has a larger coefficient of expansion. So, it will try to elongate more, but you have a nut which is sitting on top of it. So, nut will not permit the sleeve to expand to its full length, ok.

Because of that, you will have stresses developed in the sleeve. What is the nature of the stresses developed in the sleeve? Compressive, that you should recognize. On the other hand, if you look at what way the strain happens, the strain is, it is only expanding that is elongation. That elongation has two components. One component comes from thermal effect, the other component comes from stress, but for the sleeve it is getting compressed.

So, that you should recognize when you write the expression. In the case of a bolt, I have again the thermal effect, ok and bolt is subjected to tension. So, you should recognize the difference and write your expressions properly. So, that what I have discussed is summarized here. Since the materials are different, the expansion should be different.

Steel will expand less and the nut will limit the sleeve's expansion. And we have also said, there are two ways of looking at it. It is said in the problem itself nut is rigid. On the other hand, even if you consider it as deformable because the thickness is very small, any change in the length you can ignore it as a second order effect. So, in essence you are treating the nut as rigid. And because of differential thermal expansions, thermal stresses would be induced in the bolt. Bolt it is tensile and sleeve it is compressive, ok. And in the subsequent discussion, we will put the net strain due to thermal effects labeled as total in the bolt as well as the sleeve, ok. And you write the expression completely. When we look at stress strain temperature relation, write the stress strain temperature relation completely.

That means, do not write only $\alpha\Delta T$, write the stress components too. Because the problem is simple and I have only axial stress, I have in only one direction, I have directly put $\frac{\sigma}{E}$. The message here is, you should not ignore this. You should write stress strain temperature relation when there is an interaction like this, otherwise you will make a mistake.

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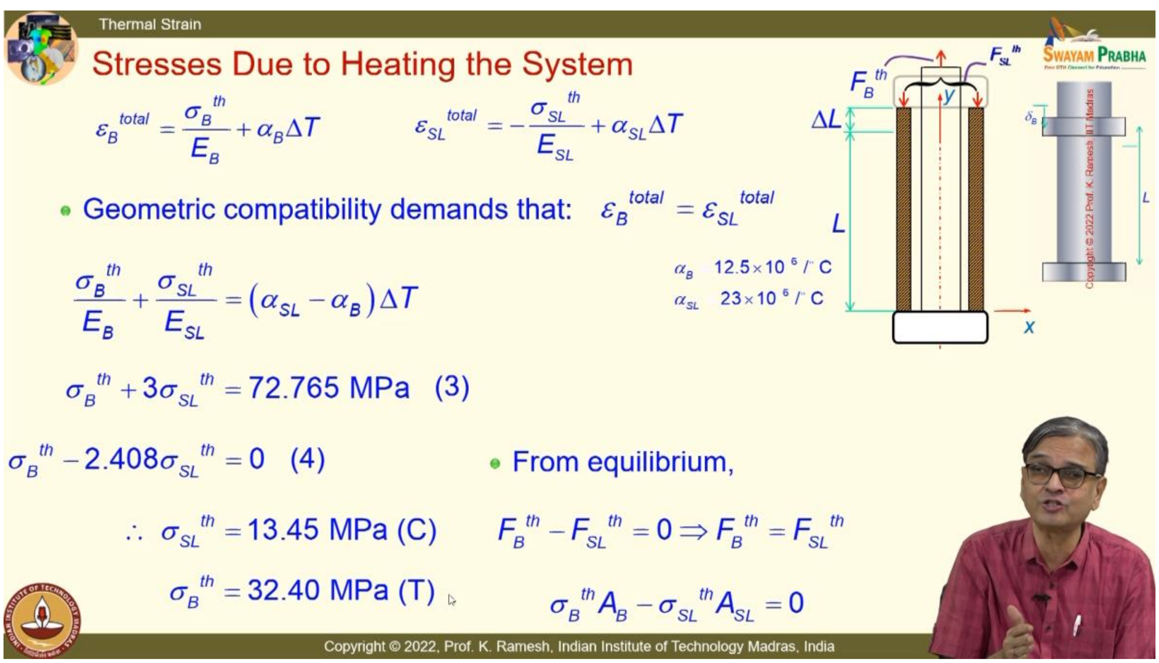

And we have already seen that ε_B^{total} is like this, it is tensile and for the sleeve, see this is compressive, you must bring in this minus sign here. See the net strain is tensile, but it has a component from stress which is compressive, but the elongation due to temperature is much larger than this. So, the addition of these two finally, give only tensile strain and from the geometry, what I have as ε_B^{total} should be equal to ε_{SL}^{total} . Is the idea clear? That is the very important geometric compatibility that you should bring in. Once you do that, rest is simple arithmetic, there is no difficulty in handling that. So, I have this as

$$\frac{{\sigma_B}^{th}}{E_B} + \frac{{\sigma_{SL}}^{th}}{E_{SL}} = \left(\alpha_{SL} - \alpha_B\right)\Delta T$$

And you are given in the problem statement what are the thermal coefficients of expansions. And I have this as $\sigma_B^{th} + 3\sigma_{SL}^{th} = 72.765 \text{ MPa}$. And what do you do from equilibrium? See equilibrium also you have to look at it. So, I have a tensile force acting on the bolt and I have the compressive force acting on the sleeve and they are equal and opposite. So, I have two equations and I can easily solve for what are the stresses in the bolt, what are the stresses in the seal and this is because of temperature change.

