Strength of Materials
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Lecture - 10
Octahedral and Deviatoric Stresses and Principal Directions

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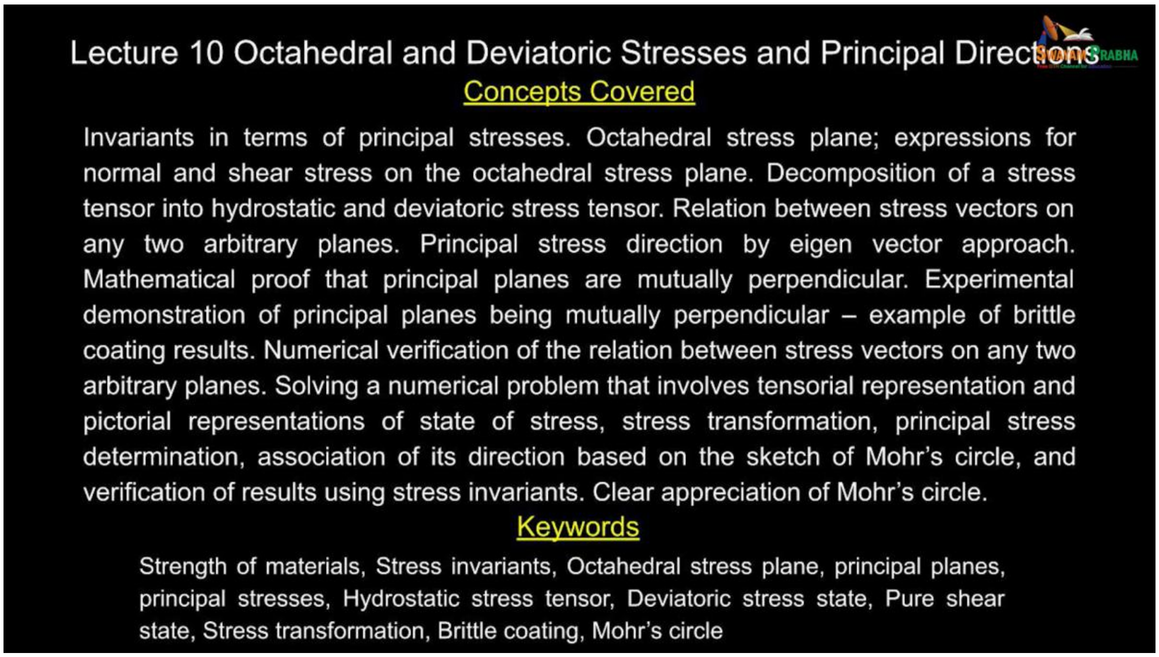

See, we have been discussing about principal stresses. And I said, if you express the stress tensor at a given point in terms of principal stresses, there are certain advantages. Your mathematics becomes lot simpler. And while developing the principal stresses, we have also looked at what are stress invariants. Stress invariants help us to have inbuilt checks in our calculations, that is one advantage. Other advantage is, it can also be used to find out and investigate whether the given stress tensor corresponds to a simple uniaxial stress or pure shear stress state, fine?

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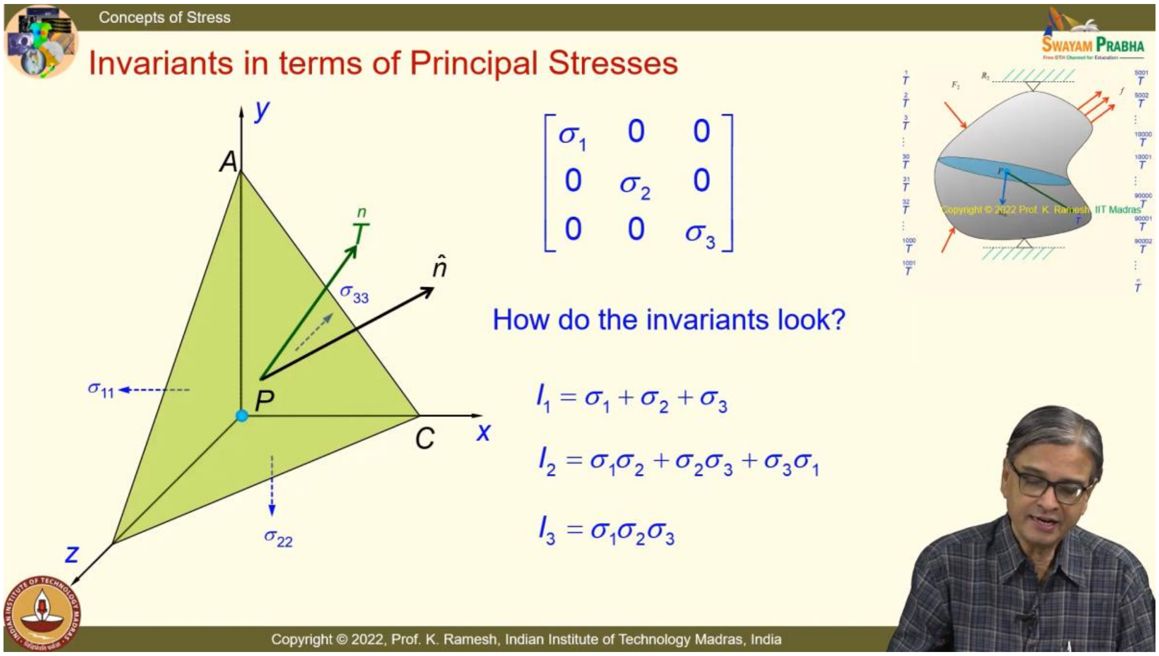

We will also look at what are the other aspects. See, when I look at the stress tensor in terms of principal stresses, and I have also said that in order to distinguish, I have put this in different color. You can also see that these are all normal stresses. And the invariants become very simple. Can you write the invariants I_1, I_2, I_3 quickly? You know definition of the invariants.

I_1 is straight forward, that is very very simple. You should be able to write I_2 as well as I_3. When I express it in principal stresses, the computation is so simple, straightforward, hardly takes a minute, fine? So, I have I_1 as $I_1 = \sigma_1 + \sigma_2 + \sigma_3$, and I_2 simply becomes $I_2 = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1$. And I_3 is $I_3 = \sigma_1\sigma_2\sigma_3$. So simple and elegant! So, there is good reason to visualize the stress at a point in terms of the principal stresses.

And I also said, many of the failure theories are essentially constructed around the principal stresses, fine? So, it has an advantage. And you can actually combine loading from various individual loads; from that individual stress tensor assemble and get the final stress tensor; from the final stress tensor get the principal stresses, and adopt one of the failure theories.

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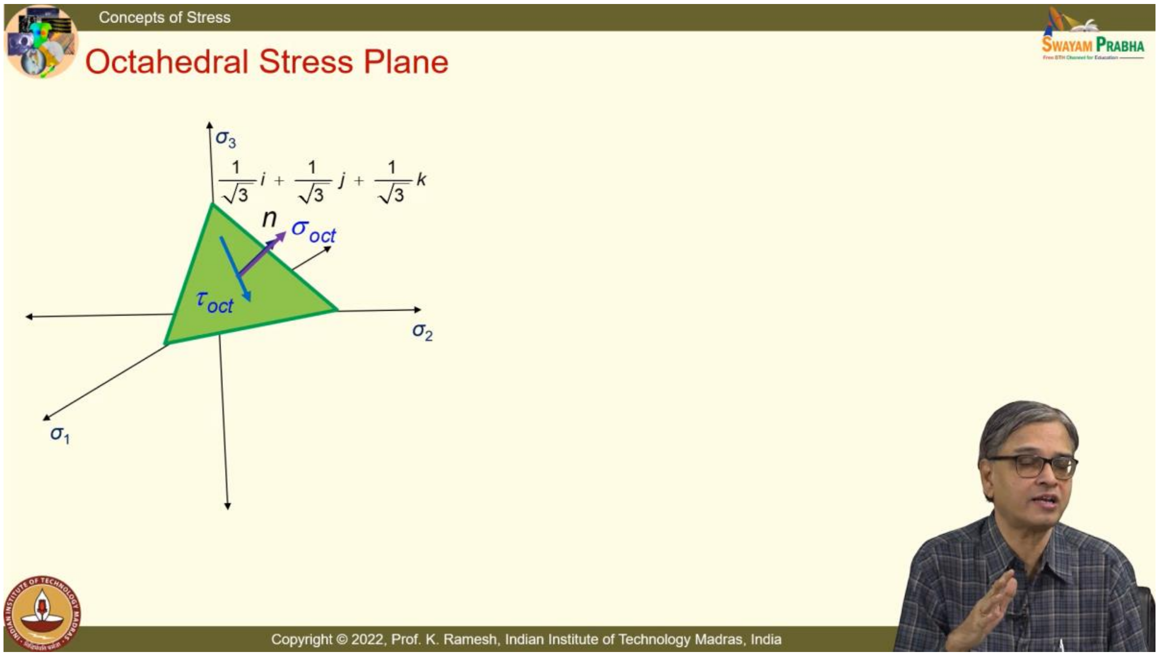

And you know, people also have looked at newer planes. I said some planes are very important. We have looked at principal stress plane, we have also looked at maximum shear stress plane, fine? From principal stresses, people also have looked at when they were working on plasticity, what are known as octahedral stress plane.

And what you have is, I have the reference axis as σ_1, σ_2, σ_3. And this plane is equally inclined to the reference axis. And we have already said, certain planes are very important, and in that list, you also include the octahedral stress plane. Why it is called *octa*? I have this quadrant, I can have four such in the top half and four such in the bottom half, then I will have eight such planes identified. And whenever you have an arbitrary plane, the stress vector what you have can be resolved into a normal stress component as well as a shear stress component.

And we have looked at brittle materials fail by normal stress, and then ductile materials are sensitive to shear stress. And once they have determined this kind of octahedral stress, they have also looked at what happens to octahedral shear stress. I said multiple failure theories have been created. So, when you understand what is state of stress at a point, whichever aspect, if it is a normal stress, what happens when the normal stress reaches a critical value. If it is a shear stress, look at what happens when it reaches a critical value.

