**Strength of Materials**
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# Lecture - 11
# Free Surfaces

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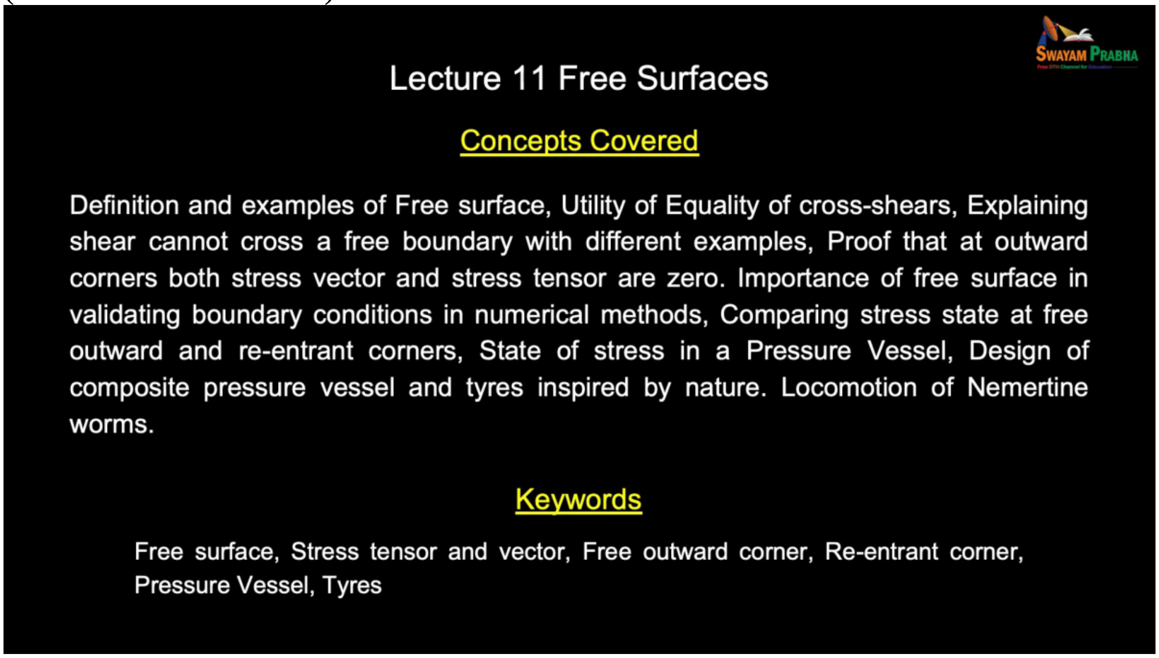


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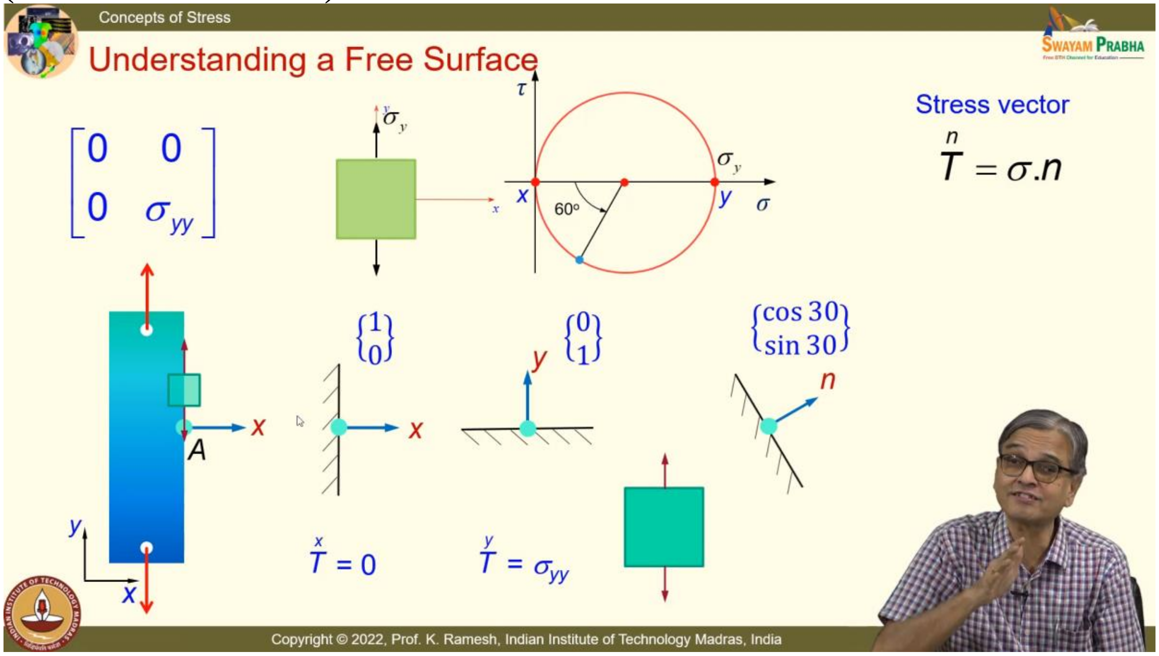

Let us continue our discussion on concepts related to stress. In fact, in this class we will try to understand the concept of stress little more better and also develop certain subtle concepts which are not normally discussed at this level of the course, but you are learning the course in 2022. So, you should also be equipped to handle the modern challenges. So, if you take up higher level courses, you need to specify the boundary condition, only then you can solve the differential equations. So, you need to understand what happens on a boundary, how do you specify the traction on the boundary. So, you need to understand certain subtle aspects that we will look it up.

So, the first concept is understanding a free surface. In fact, when Cauchy's formula was discussed in the class, one of the tutorial problem was to find out what is the stress vector acting on *x*-plane, that is what we are going to discuss. And you have the tensorial representation of state of stress. You know after learning concept of state of stress at a point, you must always develop the habit of putting the stress component in a matrix like this and give it a tensorial flavor.

If you look at $\sigma_{yy}$, it is $P/A$, $P/A$ appears like a scalar, just like a temperature, but it is not a scalar, you have to recognize that this is a tensor. And once you have a pictorial representation like this, you can also draw the Mohr's circle. You know what happens on the *x*-plane and *y*-plane, you can mark the *x* point and *y* point and you can draw the circle. And now let us look at what happens to a point, which I have shown it very big for you to appreciate very clearly, what happens on this surface. See the moment you take stress, you will also have to raise a question which plane are we talking about, fine.

And when I take this surface for you to visualize very clearly, I have shown in the surface here, it is represented by the outward normal, it is nothing but your *x*-plane. So, you can write the direction cosines. I would like you to draw neat sketches, I will give you sufficient time for you to do that. And you also have the Cauchy's formula and I have

$\overset{n}{T} = \sigma.n$ and I have this as $\overset{x}{T} = 0$. I have the stress terms are like this and when I have the direction cosine like this, this gives me $\overset{x}{T} = 0$.

By definition when the stress vector is zero, you call that as a free surface, fine. Now, let us look at the same point, but we will look at a plane which is the *y*-plane now. When you say the *y*-plane, what are the direction cosines? They are $\begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$ and you can apply the Cauchy's formula, you have the stress tensor here, you can also find out what is $\overset{y}{T}$ and $\overset{y}{T}$ comes out to be your $\sigma_{yy}$, fine. Suppose, I take an arbitrary plane, I take a plane oriented at $30^{\circ}$, you can obviously locate this plane in Mohr's circle, there is no difficulty at all, you know Mohr's circle very thoroughly. And you should also know where will I locate this plane, because this plane is anticlockwise from this by $30^{\circ}$, I have given the direction cosines.

So, that tells you this, I have not marked the angle as such. So, you should also know where I would locate this point on the Mohr's circle. I go to *y* from *x* anticlockwise like this, if it is $30^{\circ}$, I will have that fixed at $60^{\circ}$, is the idea clear? So, when I take this point, when I look at *x*-plane, when I look at *y*-plane or when I look at an arbitrary plane, you can individually find out the stress magnitude on that plane, fine. You can find out the stress vector and you can also find out if there is a normal stress, if there is a shear stress, you can find out all those quantities. Because when you have the plane like this, I have both normal and shear stresses on this plane, which I have not indicated it here.

Now, the question is, you know I have a point and then I have a stress acting on this plane, the confusion what people have is, I have this, is it a shear acting on the element? I have a point taken here and you know very well, what I have shown this as a green square, it is actually no dimensions, it can be shrunk to a point. Just because this appears tangential to your surface, you cannot construe that you have this is tangential like this and acts like a shear. Suppose, I take a point inside, you do not get that confusion, you say that it is pulled. Suppose, I take a point somewhere here, interior, there again you do not have a problem, you have a problem once you come to this. So, you should understand, it is not a shear, but it is still a normal force acting on this plane, normal stress acting on this plane.

So, your shear will go to zero, but you can still have a normal stress acting on it, is the idea clear? So, when you understand the concept of stress, you should look at on which plane am I asking this question? And if you look at this specimen, see all through this boundary is free, it is not loaded. You know what I do is, I take a specimen like this, I have a hole and then I put it in this. So, what I do is, I have the loads applied only here, I do not physically touch this boundary. So, they are physically free, this has a very important connotation. When it is physically free, what we say is, when I look at the stress vector on plane *x*, it is zero, but it can still support a stress in a plane perpendicular to it and a stress component is tangential to the surface, never forget this, this is a very subtle point, is the idea clear? We will see some more examples, ok.

