Strength of Materials
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Lecture - 13
Strain

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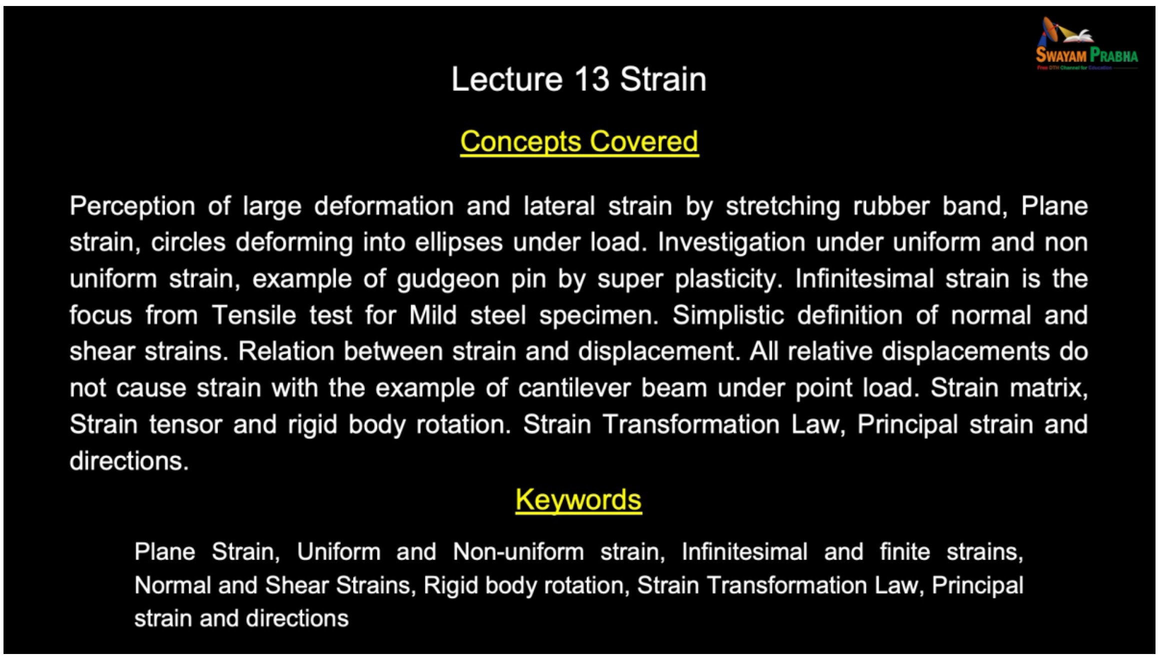

Let us learn from this class concepts related to strain. And you know, we have looked at concepts related to stress and soon we understand that strain can also be labeled as a tensor. So whatever the concepts that we have developed for stress, you will have a mirror image of it in terms of strain. And you will also have to look at, we will confine our attention to infinitesimal strain in this course. But nevertheless you should also recognize that you have situations where you have finite strain. How this needs to be handled? At least in a rudimentary manner we will try to see in our lectures on strain.

And what we will do first is, you are all given a rubber band and then I would like you to take it, identify a region where you have that uniform cross section because you know it is all made in a very crude manner. So there is no consistency in the cross sectional dimension. And I would like to hold it and pull it and make observation. See I have made sure that I have, I am holding a portion of it that can be taken as a starting length.

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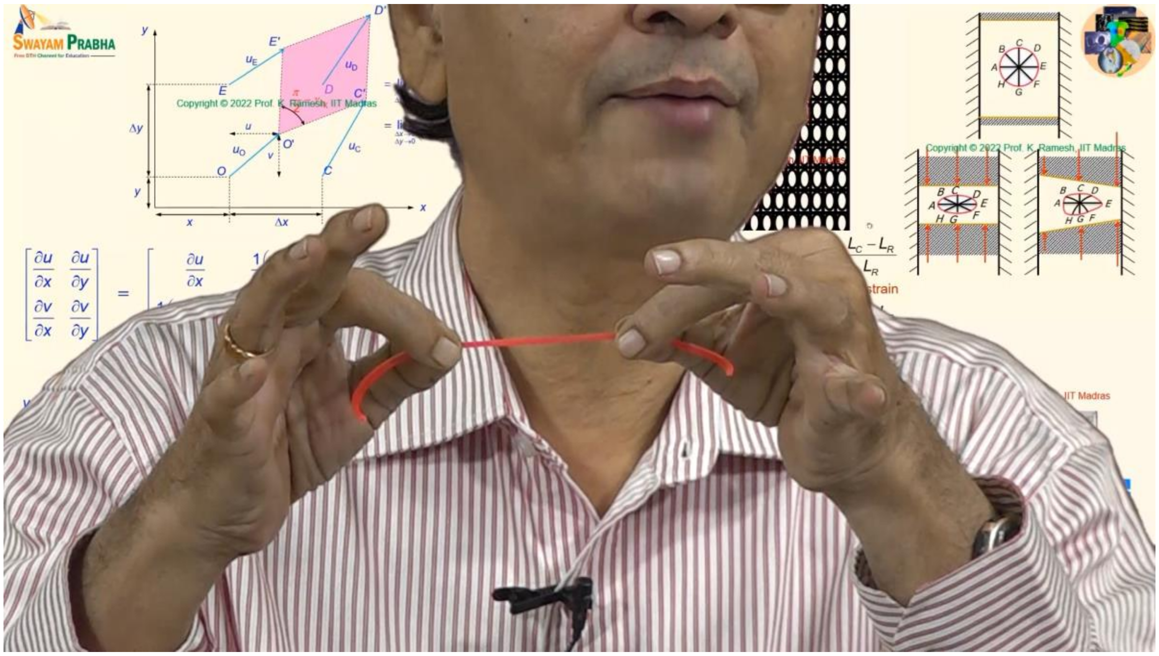

Now I slowly increase it, I am able to go double the length, I am also able to go beyond that. And suppose I release it, I can come back comfortably to the initial position. There are also other observations anybody has made. When I elongate like this, what are all the observations you make? Cross section becomes smaller. So that means, there is something happening in the lateral direction.

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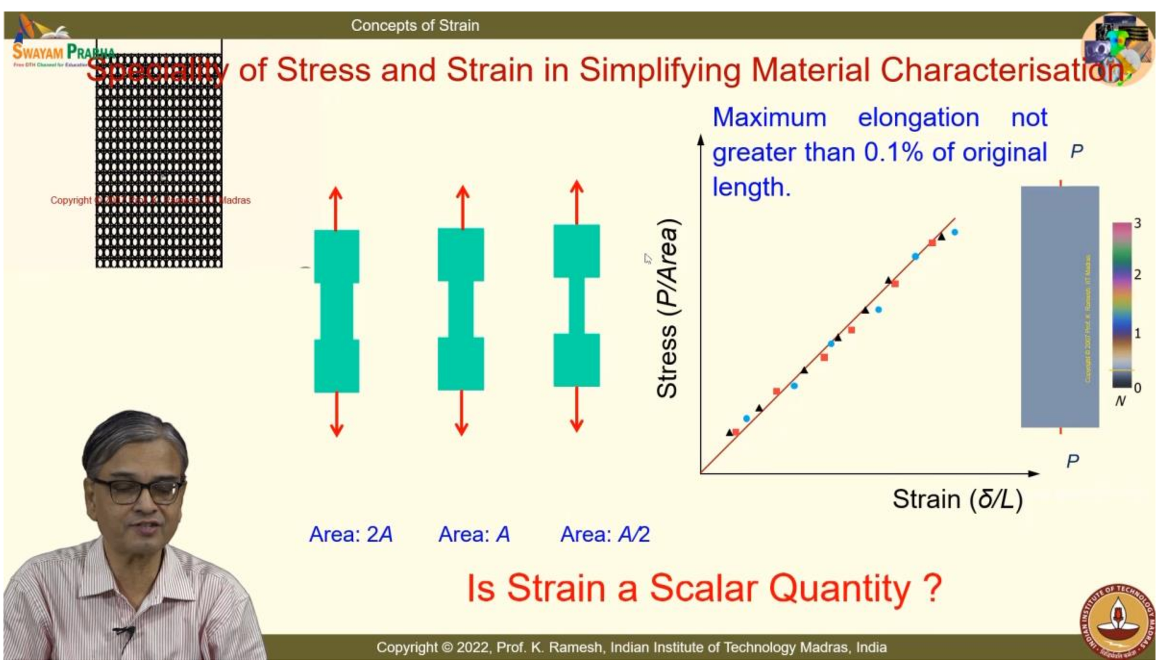

So that is what you see it in this in a very magnified version. So when I have a uniform stretching, it also has a contraction in the lateral direction. If I have a block of material, then it will also have in the perpendicular direction. Whereas in the case of stress, when we looked at it, we were able to apply uniaxial stress very comfortably. I do not have to do anything extra to apply uniaxial stress.

Simply take the object and pull it, that is all I do. Suppose I pull, if I exaggerate the deformations, I also have a lateral contraction. Suppose I push it, instead of pulling it, I compress it, what would you anticipate in the lateral dimension? Opposite of this should happen, it will bulge out. So when I have strain, it is always either biaxial or triaxial. So what I have here is, I have contraction which is exaggerated.

What I have done is, I have put circles on this and which is magnified many times, because for us to visualize and develop the concepts, we have to magnify it, there is no other goal. And while developing the concepts of stress, we have also seen that we have taken different cross sections and if you plot a quantity strain versus stress, you get only one graph and this was also the birth of the concept of stress, this is also the birth of concept of strain. And what you will have to note down is, this is all discussed when the deformation is within 0.1%, fine.

That 0.1% is very very important and we have also noted that this was not something very simple. You know, Jacob Bernoulli is credited with the beam theory and you find it was his last paper of his life. And I have also said in engineering, how do you process the data? What you plot in the x-axis and what you plot in the y-axis has really revolutionized understanding of concepts and also development of new entities.

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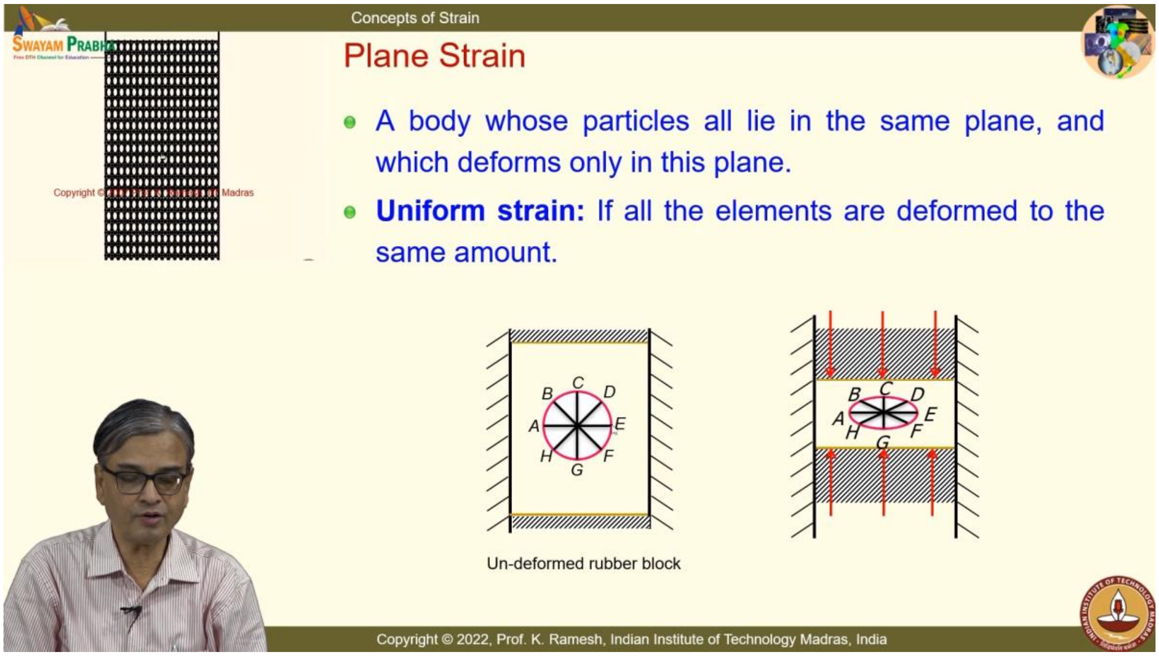

Now let us look at plane strain, which is defined in a very simplistic manner here. We will also see a different type of presentation later. A body whose particles lie in the same plane and which deforms only in this plane, that is what is emphasized. Now let me take up a simple case of uniform strain. That is, if all the elements are deformed to the same amount. So, I take a simple example and then here I have a constraint on either side. I have a rubber block because that is what you and I can apply load and then see.