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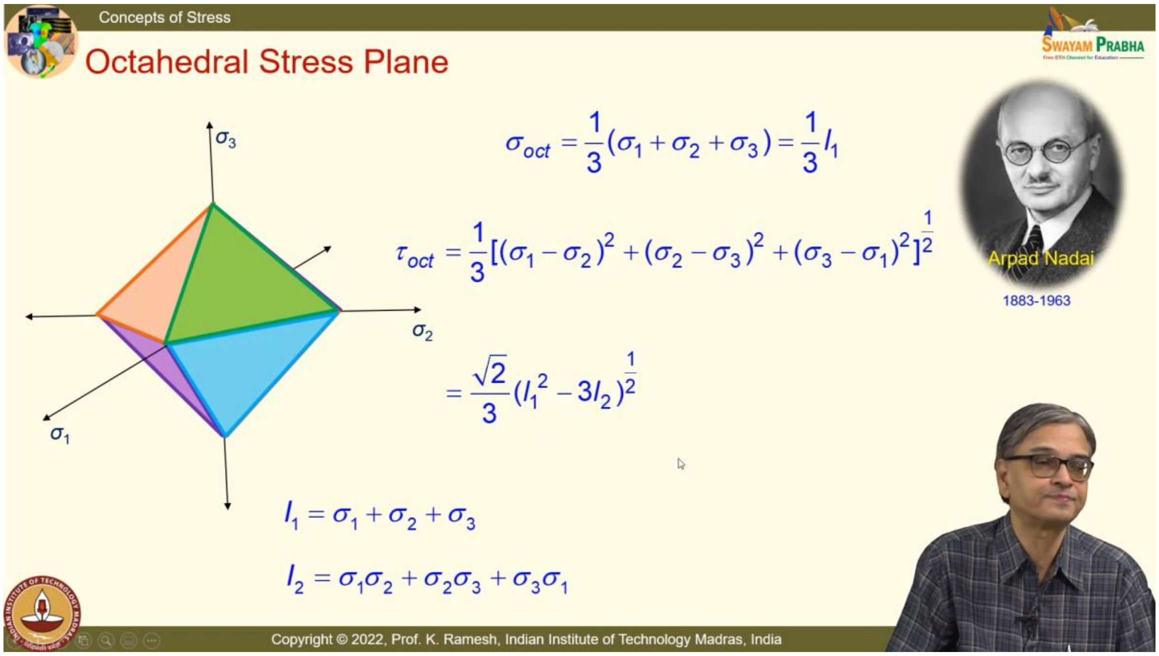

On the similar line, people also have developed theories, where what happens when octahedral shear stress reaches a critical value. So, the term octahedral, you can understand. I can have four such planes in the top half, and I can have four such planes in the bottom half. And they are also nicely colored to give you some kind of an enthusiasm in learning this course, fine? Do not think that these planes are colored, ok? These are normal planes; these are colored for you to keep your attention. And development of this goes to Arpad Nadai; he lived very recently. See, the plasticity theory really developed in the early part of 20th century, fine? And people thought when they were looking at it, so they need to have this octahedral shear stress also.

Now you can find out, because you have σ_1, σ_2, σ_3. In the last class, I said when I have a plane, the plane is now defined, it is equally inclined plane. You can find out mathematically, what is the expression for the normal stress, and what is the expression for the shear stress. Can you work it out and then check it with me? Because you are supposed to do a homework yesterday. I am not sure whether all of you have done it, because the ground rule is you tell the homework, it is never done, ok? So, I have octahedral; the normal stress is very simple to write, that is $\frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$. I get this as, in terms of invariant $(1/3)I_1$. You can also write the octahedral shear stress, and you will find that that can also be expressed in terms of the invariants, ok? And the τ_{oct} is given as

$$\tau_{oct} = \frac{1}{3}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{\frac{1}{2}}$$

And you know the invariants, I have $I_1 = \sigma_1 + \sigma_2 + \sigma_3$, $I_2 = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1$. And this expression can be rewritten in terms of the invariants. It is not very clear to see at the outlook, but you can write it. You can substitute I_1 and I_2 and simplify and make sure that this matches with this. So, I get τ_{oct} as $\frac{\sqrt{2}}{3}\left(I_1^2 - 3I_2\right)^{1/2}$.

See, scientists, when they want to do research, they also come out with newer names. And this is very mnemonical because I have eight such planes. So, octa is very nice to attach. So, octahedral shear stress and this is one of the later theories, fine? You had the early theory connected to ductile material was maximum shear stress theory. That was there for may be 200 to 300 years. Now what you will have to look at is, whether multiple theories when you have, do they converge to a single result, which we will have to wait until we go to theories of failure. But keep that curiosity in your mind, fine?

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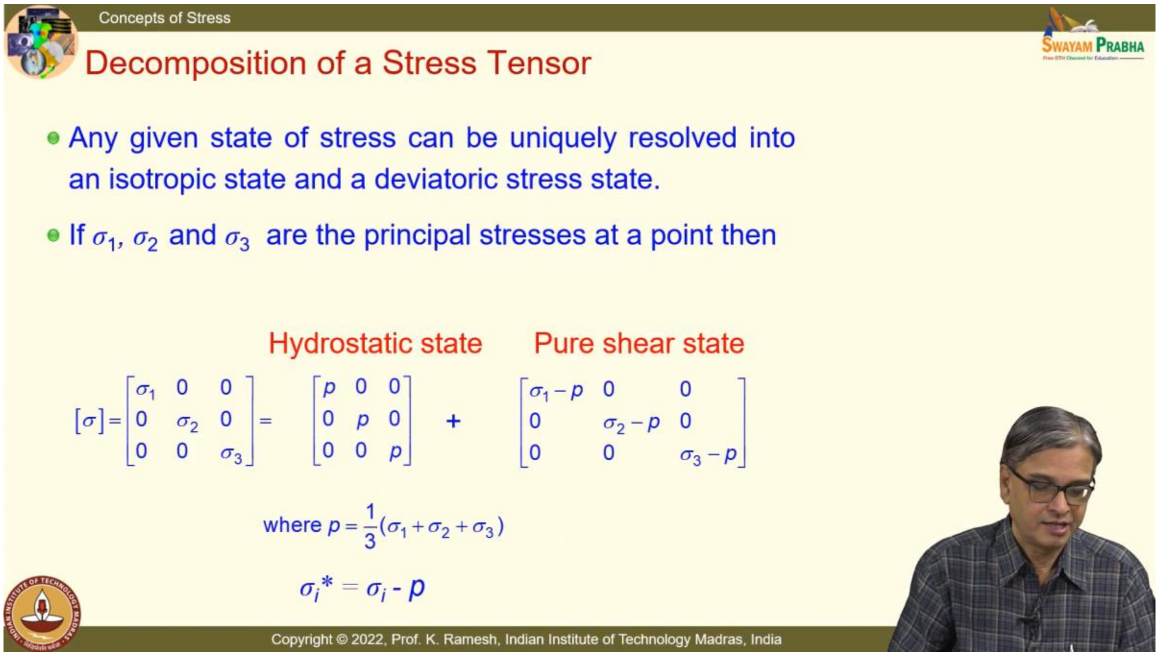

And people also thought, you know, I have been saying that we are happily living in linear elasticity. So, I can superimpose stress tensor. And any given stress can be uniquely resolved into an isotropic state and a deviatoric stress state.

So, we will have to see, you have to look at the stress tensor, the way I decompose and you have to figure out what is the meaning of deviatoric, ok? So, I have the stress tensor expressed in terms of the principal stresses σ_1, σ_2, σ_3. So, you will write the stress tensor as in this fashion.

$$[\sigma]=\begin{bmatrix}\sigma_1 & 0 & 0\\ 0 & \sigma_2 & 0\\ 0 & 0 & \sigma_3\end{bmatrix}$$

And this is written as a superposition of an isotropic state. That means, I have a constant value, the value of *P* is identical in all the three directions. That is what happens when you immerse anything inside a fluid bath. Suppose I have put it below 100 meters in a liquid, you find it is a hydrostatic compression, ok? Now, we have to choose what is *P*, what is the way you have to select *P*; that matters. And *P* is taken as $\frac{1}{3}(\sigma_1+\sigma_2+\sigma_3)$. So, can you fill what should be the stress tensor that adds up to σ_1, σ_2, σ_3? Can you fill the next one? You can easily do that, because I have separated out a part where the stress magnitudes are *P* and we also understand that this is hydrostatic state. And if I have to have the addition equal to σ_1, σ_2, σ_3 on the diagonal, the matrix or the stress tensor is simply

$$\begin{bmatrix}\sigma_1-p & 0 & 0\\ 0 & \sigma_2-p & 0\\ 0 & 0 & \sigma_3-p\end{bmatrix}$$

Can you comment from whatever you have learnt so far, what this represents, from what we have discussed? Anybody can make an intelligent appreciation of what this stress tensor is. This is definitely called a deviatoric stress state. I have not deliberately given what is the other name. I want you to find out and tell me from what we have learnt so far. I can give you the clue; use the invariant. We have looked at; invariants have some advantage. It is learnt very recently. This stress tensor represents what kind of stress state? That means you are sleeping; you are not exercising your mind. You have learnt it just in the previous class. I_1? I am asking, you have a stress tensor, when some stress tensor is given, you can find out what stress state it represents. I gave a clue that you can arrive at the stress state by looking at the invariants and this is a simple mathematics that you will have to do.

Somebody said; I am able to hear some voices. This is? "*Pure shear state*". Because I_1 is zero; you should be able to see that. See, if you do not see that, then you are not learning. It is not that you simply copy down whatever that is discussed in the class without assimilation. You are able to see now? I have separated this stress tensor into two components. One is the hydrostatic stress state, another is a stress state like this and you know how to look at the stress state and comment about it. That has been taught to you. Apply that knowledge. So, this is nothing but a pure shear state. So, what happens? When you have a hydrostatic state, it can either get compressed or elongated, expanded in all directions in similar fashion because we are looking at isotropic material.

If it is a shear stress state, the material gets distorted. So, once people have found this kind of quantities, you know, scientists do not keep quiet. So, we have also said this material is like a spring. What is the property of the spring which is very attractive to you? It can store energy. Like spring stores energy, elastic materials also can store energy. So, people thought I have a deviatoric stress state, so, what happens to the deviatoric energy associated with this stress state? You understand?

