

In fact, you graduate from rigid body mechanics to deformable solids. So, the moment you go to deformable solids, we do not make a step change. We say we will have small deformation. We will consider deformation; we have always been considering bodies as rigid. We will definitely accommodate deformation, but the deformations are small.

How small? That is another question to be answered. And if you look at, you need to have a test; that is what is shown here. And if I have a look at the curve what you had seen, mathematically how will you state that? Is it linear or non-linear? It is non-linear, fine. But cleverly if I say it is small, I can make it linear and make my life very simple.

(Refer Slide Time: 01:27)

And this also summarizes some of the results. You had spectacular failures. And you can also see the stress colorfully, ok.

(Refer Slide Time: 01:39)

Introduction to Mechanics of Solids

Failure of Common Amenities

- Many accidents involve a failure – either material separation or buckling - that causes collapse, injuring the occupant.
- Such failures are often due to design defects – choice of material, the choice of the appropriate cross-sectional dimensions or ageing.

Collapse of False Ceiling

Copyright © 2022, Prof. K. Ramesh, Indian Institute of Technology Madras, INDIA

You know there have been very simple failures which you do not want to have in your day-to-day life and particularly this plastic chairs, they are known for their notoriety. Many times, they buckle. The fall appears simple, but it can be crippling you because if there is a tendon tear, you had it, fine.

And you are sitting in a nice room with a false ceiling. Imagine if the false ceiling falls like this. This has in fact happened, fine. You do not want that to happen. That means you need to find out; have I made the selection of the cross-section appropriate? And you go to the garden and you want to sit, you do not want to have a surprise like this.

See if you look at all of this, you find its choice of material, poor quality design and also due to aging because if you see a garden chair that is exposed to natural elements, over time the strength degrades, fine. So, after learning this course, you should be able to sit comfortably in a chair and you should also have your room well protected.

(Refer Slide Time: 03:05)

Introduction to Mechanics of Solids

SWAYAM PRABHA

Tacoma Narrows Bridge Collapse

Wind speed 42 miles/h
Period of oscillation 5 s



Specifications:
 Length of the center Span = 853.4 m
 Width = 11.9 m
 Depth of stiffening girder = 2.4 m

History:
 Start of construction: 23rd Nov 1938
 Opened for traffic: 1st Jul 1940
 Collapse of bridge: 7th Nov 1940

Started vibrating at 10:00 am
 Collapsed at 10:40 am

Failed by Torsional mode of vibration

Copyright © 2022, Prof. K. Ramesh, Indian Institute of Technology Madras, INDIA

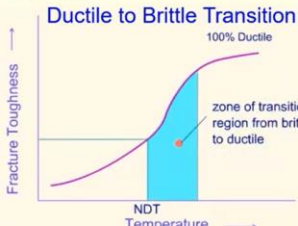
And you also had spectacular failures. This is a bridge made of concrete and if you look at the history, this was exposed to wind loads of the order of 42 miles per hour and you can see that it is actually twisting like this, fine. And if you look at when was this constructed and when the failure occurred; within a span of 6 months! So, that means, something grossly missed in the design of the bridge. So that led to development of cross-bracings to sustain the wind loads. So, what you had was, it started vibrating at 10 o'clock, vibrated for about 40 minutes and it failed by torsional mode of vibration. Fortunately, there was no major casualty except one car was falling into this. And there have also been failures precipitated by temperature change. Imagine people have built ships, this is during World War II and you find thousands of them having failures, not one or two.

(Refer Slide Time: 04:16)

Introduction to Mechanics of Solids

SWAYAM PRABHA

Failures Precipitated by Temperature Change




Fracture Toughness

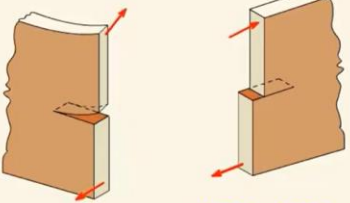
100% Ductile

zone of transition region from brittle to ductile


NDT Temperature



1000 ships had failures!