Now I have drawn a circle and the circle also has several lines drawn for us to visualize what would happen to this when you apply a uniform deformation. And mind you here, I have constrained the sides and I am going to apply a compressive load. So, I have to take special care for me to have a uniaxial strain. Here I have not done any special attention, so this is a biaxial strain. And when I apply the load, when this rubber block is compressed, what would happen to the circle? It will translate into an ellipse.

So, what you find is, circle has deformed into an ellipse and you can also see what has happened to the lines drawn, what happens to the line drawn *AE*, *CG*, *HD* as well as *BF*. They still remain straight because it is uniform strain and you can also understand in like in the case of stress. So when this line element *AE* shrinks to zero, we get strain at a point. I raised a question whether strain is a scalar or it is something much more. We have raised a similar question when we develop stress.

It appeared like a scalar to start with because when you look at components, they are always numbers, they will appear like scalar. But that entity is not a scalar entity, like stress strain is also a tensor and you have all that understanding here. So, when at this point, if you take the center as the reference, when you look at what happens on all the directions, you find it changes from direction to direction. You get the idea, very similar to stress. You will also have to look at instead of the planes, you have to look at directions which are infinite when I go around the point of interest. So, we will also say strain is also a tensor of rank 2.

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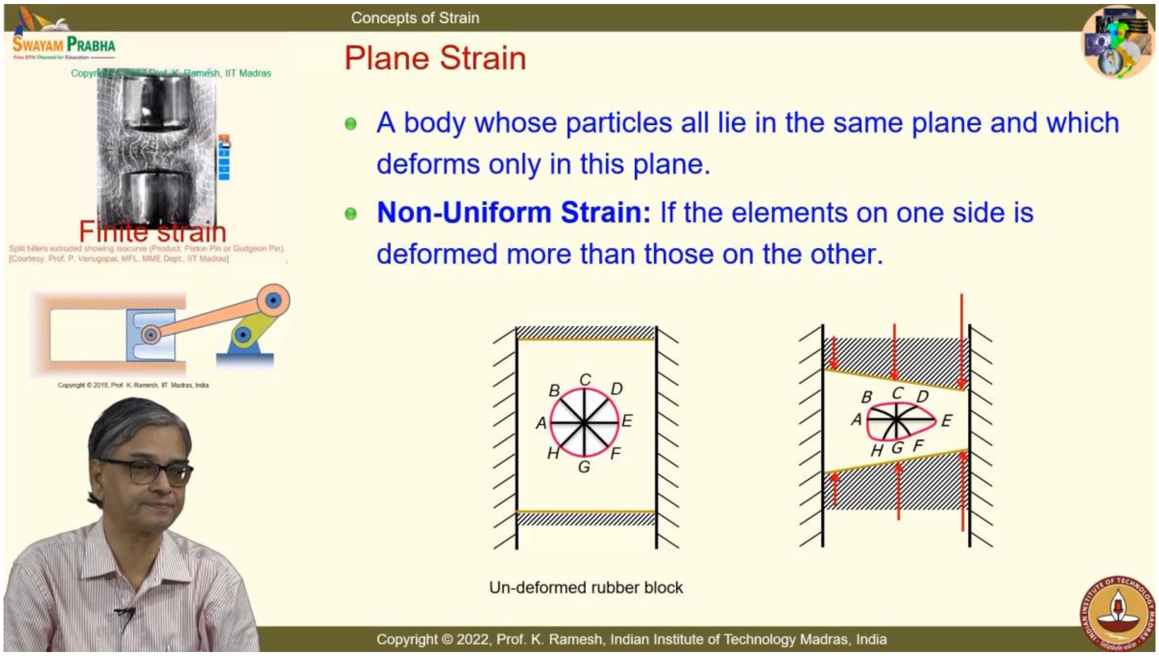

And you know you can also have a non-uniform strain and what I have shown here is a very interesting example. A Gudgeon pin is developed by a punching operation and this is done by super plasticity. This is again done in IIT Madras by Professor K. Venugopal. And you find visually whatever the lines which are like square to start with get distorted in the process. And here we are talking about very high values of strain. And what a Gudgeon pin is, when you have a IC engines, I have this cylinder is connected to the connecting rod, the pin that forms in this joint is a Gudgeon pin. I have a horizontal section and a vertical section. So, what you see here is, what is the level of strain? The level of strain is so high.

So, whatever the lines drawn you can visually perceive that these lines are distorted. And this is the case of a non-uniform strain, strictly does not come under plane strain. For the purpose of illustration this is good for a non-uniform strain. And what I find is, if I apply variable load on the square block, I do get distorted image of the circle and the lines also become curved. This is what I have in this diagram, *BF* and *HD* are curved.

So, when I have a uniform strain, the process is very simple and usually you come across non-uniform strain. In special cases you may have the situation of uniform strain. And you will also have to recognize what is the level of deformation we are talking about.

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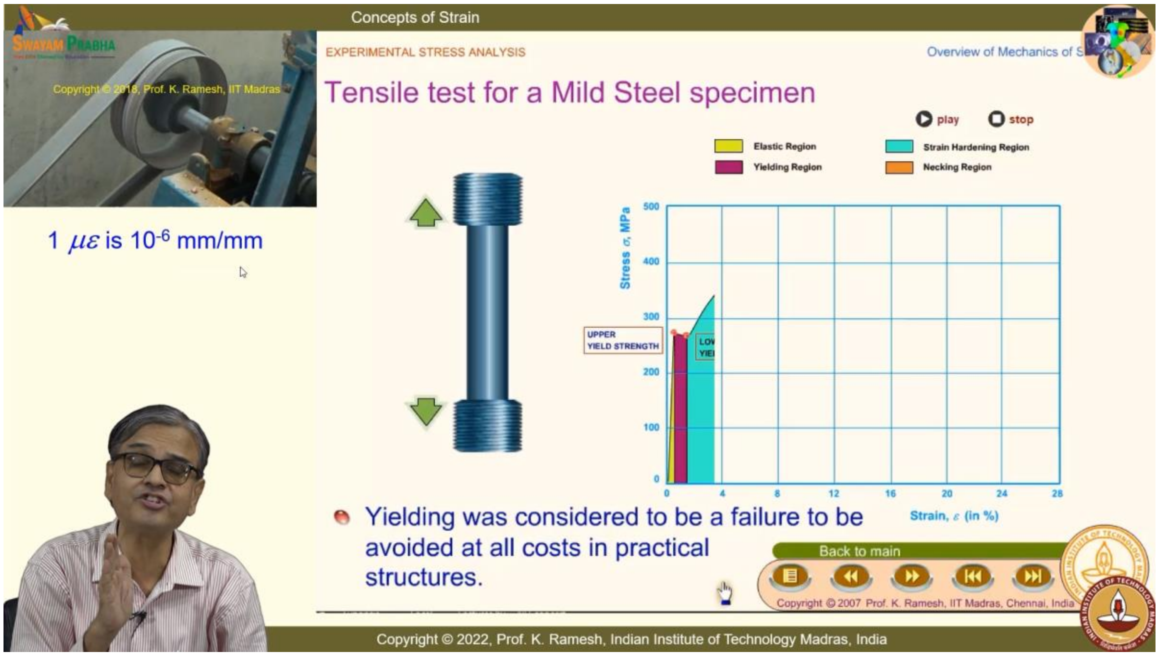

And that you can understand when I look at the simple tension test. See you have a Attachachki where you have a shaft is rotated and supported on bearings. And when I have a mechanical structure like this, it has to remain elastic and this is a very very small region. This is yellow region shown here. And we have already seen that the maximum deformation is limited to 0.1% strain. 0.1% translates into, when you look at in terms of micro strain which is defined as 10^{-6} mm/mm, 0.1% reduces to $1000\mu\varepsilon$. $1000\mu\varepsilon$ is the maximum level of strain permissible in most of the practical structure. You never touch $1000\mu\varepsilon$ when I have a mechanical element like this, you get the point. Because when the loads are removed, they have to come back to its original position.

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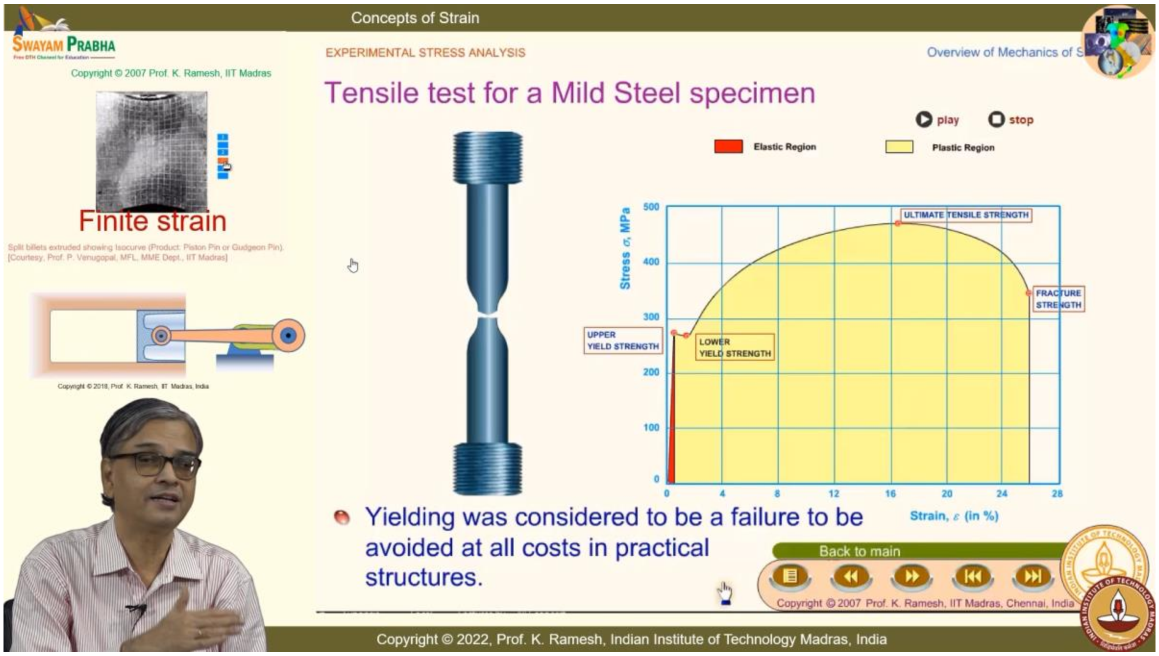

If they deform excessively, a shaft cannot rotate inside the bearing. So, in practical applications you look at deformations which are extremely small. And mind you, when I say micro strain is 10^{-6} mm/mm, it is going to be extremely difficult to measure, fine. On the other hand, when I look at the tension test and I look at what happens to the material, the *x*-axis is drawn up to 28% of strain. And when you live in this region, you get this material deformed into a Gudgeon pin.

So, when you have a situation like this, strains are way beyond, it is up around hovering around 20 to 22 %. It has not reached the stage of fracture and you call this as a final strain. So, you need different mathematics to express what are the strain behavior and so on and so forth. It is not simple mathematics.