See, if you look at, if there is truth, whichever way you start and investigate, I must get one answer. Only that is truth, no matter which way I have looked at. See, while developing the state of stress at a point, we looked at a graphical representation like Mohr's circle. We understood how to evaluate principal stresses. Then we did mathematical development. That also gave the identical expressions, isn't it? We will also see some experimental information which will add up to this knowledge. So, if you look at any of these branches developing, people have verified it again and again. Because you are developing new body of knowledge, it has to be verified. It is not that you wake up from your dream and then say this is the result and everybody should accept it. Science does not go that way.

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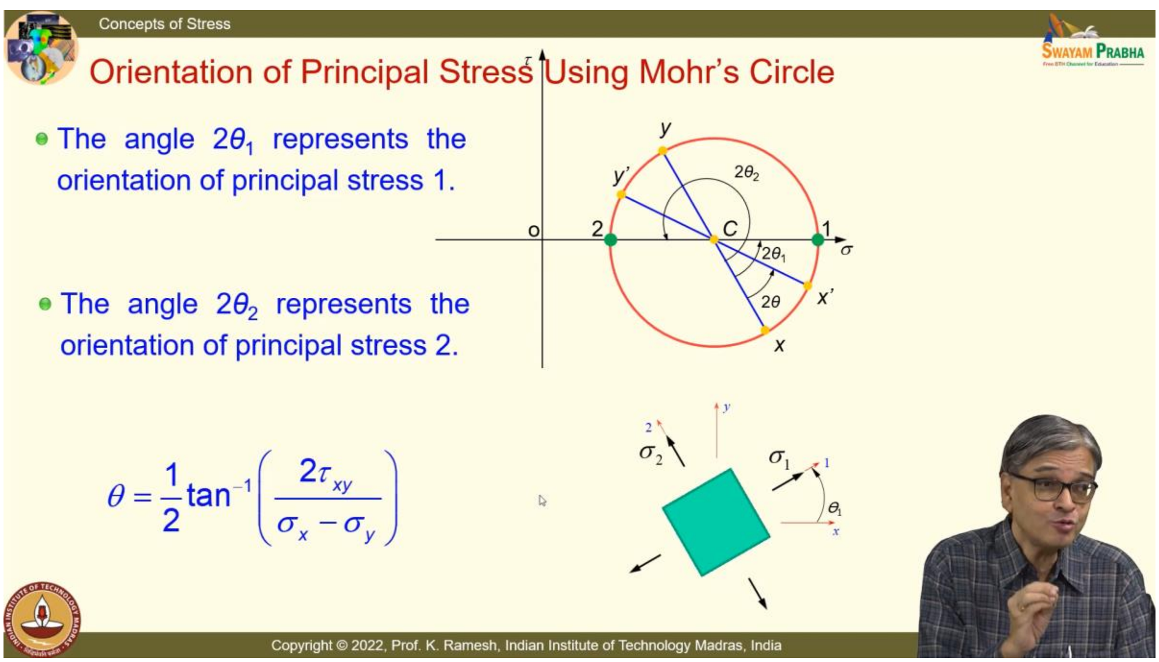

And let us go back to the orientation of the principal stress using Mohr's circle. This is a very useful utility of a Mohr's circle. You do not have to sketch it with geometric accuracy. It can be used to find out the principal stress orientation unambiguously, ok? And you have this principal stress 1. It is given as $2\theta_1$. This is an acute angle and you have an obtuse angle with respect to the x-axis, the principal stress direction 2. You label it as $2\theta_2$. And why do you need this? When you have the expression

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right),$$

you have to recognize that this is a multi-valued function. It is not a single valued function. So, once you get the result, you will have to investigate which of them represents the principal stress 1 direction, which of them represents the principal stress 2 direction by making a simple sketch. And you can quickly find out. I would like you to develop that habit. When you are having σ_1, σ_2, σ_3, you arrange them algebraically. You cannot do the same for θ_1, θ_2, θ_3, ok?

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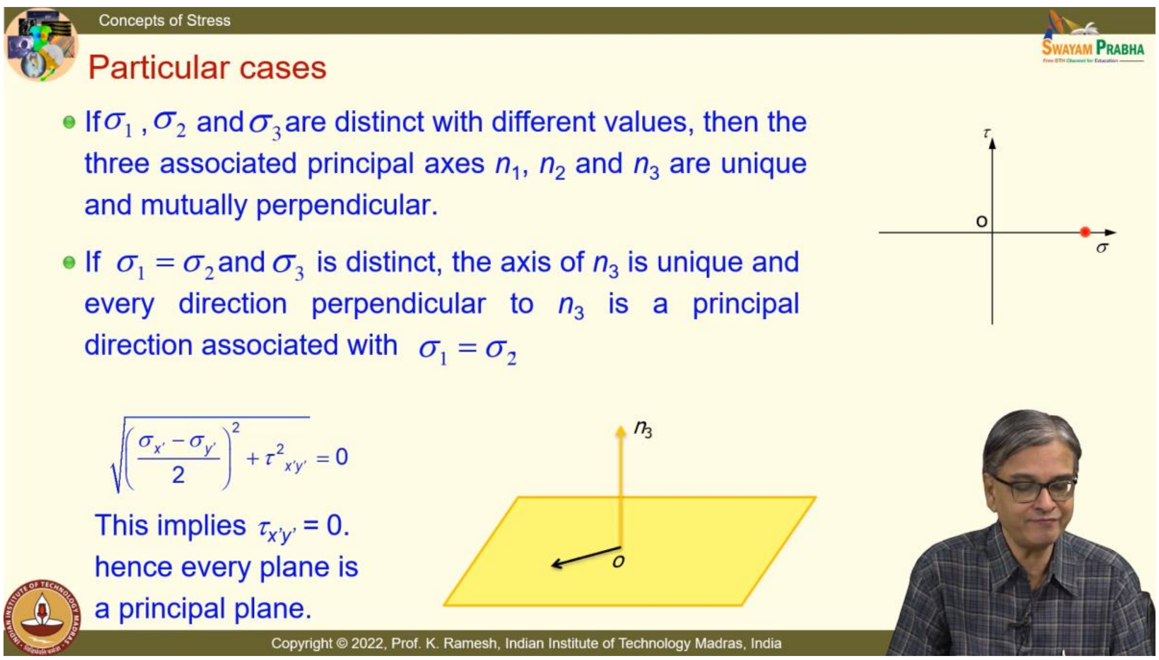

And you know when I have σ_1, σ_2, σ_3 distinct, I will have different directions associated with the principal stresses and they are mutually perpendicular. We have seen mutually perpendicular from Mohr's circle, fine? We have still not established it as mutually perpendicular from mathematical perspective. I will also try to give an experimental perspective which is done in the early part of 20th century, so that you can get convinced when I say principal stresses, they are mutually perpendicular.

These are all very celebrated results. The celebrated results have been verified multiple times in the history of evolution of engineering, ok? Suppose I have σ_1=σ_2, and σ_3 is distinct, what happens to your Mohr's circle? It becomes a point. Is the idea clear? When you have that as a point, the expression for the radius goes to zero. When this goes to zero, it implies shear stress is zero. And shear stress zero is the definition for your principal stress planes. So, that is the reason why every direction perpendicular to the σ_3 direction is a principal stress direction. On this plane, every direction is a principal stress direction. So, these are all special cases, fine?

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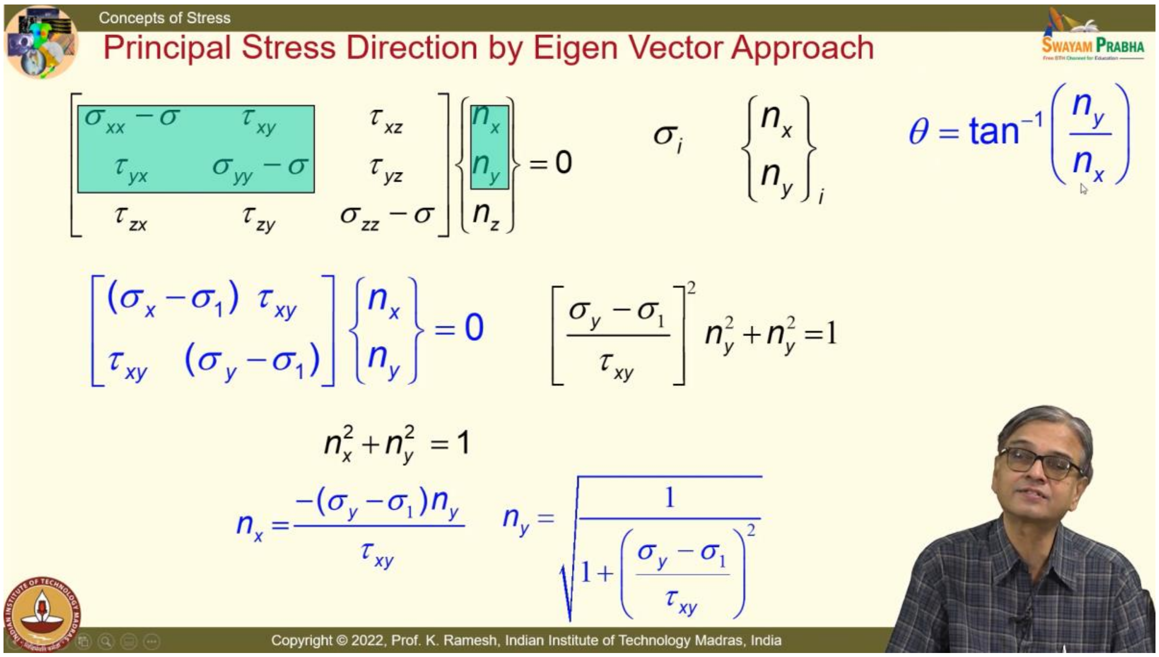

See, while developing the principal stresses mathematically, I made a statement this can also be solved as eigen value, eigen vector approach, but we did not solve it like that. I got the determinant as zero, I pulled out only two-dimensional case and it was a quadratic equation, so it was tempting to solve by simple quadratic approach and we got the numbers. We got σ_1, σ_2, σ_3; we never worried about the directions.