-17°C to 4.5°C



15 mm thick steel

Birth of Fracture Mechanics

Copyright © 2007 Prof. K. Ramesh, IIT Madras, Chennai, India

Copyright © 2022, Prof. K. Ramesh, Indian Institute of Technology Madras, INDIA

If there is one or two, you can say that there is some kind of a manufacturing defect that has led to the failure. People also switched over from riveting to welding and welding was in the nascent stage in those days. And when you had thousands of failures without your enemy bombing your ship, that is a cause for concern. And you also had in the 1920s, you had this famous molasses failure in Boston. If you look at this, there was a temperature change, it was made of 15 mm thick steel and you all know it is not made of glass.


You can understand glass is brittle and steel is ductile. If that breaks like a brittle material, it is a cause for concern. And you also have in your material science, you will probably go and learn there is a ductile to brittle transition, it happens at low temperatures. And there was commonality between the two, when the temperature was about 4.5 degrees, the steel gave way and behaved like a brittle material. And this is accommodated with the development of a new set of mechanics called fracture mechanics, fortunately beyond this course.

(Refer Slide Time: 05:57)


Introduction to Mechanics of Solids

SWAYAM PRABHA

Failures Precipitated by Corrosion and Fatigue



- In 1988 an Aloha airline's Boeing 737-200 airplane suffered a partial in-flight disintegration.
- An 18-foot crown section of the fuselage tore apart while in flight.
- The root cause was found to be the presence of multiple cracks in the structure.



Courtesy: aoha.net

Copyright © 2022, Prof. K. Ramesh, Indian Institute of Technology Madras, INDIA

And we also had another very devastating failure, Aloha airline failure. The plane was flying at 30000 feet, all of a sudden, a portion of the roof blew off and the pilot landed the plane safely. You have to congratulate the pilot, there was only one casualty. The people understood that you have to design the aircraft, so that it drains the water, otherwise it leads to corrosion. And you also have fatigue that has precipitated this kind of a failure.

(Refer Slide Time: 06:34)

Introduction to Mechanics of Solids

Idealization

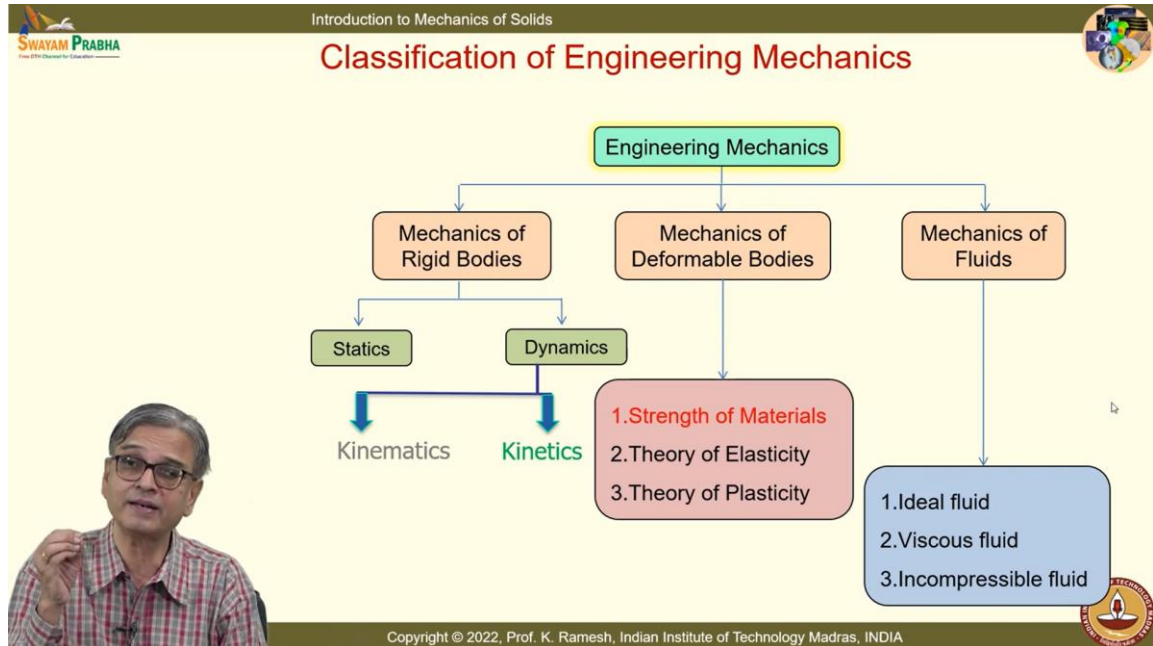
- Crucial step in engineering approach for the purpose of analysis.
- Actual behavior of systems is complex.
- Consideration of all features is difficult or impossible.
- Mathematically ideal model: simple to analyze yet exhibits the phenomena under consideration.
- Model is valid if analytical solution checks well with results of experimentation or observation.