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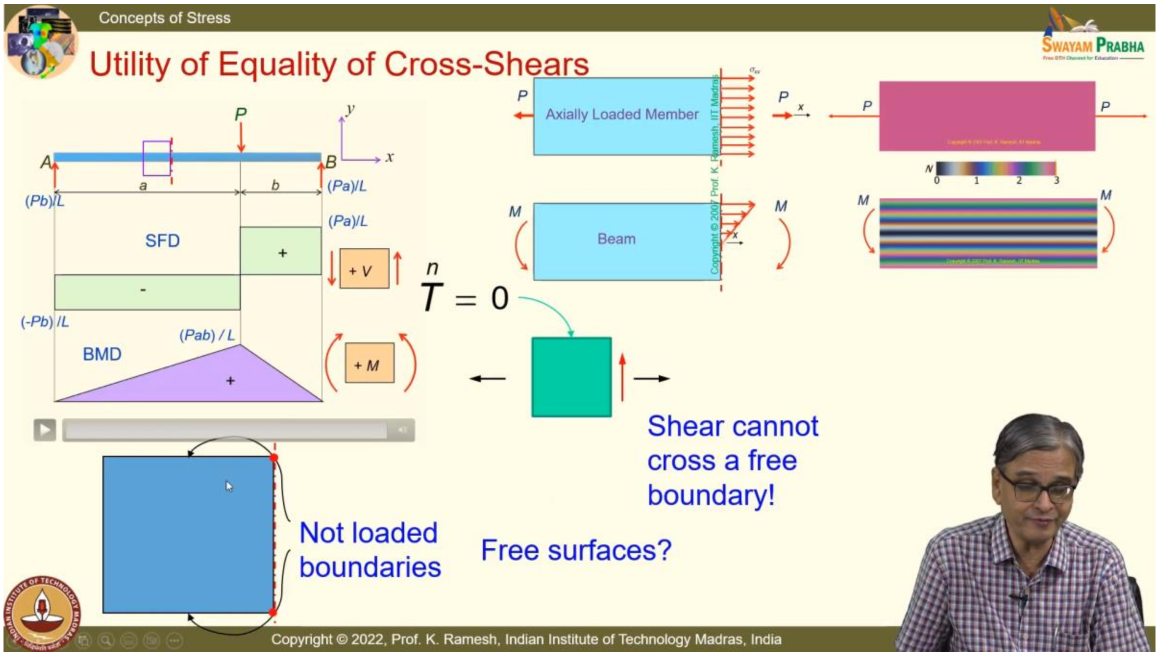


See, one of the important learnings in this course is, how to find out the variation of the resistance over the cross section? And I have said, by inference you will be able to see what happens in a beam, what happens in an axially loaded member. Now, I take up a problem of a three point bending, the load is shifted to the right, it is a simple problem where you can write the shear force diagram and bending moment diagram. Do you know why you have drawn that? Do not tell me that you have drawn it to get grades in that course, fine. See the whole idea is, when you want to design or select what should be the cross section of this beam, I should find out where the bending moment is maximum, so that I select a cross section which withstands the maximum load that the member is supposed to have experience when the actual loading takes place. So, that is why you have drawn it.

And from the fringe patterns you know that this is uniformly distributed, because you are now tuned to see that you have a constant color. Whereas, in the case of a beam, the color varies which we interpreted this as a triangular variation. And we have also said that the distance between the fringes are constant for a given load. And you also see that this stress goes to zero. See there is a distribution here and obviously, the distribution should satisfy the boundary condition. I am applying a bending moment. So, this distribution when you find out, it will also generate a bending moment. But there is a variation over the depth of the cross section and this goes to zero at the center that we have seen for the bending stress. And we will also say that this solution is applicable even though in this beam in addition to bending I have a shear that is transmitted by the beam. Definitely, once we take up bending, we will take up a discussion to find out what is the distribution of the shear stress over a cross section.

Let us take a cross section which is far away from the load application points. Now, you know very well, you have been exposed to what is the Saint-Venant's principle. So, if you work far away from it, you know our visualization is very simple as what is the intended load is realized in that far away from the load application point that also makes our life simple. From the background which we have developed so far, you can answer at least what happens to the shear stress value at selected points that you can answer, if not for the distribution. I take a magnified view of this and I have the cross section shown and you can see very well here that whatever the boundary I have on the top and the bottom, they are not loaded boundaries.

Is the idea clear? And we have to know what is the variation of shear stress over this cross section which is not simple as constant or linear as what the bending stress is, it is little more complicated. That we will postpone for a later discussion. Can I comment what happens on the top point and the bottom point? With your background in concepts related to stress, can I find out what happens just at the top point as well as the bottom point? We will take the top point, fine. When I say the top point, I show that as a square block whose dimensions are zero. And you know very well I have applied a shear stress, I have applied a shear force, fine. I have applied shear on this surface of the, on this surface of the beam. What happens at the top point is what we are going to investigate. For the purpose of investigation, let us have that this has a shear stress like this. In some form, I am not looking at my mathematics will tell me the sign. We will not worry about the sign and things like that. I have just taken on a positive surface, positive direction is positive. Now, the question is on the top surface for the point here, can I have this shear stress? I have given you a clue, they are not loaded boundaries. I have raised a question whether it is a free surface. Do I do anything with the surface on this? I do not do. So, this quantity has to go to zero.

So, in a generic problem, we simply say the plane as n. You should qualify what is the plane. In this case, it turns out to be the $y$-plane, but it can be anything because we are going to take certain examples where you have the boundaries are curved. So, I cannot have this. Can you say anything about this shear stress? Because you have learned something about shear stress interrelationship.

You have learned $\tau_{yx} = \tau_{xy}$. So, if I cannot have a shear stress on this surface, I cannot have this shear stress. When I cannot have this, I cannot have these stresses either. So, at this level of course, development you can confidently say yes, I am going to have a shear stress distribution on the cross section. It has to be zero at the top point. Extending the same logic, it has to be zero at the bottom point. In between what happens, let us postpone it. At least this I can confidently say from whatever we have developed or when you develop the distribution, if this is violated, then you can say there is something wrong in your calculation of the distribution. Either way you can look at it. When I come to this point, can I have the stress in the plane perpendicular to it? Can I have a stress component like this? Can I have or not? See what is shown here is, what is blinking is shear cannot cross a free boundary.

It is a very famous axiom which you can use it and find out and simplify many of your boundary conditions. That is why I put this as blinking. It is not existing and this is like your Lakshman rekha. You cannot cross. You should not cross was the requirement in Ramayana.

What happened you know the story, but here it cannot cross. Can I have this component of stress perpendicular to it on this point? You have enough clue in this slide. I have already said whatever this is developed for a beam under uniform bending, there is no shear stress acting on it. I said this solution is applicable to this problem as well which we call that as engineering analysis of beams. Now, the question is, I am looking at a point which is sitting on the surface. It cannot have a shear stress even though I am applying a shear force. The distribution has to be such it should start at zero, but the resultant will match with the shear force. Resultant has to match because the equilibrium should exist. That should not be violated, fine. I am asking a question can I have a stress in this plane perpendicular to it? Tell me.

No. Why do you say no? See this beam is transmitting shear force. The beam is also transmitting bending moment. When I look at this cross section, I have a shear force magnitude. I also have a bending moment magnitude. From the shear force perspective, I will have shear force, but the distribution should be such it should be zero at this point and it can have some variation later. Is the logic clear? I have also drawn up this. Here it is zero. I have a distribution of normal stress, but it is zero. So, when I say a distribution can have zero also as a value at a particular point.

There is no harm about that. So, when it has to transmit the bending moment, we see here on the top surface it is actually maximum. So, when I sit on this as a free boundary, shear cannot cross a free boundary. So, shear stress cannot exist, but I can still have a normal stress in a plane perpendicular to it. See this point is same. If it is temperature, you have only one temperature whether this plane or that plane, you have only one temperature. It is a scalar. When I come to a tensorial quantity, you must filter it out and understand what can happen in different planes. That is the subtlest aspect of stress. Whatever the tensor, when we graduate from stress we will go to strain. Strain is also a tensor of rank 2.

So, what you can, I am showing you evidence. I have bending stress. So, it can have stress perpendicular to the surface. You cannot have shear crossing the boundary, free boundary. We have also seen for the simple case of axial tension. Is the idea clear? We will see this for many more examples and I am going to take up a problem of a plate with a hole.

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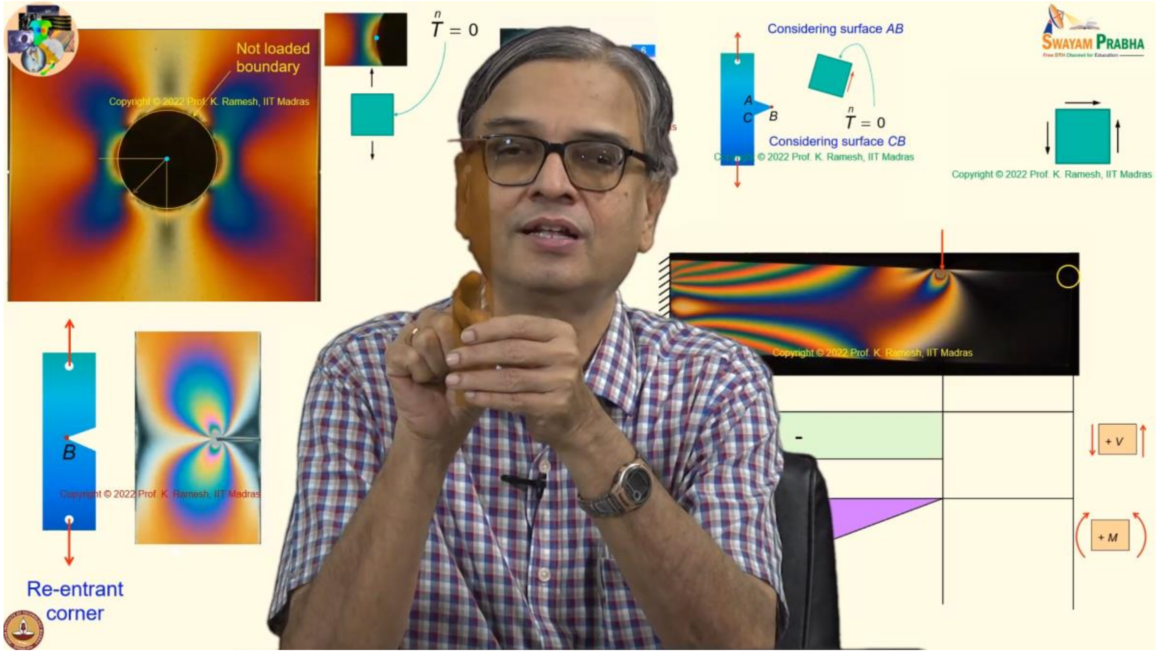


See what you will have to look at is when I have a plate, I am not sure whether you are able to see this. Can you see there is a very small hole here? You cannot even see it. That is a good thing because you know mathematically I can idealize this as an infinite plate with a very small hole. For this, you have closed form expression exist.