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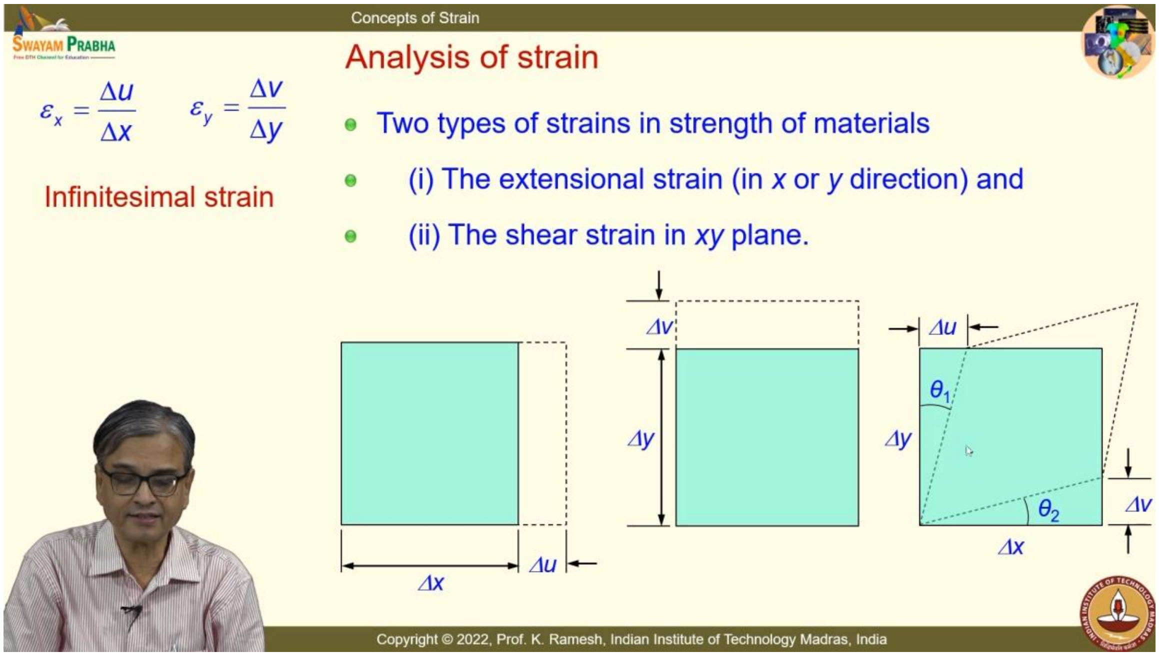

And in the case of strength of materials, we start with simple definitions. You have an extensional strain which you all can easily decipher. I have a small element of size Δx and Δy. We have taken a small element Δx Δy. In the limit, we will say Δx goes to 0 and Δy goes to 0. And whenever there is an extension like this, I can have this as Δu change in length divided by the original length.

I can call this as axial strain in the *x* direction and you will also have two subscripts. That subscripts share the similar nomenclature what we have looked at. It will represent direction in sort of the plane. First one will show the direction, ok. And you have the extensional strain in the *y* direction. We have shown in such a manner that there is no lateral contraction. In fact, this happens in certain type of material. In the case of cork, it does not have any lateral strain. It is very interesting to see.

And these are all infinitesimal strain. We are talking of very small values of deformation. We have already said in the beginning of the course, we will confine our attention to small deformation that makes our stress strain relationship linear and it also gives the host of other advantages. So, you should never forget that we are dealing with extremely small quantities. And you also have a shear strain. A shear strain, we will also look at the definition a little while later. It is a change in the initial rectangle.

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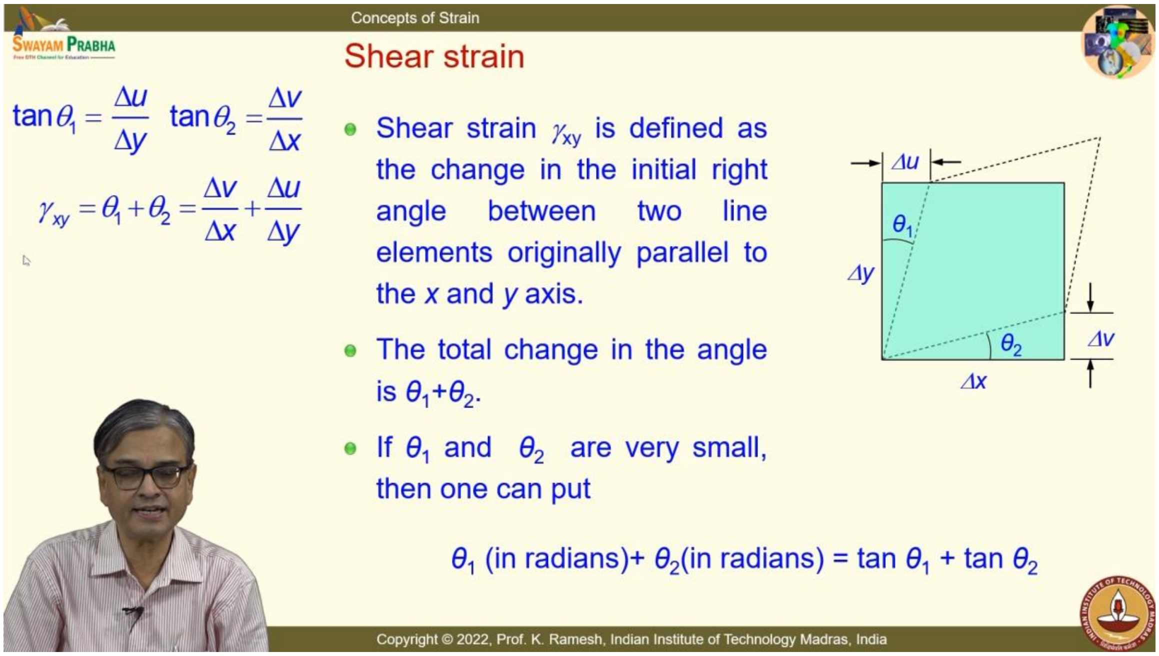

So, I have this edge rotated like this, this edge rotated like this and that is what we have in the next slide. And you have shear strain is defined as a change in initial right angle between two line elements originally parallel to the x and y axis. For the purpose of illustration, I show these angles as very big, fine. From your analysis perspective, these are all extremely small less than a degree. I have shown this as may be 20° on each side, but it is less than a degree.

And when I have this, the angles $\theta_1 + \theta_2$ gives me what is the shear strain. And when the angles are small, I can simply write this as for me to express them in terms of Δu and Δy , Δv and Δx as summation of $\tan\theta_1 + \tan\theta_2$. And you know how to calculate $\tan\theta_1$. So, when I do that,

$$\tan\theta_1 = \frac{\Delta u}{\Delta y} \quad \tan\theta_2 = \frac{\Delta v}{\Delta x}$$

And you have γ_{xy} and I should also tell you that this is labeled as engineering strain, ok. It is

$$\gamma_{xy} = \theta_1 + \theta_2 = \frac{\Delta v}{\Delta x} + \frac{\Delta u}{\Delta y}$$

We would soon see later that we will use only half of γ_{xy} when we express this as a tensorial quantity. And what is the convention? See, we have looked at the convention for what is positive shear stress. A positive shear stress will introduce a positive shear strain. And what you have is you make a neat sketch of this, please make a neat sketch.

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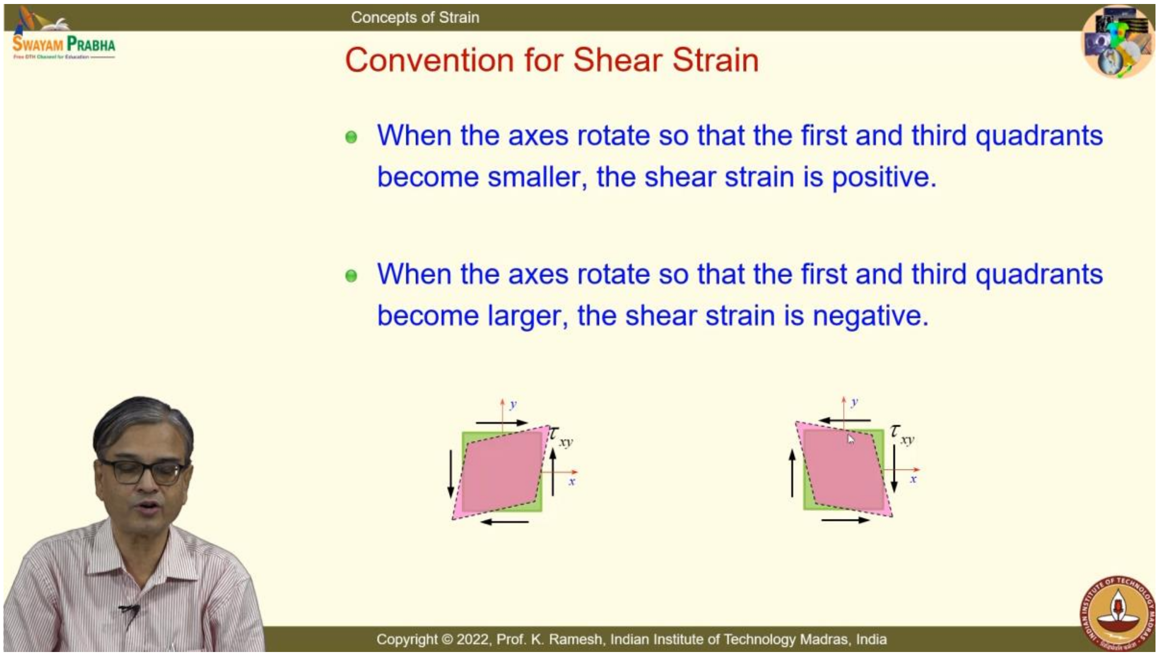

But when you express it, the expression looks little complicated to visualize. When the axis rotates so that the first and third coordinates become smaller, that is seen very clearly in the diagram, first and third coordinates become smaller, the shear strain is positive. The opposite of this, you call that as negative shear strain. So, you have the diagram drawn and diagram is very illustrative of what we are meaning as positive and negative shear strains. So, strength of material simply states this and then carries on with all the analysis.

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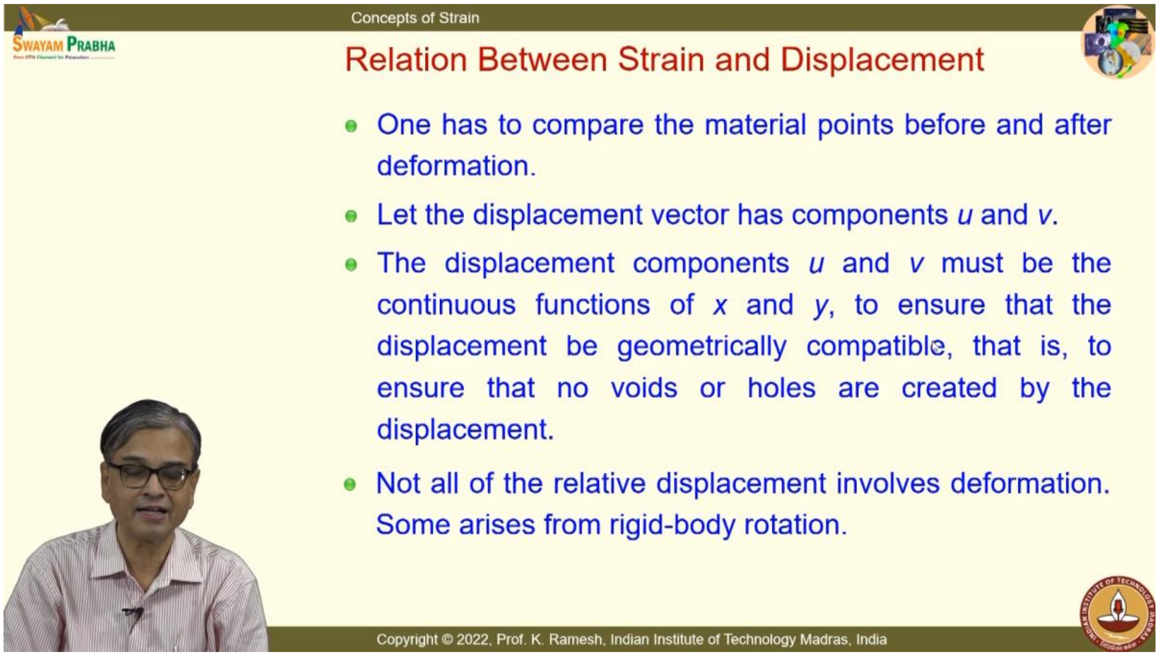

We will also look at from a displacement perspective. See we have to look at what happens before deformation and after deformation and let that be dictated by a displacement vector. When we look at a planar situation, it has components u and v. And how do the displacement component should be? They should be continuous functions of x and y so that when the material deforms, I do not have voids, I do not have separation, I have compatibility in the deformation, all that is taken care of. And you will also have to look at, it is not the relative displacement always causes strain.