Copyright © 2022, Prof. K. Ramesh, Indian Institute of Technology Madras, INDIA

And there can be no engineering without idealizations, why we do the idealization? We would like to make the problem simple enough for us to analyze, because all natural systems are non-linear and complex. And when you want to mathematically analyze, it is difficult for you to accommodate all aspects. Those aspects that are primarily important for the phenomena under study, you would like to capture. And what is the way that you can be satisfied that the mathematical model is sufficiently represented the physical phenomena, the only way is, it should satisfy the observations or experimental measurements.

Without experimental verification, you cannot rely on the mathematical model. So, you have to always understand if DGCA has to permit the aircraft to fly, it should satisfy the safety norms, not just your design is sufficient. So, that is taking safety into consideration. So, always you need to have an experimental information.

(Refer Slide Time: 07:51)

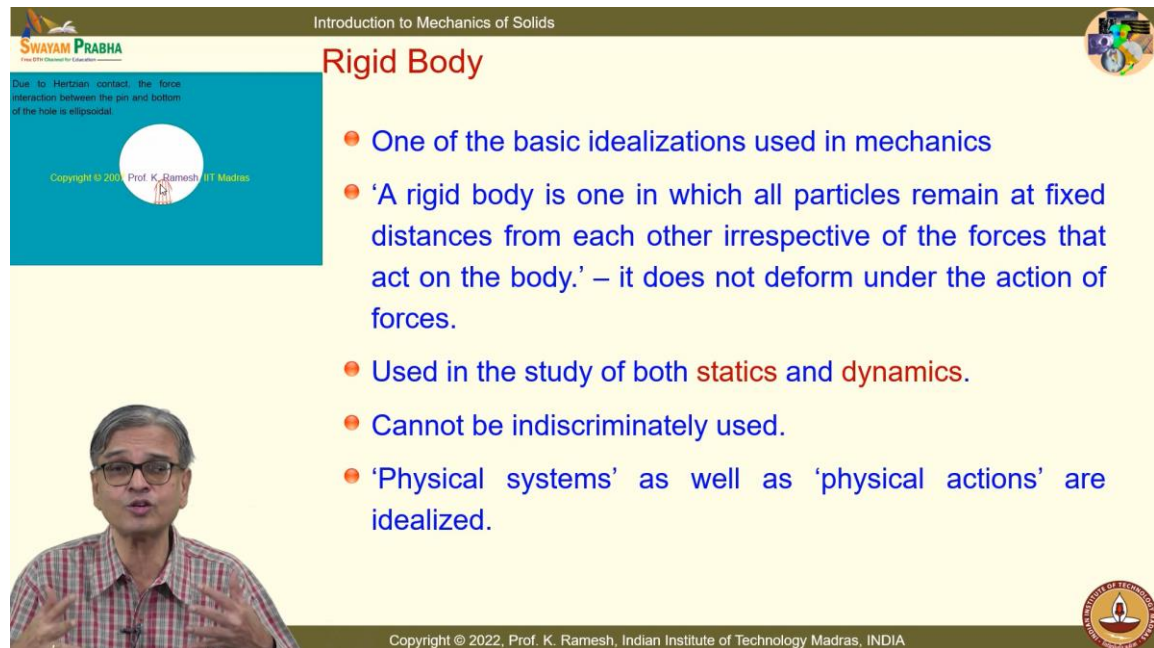


And when you look at Engineering Mechanics, you know you had done the first course – Mechanics of Rigid Bodies and that was divided into Statics and Dynamics, that was subdivided into Kinematics and Kinetics.

So, you could do one full course considering the body as rigid. No longer you can live in that domain, you have to graduate. So, now you go for Mechanics of Deformable Bodies. And here, you start with Strength of Materials, where you cleverly conjecture the deformation, select cross-sections, make idealizations. So, you avoid solving differential equations.