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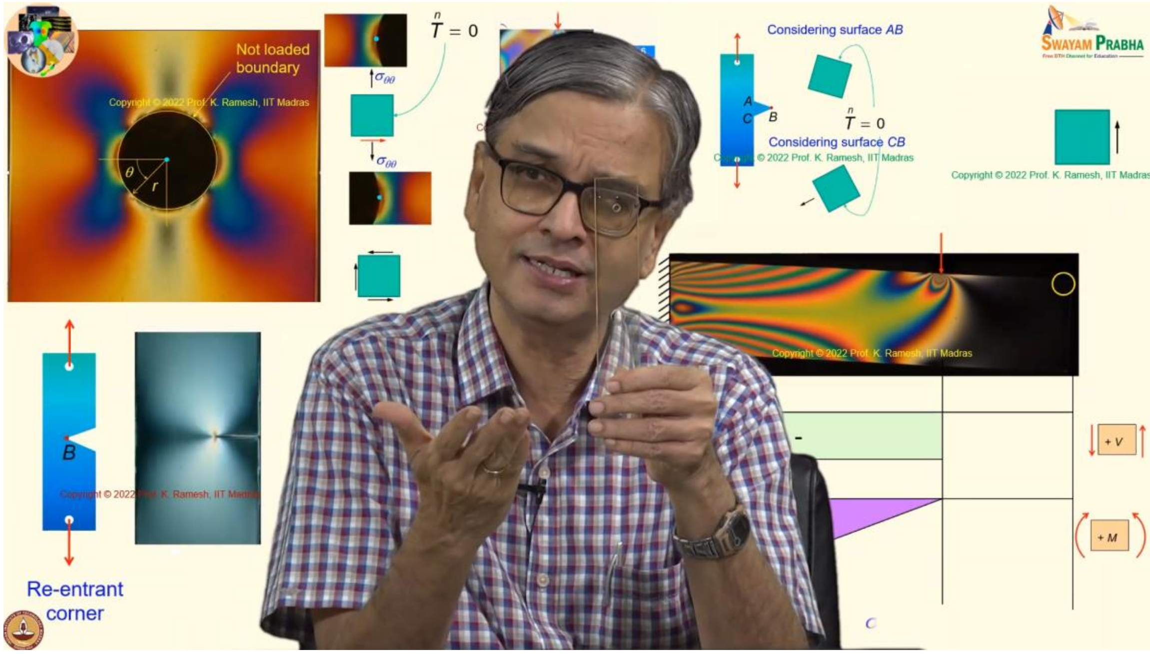


On the other hand, when I take a problem where I have a hole like this, the hole is perceptible.

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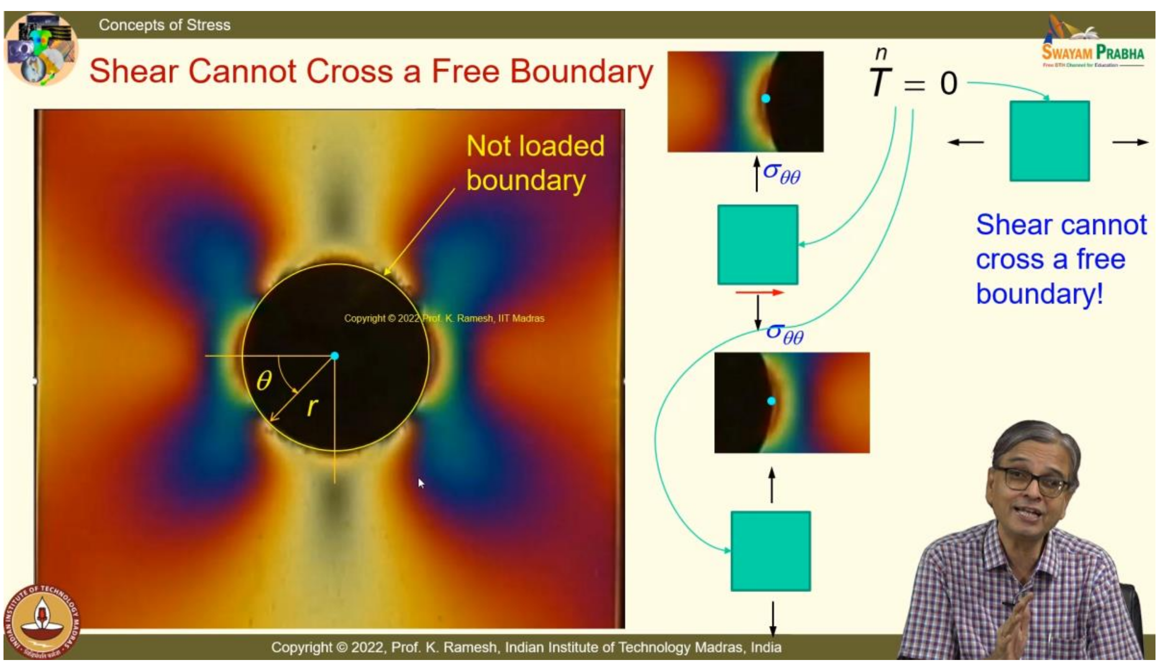


What fringes I am showing corresponds to this hole. See you have a problem when I have a plate without a hole, you have uniform color. Every point has same value of stress $P/A$

The moment I put a hole, you have a flowery pattern that gets developed. Not only that, you have the fringes are also very high. The fringes are also very high. I will just stop this animation, ok. I have this here and you can see from the color code, this is around this color in the region away from it, but when close to the hole, the colors are somewhere here. So, you have very high stress developed because of a hole. In fact, in earlier development of stress concentration, people had, celebrities had fist fights in the conferences. Mathematicians would come and say infinite plate with a small hole is infinite plate without a hole.

So, nothing happens. You can ignore the hole, but whereas, all the failures start from the hole. And it took sufficient discussion to appreciate that presence of a hole introduces the concentration of stress, which you can see very clearly. I have the color code. The color code shows, says that this is very high number. And this turns out to be three times the *P/A* for a infinite plate with a small hole.

If I have to go and solve that problem, I should know how to specify the boundary condition on this inner boundary. Now, let us see whether we can do that from what we have understood from this. Concepts of shear stress cannot cross a boundary, can be extended and understand. And I have already said that you should understand shear cannot cross a free boundary and I am showing it as an animation.

So, you have $\overset{n}{T} = 0$ and these shears also go to zero. So, shear cannot cause a boundary that does not imply everything is zero on that surface. It can support a stress perpendicular to the surface, if the situation is like that. We will also come to a situation where it cannot support, it will also go to zero. We will see that counter example also, fine.

But in general, you cannot say everything is zero. When free surface, if you say $\overset{x}{T}$ or $\overset{n}{T} = 0$, it is not everything is zero there. Stress tensor still exist. When you say stress tensor still exist, you have to look at different planes. What is the quantity or what is the magnitude of stress vector on all the possible infinite planes, ok.

Now, I take the boundary and I have shown you this. I am going to only pull this. So, what happens to this boundary? You are not doing anything. I am not putting a, I am putting a pin here to pull it, but this surface is not loaded, ok. So, that means, it is a, it is a free boundary, ok, not loaded boundary. So, now I take up a small segment from this and I take up a point.

Let us investigate what kind of stress state can exist at this point. Is the idea clear? I can take up a point and then I have shown this as a square of zero dimensions. For argument sake, I take a shear stress on this. Can you tell me of these quantities, which shear stress will go to zero from the understanding? See earlier we had seen straight boundaries.

Now, we have come to a curved boundary. Which surface is free? I look at this surface, not loaded boundary. So, this shear cannot exist. So, this is a free boundary. If this shear cannot exist, I cannot have this component of shear by equality of cross shears.

So, I cannot have the other two shear stresses. So, shear cannot cross a boundary. I show that as a red blinking arrow. That means, it is not there, but you have to understand the red signal. It is not there, it is highlighted. It is not that I have, I will have only this shear stress component, all other shear components are zero.

Do not interpret it that way. To emphasize the point that this is the free surface and this is not existing, I have changed the color. But in principle, you can have this. Do you agree that I can have an axial pull? Now, you know coordinate system is your choice, whether you take a *xy* reference frame system or $r\theta$ coordinate system. If you want to simplify a problem, depending on the geometry, you select the coordinate system. When I have a circular hole, it is easier for me to define the circular boundary in polar coordinate system.

Do you agree with me? So, I have any generic point is specified by the distance $r$ and this is your $\theta$. Can you specify what is the component of stress that you can label? Because now you know how to label it. If we have an infinitesimal element, this infinitesimal element is taken from this point and in this coordinate system of $r\theta$, how can I label these two stress quantities? Any guess? First of all, you should recognize that you can have a normal stress like this. Now, you should label it.

In the earlier cases, you labeled it as either $\sigma_y$ or $\sigma_x$. In this case, how do I label it? Somebody? I am hearing louder please. You are right. $\sigma_{\theta\theta}$. I can label it as $\sigma_{\theta\theta}$.

Now, let me take up a point on this portion of the hole, ok. I take up a point, you do the mental exercise, make notes. Do not just listen. I give you sufficient time to write down. So, in this which surface is free? Which surface is free? I have this as free.

So, I have this as $\overset{n}{T} = 0$. See the other shear cannot exist. So, you have all the shear stress are extent and you have possibility of this normal stress. Because I have taken this as $r\theta$ coordinate system, this can again be labeled as $\sigma_{\theta\theta}$. So, on the boundary of the hole, I can have $\sigma_{\theta\theta}$. I cannot have any shear stress.

In this coordinate system, you will call the shear stresses $\tau_{r\theta}$. So, that is the boundary condition that you have to specify. To specify the boundary condition, you have sufficient background. But you do not have a background. Once the boundary condition is specified, how to solve the problem? That can be done only in the next level course, fine.

Because you need to develop semi-inverse method to do it. Let us take up another simple problem which we have looked at, which we developed the stress magnitudes without even a great discussion on the displacements. Have you not done that? It is simple *P/A*. You just rotate it and then tie it.