Now what we will do is, we will go and see how I can evaluate the orientation from mathematical approach. Because you are living in the computer world, if you have to go and depend only on a graphical approach, it will not help in these days. So, you should also have a mathematical methodology that helps you to do that. And it is very simple; it is very straightforward. I just take a two-dimensional case. So, in this case, I have the eigen values as σ_i representing 1 and 2 and the direction cosines associated with this is given with a subscript *i*.

And if I take a two-dimensional case, I already find out the principal stresses using the quadratic expression. So, substitute instead of σ, σ_1; and I have this expression. Can I use this to solve for n_x and n_y? You have two simultaneous equations. You can easily solve for two unknowns; it is straightforward. And I use the second equation to get the expression for n_x and we also know $n_x^2 + n_y^2 = 1$, ok? So, I have

$$n_x = \frac{-(\sigma_y - \sigma_1)n_y}{\tau_{xy}}$$

And I can use this for me to find out what should be the expression for n_y. So, I have this substituted in this, ok? And when I simplify, how would I get? I get n_y as

$$n_y = \sqrt{\frac{1}{1 + \left(\frac{\sigma_y - \sigma_1}{\tau_{xy}}\right)^2}}$$

See, I have always said when you use mathematics, you should use mathematics very precisely. When I put square root, what are the roots? Both positive and negative are roots, isn't it? So, you should investigate what happens when n_y is positive and n_y is negative from this. And I get the θ orientation in a simplistic manner as

$$\theta = \tan^{-1}\left(\frac{n_y}{n_x}\right)$$

So, take up a simple stress tensor and then find out the principal stress direction using your conventional approach of Mohr's circle. And also use this and substitute for both positive root and negative root and have a clear understanding how this expression helps you to find out the associated direction. If I have taken σ_1, this will invariably give you the direction of σ_1 only. You do not have to go to Mohr's circle and then verify. Convince yourself by solving test problems created by you. Is that idea clear? Ok.

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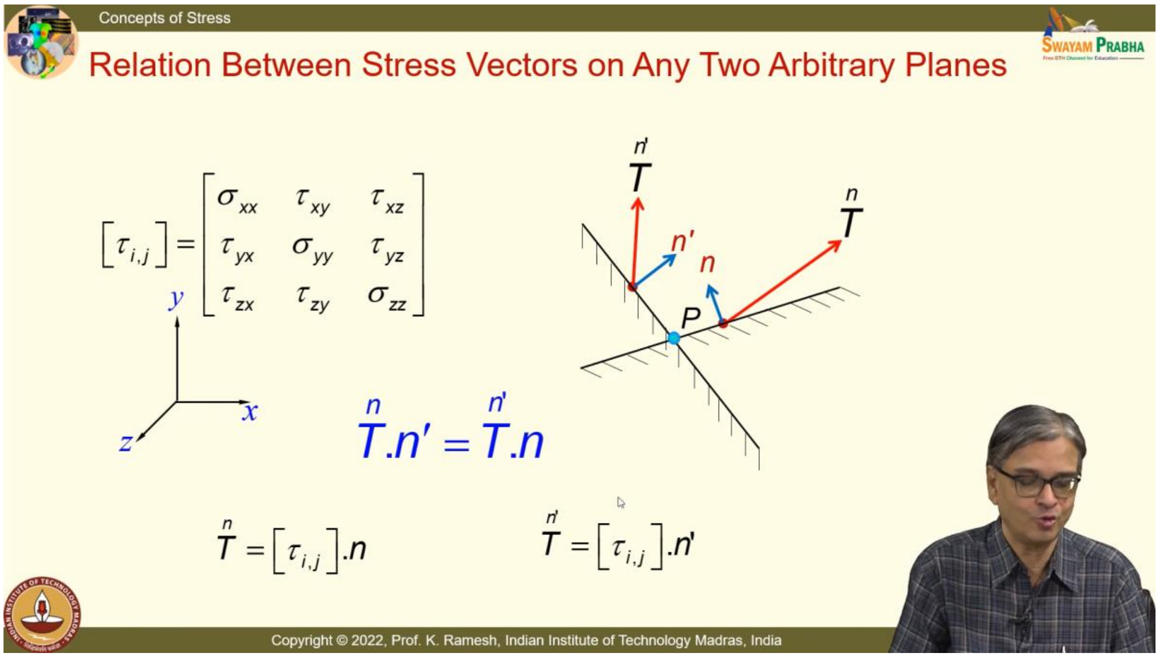

See, this also I asked you to try yesterday as a homework. How many of you tried? See, once people develop certain concepts, the mathematicians do not keep quiet. They try to find out some identities and what you have is, suppose I take a point, I take an arbitrary plane *n*. And if I know the stress factor is $\overset{n}{T}$ and I take another arbitrary plane which is *n*', with our symbolism we will say the stress factor acting on that is $\overset{n'}{T}$. And mind you, these are vectors. We have already understood. We will not put the vectorial notation. It is easy for us to interpret once you understand the nomenclature that we have mentioned in the earlier classes. You have an identity $\overset{n}{T} \bullet n' = \overset{n'}{T} \bullet n$. See, this is a vector, this is a vector and this is the representation of the dot product. Whatever the software that we have used to write the equation, it puts the dot product only like this. Earlier I tried to show it with a big dot. Later on, I gave up. It is because it is becoming difficult to edit every equation.

So, you understand that this represents the dot product. This identity, we will verify and we will also use it to show that principal stresses are mutually perpendicular. First, we will take this identity. If you have done the long hand mathematical expressions, if you have calculated what is $\overset{n}{T}$, what is $\overset{n'}{T}$ and if you have done the dot product, you would have got both the expressions are identical, which you should have done as a homework.

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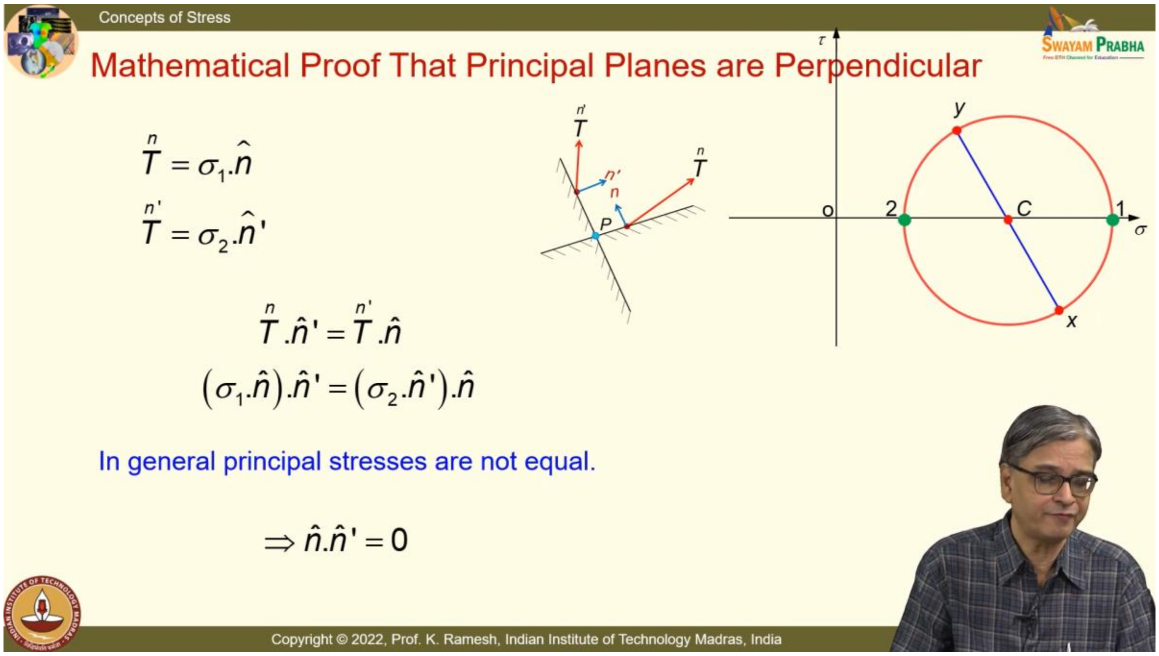

Nevertheless, we will verify it in a different manner. But before that, I would like to show mathematically that principal stress directions are mutually perpendicular. So, I take this identity and I say that I have principal stresses as σ_1 and σ_2. And from your Cauchy's formula, I can find out what is stress vector $\overset{n}{T}$ and $\overset{n'}{T}$. So, if I look at the identity, they have to be equal like this. So, if I substitute for $\overset{n}{T}$ and $\overset{n'}{T}$, I get

$$(\sigma_1.\hat{n}).\hat{n}' = (\sigma_2.\hat{n}').\hat{n}$$

In a generic problem, σ_1 and σ_2 are non-zero. But if you have to maintain the identity of this, what should go to zero? You get the point? The dot product of $\hat{n}$ and $\hat{n}'$ should go to zero. That is what this equation says. When I have this identity to exist, if the dot product is zero, the identity is validated. When does the dot product goes to zero? When the directions are mutually perpendicular, ok? So, that means, I have this one on a diameter, that is what you have, which we have already learnt it while doing the Mohr's circle.

And you have these two planes are perpendicular to each other. The principal planes can only be perpendicular. It is a mathematical development, fine? Now, we go and take a simple stress tensor and then verify mathematically, whether this identity is existing, ok?