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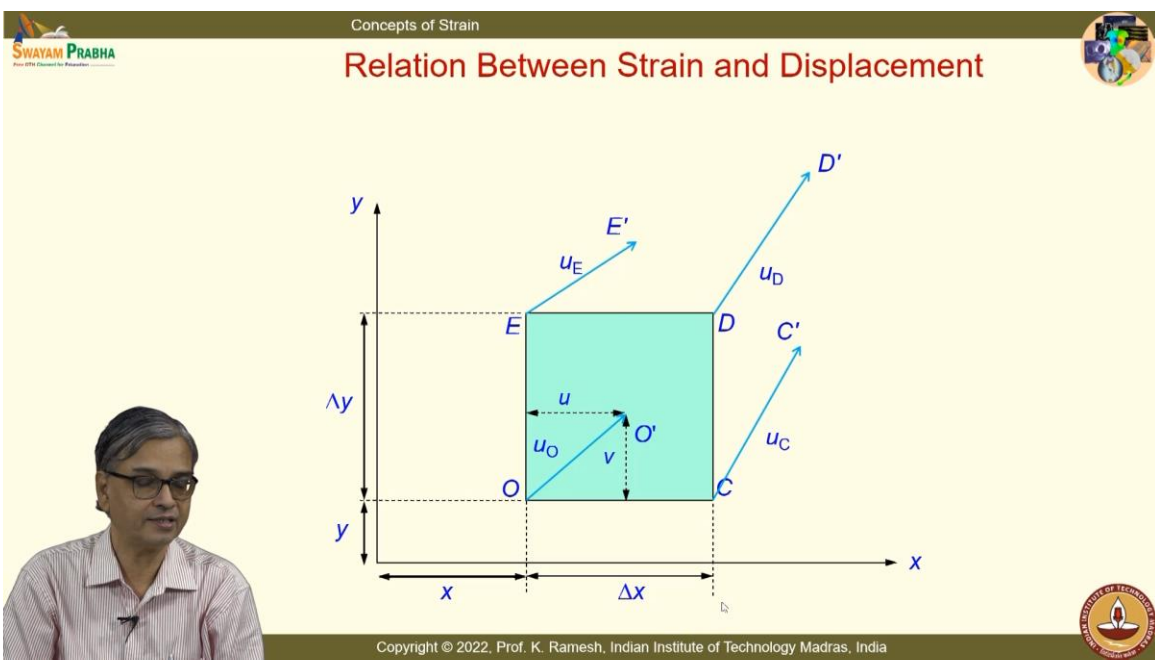

You could have a relative displacement that arises from rigid body rotation will not contribute to strain. This we will understand in stages. Now we will look at, when I say this I have a displacement field, then I take a simple element which we have taken as Δx and Δy. And we would say after applying a deformation, this gets distorted. Let each of these points have some displacement vector, fine.

And these are all exaggerated. I have taken Δx comparable to *x*, but I call it as Δx. You should understand unless these diagrams are big, you are not in a position to visualize what happens. So, I have a displacement vector, the point *O* move to O'. Similarly, point *C* has moved to C', point *D* has moved to D', point *E* has moved to E' and I can have individual displacement vectors u_o, u_c, u_d, u_e so on and so forth. And when I join this, I get this as a in general some distorted shape.

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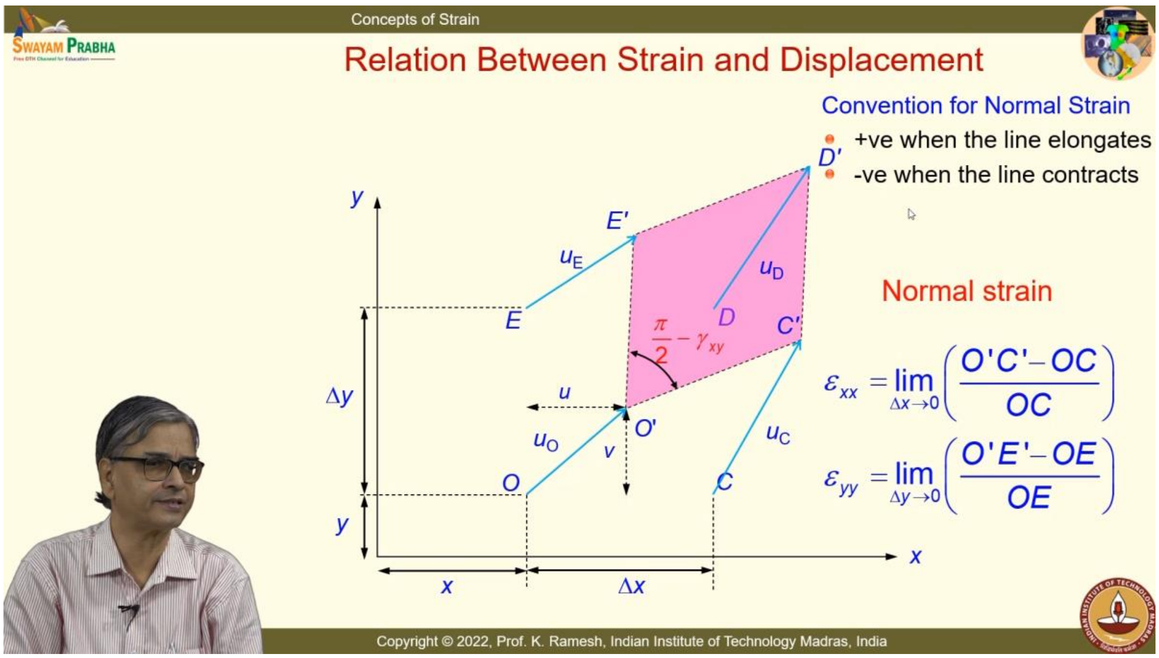

And I can also define the strain quantities based on the change in the length. If there is a change in the length in the *x* direction, I would say it contributes to ε_{xx}, ok. And when I have a normal strain, you have a definition ε_{xx}. This is nothing but in the limit delta x tends to 0. This helps us to understand what is the state of strain at a point.

When the limit goes to 0, we also exploit the continuum nature of the material. That is the assumption that we have made, it is an elastic continuum. Otherwise, I cannot write this limiting process. So, the change in length

$$\varepsilon_{xx} = \lim_{\Delta x \to 0}\left(\frac{O'C' - OC}{OC}\right)$$

And ε_{yy} comes to

$$\varepsilon_{yy} = \lim_{\Delta y \to 0}\left(\frac{O'E' - OE}{OE}\right)$$

And how do we say when the elongation, when it elongates, we call that as positive strain, when it contracts we call it as negative strain. Like you have calling tension and compression, it is very similar to that.

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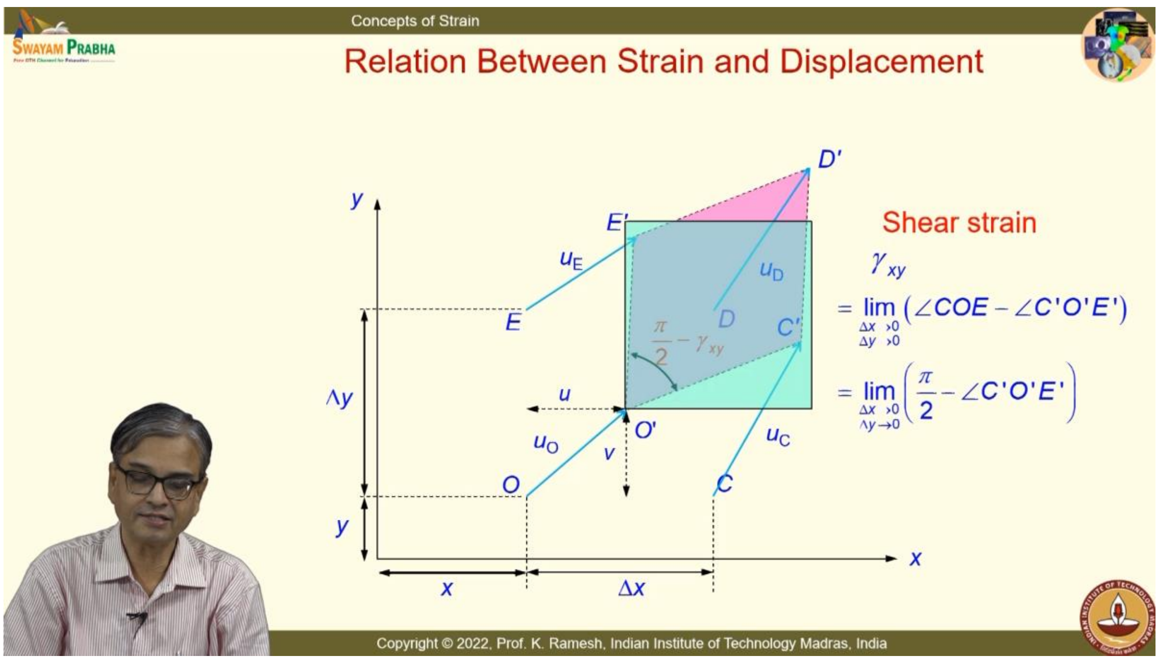

And you can also define the shear strain and shear strain is nothing but the change in the original rectangle. So, whatever the original angle is *COE*, it was initially $\pi/2$ and the final angle is $C'O'E'$. So, this is the definition of your shear strain.

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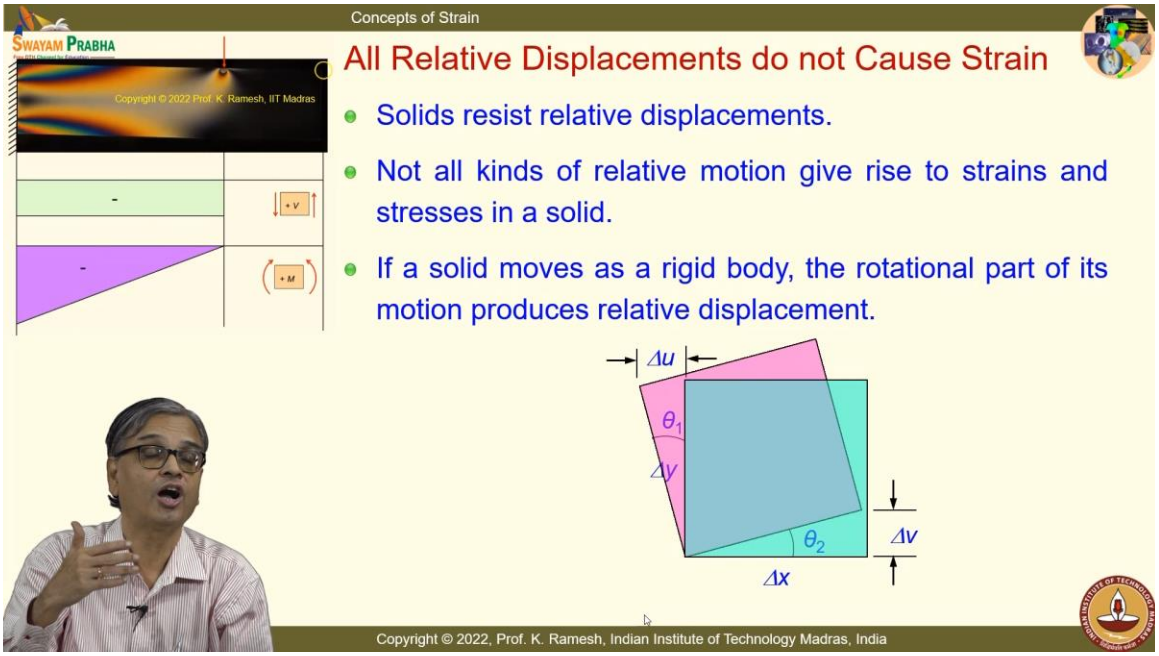

And you know I have said that all relative displacements do not cause strain which we will have to understand. And this is a very nice example, you know you have a cantilever where I have put the load in between and you know how to draw the shear force and bending on diagram. You have a segment where there is no load at all and you also have a photo elastic pattern which shows which portion is experiencing stresses. And stresses are there strain will also be there, when stresses are absent, strain also will be absent. And this is nicely summarized here, solids resist relative displacements.