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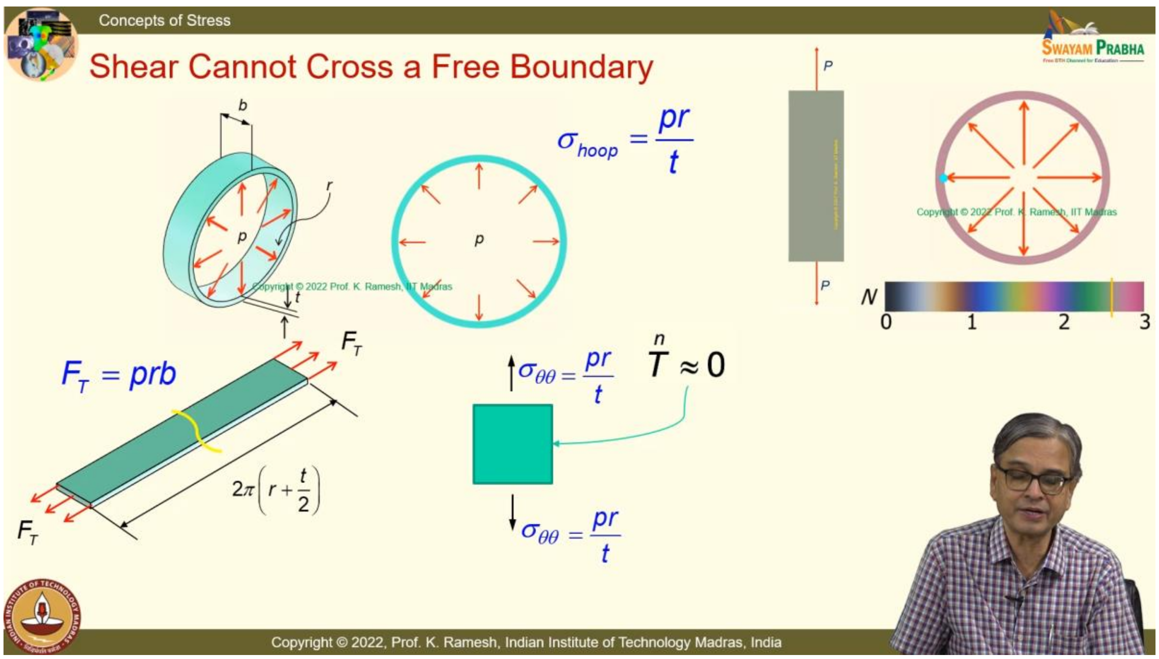


And we also seen that this as the complete length of the hoop and we got the force as $F_t = \Pr b$ and we got the stress as $\frac{\Pr}{t}$.

It is a very famous result. Now, let me take up a point. I take a point on this. Now, how you have loaded this boundary? You have loaded this boundary with a internal pressure. And when you have internal pressure, it is all radial to the surface. There is nothing tangential to it. Suppose, I take up a point and investigate, can there be shear on this? Can there be shear on this? There is no shear.

It is only axial force is applied there. Radial force is applied on this surface, fine. But I can still have a normal stress and that is what you calculated.

You have this as a free boundary. I would say that shear is zero. We have to investigate. $\overset{n}{T} = 0$, we have to investigate. $\overset{n}{T} = 0$ may not be strictly correct because in many problems we ignore the radial stress. But there is no shear on this boundary and shear cannot cross that boundary because shear is not there.

So, shear cannot cross. But I can have a stress in this direction. This is nothing but what you learnt as hoop stress, ok. This is what you have learnt it as hoop stress. $\overset{n}{T}$ may not be zero here. Shear is not there. That is the definite answer because when I consider even the pressure, I will have a compressive load due to pressure acting on it, ok.

You can correct the information in that light. Now, let us take up another problem. So, you have this as $\frac{\Pr}{t}$ and $\frac{\Pr}{t}$.

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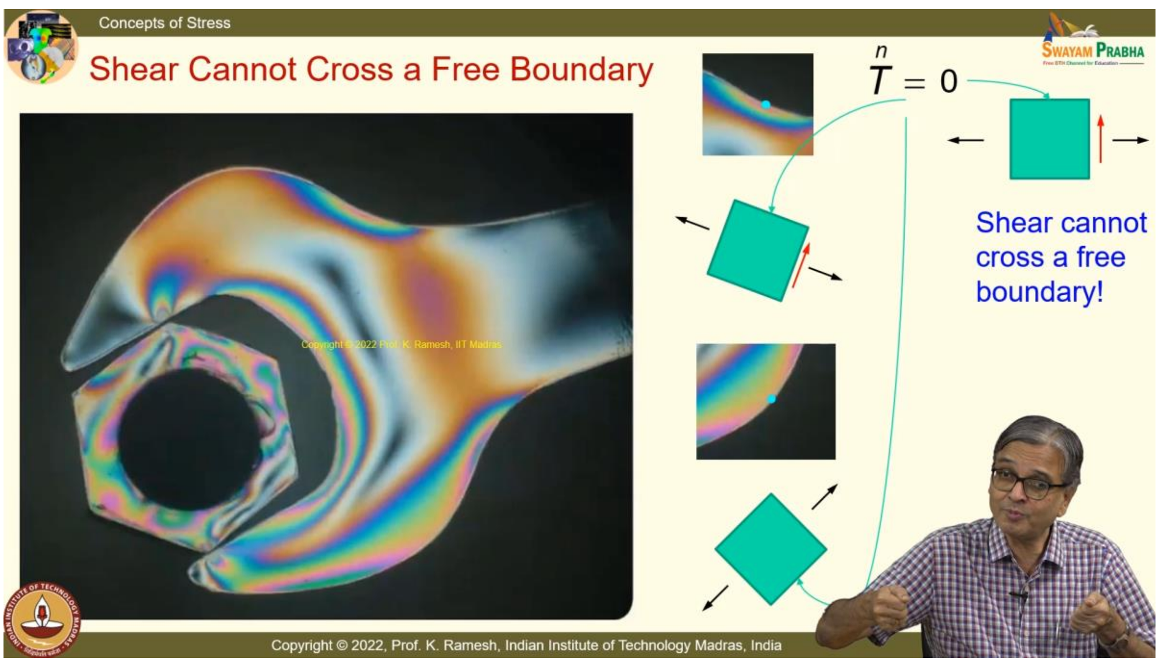


Now, I take up a problem where I do not have a circular boundary. I have a generic curve. So, I can also look at a curve with normal and tangential coordinate system, ok. And you have a luxury because normally when you load a member, you can see only displacement because you have exposure to photoelasticity which I have been showing in from class after class.

You are now in a position to see the stresses in beautiful colors, ok. And you are able to see that this is a stressed member. You have an advantage. It aids your visualization. Now, I take up a portion here.

This is a generic one which shows that shear cannot cross a free boundary. And but in general, it can have a axial stress that is the message there. Now, I take up a portion from this.

I take up a portion. This is a concave portion. I will take a convex. So, to remove the mental block, we graduated from straight lines to circles. Now, to non-circular geometry, even here if I take up a point like this, you have enough information to specify what can happen on that surface. Suppose, I take an element from here of zero dimensions. This is your free surface because there is nothing, no force acting on this, ok.

So, this shear has to go to zero. So, you have all the other quantities go to zero, but it can support a normal stress. So, this will be labeled in the coordinate system. I can have $\sigma_{nn}$ and $\sigma_{tt}$, ok.

So, this will be tangential to this. So, likewise you can label the stress. Now, I take a convex portion. I take a convex portion. Here again I can repeat the same exercise. I take a point and then I look at what happens on this and which surface is free.

You have this surface is free, ok and this shear cannot exist. So, other shears also cannot exist, but it can support a stress component in a plane perpendicular to it which is normal to that. It can exist in principle. In this problem also it can exist because you see the colors, Ok.

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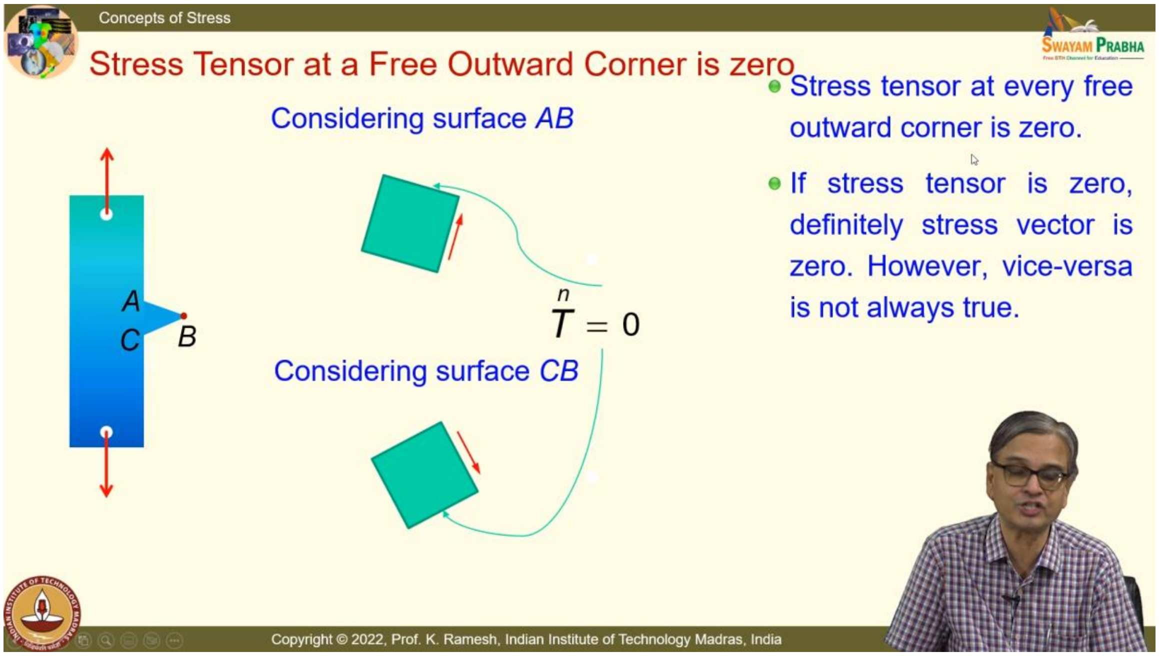


Now, let us take up a very special point. I have a simple tension specimen, but it has a projection and what is important here is this is a very sharp corner.

It is very pointed edge. It is like a knife edge. If it is not a knife edge, you cannot develop the logic. It is not you do not have a fillet there. Now, let us take up a point there. Now, let us look at what kind of stress state can exist at this sharp corner.