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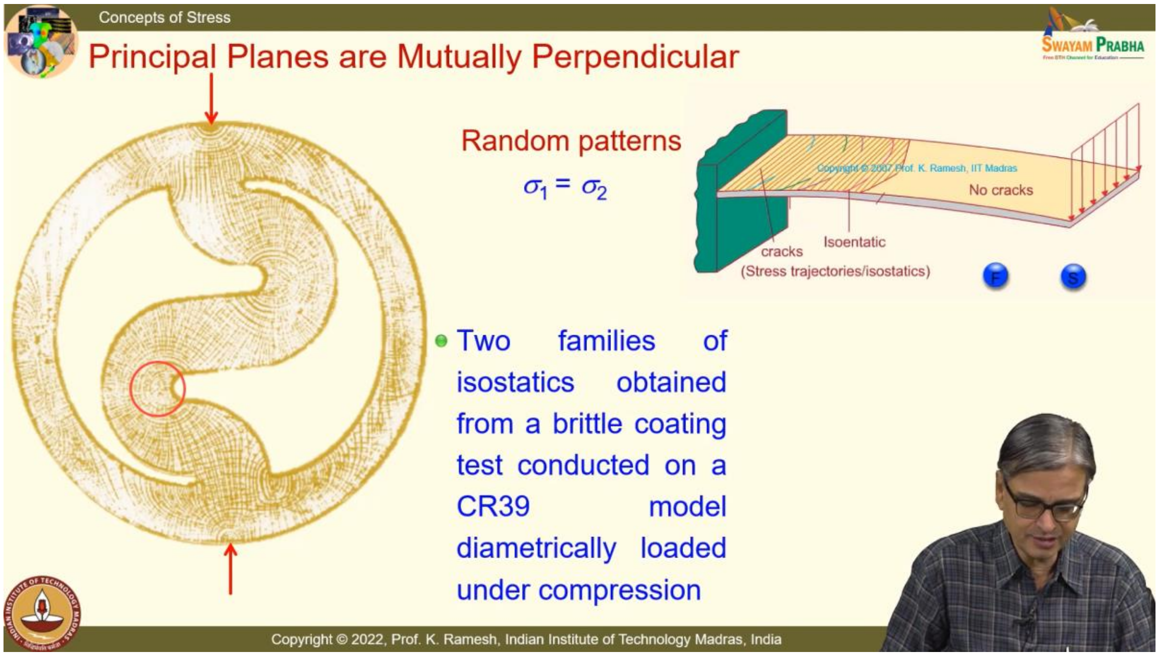

Before that, I think I have experimental verification of this. See, you are all now familiar with failure of a chalk. How does a chalk fail? When you have the normal stress reaching a maximum, it fails, ok? And we have also classified that kind of failure associated with a material which is brittle. And what is done here is, very beautifully, this is a cantilever beam on which you spray a paint which is a brittle material.

And when I apply the load, what you find is you have cracks getting developed, you have cracks. Can you associate the crack to a physical reality? That means, the normal stress at that point has reached a maximum, that is why the crack has formed. So, tangential to the crack is the principal stress direction, one of the principal stress directions; perpendicular to that is what I am applying. See, principal stresses are mutually perpendicular. So, the direction perpendicular to the crack as well as tangential to the crack are principal stress directions. And what is done is, people have taken it; it is a very complicated experiment because this is done in compression. So, I have to spray the paint when the object is compressed. Then slowly release it, I will get first set of cracks. And then apply the load again and then slowly release it, I will get the second set of cracks.

If you see this here, do you see the two families of crack patterns? They are orthogonal patterns, two families of isostatic. These are called isostatics and this technique is called brittle coating. This was done in the early 1930s or so. This is the only experimental proof that gives a demonstration that principal stress directions are mutually perpendicular.

But before that, I will also like you to see one region here. Here if you look at, the cracks are random, ok? We have also seen when σ_1 equal to σ_2, every direction is a principal stress direction. So, that means, crack can appear in any direction depending on the local weakness of the material, fine? And I also show a blown-up image of this.

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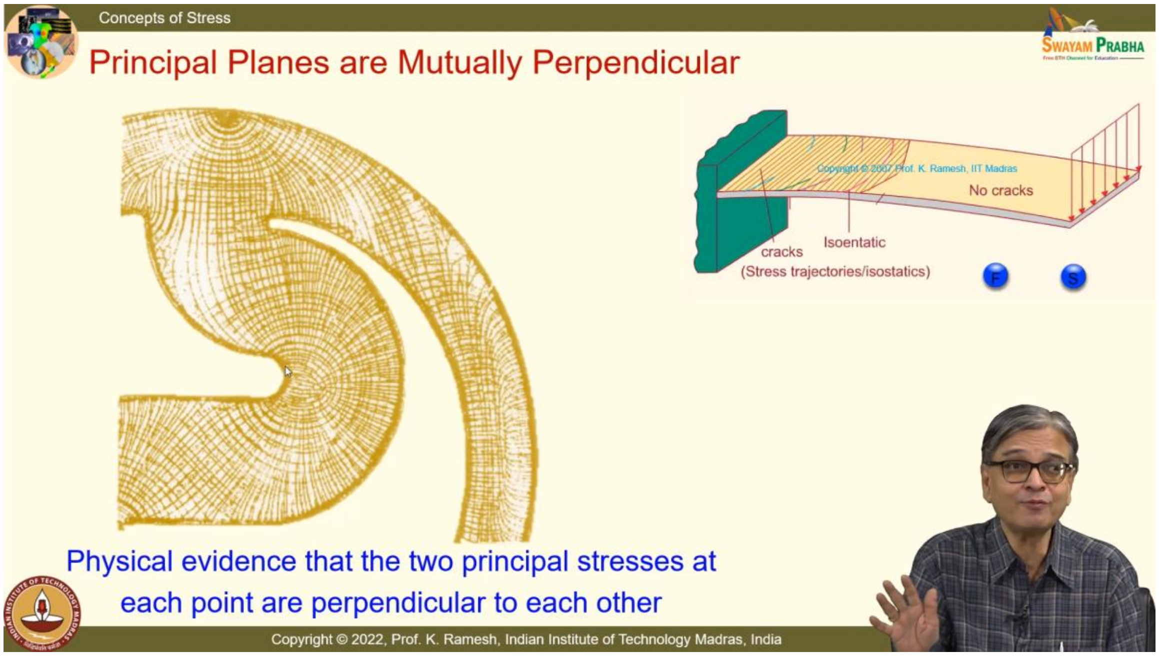

Here you can see that you have two orthogonal families. Are you getting convinced? Can you make at least a few lines sketch of this region? I have one family of crack pattern, another family of crack pattern, they are mutually orthogonal and this is demonstrated from point to point.

See, you are actually discussing state of stress at a point of interest. When you go to whole field experimental techniques, you get a picture what happens on the field. You need that kind of information also. Imagine, stress tensors were developed in 1822 and your verification whether principal stresses are mutually perpendicular experimentally happened in 1930s. Until then, they did not have this kind of an experimental approach. And this convincingly proves that principal stresses are mutually perpendicular.

So, in science, when people make statements, they verify it from various perspectives. So, when you do your mathematical calculations also, I want you to use this stress invariants and effectively use it to verify your results. You can verify it yourself. It is not that only when your answer book is corrected by me, I verify whether your answer is right. You can verify yourself whether you have got the correct answer when you do stress transformation, when you do principal stresses. So, certain aspects of a calculation, you can verify. If there is a possibility to verify, utilize that. Do not be in a hurry, you followed it, you got some number and you just write it down. I expect you to verify.

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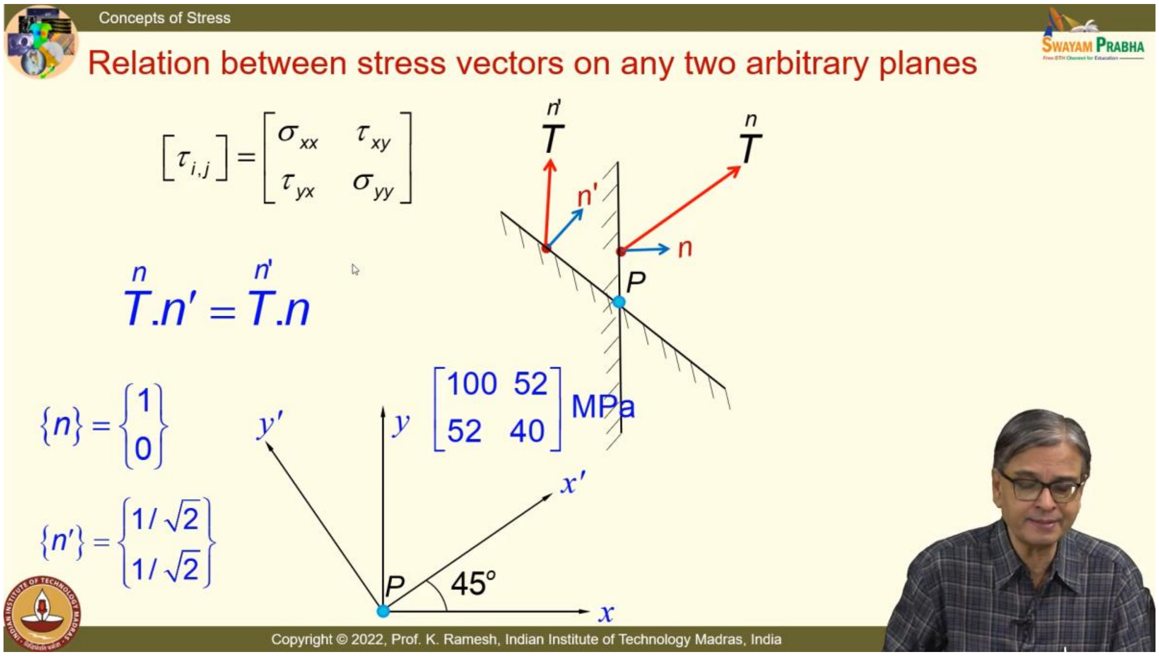

So, now what we will do is, we will do a class of problems which will verify whatever the identities that we have developed and also how to get the principal stresses, stress transformation, etc.

Now, I want to take two planes. I have the plane like this, $\begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$ and I have a stress tensor.

And I want to take another plane which is at 45°. It is easy for me to write the direction cosines. So, *n*' is nothing but $\begin{Bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{Bmatrix}$. Can you go and verify the identity $\overset{n}{T} \bullet n' = \overset{n'}{T} \bullet n$? I have given you the stress tensor.

Please take out your calculator and you have the Cauchy's formula to find out the stress vector. Find out the individual stress vector and take the dot product. Then verify with my slide, fine? That is simpler way of verification of these identities. One long hand way is put the expression and make sure that the expressions are correct which you ought to have done. If you have not done it, go back and do it at least today.

So, I have to verify this identity. Very simple to do. It is a very simple calculation. The idea of doing the calculation is that you should retain in your memory that such identities exist. So, if you solve a problem, you remember them better.