This is not the complete stresses developed on the system because the system was initially tightened by a nut, then a temperature change was given. We have analyzed the problem at two stages and when we substitute these values, please do the arithmetic at home and alert me if there is any change. So, I have the second equation as $\sigma_B^{th} - 2.408\sigma_{SL}^{th} = 0$. So, solve these two and get the stresses. I have the stress on the sleeve as 13.45 MPa compressive and sigma b theoretical as 32.40 MPa tensile. So, now if I have to find out the total stresses, this is where we use linearized elasticity. If the material behavior is non-linear, you cannot add because the material behavior is linear and also the temperature change is small enough so that the temperature change also can be considered as linear. I can add them and simply solve the problem.

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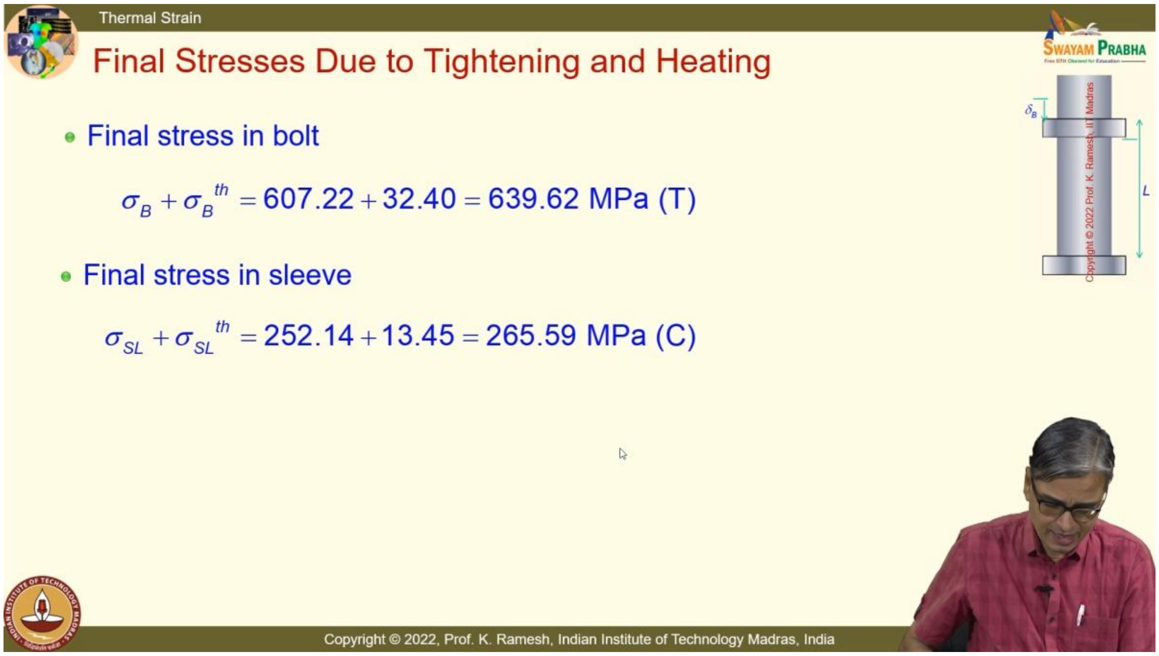

So, the final stresses are, because of tightening, the nut has moved down. Because of temperature change, it has moved slightly above and what is the effect of it? Because of the temperature change, this is the original stresses which you have calculated because of tightening. Due to temperature change, additional stresses are introduced. See, suppose you are operating a system in a situation where you are not accommodated with the temperature change and you have taken factor of safety is very small, you may eventually end up in a failure and unless you accommodate the temperature change or increase the factor of safety, either of the two you have to do it. So, you should not ignore, even though the temperature stresses are reasonably small when you compare to the stresses due to tightening. The point here is when there is a temperature change, additional stresses are introduced into the system. So, you need to have that estimated so that your design functionality is achieved. You should not have a failure.

So, I get the total stress on the bolt as 639.62 MPa tensile and for the sleeve as 265.59 MPa compressive. So, the problem brings out how to handle problems dealing with temperature change as well as mechanical load, fine.

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And you know, I was interested that glass is a green material. It is also very interesting to see. People have done stresses on glass which is a gorilla glass and when you compress it like this, it withstands a very high level of stresses. Probably it is an MPa, I believe 120 MPa it is able to withstand, whether other glasses which have also gone through some kind of heat treatment has not withstood.

So, this is one aspect of it. Normally, you think glass is brittle. See, you have fiber composite, you have glass fibers *E* glass, *E* to *Z* you have variation of glass fibers that is used in polymer composite and you also have all your data is glass fiber. That means, glass also can be processed to be flexible and Corning may not reveal the secret, but you can at least enjoy what is the flexibility. Imagine, you are able to bend so much without any failure of it. So, people have tamed glass, it is environmentally friendly now and you use thermal effects to do that.

The details may not be available to us, but people have tamed glass for the modern world. So, in this class, we have looked at what way the temperature modifies the system. You can have only strain introduced. If the system is constrained, you also have stresses developed and one of the ways engineers have mitigated thermal effects is, wherever possible, if there is a thermal issue needs to be accommodated, if you can allow in a design expansion provide rollers on the supports, allow it to expand. So, you do not have to worry about additional stresses developed. That is one of the simplest way to handle thermal stresses, but if you have constraints, you also have the stress strain temperature relation for you to evaluate the thermal stresses as well as thermal strain.

And the recommendation here is when you write the stress strain temperature relation, do not simply say strain as $\alpha\Delta T$. $\alpha\Delta T$ may be special situation when there is no constraint. So, these are all the ways you know you can make a mistake. So, write the complete expression including the stress terms plus the temperature terms and knock off those terms which are zero. Thank you.