So, I take up the point as *B*. I look at the surface *AB* and you know very well now by looking at the physics of the problem, it is not a loaded boundary. And if I take an element like this which is of zero dimension, this shear cannot exist. So, that is $\overset{n}{T} = 0$. So, I cannot have this shear and all other quantities go to zero.

But in principle, it can have an axial stress like this. If I take any point on this and then I come to the edge here. Do I have any material point next to this to exercise this? So, this cannot exist. So, this also cannot exist. Now, let us investigate what happens to point *B* by looking at surface *CB*.

Here again, I take a hypothetical shear stress for us to investigate. This shear cannot exist and in principle, it can have a normal stress. And if I extend beyond *CB*, I do not have a material point to provide this. So, this goes to zero, this also goes to zero. It gives a very interesting result that stress tensor, on a free corner is zero.

We have seen on a free surface, stress vector is zero. When stress vector is zero, it is not necessary stress tensor is zero. I can still have some component of stresses acting on different planes. But the moment I come to a free outward corner, from the basis of the argument that I have put forth, you get a very interesting result.

If stress tensor is zero, definitely stress vector is zero. That is the, they go together. However, vice versa is not always true. What we have seen is stress tensor at every free outward corner is zero. See, these are also free outward corners, which is labeled in the next slide.

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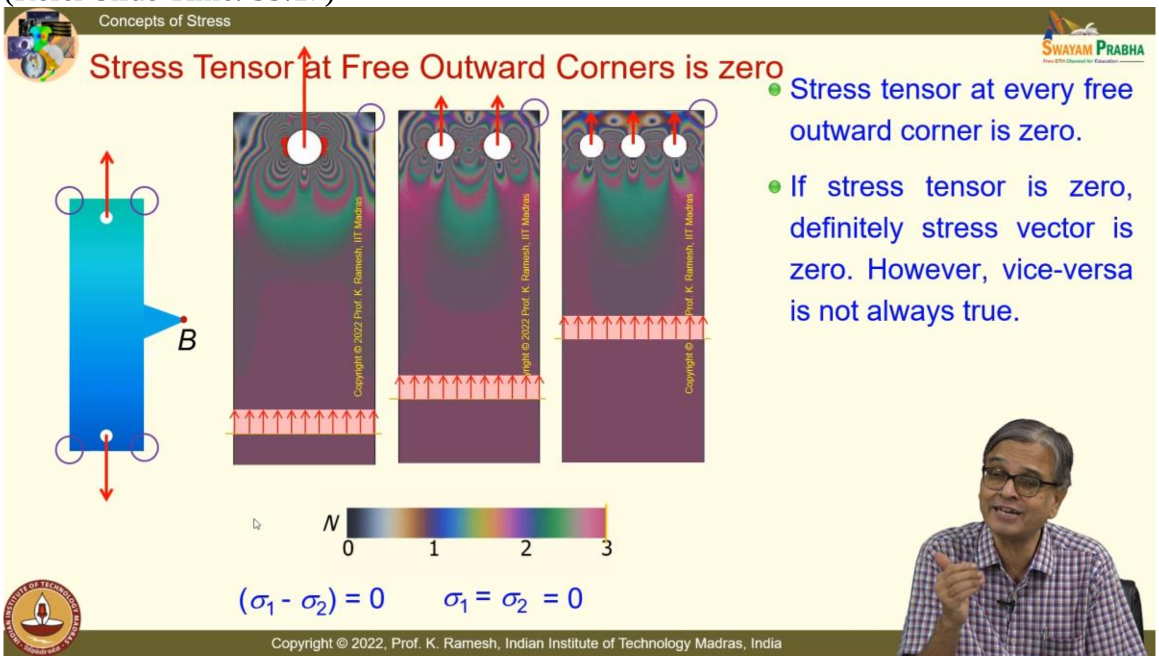


We will go to the next one. You know, I have taken this up because we have looked at what is the Saint-Venant's principle and that concept you should understand.

If I have a resultant force is same and if I see when this force application becomes uniform that takes at different distances. If I make my load application more and more equivalent to the distribution I want, the distribution becomes uniform earlier than the other case. But if you look at here, I have these corners are black and what you see black in this color code, black starts at zero. And we have also looked at when I say a fringe order, it only indicates $\sigma_1 - \sigma_2$.

It does not say $\sigma_1$ or $\sigma_2$. So, when I have this, I will have $\sigma_1 - \sigma_2$ equal to zero. It only implies $\sigma_1 = \sigma_2$. And in this specific case, when I am looking at the free outward corner, individually these magnitudes are also zero. This is an indirect check for you to verify your experiment. Also when you want to model this problem numerically, your numerical analysis should give these values as zero.

It is a very powerful yet a very simple concept. People ignore it. And what happens near the hole? It is too complicated. See in this course, we are worrying about what happens away from the point of loading. Because I have shown you the experiment, you also see a very complex play of stress magnitudes near the hole. You see different levels of stress concentration. Unless you go to numerical analysis or experimental analysis, you cannot handle it, fine.

So, like I have this, see these are also free corners. These are also free corners. All these free corners, whether you do it by analytical approach, mathematical approach, experimental approach or numerical approach, you have to get the stress tensor as zero. This is one of the indirect checks.

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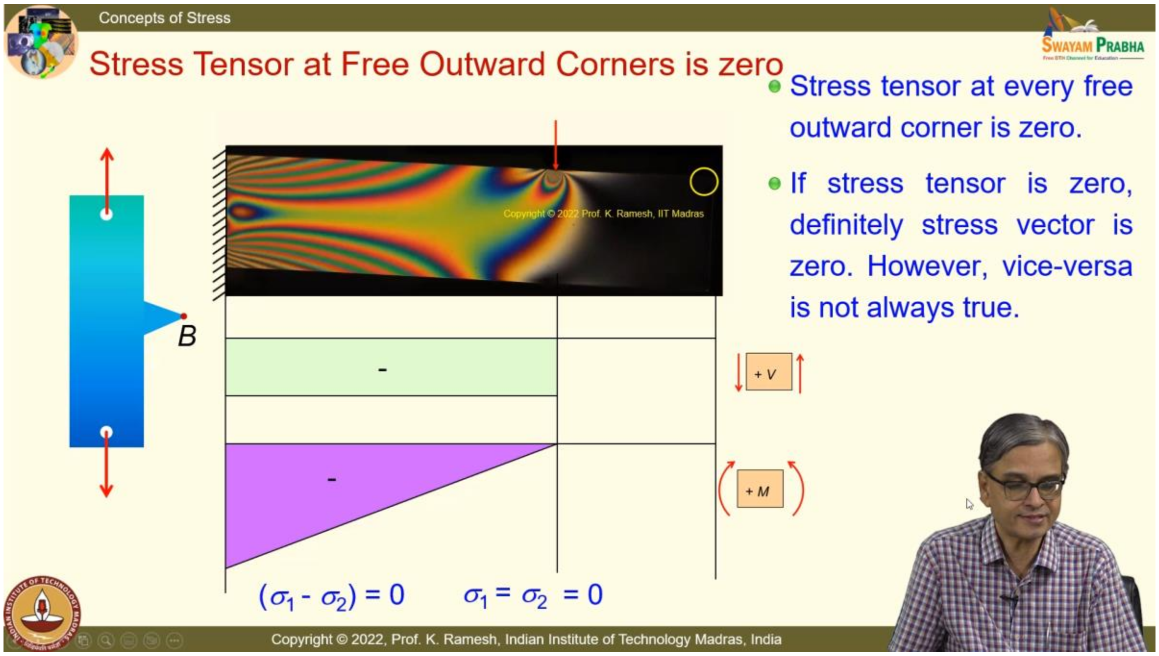


The same thing you know you can see in a cantilever beam. I have a cantilever beam. I have applied the load here. Can you draw the shear force and bending moment diagram for this? This is fixed end here. You have a cantilever. The cantilever is protruding up to this.

I am applying a load in between because I want to bring it to your idea. If you do not load it, you will have everything as zero. You should also understand that. You know what is the shear force diagram and bending moment diagram for this. Can you draw and verify it with what I have in my slide? I suppose you understand the problem.

I have a cantilever beam. I have a load applied in between, not at the free edge. So, when I draw the bending moment and shear force diagram, I would definitely show that this zone is zero. There is no load at all and which is also verified in your photoelastic results. I get this zone as completely black and the corner is merging with the background.

The background light is zero. The corner is very difficult to see and we have seen on a free outward corner, stress tensor is zero. From your understanding of shear force and bending moment diagram, nothing should exist here, ok. That is also you are getting the clarity. See the same concept I am trying to illustrate through graded examples so that you will appreciate the concept better.

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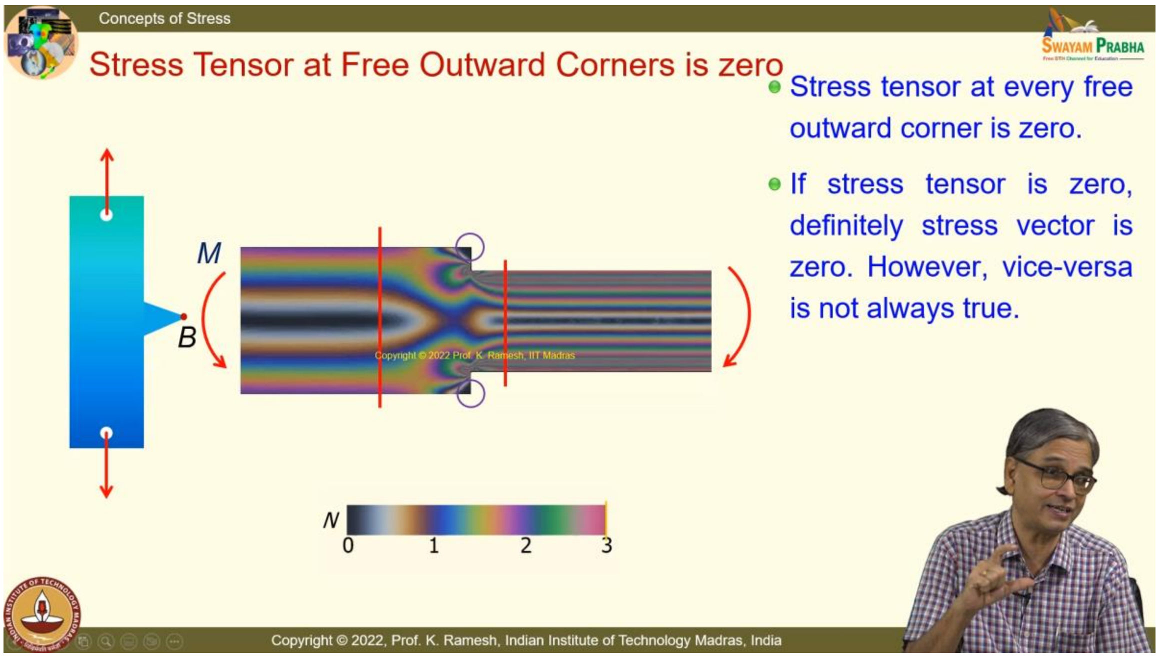


Now, let me take up the another problem. I do not know how many of you have noticed it when I shown. We have also taken up a stepped beam. I said if you take a shaft, you need to have steps, but in this course, we will simply take it as a slender member of uniform cross section. We will make our life simple. Even that is back breaking when I have to find out what is the distribution of shear stress, but you cannot avoid.