Then you graduate to the next level of course, where you solve the differential equation and solve little more complex problems in Theory of Elasticity and much complex problems in Theory of Plasticity. In contrast to that in Mechanics of Fluids, even when you analyze the ideal fluid, you solve differential equations. So, there is a fundamental difference. And in this course, we would do thought experiment, we will try to get it from inference, we will also develop the equations mathematically, rigorous mathematical approach will also be there.

(Refer Slide Time: 09:16)



Introduction to Mechanics of Solids

Rigid Body

Due to Hertzian contact, the force interaction between the pin and bottom of the hole is ellipsoidal.

Copyright © 2009 Prof. K. Ramesh, IIT Madras

- One of the basic idealizations used in mechanics
- 'A rigid body is one in which all particles remain at fixed distances from each other irrespective of the forces that act on the body.' – it does not deform under the action of forces.
- Used in the study of both **statics** and **dynamics**.
- Cannot be indiscriminately used.
- 'Physical systems' as well as 'physical actions' are idealized.

Copyright © 2022, Prof. K. Ramesh, Indian Institute of Technology Madras, INDIA

And you know in the previous course, you considered the body as rigid. When do you say the body as rigid? What is the way you idealize the body as rigid? What is a rigid body? No deformation, otherwise the distance between the particles remains same. Is it really so? It is an abstraction, fine. You have not only talked about the physical abstraction, but also the physical action.

See, in the previous course, you have left and right used concentrated load. The concentrated load concept goes hand-in-hand with rigid body. So, what is shown in this insert is, you have a distribution; from the distribution you find out using your composition and resolution of forces as a concentrated load. So, you keep your eyes half closed, reality is 'always deformation'. However small, there is deformation. At suitable magnification, you will be able to perceive, but we have ignored it, we could solve a variety of simple problems.

We could find out what are the forces happening on the system under consideration. So, it was useful, it is not that it was absolutely useless. So, this is what is summarized here, I said it, in Statics as well as Dynamics, you cannot use it indiscriminately. Physical actions; that is concentrated load is an abstraction goes hand-in-hand with rigid body idealization.

(Refer Slide Time: 11:03)

Introduction to Mechanics of Solids

SWAYAM PRABHA

Structures that Support Transverse Loading

The rafters or purlins are only supported at the joints

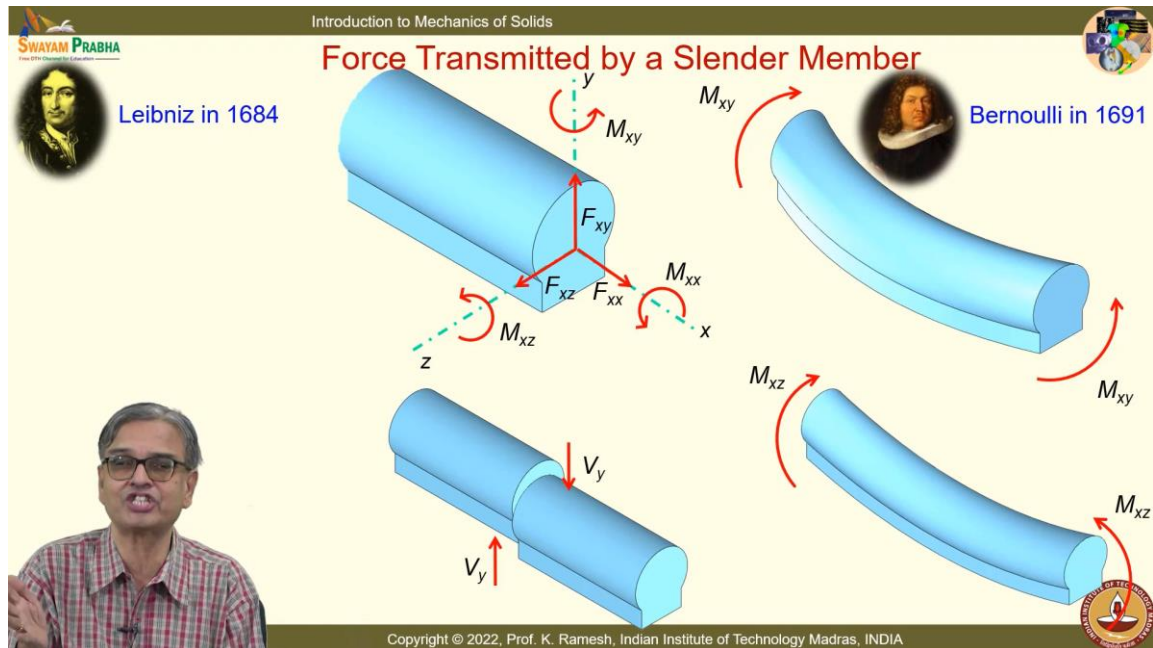
- The rafters are beams in bending which carry the wind load and weight of the roof.