Not all kinds of relative motion give rise to strains and stresses in a solid. The solid moves as a rigid body, the rotational part of its motion produces relative displacement, but that does not cause a strain. I take the small element, when I have a translation no problem. On the other hand, if I have a rotation, if you look at there is relative displacement, but this is a pure rotation in the about the z-axis. And if I take a cantilever like this, let me see how many of you have a look at it.

See this is a very soft material, so it sags even with the applied load, this is the free end I put a load somewhere in between. So, what happens to the region from the front to this? It is subjected to resisting the applied load by bending moment and shear force. And in this section, this is not experiencing any load, but still it is having a rotation. Can you see that? And this rotation is a rigid body rotation. On the other hand, in this section whatever you have, I have strain as well as rotation of various degrees.

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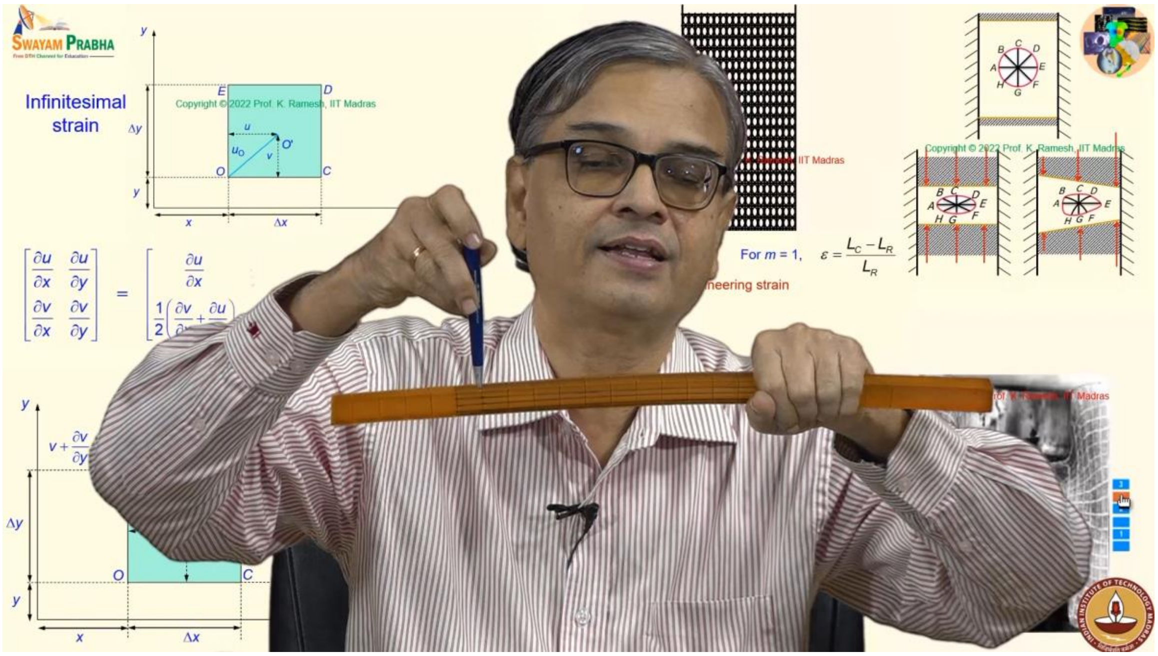

So, relative displacement in a sense causes strain, but not all relative displacement contributes to the strain. So, you have to understand. We will also look at when we see the thermal effects. So, I have these angles are also shown, fine. We will also find a mathematical expression, how to define rotation.

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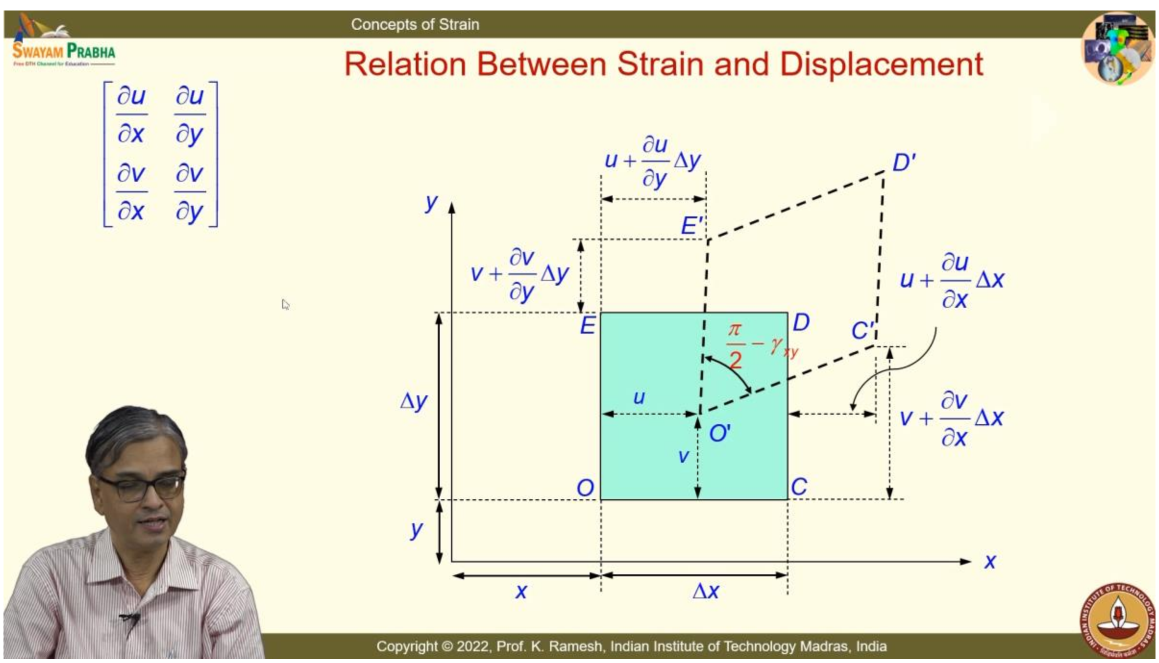

Now what we will do is, in a while we develop the equilibrium condition when we wanted to do the stress analysis. We have learnt when the quantities are changing as a function of position, how do I write it using Taylor series? We have looked at that. Now we have taken a small element Δx and Δy .In the limit this goes to 0, but this has a finite distance now. So, if I have a quantity as *u* at *O*, when I go to *C*, this is separated by a distance Δx .

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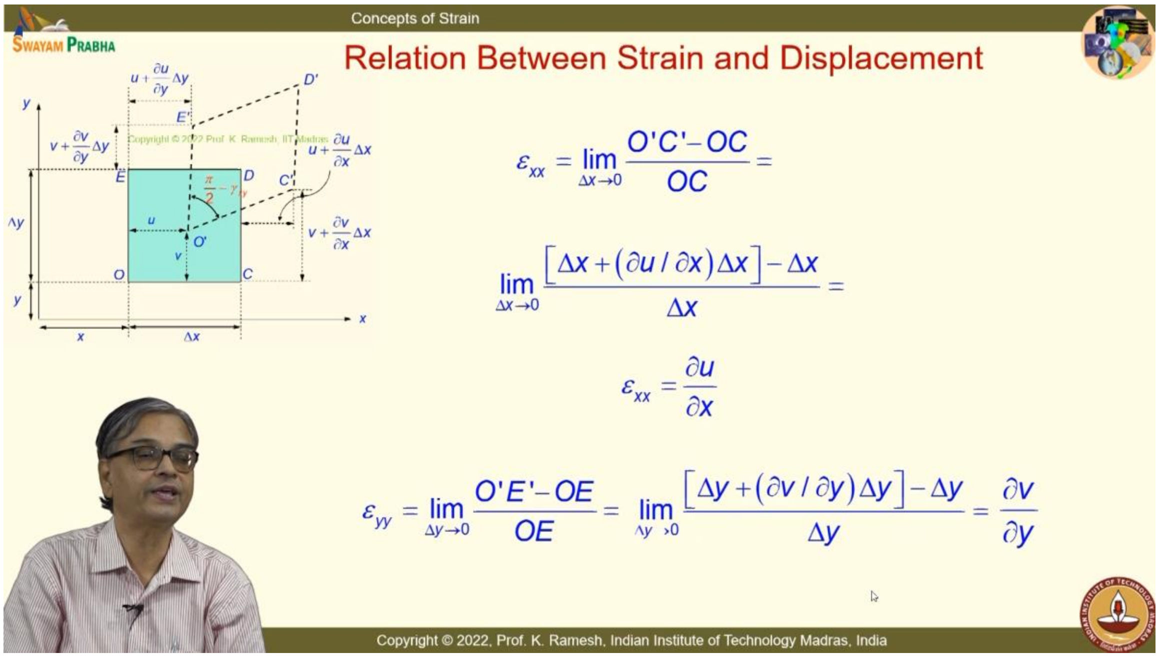

So, I will have u plus what is that we are going to have? The same thing I am going to express it in terms of the displacement *u* and *v*. So, I have the distorted element like this. Please make a neat sketch and you would get expressions in a very nice manner. What we have defined, we will see right here and I want you to tell me what is this distance.

I have moved by a distance Δx . The point *O* has moved horizontally by a distance *u* and the point *C* has moved to C'. Can you tell me what is this complete distance from the way I have drawn the diagram? Can you tell me? Can you guess? Make a mistake, no problem. I will have *u* + what happens to the Taylor's component? Taylor series component. So, $\frac{\partial u}{\partial x}\Delta x$. So, similarly we should write this for the other points *D*, D', $E'$and so on. And even for C', there is also a shift of this in the vertical direction. So, the next one is how do I write this distance? This distance we have written as *v* and this distance how do I write it? This is again happening because of the change in the *x* direction. So, you have to write this as $v + \frac{\partial v}{\partial x}\Delta x$. Do you want, do you get the idea? How to apply this Taylor's approximation systematically? We have to see *u* is a function of *x* and *y*, *v* is also a function of *x* and *y*. Now we are looking at what happens when *x* changes. And similarly when I come to E', this because *y* has changed or *v* has changed, whichever way you can look at it. y has changed is the better appreciation. So, I have this, the point O has moved to O', E was also at the same position of x, E has moved to E', they are separated by Δy. So, I have this as $u+\frac{\partial u}{\partial y}\Delta y$. And how do I write the vertical distance? I will have this as, what is this, what is this quantity? $v+\frac{\partial v}{\partial y}\Delta y$

So, if you understand and put all these labels, we have already looked at how we are going to define the normal strain. We have also seen how to define the shear strain. If I take that formula and plug in these values, I get expressions for normal strain, I also get expressions for shear strain. And in the process what we have looked at is, we have also looked at a displacement gradient that can be put in the form of a matrix.

I have this as

$$\begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{bmatrix}$$

I have a displacement gradient matrix. We will see, it is very interesting to see how they are related to strain and rotation. So, now we are going to look at how to get the normal strain, we have the expression

$$\varepsilon_{xx} = \lim_{\Delta x \to 0} \frac{O'C' - OC}{OC} =$$

So, when I do this, this is nothing

$$\lim_{\Delta x \to 0} \frac{\left[\Delta x + (\partial u / \partial x)\Delta x\right] - \Delta x}{\Delta x} =$$

So, this gives me ε_{xx} as $\frac{\partial u}{\partial x}$. Similarly, I get an expression for ε_{yy}

$$\varepsilon_{yy} = \lim_{\Delta y \to 0} \frac{O'E' - OE}{OE} = \lim_{\Delta y \to 0} \frac{\left[\Delta y + (\partial v / \partial y)\Delta y\right] - \Delta y}{\Delta y} = \frac{\partial v}{\partial y}$$

So, change in length divided by the original length, that is the simplest definition of strain. When I substitute the quantities, this reduces to $\frac{\partial v}{\partial y}$. Is the idea clear?