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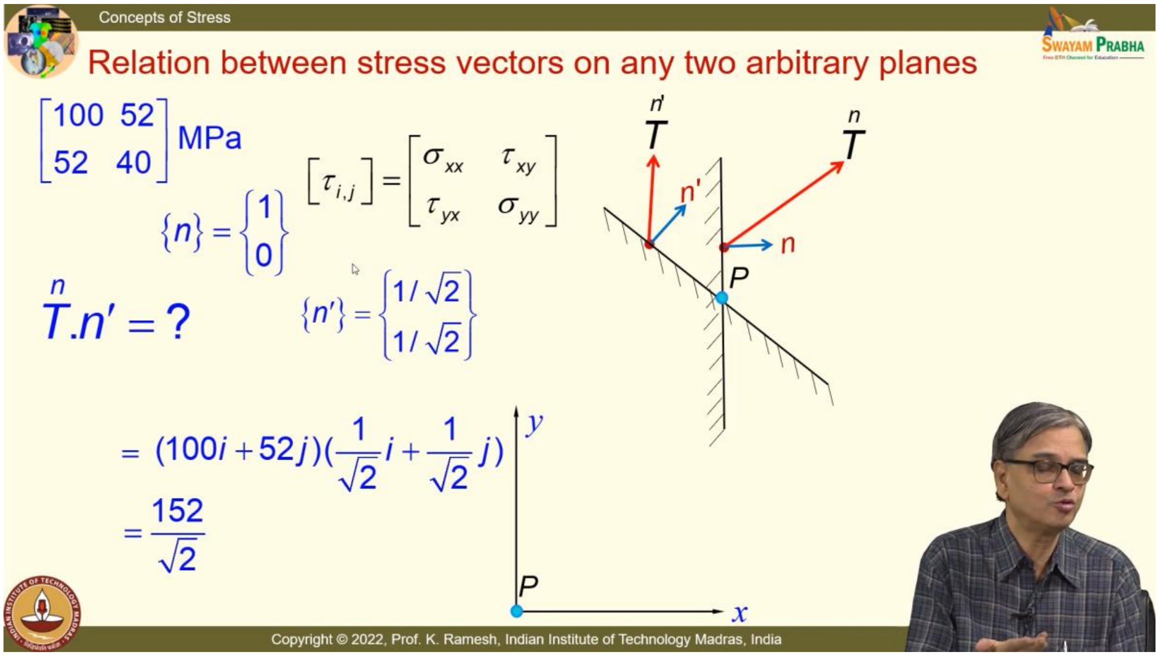

So, I have this, I have the stress tensor given, I have $\begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$ is the plane and I have this $\overset{n}{T}.n'$, if I want to calculate. Can you get $\overset{n}{T}$ from Cauchy's formula? And then we will, you can verify what is $\overset{n}{T}.n'$. Very simple to do.

I have n' is $\begin{Bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{Bmatrix}$. I have all the quantities that I need. So, when I do this, I get this as $(100i + 52j)\left(\frac{1}{\sqrt{2}}i + \frac{1}{\sqrt{2}}j\right)$ that gives me $\frac{152}{\sqrt{2}}$. Can you do $\overset{n'}{T}.n$? When you do that, you will find that this is also identical.

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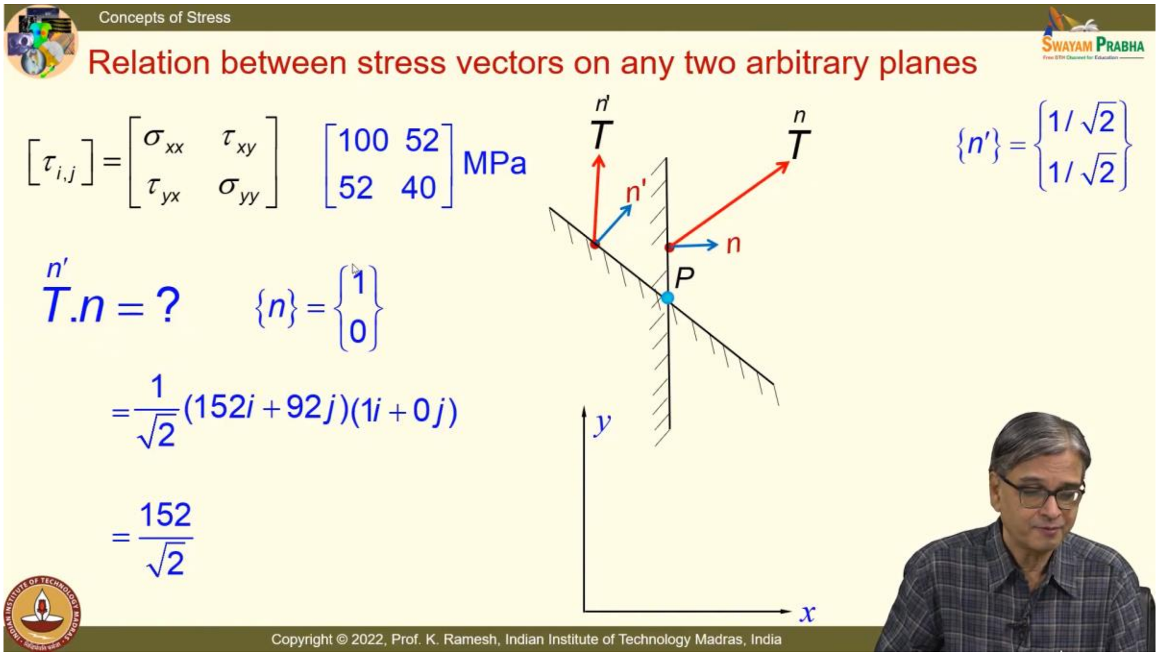

Very simple calculation, it is not very complicated or anything like that. But if you do that, you will remember the identity $\overset{n}{T}.n' = \overset{n'}{T}.n$. I have the direction cosine, I have to get $\overset{n'}{T}$ and I have *n*, all the quantities are there. So, when I do this, I get this again as $\frac{152}{\sqrt{2}}$. So, this shows that you have the identity. We have already utilized the identity to prove that principal stress directions have to be perpendicular mathematically. And we have also seen by a beautiful experimentation called brittle coating. You have two families, you have also seen the whole field representation, not just at a point that is very difficult to do. That is why it took so much time and it was done by Durelli, fine? He was the proponent of that technique.

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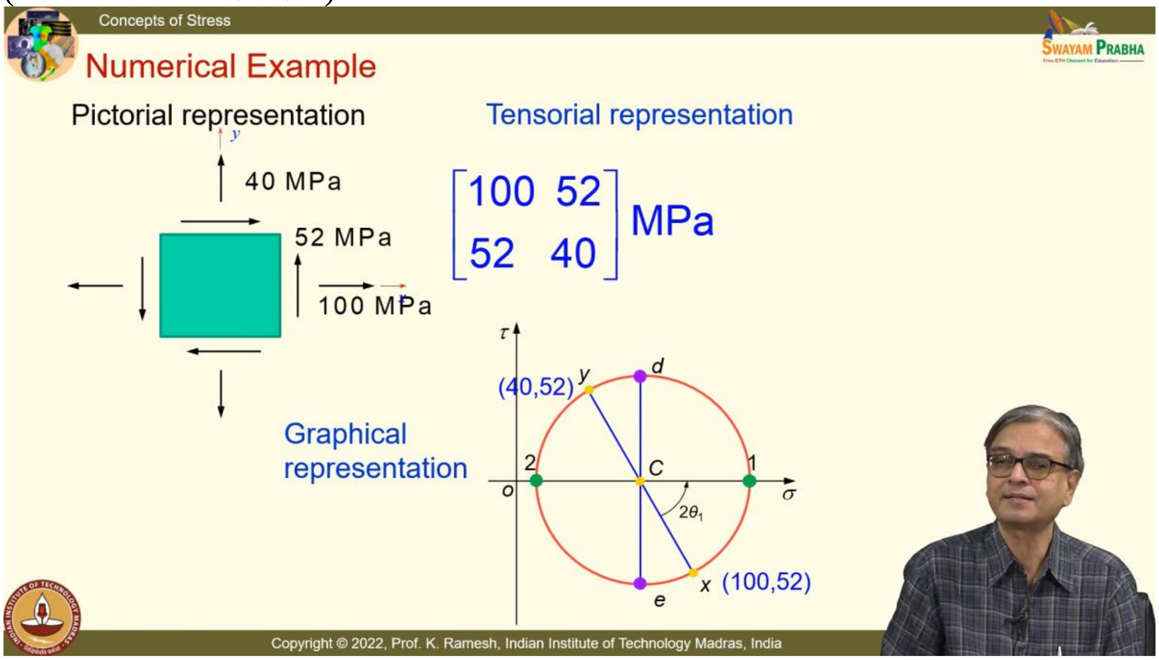

Now, we will go for stress transformation, principal stress direction, so on and so forth. I have a pictorial representation. We have already seen you have a pictorial representation. Can you write the tensorial representation? If the pictorial representation is given in this manner, can you write the tensorial representation? Once you write the tensorial representation, can you go and give the graphical representation? You should be able to do it quickly, because we have spent sufficient time on developing the fundamentals. Stress tensor can be given pictorially, can be written in the tensorial fashion, can also be given to you in a graphical manner.

I can also give you Mohr's circle and ask you to do calculations. So, the tensorial representation is the same one; $\begin{bmatrix} 100 & 52 \\ 52 & 40 \end{bmatrix}$. And can you draw the Mohr's circle? Mohr's circle is very straight forward and you have to mark the x and y plane correctly. If you draw it with the convention, if you rotate in the clockwise direction in the Mohr's circle, you can also rotate in the similar direction in the physical plane to locate the plane of interest, whether it is clockwise or anticlockwise, whichever way.

So, I have the graphical representation like this. I have the σ and τ axis. So, x plane will come in the first quadrant or which quadrant? Fourth quadrant, ok. So, I have the x-plane, I have the y-plane, join them and I get the center and draw the circle. So, I have everything, any plane of interest if you say, I can go to the Mohr's circle and get it, but we would also like to do it mathematically. You also know that you have the principal stress 1 and principal stress 2. And from this diagram, you can say the acute angle is σ_1 direction and obtuse angle is σ_2 direction.

If I do the calculation, if my multivalued function gives me acute and obtuse angle, I should correctly filter out which is θ_1 and which is θ_2. And you have the maximum shear stress representation. So, what we will do is, we will go and find out the principal stresses mathematically. You have the expression; can you get me the principal stresses?