Copyright © 2022, Prof. K. Ramesh, Indian Institute of Technology Madras, INDIA

See, in engineering you will find, you have to support transverse load. You have studied trusses in your previous course. Can you recall? What is the kind of forces that was resisted by the truss members? It is resisting the transverse load acting on it, but what was the resistance developed in the members? You have determined that by method of sections and method of joints, can you recall what is it that you found out? You will also have to find out the sign and then say whether it is. So, essentially axially loaded, you understand? See, I have a transverse loading happening on the structure, but by the way you have constructed the truss and you also have restriction, the load should be applied only at the joints and joints have to be constructed in a particular manner. All that you do, because you get an advantage, the members transmit only axial load, but to do that you also had rafters, these rafters support bending. See, tension or compression is easy to comprehend, bending is not so.

It has taken close to 400 years to understand. We are going to complete it in a few lectures, fine. And you also have bridges. So, many of these bridges you have truss construction and you have the floor beams. The floor beams take the transverse load, they resist it by bending, fine. But in your previous course, you have analyzed only trusses and you have been able to find out that it transmits only axial loading. And this again has, you know, transverse loading like this.

(Refer Slide Time: 13:19)

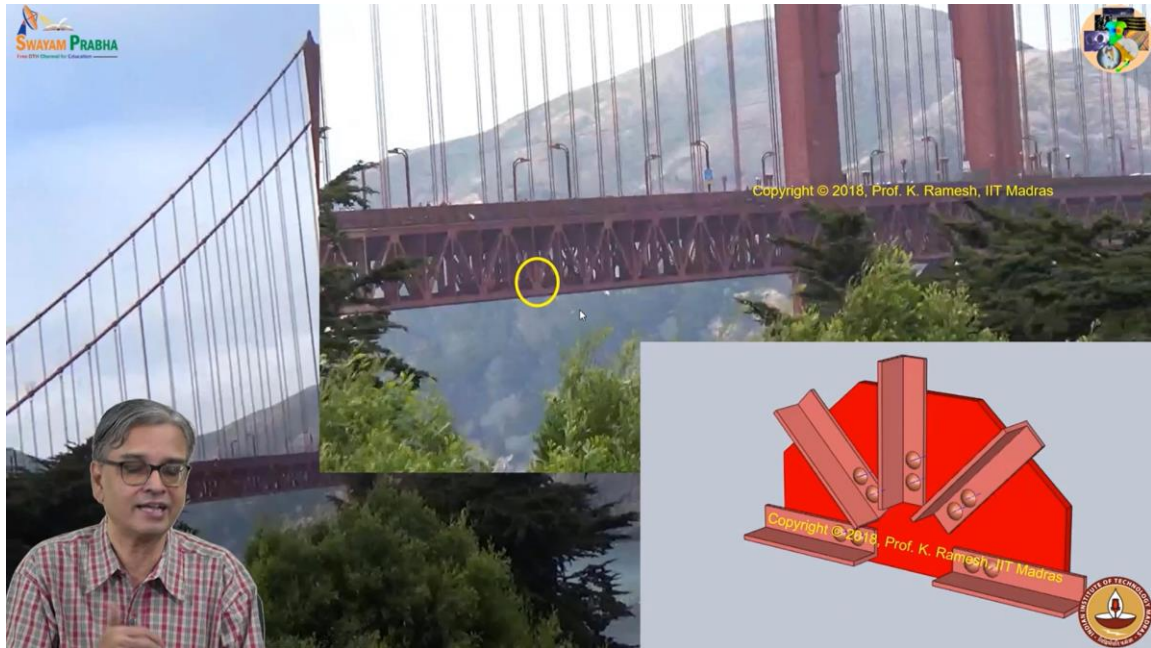


And another important assumption also you had made; we will confine our attention to slender members. What is the definition of a slender member? The length of the member is much larger compared to the cross-sectional dimensions. See, we make these idealizations to analyze structures with deformation, which can be captured mathematically, fine.