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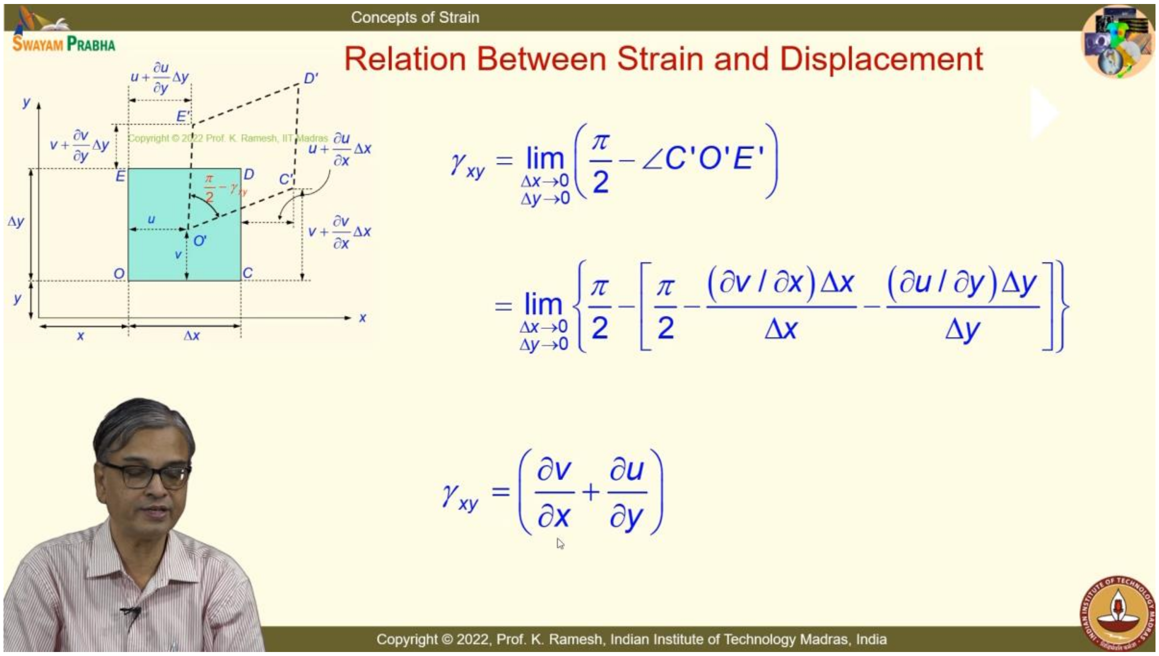

I can extend the same approach for me to get a mathematical expression of shear strain. We have said that this is

$$\gamma_{xy} = \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \left(\frac{\pi}{2} - \angle C'O'E' \right)$$

and from the geometry, I can write this as

$$= \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \left\{ \frac{\pi}{2} - \left[\frac{\pi}{2} - \frac{(\partial v / \partial x)\Delta x}{\Delta x} - \frac{(\partial u / \partial y)\Delta y}{\Delta y} \right] \right\}$$

And when I simplify this, I get the expression as

$$\gamma_{xy} = \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)$$

which we have simply said as a definition to start with. Now, what we have achieved is we have recognized there is going to be a displacement field. From the displacement field by applying Tyler's approximation, we are now able to express the quantities like ε_{xx}, ε_{yy} as appropriate partial differentiation of u and v. Now, I said in addition to the strain components, you will also have to recognize what contributes to rotation and you have all that available in this diagram.

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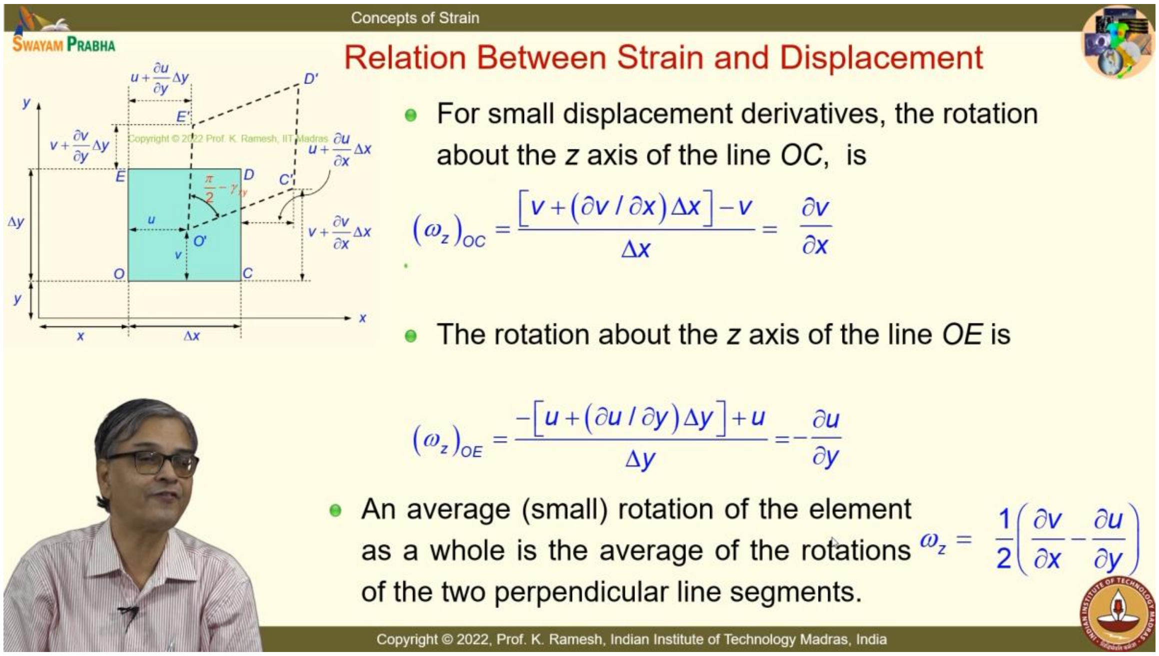

So, we will look at the rotation. How do I express the rotation? So, we have also recognized that this is rotating about z-axis because I have this as x and y, z is perpendicular to this. So, the line OC has rotated by a small angle and that angle you can find out, we have already looked at as θ_1 and θ_2. And here I say anti-clockwise rotation is positive and clockwise rotation is negative.

I should apply the sign convention. I should apply the sign convention. So, when I do this, I get the rotation about the z-axis of line OC reduces to

$$(\omega_z)_{OC} = \frac{\left[v + (\partial v / \partial x)\Delta x\right] - v}{\Delta x} = \frac{\partial v}{\partial x}$$

and this has rotated anti-clockwise. So, this is positive. And what way the line OE has rotated? It has rotated in the clockwise direction. The sense is also very important. So, when I have the line OE, I get this as

$$(\omega_z)_{OE} = \frac{-\left[u + (\partial u / \partial y)\Delta y\right] + u}{\Delta y} = -\frac{\partial u}{\partial y}$$

You have to recognize that this has rotated in a clockwise direction and for an element we take an average of this, rotation of the element as a whole is the average of the rotations of the two perpendicular line segments. So, I have

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

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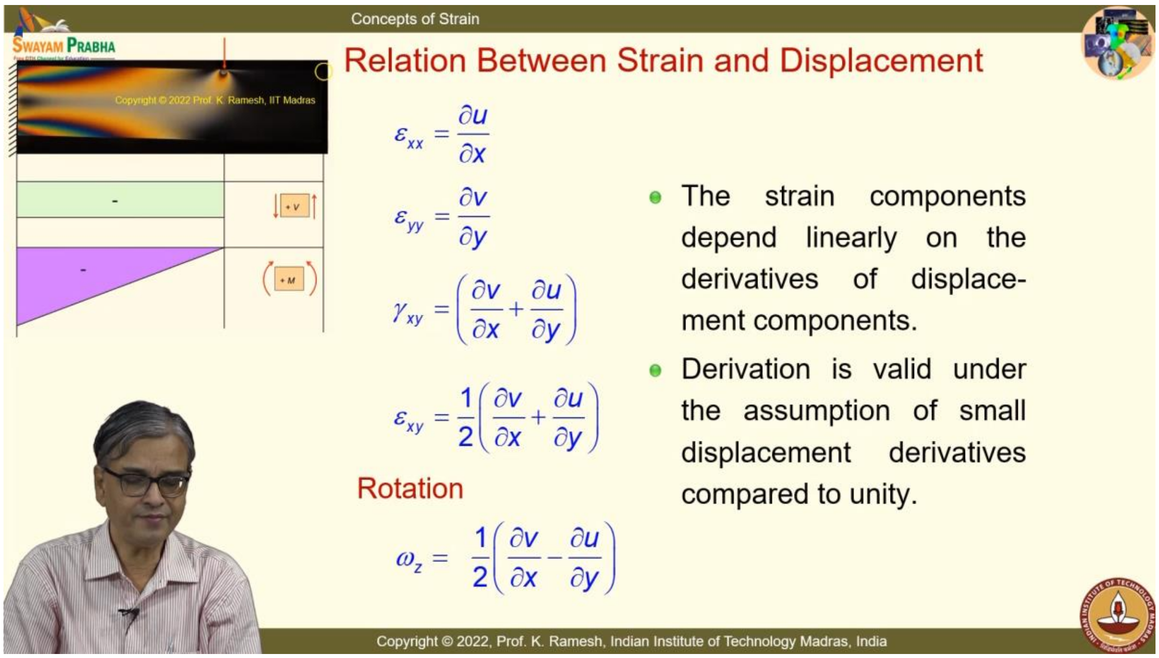

And let me summarize these quantities. And why I show this is, this is a very nice example which has rigid body rotation in some segment which has strains as well as rotation in one of the segments. Because you are all tuned to photoelasticity, you can see that this is the portion which is stressed.

I have ε_{xx} is $\frac{\partial u}{\partial x}$, ε_{yy} equal to $\frac{\partial v}{\partial y}$ and this is the engineering shear strain, $\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$. And what transforms like a tensor is only ε_{xy} which is defined as $\varepsilon_{xy} = \frac{1}{2}\left[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right]$ And what is the rotation? $\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$. And we have already seen, see these are all infinitesimal strain components, very small.

We are talking of the order of 0.001, it is a number because you have ratio of distance as you have this as a number. And the strain components depend linearly on the derivatives of displacement components. Derivation is valid under the assumption of small displacement derivatives compared to unity. We have this maximum value is 0.001 which is very very small when you compare to unity.

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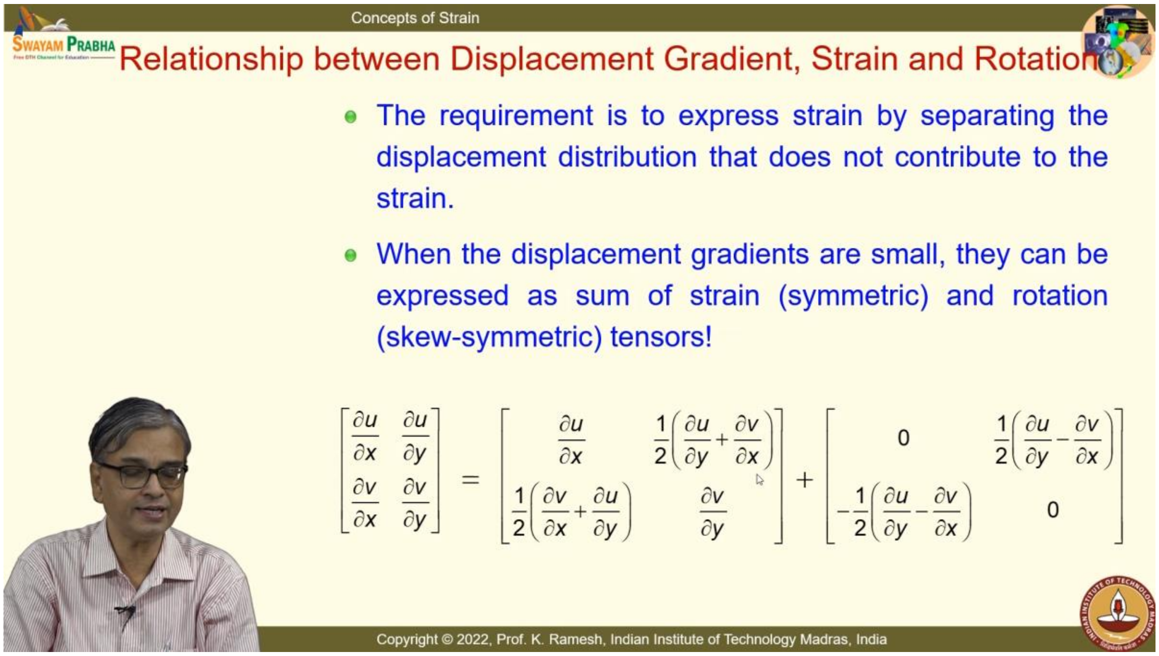

And there is also a very interesting interrelationship. The requirement to express strain by separating the displacement distribution that does not contribute to the strain. So, if you look at the displacement gradients, when they are small, they can be expressed as some of a strain tensor and a rotation tensor. And strain tensor is symmetric, we have seen stress tensor is symmetric from the equilibrium condition, we got τ_{xy} equal to τ_{yx}, all those ideas are borrowed.