You have to have steps. I have a free outward corner. Now, you know what should happen in a free outward corner? Stress tensor is zero, stress vector is also zero. Stress vector is zero on this surface, but especially at the corner, you have a very special situation. Stress tensor is zero and we have also noted the idea of St.Venant's principle. Away from this disturbance, this is horizontal, this is also horizontal and that differs depending on the width, ok. So, you are also learning more of engineering, not just stress analysis, not just concepts related to stress.

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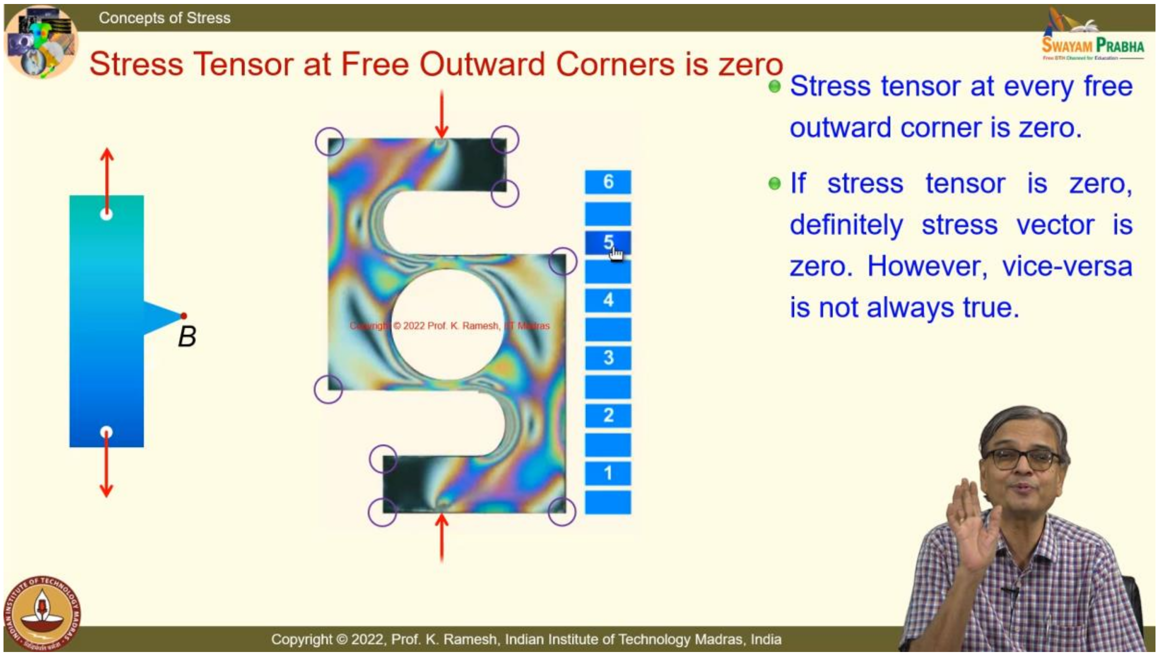


Now, I take up another very interesting problem. See this is a very complicated element which is available in all your load cells, the very common spring element used in load cells and I have so many corners 1, 2, 3, 4, 5, 6, 7, 8.

All these corners you have to get that as zero. This is shown as an experiment. Are you not seeing all those corners have black color? The experiment is well conducted. Suppose I analyze the problem numerically and plot photo elastic fringe pattern, I should also get these as stress tensor being zero. If stress tensor is not zero at the corners, something wrong in your boundary conditions, ok.

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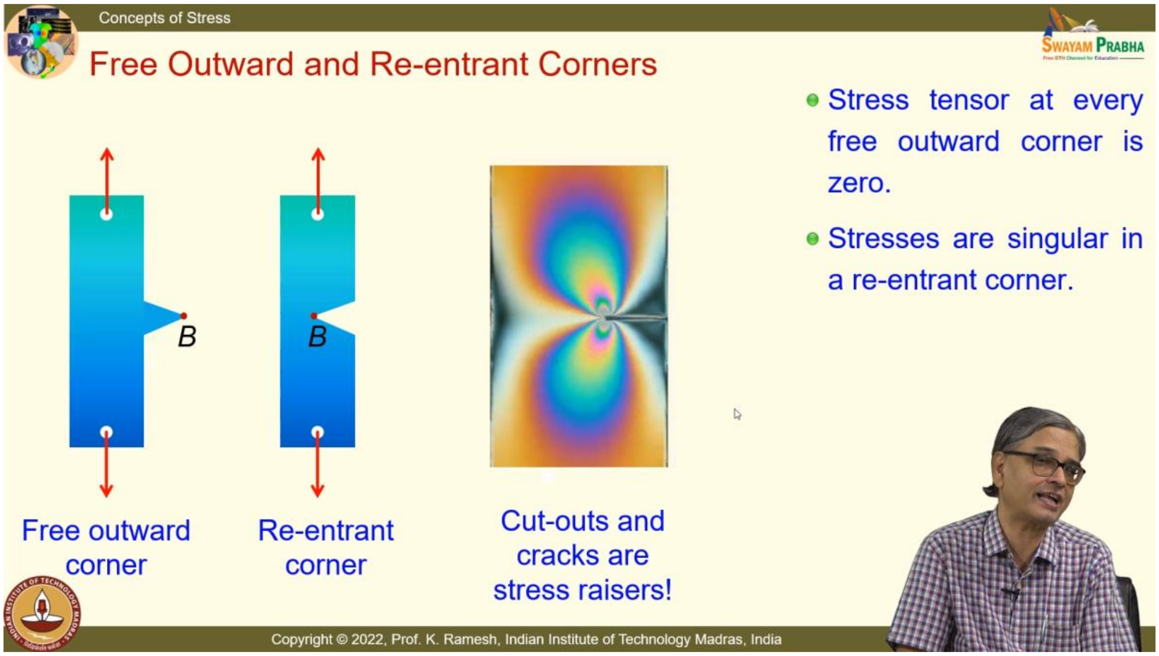


Now, I have taken up a problem like this. Suppose I do not have a projection like this, I have a situation where this is opposite. This is called as a re-entrant corner. You make a wild guess because we have learnt concepts related to stress. We know what happens on a free surface, we know what happens on a outward corner and is it a loaded boundary here? It is not a loaded boundary.

So, it is still a free surface. We have learnt what happens at this outward corner. Can you guess what could happen in a re-entrant corner? Can I say it is zero? You cannot say it as zero. If you recognize that, I am happy because when one place you say zero, other place also should be zero. I have a material point here and mathematically this goes to singular values, but physically it reaches the plasticity and I do not have a fringe patterns for this V-groove. My students will work it out and then we will show it in some class later.

I have a limiting case when it becomes a crack and you know very well from photoelasticity. When I see more and more dense fringes, what happens? It means stresses are very high. So, do not jump to conclusion this is zero here. I see a free surface, I have a corner that is also zero.

You should learn the subject thoroughly. So, whenever I have a cut out, whenever I have a crack, they are all stress raisers. So, stress tensor at every free outward corner is zero. Stresses are singular in a re-entrant corner. So, you have to be very careful when I come to the re-entrant corners and that is very important learning that you get and you have a whole course of fracture mechanics deals with that.

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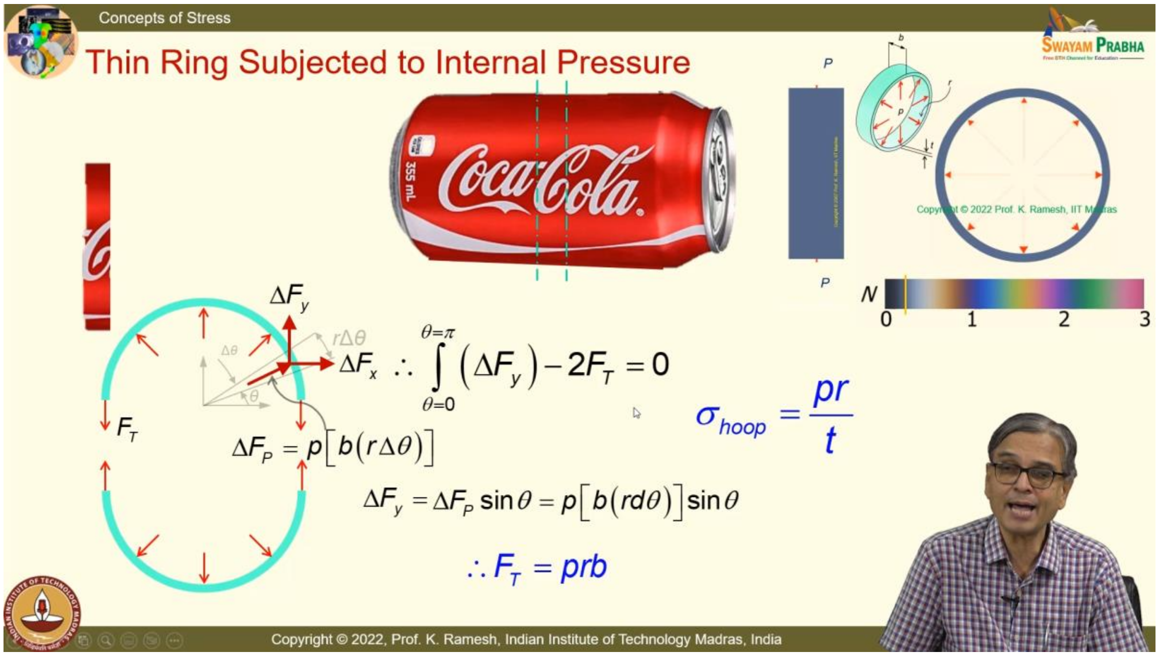


It is a very important concept, fine. You know this is a recapitulation. So, I am going to go quickly. We have already seen if I take a thin section and if you want to find out what is the stresses developed, we have seen that this has a hoop stress, ok. I am not going to spend time on it because my interest is to complete this problem because we have taken this problem which is closed ends and you know Saint-Venant's principle I will not worry about how this is closed, how this is closed. I will stay away from the ends, ok. Now the interest is what happens in the longitudinal direction or the axial direction the way that I have drawn.