And you also learned in the previous course that I can take an imaginary cross-section and this imaginary cross-section in fact can support a force and a moment. This force can be visualized as components; three components and moment also can be visualized as three components. And whenever the component is tangential to the surface, you call it as a shear force. And you have done in the case of beam, how to find out the shear force diagram. Then you have also learned, if there is a moment, how the beam bends, it can bend in one axis.

If I have the moment in the other direction, it can also bend in the other axis. And in addition, it can also have another shear force as well as axial tension or compression or a twist. This is what we understand now, but if you go back to history, at that time they were really feeling that these arbitrary sections can support tensions. That is the way they started and the credit goes to Leibniz in 1684 and Bernoulli in 1691.

(Refer Slide Time: 15:08)



Look at this bridge. See from a distance, I can simply consider this long bridge as a slender member. I can find out what should be the natural frequency of it. So, it gives you a via media to analyze complex problems. The real situations are very complex. We mathematically model it and we come with a result closer to reality.

That is the difference between Engineering and Mathematics. We bring in correction factors, we bring in stress concentration factor and somehow wriggle out of the situation. We want to provide solution to complex problems. So, if I have a closer look, I can see the truss and I can also see a joint how it is constructed and you have learnt that at this joint, you should have a concurrent force system. You have a gusset plate; you join the members.

All that you do carefully so that none of the members forming the truss are subjected to bending. Even if it bends, even if you have to consider the self-weight, you simply lump it in the joints and then proceed as an approximation. Because now you have software which will find out what is the actual stresses and then, it is all very efficiently designed. Do not carry the impression that you do a crude analysis in your class. The same thing is practiced in the real life, it is not so.

(Refer Slide Time: 16:43)

Introduction to Mechanics of Solids

Various Cross-sectional Members Possible for a Truss construction

- The cross-section of members can be of I-section, channels, angles or circular rods.

Copyright © 2022, Prof. K. Ramesh, Indian Institute of Technology Madras, INDIA

And you also find the various cross-sections. See as far as axial load is concerned, whether I use an 'I' cross-section or channel or 'L' angle or a solid rod or hollow, it really does not matter. It is based on convenience. What matters in the case of axial loading is only the cross-sectional area, that knowledge you all have. The situation is quite different when the member has to support bending or torsion, cross-section matters; ok.

(Refer Slide Time: 17:32)

Introduction to Mechanics of Solids

Classification of Beams

- ❖ Cantilever
- ❖ Continuous Beam
- ❖ Simply Supported Beam
- ❖ Propped Cantilever
- ❖ Overhung Beam
- ❖ Fixed Beam

Statically Determinate Statically Indeterminate

Copyright © 2022, Prof. K. Ramesh, Indian Institute of Technology Madras, INDIA

It is not only the cross-sectional area size; how the cross-section is made? It matters. And you have also seen that by considering the body as rigid, you have happily solved problems after problems on beams. And I have two families; I have shown the first family, I am

showing the second family. Can you recall? Is there a distinct classification in the first column and the second column? Is there any distinct difference, do you perceive? Very good, very good. So, the first column you say it is statically determinate, because I can find out all the forces in the system with the help of equations from Statics.

The second set, the equations are not sufficient for me to evaluate the unknowns and you label it as statically indeterminate and you close your eyes. But in reality, you have to find solution for that also; Unless I bring in the deformation, it is not possible for me to solve these problems and bending deflection is very complex. So, we will postpone it. So, the idea is - you cannot live on the comfort of rigid body mechanics, you have to graduate to deformable solids.