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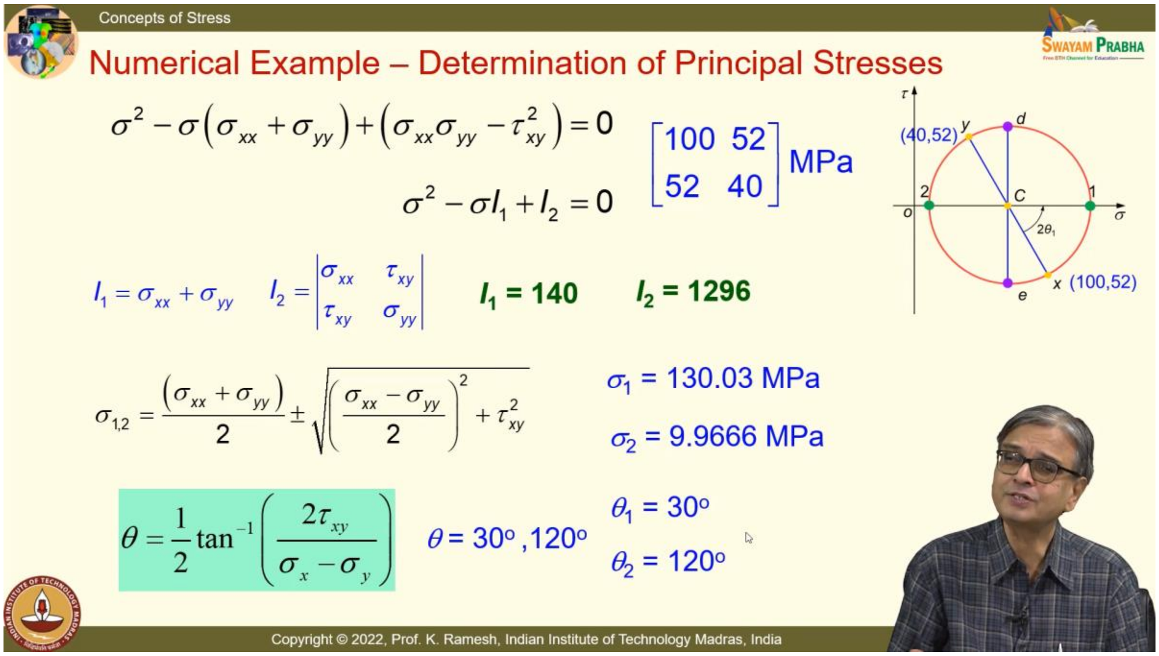

You have the mathematical expression; you have the identities. Now what I would request is, when you get the principal stresses, also verify from the invariants. You develop that habit right now. So, I should get I_1 and I_2, wherever that is relevant. In many cases, even I_1 should be sufficient, but while we develop the practice, it is better that you calculate I_1 and I_2. Then it will become familiar when you want to do the expressions that require I_2 to be determined.

So, what is the value of I_1? Louder. 140. So, what is the value of I_2? Calculate. I get I_2 as 1296. So, the habit is getting the invariants calculated, find out the principal stresses. Anybody has found out the principal stresses by using the quadratic expression? Ok.

So, you have this basic expression and you get 130.03 MPa and σ_2 as 9.96 MPa. I have to give, you know, the decimal places, because when they add up, it adds up to, does it add up to 140 MPa? It adds up to 140 MPa. So, that is an inbuilt check, fine? And you have the expression for θ and when I get this, first write it as like this. θ can have 30°, 120° and all that. Do not jump and write whatever the first value you have called as θ_1, second value what you have got as θ_2, do not do that.

We have to qualify and for this you can take the advantage of Mohr's circle. You have the acute angle. So, I can label the acute angle as θ_1 and the obtuse angle as θ_2. I am telling you a recommended procedure; verify the invariant and also use the Mohr's circle to label the θ_1 and θ_2 directions. It is very fast and you also use the Mohr's circle in a very effective manner.

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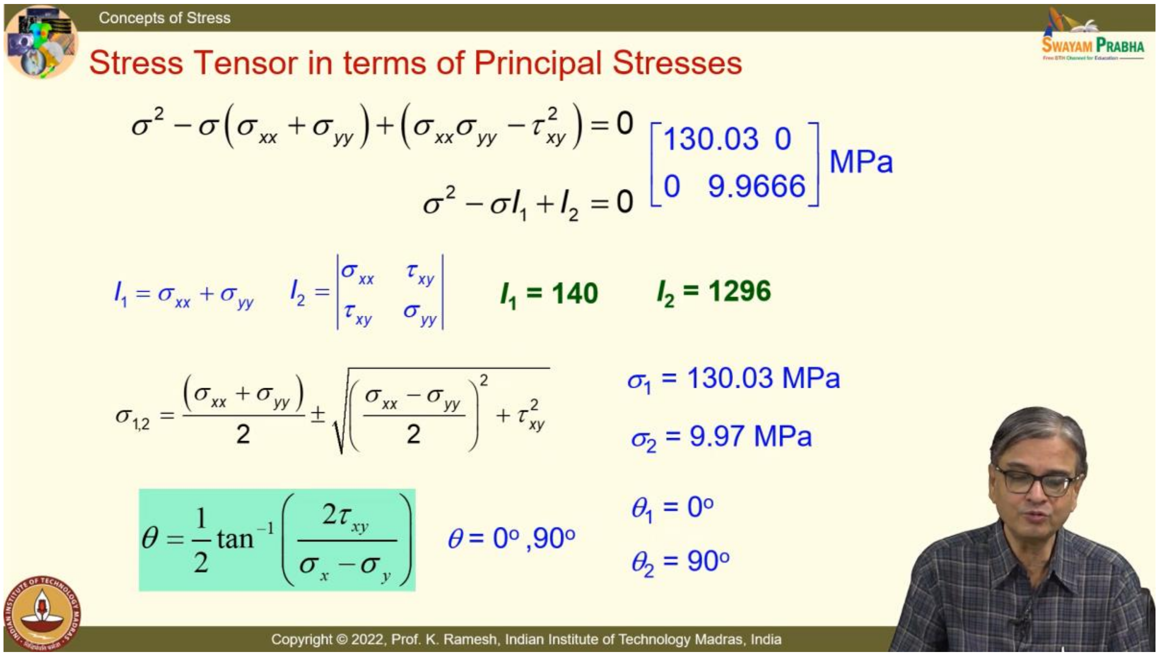

Suppose I ask a question, you know I can also represent the same stress tensor in terms of principal stresses. The same stress tensor, state of stress at the same point can be expressed in a different manner. I want you to recognize that. In this representation, I have zeros that makes my mathematical development very simple, ok? And this is trivial; let me not spend time on this. Because I can get I_1, I_2; I can also say that this is the principal stresses again and then say θ_1 and θ_2 are 0° and 90°; that is a straightforward representation.

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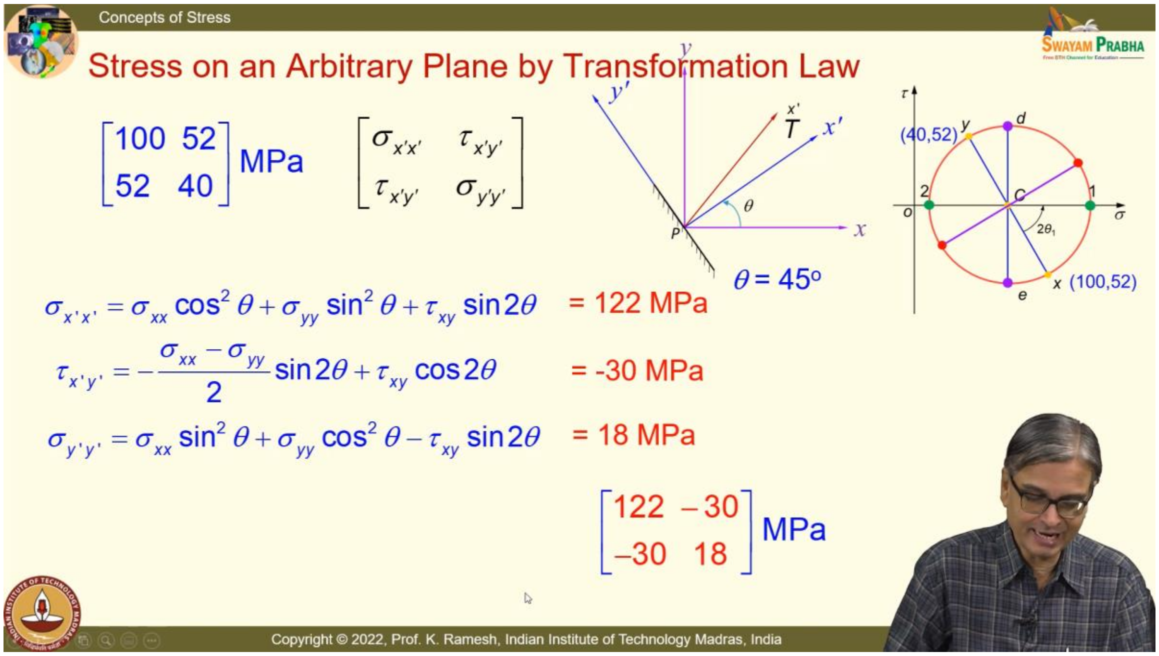

But what we will do is, we will find out for an arbitrary plane, the stress tensor. For us to make the life simple, let me take a plane at 45°; $\theta = 45°$. You know I have told you how to write the expression based on the indicial notation. Can you write the expression for $\sigma_{x'x'}$, $\tau_{x'y'}$, $\sigma_{y'y'}$? Can you write it down? Can you write it down? It is a practice, see you have to pick up speed in some sense, fine? You have to pick up speed in some sense. But verification, inbuilt verification, wherever it is possible, you should be able to do that, you should not shirk from that; that is the idea behind it. So, you have the expression for $\sigma_{x'x'}$.