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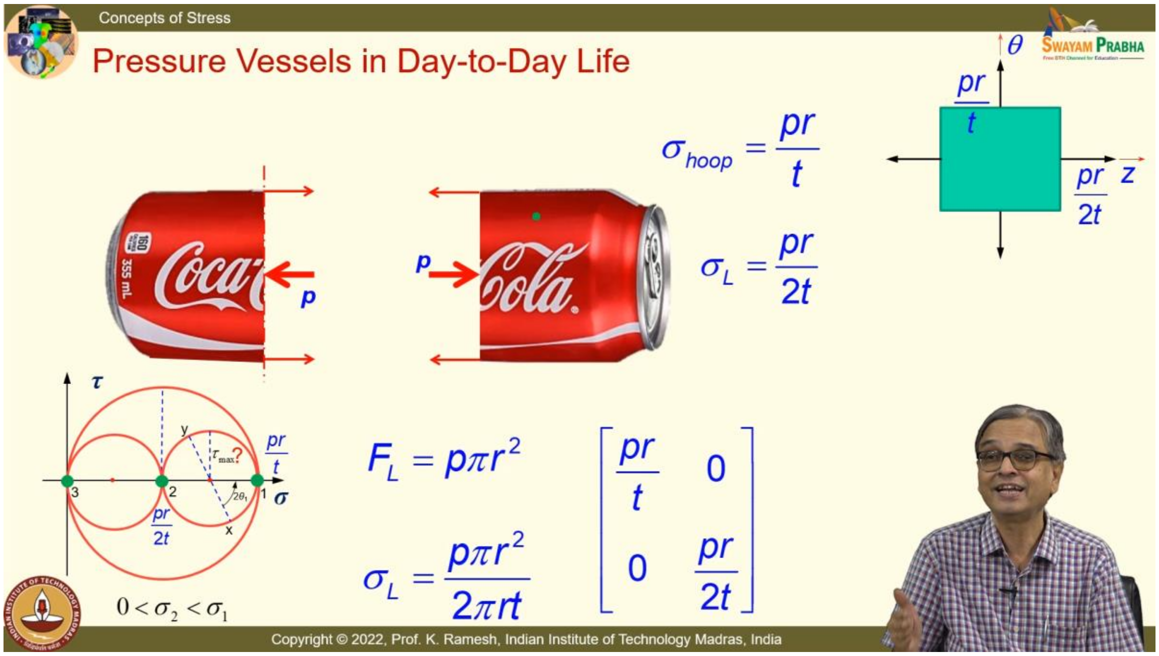


So, I am going to take this and I am going to cut this separated into two and you will have a force interaction on the complete circumference. I represent it as a arrow. I can find out the resultant, fine and that resultant can be equated to what is the pressure? Acting on which area? Pressure is acting on the complete area. So, if I do the force equilibrium, I have the force $F_L$ which is the resultant of this that is equal to $p\pi r^2$ and if I divide it by the circumferential area $2\pi rt$, I will get this longitudinal stress as $\frac{pr}{2t}$. It is a very famous problem. We have already seen what is happening on the circumferential direction because suppose I take the axis as $z$ direction and the radius as radius denoting the circle and $\theta$ in the other direction, I can take the element, I put this as $z$ direction, I put this as $\theta$ direction, I put the hoop stress that is $\frac{pr}{t}$ and I have this as $\frac{pr}{2t}$ and I have the stress tensor written down like this.

$$\begin{bmatrix} \frac{pr}{t} & 0 \\ 0 & \frac{pr}{2t} \end{bmatrix}$$

Can you investigate what happens to $I_1$ and $I_2$? You can very clearly say $I_1$ and $I_2$ are non-zero. So, this is the true case of a biaxial stress and to develop the stress components, have we done any circus? We have not done any circus. It is so simple, the problem taken is straight forward with the knowledge of your rigid body mechanics, we have been able to get these quantities. Can I also draw the Mohr's circle? Can you draw the Mohr's circle for this? And do you know your coca-cola can is made of what material? It is made of aluminum. Aluminum is a ductile material, is it not? And we have all along seen that ductile material fails by shear.

Suppose, whatever you have learnt now, can I take this as a maximum shear and design my coca-cola can? Yeah, I get the answer I want. I want somebody to say yes. It is a trap. I have put a trap and you have caught into the trap.

It is a very, it is a pure biaxial problem and both the principle stresses are positive. When you graduate, you want zero. One after several zeros are your salary. So now, the zero is a nuisance value. I will have to find out this and say when I want to design this, I have to look at what is the role of the $\sigma_3$ playing and I have to find out the maximum shear stress.

It is actually twice this value. How many of you have travelled in an aircraft? Very good, many of you because now you have a Bhoodan scheme. So, people have connected all the cities. How many of you recognize that it is a pressure vessel? Do you know that it is a pressure vessel because it is pressure vessel, otherwise you cannot breathe and you have nice windows on the aircraft. Will you sit in an aircraft which is completely closed? From stress analysis point of view, it should be completely closed and then only you have to travel, nobody will step into the aircraft and you want to have a nice window, fine. And if you really look at one of the first jet aircraft was the comet aircraft that had a huge window for people to enjoy.

Even queen victoria have travelled in that aircraft because it is a prestigious travel in the aircraft that was a jet airplane and it was permitted in UK. It was never allowed to fly in US because the US DGC has said those openings are not designed properly that is the problem for that could cause potentially a failure. In fact, the aircraft has to be withdrawn because of that issue. Now, you have a very small window which is also curved. So, any opening that you have either in your buildings or your aircraft which is a pressure vessel, you have to design those openings properly.

And this is a very nice example, if you simply say maximum shear stress theory and then investigate the failure, you can burn your fingers, fine. So, you have to be very cautious, ok. Then I will say because you know I will take few more minutes and then show you a very interesting application. I can have pressure vessel generated by various ways.

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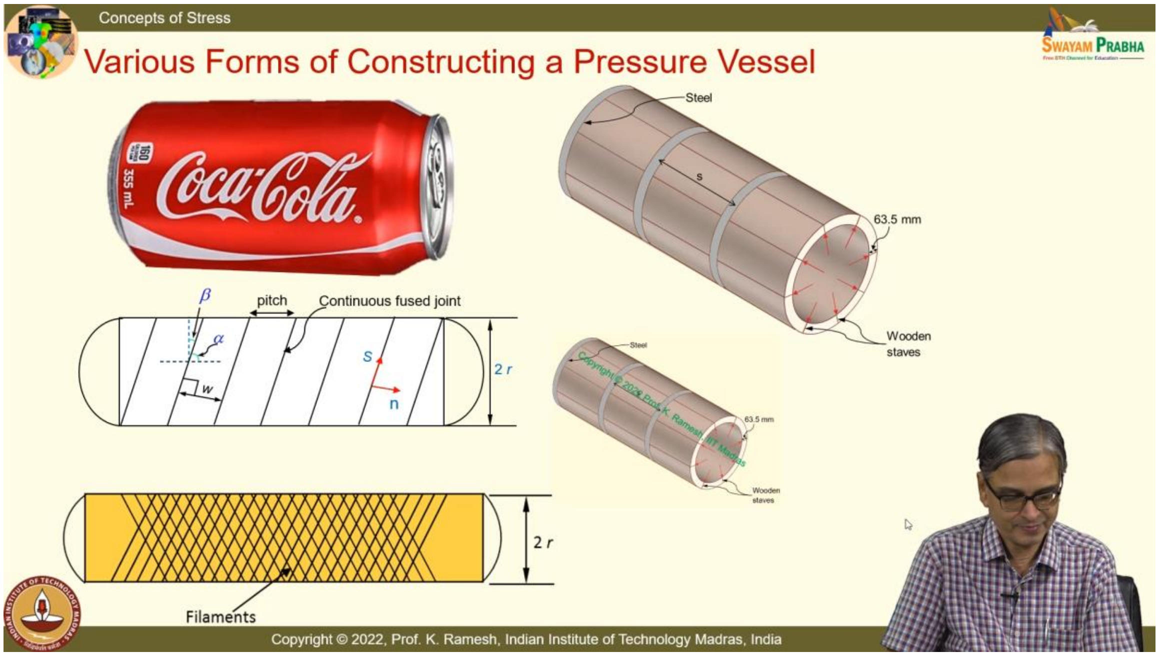


This is all interesting problems that you can solve. This is a nice problem where you have to apply concept of stress transformation to decide its strength. I can also have a pressure vessel made of filaments which is very light, we will look at that, ok.