Similarly, ε_{xy} will be equal to ε_{yx}. And you have a rotation and if you see it in black and white, it is very convincing. I have seen the displacement gradient matrix

$$\begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{1}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right) & \frac{\partial v}{\partial y} \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{2}\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right) \\ -\frac{1}{2}\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right) & 0 \end{bmatrix}$$

These are all equivalent values, though the order is written in a different fashion here. One is ε_{xy}, another is ε_{yx} and So, this quantity transforms like a tensor and you call that as a strain tensor.

So, you should know the distinction between engineering definition of shear strain, γ_{xy} is used repeatedly, both the symbols are used, when it is used as γ_{xy}, only one half of γ_{xy} to be taken as a component in a strain tensor. Only this obeys the loss of tensorial transformation. So, I can look at the displacement gradient as sum of strain tensor plus rotation tensor. This happens for infinitesimal strain and we have already noted strain is a number, it usually expressed as micro strain and if you look at $1\mu\varepsilon$, it is 10^{-6} mm/mm. And we have said point, it is also expressed as percentages and 0.1% corresponds to a strain level of $1000\mu\varepsilon$.

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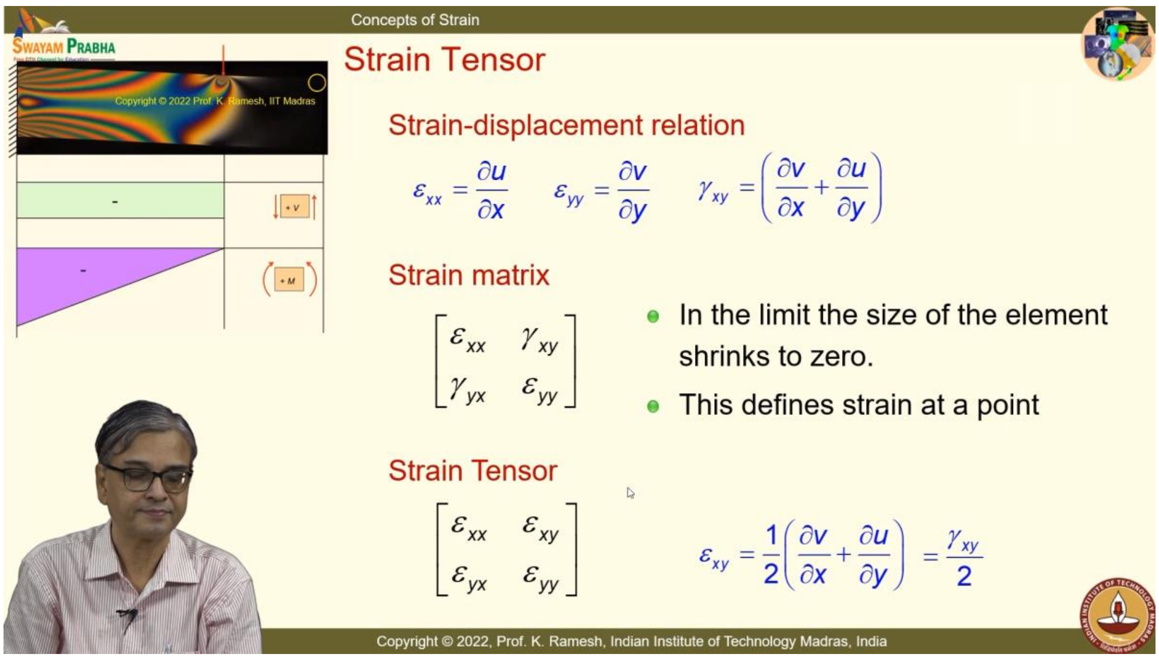
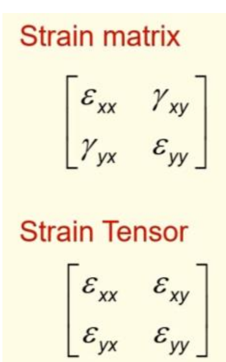


So, now we look at what is a strain tensor? I have a strain displacement relation, ε_{xx} is $\frac{\partial u}{\partial x}$, ε_{yy} equal to $\frac{\partial v}{\partial y}$ and $\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$. Strain matrix and Strain matrix is



And $\varepsilon_{xy} = \frac{1}{2}\left[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right]$ you can express it in any other way. So, when I go and plot the Mohr's circle, I will have to plot normal strain.

When I put the shear strain, I should plot only ε_{xy} not γ_{xy}. γ_{xy} comes from historical evolution, people have looked at this quantity as γ_{xy} when they initially started understanding what happens in the deformation. But from a tensorial perspective, you will have to have only ε_{xy}. So, that distinction you have to consciously make, do not make the mistake. So, whatever that stress transformation law we have looked at, I will have a counterpart of strain transformation law.

If you have looked at principal planes, I can also have principal directions here. Instead of a plane, you have a direction in which the element only elongates or contracts, it does not go any angular changes. So, all these are similar concepts.

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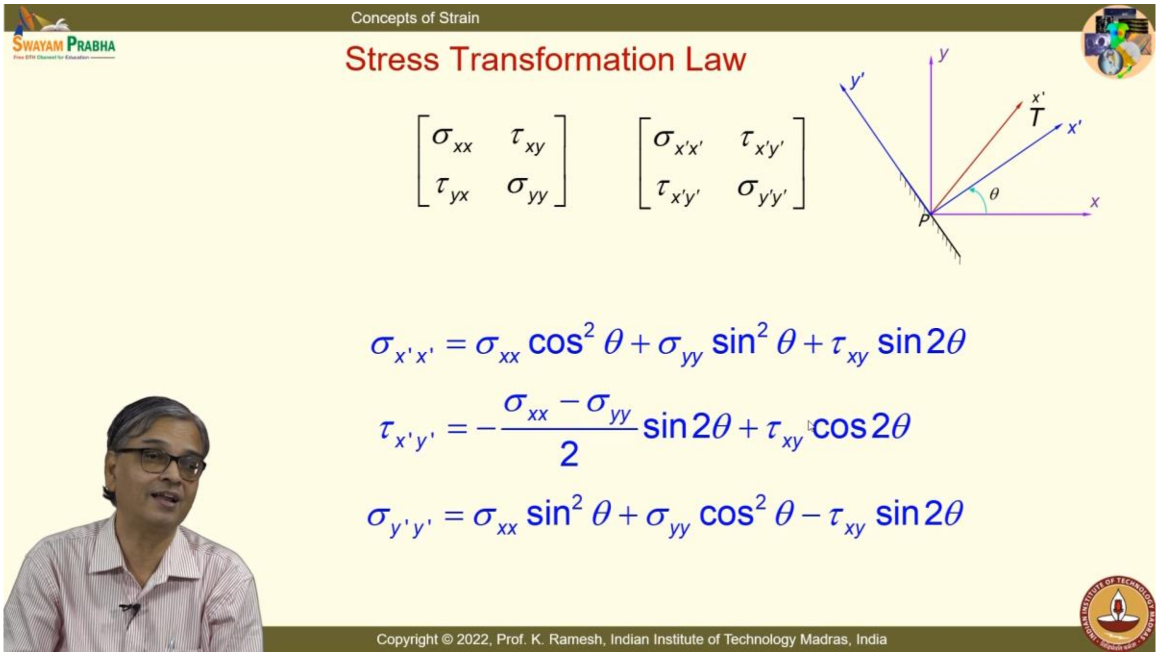

And you know the stress transformation law, I have a reference axis as x and y, I have the other reference axis as x' and y'. And if I want to get $\sigma_{x'x'}$, we have done it from a first principle and you have also learned how to write it from indical notation. All that what you have learned you can apply to strain, you can replace σ_{xx} by ε_{xx} as σ_{yy} by ε_{yy} and τ_{xy} should be replaced by ε_{xy}, not γ_{xy}.

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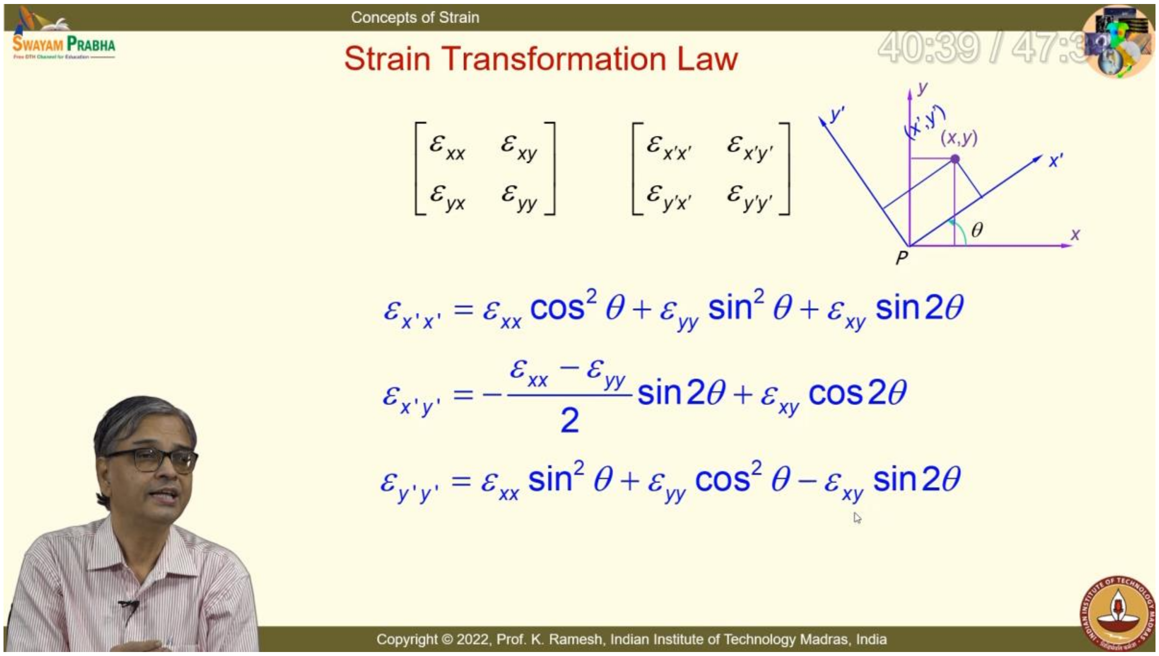

That is the only distinction that you have to make. And you know you have to write that rotation matrix, if you know how to determine $x'y'$ from xy, if you write that rotation matrix correctly, like you have written down the stress transformation law, can you write the strain transformation law? You have you know how to write it from indical notation, you can also write from $\varepsilon_{x'y'}$, I have the diagram given from that you can easily write the rotation matrix. You will have identical image, because all tensors transform in the similar manner. So, I have is

$$\varepsilon_{x'x'} = \varepsilon_{xx}\cos^2\theta + \varepsilon_{yy}\sin^2\theta + \varepsilon_{xy}\sin 2\theta$$

$$\varepsilon_{x'y'} = -\frac{\varepsilon_{xx} - \varepsilon_{yy}}{2}\sin 2\theta + \varepsilon_{xy}\cos 2\theta$$

$$\varepsilon_{y'y'} = \varepsilon_{xx}\sin^2\theta + \varepsilon_{yy}\cos^2\theta - \varepsilon_{xy}\sin 2\theta$$

So, from strain transformation stress transformation law, we have gone to strain transformation law. In stress analysis, we have determined the principal stresses and their associated direction.