(Refer Slide Time: 18:52)

The slide is titled "Useful Diagrams for Design" and is part of a video lecture. It features a list of four types of diagrams used in design:

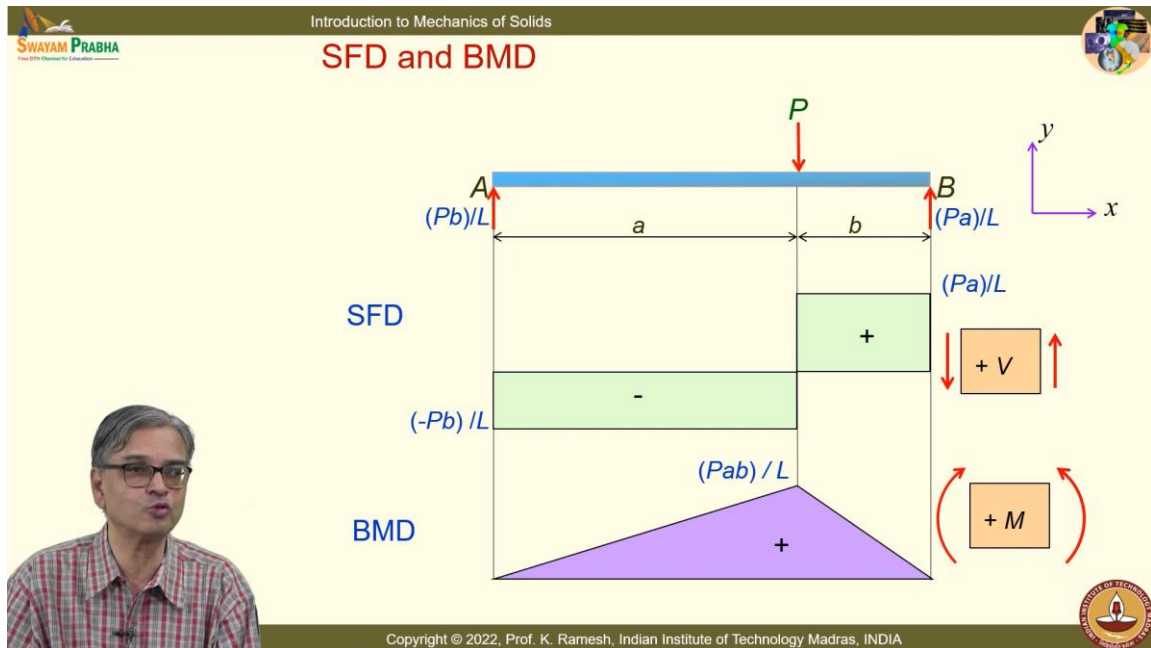
- Axial Force Diagram (AFD)
- Shear Force Diagram (SFD)
- Bending Moment Diagram (BMD)
- Twisting Moment Diagram (TWD)

The slide also includes the "SWAYAM PRABHA" logo in the top left, the "Introduction to Mechanics of Solids" text at the top, and the IIT Madras logo in the bottom right. A small inset video of Prof. K. Ramesh is visible in the bottom left corner of the slide area. The copyright notice at the bottom reads "Copyright © 2022, Prof. K. Ramesh, Indian Institute of Technology Madras, INDIA".

And you can also write and draw axial force diagram, shear force diagram, bending moment diagram, twisting moment diagram; Why do you do all that? So, in a cross-section, you find out where these are maximum.

So, you find out; the hope is that you will determine the cross-section based on the maximum of this, so that it will withstand my requirement.

(Refer Slide Time: 19:21)



And you have also done SFD and BMD; and those who have looked at my Engineering Mechanics course, in fact I discuss, at length, on what are the possible sign conventions, I used one sign convention followed by Meriam for the purpose of the class. But in this course, we will follow a sign convention - for shear, on the positive face, positive direction is positive; on the negative face, negative direction is positive. Bending moment, there is no change; because Engineering is one which requires conventions to be followed; without conventions, there is no Engineering. So, you have the shear force diagram like this and you have the bending moment diagram like this. So, you may have to brush up these fundamentals for you to follow the class.

(Refer Slide Time: 20:11)



And you have a down-to-earth example, this is there in every city. You have Atta-Chakki and we would see a shaft that is transmitting torque and if you look at this, this is also supported by bearings and there are also things that oscillate from this. So, this shaft is subjected to torque, subjected to bending and in some cases, you can also have axial loading. And if you look at this; when I have to mount a pulley or when I have to put it in a bearing and so on, I cannot have a uniform member; I cannot have a uniform member, I would have steps because I need to locate various members.

But in this course, we would close our eyes; we will have a slender member, even that when you consider, deformation is back-breaking.

(Refer Slide Time: 21:20)

The slide, titled "Variation of Internal Resistance", compares the internal stress distribution in three types of slender members under different loads. It features a small video inset of Prof. K. Ramesh in the bottom left corner.