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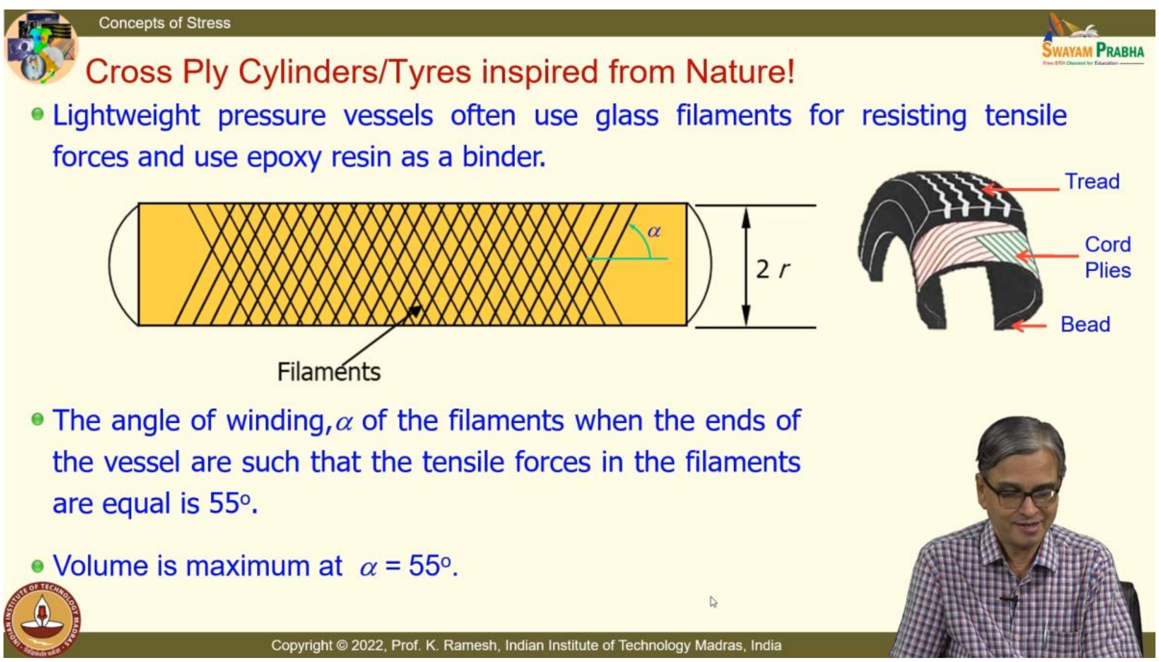


And when you want to have a pressure vessel which is of light weight, I can have the fibers oriented. One of the questions here is what should be the orientation of the fiber?

See you know when I have a stress applied, I have $\frac{pr}{t}$ in the tangential direction and $\frac{pr}{2t}$ perpendicular to that.

So, $\frac{pr}{t}$ is larger or $\frac{pr}{2t}$ is larger? So, I should have the fiber oriented towards the vertical axis that is what is shown here. So, from that point of view it is fine. You will also do a tutorial problem to find out what is the optimal angle so that the fibers take the similar load and that angle find I mean comes out to be $55^{\circ}$, ok. Please note all this, it is very important for our discussion and if you go and look at a tire, see tire is a very complicated design.

Do not think just a tire, fine. We have a tire technology center in IIT Madras and people do lot of research and it is all rubber material is very difficult to handle. This also has fibers oriented at $55^{\circ}$like this cross plies that is how the tire is constructed. So, you find that this is a very useful application, fine. And the volume of the cylinder is maximum and $\alpha$ is $55^{\circ}$. See, we normally think that humans are very smart, we have the sixth sense and then we rule all the planet.

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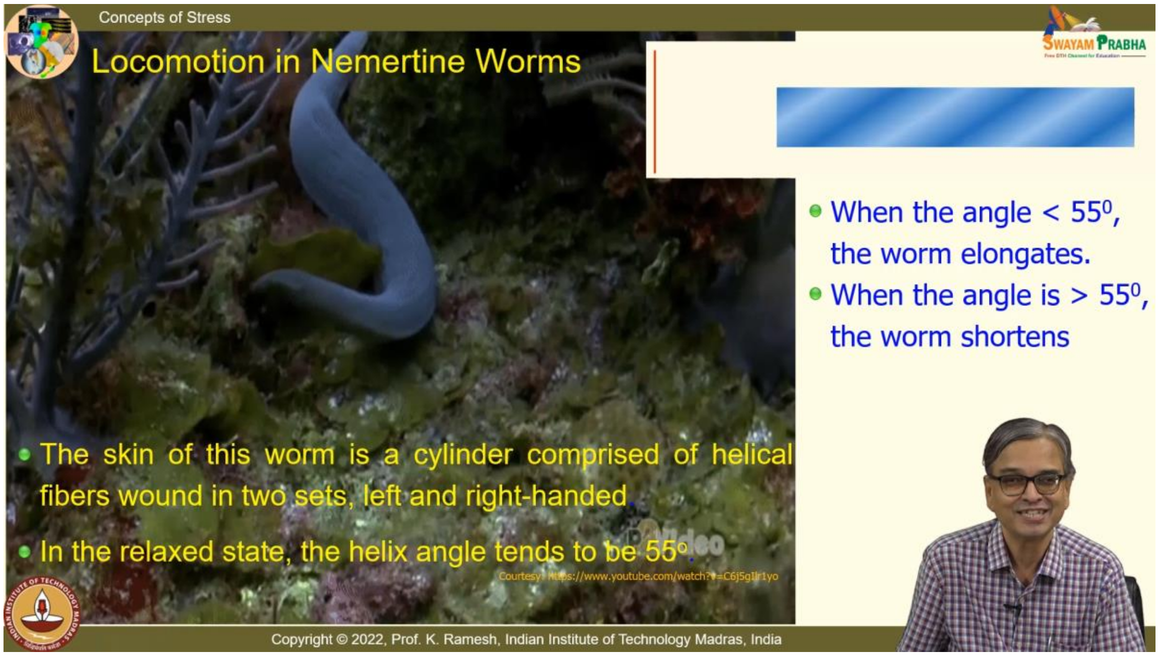


But let us look at what happens to a worm that is below the sea, how it has a locomotion? You see the locomotion simulated here. It bulges out, stretches, it does not have legs like you, it cannot go from one place to another place by moving the legs. And you see very clearly here, it bulges out and then moves. This is how the locomotion takes place. Very interesting, see understanding the locomotion of all these organism is a very important research. And it is so interesting, the skin of this worm is a cylinder compressed of helical fibers wound in two sets left and right handed. Much before your composite cylinder is developed, nature has created this worm. It has also taught the worm how to navigate in the sea. And it is very interesting in the relaxed state, the helix angle tends to be $55^{\circ}$, fine. See, you all have done conic section, fine.

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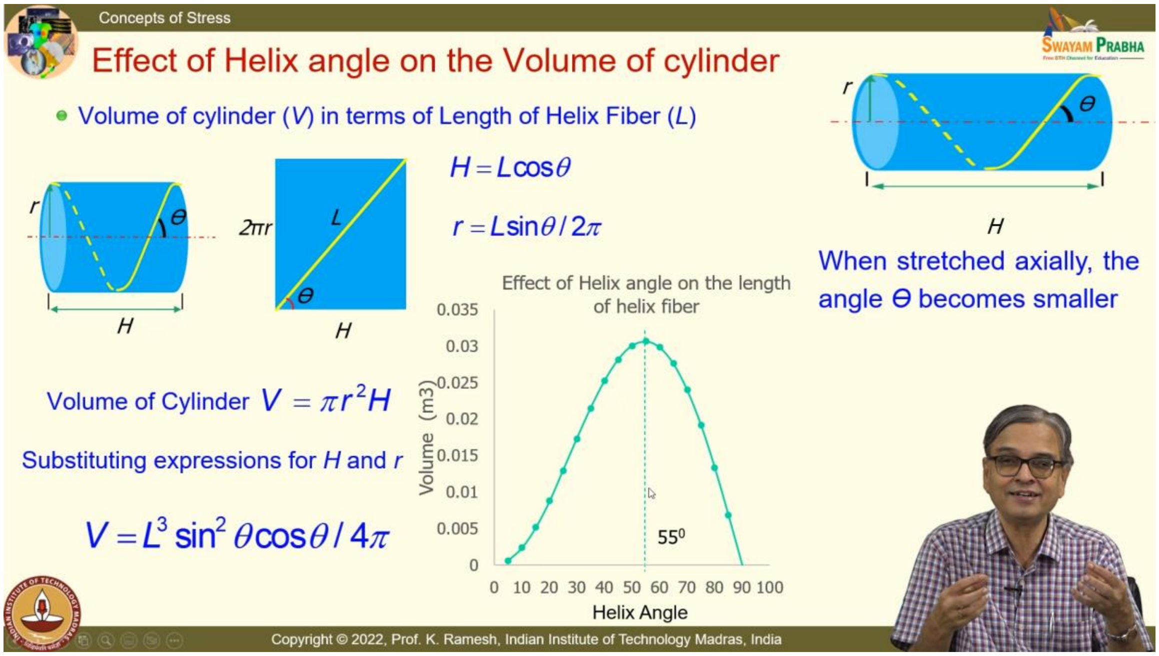


Let us go and see the relationship. You can find out the volume of the cylinder. I can make this as a unwrapped one and then draw the distances. I can find out what is the volume of the cylinder in terms of the angle $\theta$ and also the geometric quantities $H = L\cos\theta$, $r = L\cos\theta / 2\pi$. And the volume of cylinder comes out to be $\pi r^2 H$.

And if I substitute, I can also get it in terms of $\theta$, my interest is to show at $55^{\circ}$, at $55^{\circ}$it has a maximum volume. See, the worm has a fluid in it, it is in a relaxed state at $55^{\circ}$. When it has to move only it has to exert and the fiber angle changes. So, you calculate as part of the tutorial, you will also calculate that as $55^{\circ}$, fine.

You think that you are doing it in 2022, such a great learning. You find whatever the learning that you find, we are still trailing behind what nature has beautifully done. In a simple worm, worm does not have the kind of brain that you and I have, is not it? And its locomotion is dictated by this. Surprisingly, it has helical fibers which are the angles changes depending on what is the way, it is if when it is stretches the angle reduces and so on. So, in this class we have looked at certain subtle concepts related to concepts of stress. These are needed for finding out what are the boundary conditions on a given problem, because all the problems you have in stress analysis are boundary value problems.

So, it will also help you to graduate to the next level course comfortably with this knowledge. And we have also looked at the problem of pressurized cylinder, which is a very common application whether you took a look at your homemade cooker, pressure cooker that is that is a pressure vessel. If you have the cylinder for your gas, it is a pressure vessel. So, pressure vessels are very common, you have to deal with that. If you want to have your scent that is also pressurized.

So, there are many many applications and you find in that both the principle stresses are positive and you have to be very careful about $\sigma_3$ being zero. Using that only you should find out the maximum shear stress. So, whether it is a local maximum or global maximum in the shear stress value becomes very important in a day to day problem. It is not an exotic problem somewhere in your space structure, some structure has a difficult situation where you will have to apply your ultra modern theory not like that. Even day to day structures you have to understand the concepts clearly and apply them. Thank you.