We have seen it several times in the class, $\tau_{x'y}$, and you also have $\sigma_{y'y'}$. And when I substitute the quantities, the stress magnitude as well as θ =45°, this comes out to be 122 MPa and $\tau_{x'y'}$ is -30 MPa and $\sigma_{y'y'}$ is 18 MPa. Can you verify whether our computation is correct? Check the I_1, what is the value of I_1? Again 140 MPa. So, that means you have done the stress transformation correctly. So, that aspect I want you to absorb. Can I find out what is the principal stresses based on this representation of stress tensor at the same point? Ideally what I should get? I should get the same result like what we have got earlier from a different stress tensor.

Do the computation. I am going to show that you can easily verify it. You should be in a position to do that and also verify whether your I_1, I_2 are identical and how do you label the principal stress direction. Principal stress direction will be different now. Principal stress magnitudes will be same, but I am looking at from a different plane. So, principal stress directions will be different.

So, with that idea, if you look at it, and utilize your Mohr's circle effectively, you will find it useful. So, I have the new stress tensor is referred with respect to 45° from the original. That is why this is perpendicular to that. And you have a clue, what should be your principal stress direction with respect to this stress tensor. Anybody has results? Because you know, next slide, I am going to show my results. I am not going to spend time on it.

At least you calculate the principal stress direction because principal stress magnitudes will be identical. You can easily calculate $\frac{2\tau_{xy}}{\sigma_x - \sigma_y}$, calculate θ. Can you calculate θ?

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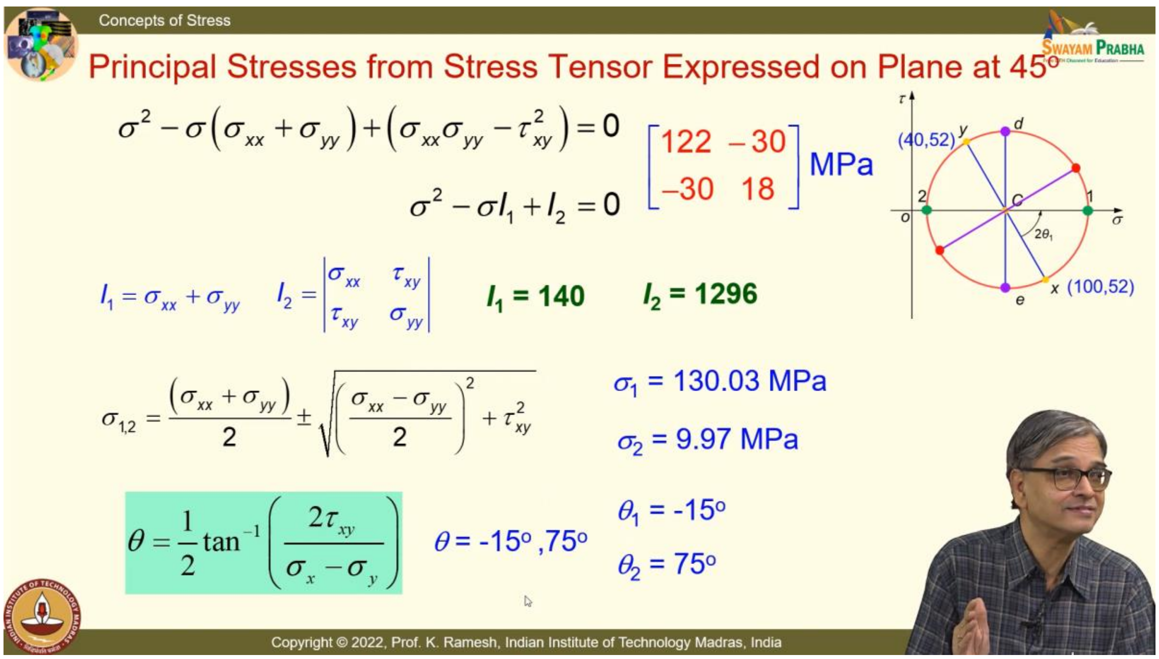

By the time I will have the other computations shown on the screen. So, this is just to make you recognize the steps. I have the I_1 and I_2. And then when I use the quadratic expression, I do get the same magnitude of σ_1 and σ_2. Tell me the principal stress direction. What are the values that you have got now? Anybody? At least, I should have one student completed the computation. Is the idea clear? You have the expression for θ, $\frac{2\tau_{xy}}{\sigma_x - \sigma_y}$. My suggestion is, write all the values from that as θ and compare it with the Mohr's circle, label that as θ_1 and θ_2.

Anybody has got the values now? I think it is taking little more time than what I anticipated. Ok, I will give you the values. How many of you have got -15° and 75°? Then why did you

not tell me? Ok. So, I look at the Mohr's circle.
So, from the Mohr's circle, I can say θ_1 is -15°. Why it is minus? It is clockwise, fine. I have this as the reference plane. From this plane, I have obtained the θ direction. Is the idea clear? Do you know how to do the computation? So, you have seen for yourself whether I take the stress tensor with respect to x y plane or this plane, I get the same σ_1 and σ_2, ought to be. But you have seen it black and white in this.

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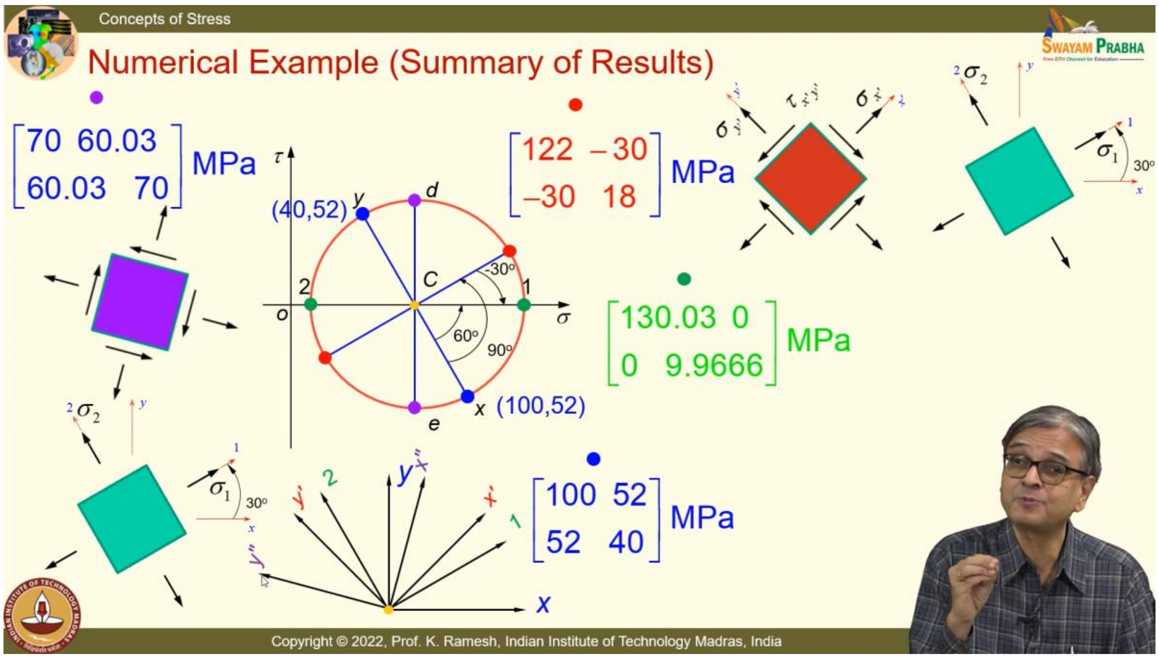

And this is summary because this gives you what is the way that you should look at the stress tensor. State of stress at a point. It can have multiple avatars. All are identical. They look different, fine? See, Hindu gods are different, but the concept of god is one. You have that expression here. The stress tensor looks different from different planes, fine? But they all represent the state of stress at a point.

The principal stresses are identical. So, I have this $\begin{bmatrix} 100 & 52 \\ 52 & 40 \end{bmatrix}$. Let me mark that as a big blue dot on the Mohr's circle. I have the Mohr's circle drawn. Then the second representation is, it can be represented with respect to the principal stresses. The stress tensor appears different.

The principal planes are beautifully shown here. So, I have the principal stress direction 1 and 2 shown. I have referred with respect to the x-y axis $\begin{bmatrix} 100 & 52 \\ 52 & 40 \end{bmatrix}$. With respect to 1-2, I have this as $\begin{bmatrix} 130.03 & 0 \\ 0 & 9.9666 \end{bmatrix}$. The stress tensor representation appears different, ok? But we have seen that it represents the same state of stress for the point of interest.

Then we have also looked at the plane at 45°. And what are the magnitudes that we have got? We have got that as $\begin{bmatrix} 122 & -30 \\ -30 & 18 \end{bmatrix}$. See, here, everything is positive. Some quantities are negative here. Some quantities are zero here. It is again state of stress at the same point. It can appear differently. But from a Mohr's circle, it is very easy to see. But from a number, it is difficult to comprehend, ok? So, this also you have the axis oriented here.

I would appreciate that you draw this sketch, ok? You have this. I have this represented in this fashion. Stress tensor is represented with respect to the x'- y'. I can also represent the stress tensor based on the maximum shear stress plane. I can also do that. I have all the numbers from the Mohr's circle.

From the Mohr's circle, I can pick out this and this is the stress tensor with respect to the maximum shear stress plane. So, what we have done is, we have represented this stress tensor from x-y, 1-2, x'-y' as well as x''-y''. All are giving the same values of principal stresses. And using Mohr's circle and the expression for θ, you can correctly fix the principal stress direction 1. Whether it is θ_1 or θ_2, you have to filter it out from your result. Not the first result is θ_1 and second result is θ_2!

So, in this class, you know, we have looked at what way the principal stresses have encouraged the researchers to come out with new ideas. One of the new ideas which we saw was identification of octahedral stress plane. Other idea was splitting the stress tensor into a composition of a hydrostatic state plus a pure shear stress state. Instead of deviatoric stress state, we will call this as pure shear state because we know what happens pure shear.

That you are able to identify by utilizing the invariants. Then we have also proved mathematically, that principal stress directions have to be mutually perpendicular. And also saw a beautiful experimentation by brittle coating that gave two families of curves on the entire body that show that every point on the object, you have principal stress directions at that point mutually perpendicular. That is seen at every point in the whole field. And we have also learned how to do the computation systematically as well as to do the inbuilt checks. I expect you to do the inbuilt checks and then report the results. Thank you.