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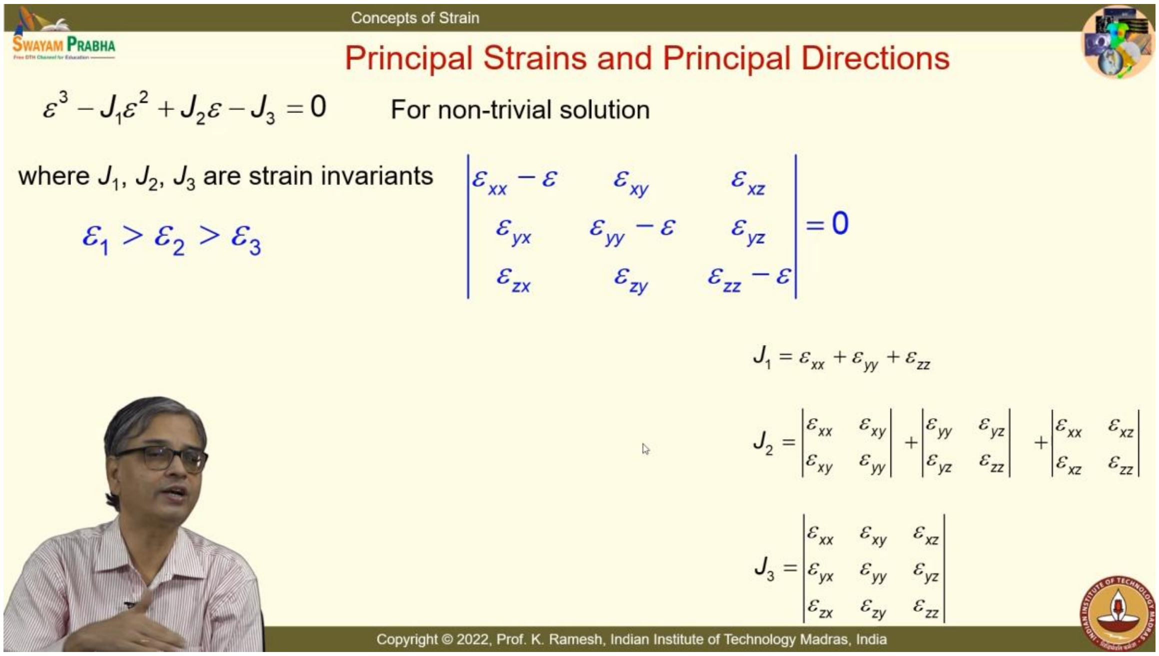

Here, we will find the principal directions and what is the angular orientation. Similar way, so I have to coin this as a Eigen value and Eigen vector problem. So, I have to find out what is ε_1, ε_2 and ε_3 and to make your life simple I will also give you this as a expression,

$$\varepsilon^3 - J_1\varepsilon^2 + J_2\varepsilon - J_3 = 0$$

Like you had seen I_1, I_2, I_3 as invariants. In this strain discussion, we label them as J_1, J_2 and J_3. And what is the definition of I_1? Sum of the diagonal. What is the definition of J_1? J_1 is also

$$J_1 = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$$

Like what you have seen in principal stresses, here again when I get three roots of this equation, arrange them in the algebraic manner, you get ε_1, ε_2 and ε_3. And you also know how to write I_2, you can similarly write J_2 and J_3. I have shown one element of J_2,

$$J_2 = \begin{vmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_{yy} \end{vmatrix}$$

You have to get the determinant of this and this will have three elements. Can you write the second element? We have taken the 2 by 2 matrix from this. So, determinant of this,

ε_{yy} , ε_{yz} and ε_{zz} . And similarly, all the four corner elements contribute to the third element of J_2.

$$J_2 = \begin{vmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_{yy} \end{vmatrix} + \begin{vmatrix} \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{yz} & \varepsilon_{zz} \end{vmatrix} + \begin{vmatrix} \varepsilon_{xx} & \varepsilon_{xz} \\ \varepsilon_{xz} & \varepsilon_{zz} \end{vmatrix}$$

So, it is a mirror image. Whatever you have learnt in stress analysis, similar quantities you look at in strain. And what is J_3? It is the determinant of the complete matrix. And like what we have seen in stresses, I have a cubic in a three-dimensional situation. I simplify when I look at only in the *xy* plane, knock off all quantities related to *z*.

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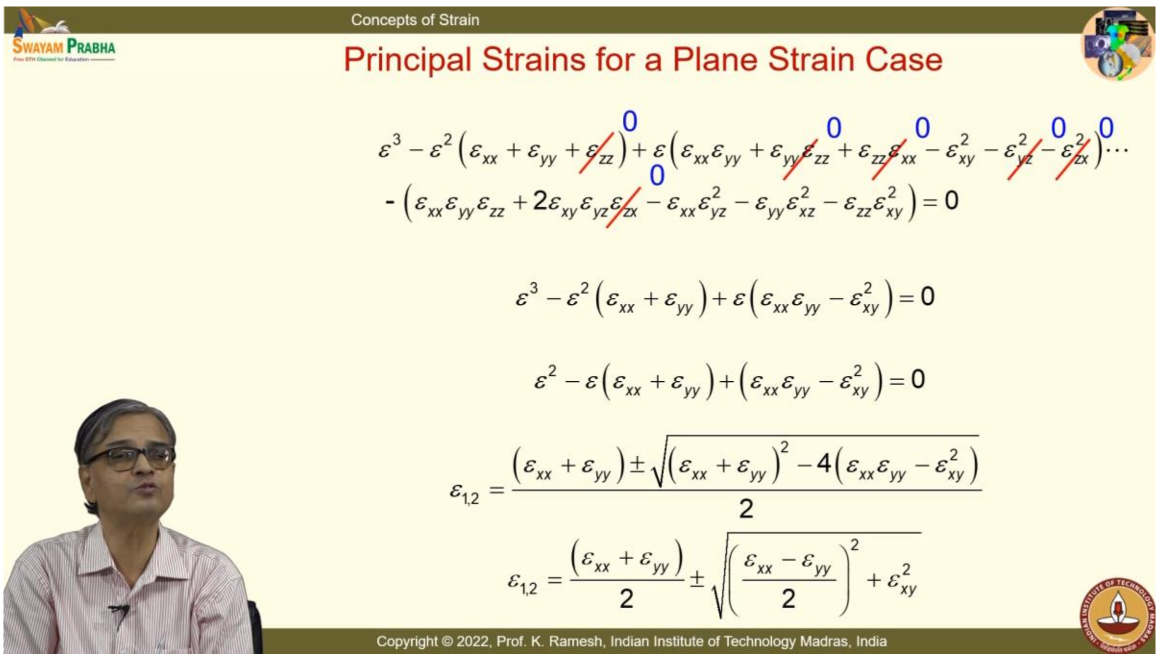

Same thing I do this here also. When I write this J_1, J_1, J_2 and J_3, they are expanded in this form. And J_2 is

$$\varepsilon^3 - \varepsilon^2\left(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}\right) + \varepsilon\left(\varepsilon_{xx}\varepsilon_{yy} + \varepsilon_{yy}\varepsilon_{zz} + \varepsilon_{zz}\varepsilon_{xx} - \varepsilon_{xy}^2 - \varepsilon_{yz}^2 - \varepsilon_{zx}^2\right)\cdots$$
$$- \left(\varepsilon_{xx}\varepsilon_{yy}\varepsilon_{zz} + 2\varepsilon_{xy}\varepsilon_{yz}\varepsilon_{zx} - \varepsilon_{xx}\varepsilon_{yz}^2 - \varepsilon_{yy}\varepsilon_{xz}^2 - \varepsilon_{zz}\varepsilon_{xy}^2\right) = 0$$

Suppose I knock off terms related to z, since we confine ourselves to two dimensions, you have this quantity also will go to 0, this will also go to 0, this will also go to 0 and the complete J_3 will go to 0. And I have a quadratic expression. From the quadratic expression, you can find out the roots and the roots turn out to be the same form. So, what you need to do is instead of σ_{xx}, σ_{yy} and τ_{xy}, you have to replace them as ε_{xz}, ε_{yy} and ε_{xy}.

You should not make a mistake that you substitute γ_{xy} there. γ_{xy}, traditionally we use a symbol γ_{xy} that is how shear strain was originally understood. So, I can get this. So, this will give me what will be the magnitude of the elongation or contraction. All that you have seen, I can have both positive, both negative, one is positive, one is negative, all the ideas what you have seen in stress are applicable here.

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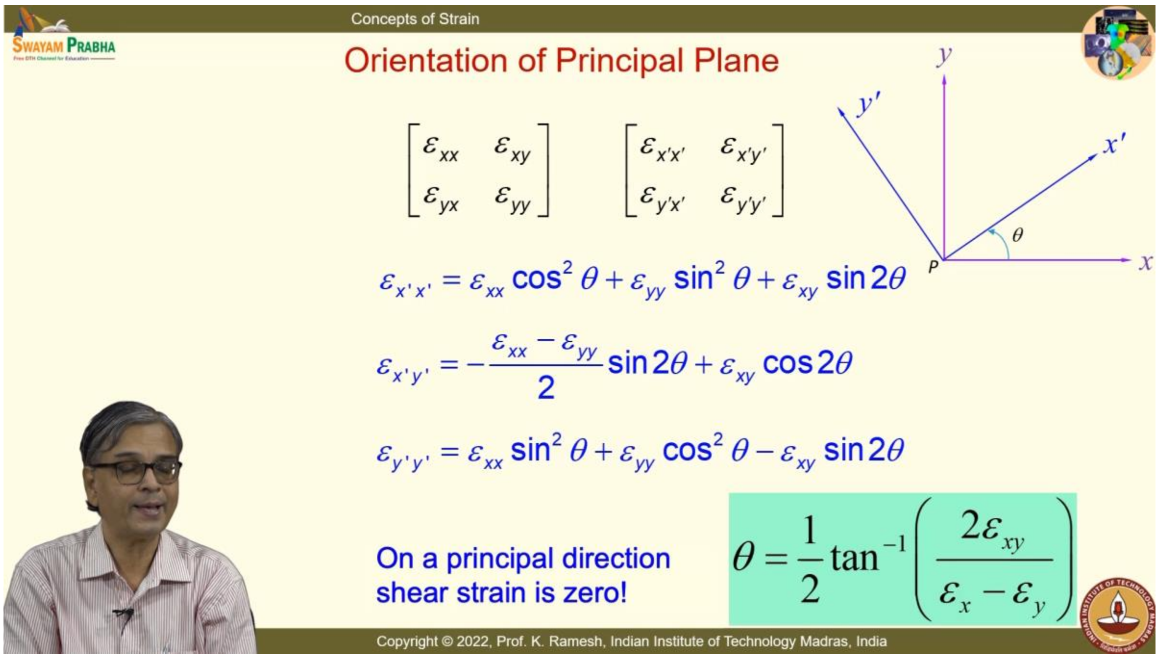

Now we will also have to find out the angular orientation. You know from strain transformation law, strain transformation law you define principal directions are those where you do not have any shear strain. So, that again reduces to what? When I say $\varepsilon_{x'y'} = 0$ gives me the expression

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{2\varepsilon_{xy}}{\varepsilon_x - \varepsilon_y}\right)$$

So, what you have to understand is we have looked at stress from the original simple experiment of taking the material with three different cross sections. If people plotted on the *x*-axis change in length divided by original length and in the *y*-axis, force divided by area, you could get only one line for the given material.

Then we developed concept of state of stress at a point. Similarly, we have looked at from that graph started with change in length by original length of strain and we have said if you have ε_x, ε_{xy} and ε_y, it behaves like a tensorial nature and whatever the concept that we developed for stress you can find in strain. We have looked at principal strain magnitudes and principal strain directions. When we go from one axis of reference to other axis of reference, the same transformation law is applicable and we have also recognized that you can have a relative displacement that is not contributing to strain. We recognize that as a rotation and while looking at the rotation, we have assigned if it is a rotation, the anti-clockwise direction is positive and clockwise direction is negative. So, we also have a expression for the rotation and in future we would also see similar to Mohr's circle of stress, we will also see Mohr's circle of strain. Thank you.