- Axially Loaded Member:** Under a uniform axial load P , the internal stress σ_{xx} is uniform across the entire cross-section. The text states: "Cross section is fully utilized in load transfer".
- Beam:** Under a bending moment M , the internal stress σ_{xx} varies linearly from compression at the top to tension at the bottom. The text states: "Central core does not take any load".
- Shaft:** Under a torque M_t , the internal shear stress $\tau_{z\theta}$ varies linearly from zero at the center to a maximum at the outer radius r . The text states: "Central core does not take any load".

Copyright © 2022, Prof. K. Ramesh, Indian Institute of Technology Madras, INDIA

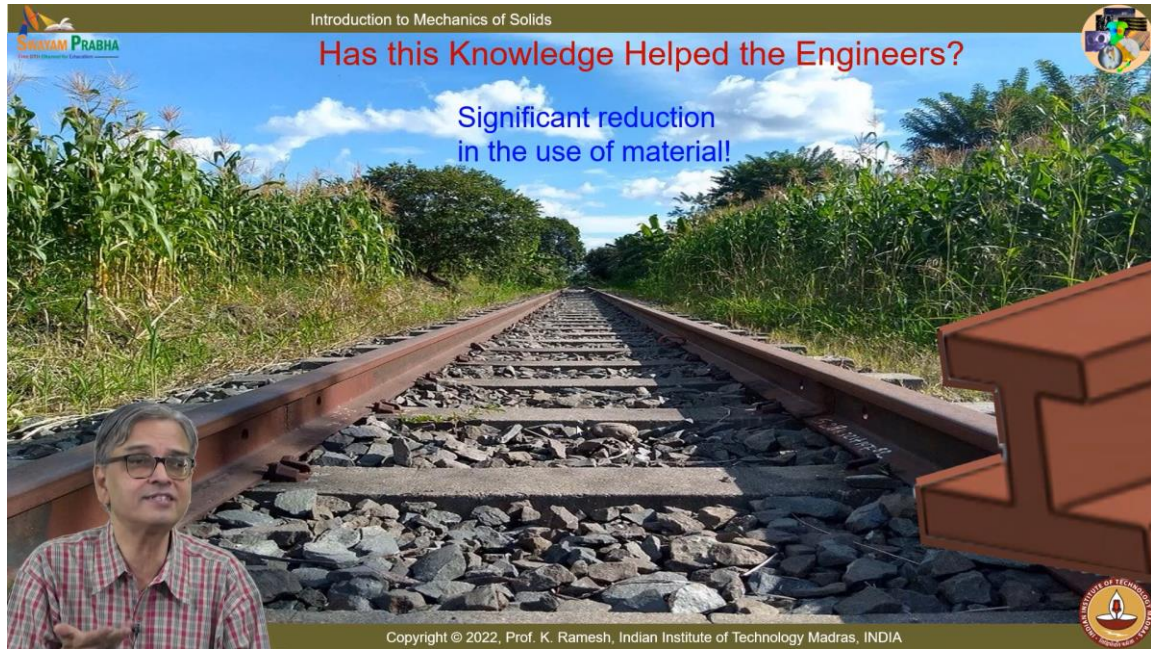
And you know, you all know, once I bring in deformation, there is resistance to deformation. What is the typical variation? The resistance is termed as a terminology 'stress'; and even before we get into the full course, I thought it is desirable that we look at the result that we are going to get. In this class, I will prove this result by inference; we will do a mathematical development in the subsequent classes. Now you know, I have shown it for convenience, a very large cross-section, it is only a portion of a very long slender member.

Imagine it like this because our focus is what happens in a cross-section, that should be lot more clearer. Imagine that the axially loaded member, beam or shaft are slender long members and I have taken a particular portion, zoomed it here. And when I have an imaginary section, if I look at this the result is like this; it is uniform. Suppose I go to beam and then find out what way the resistance is; it is not uniform like this but it is a triangular variation. And you also see a symbol like σ_{xx} , so we need to develop the mathematics behind it, understand what are these symbols; that we will postpone it for the time being.

The message here is, in the axially loaded member, the resistance is uniformly developed all across the cross-section. What happens to torsion? Torsion and bending shares a similar

feature, the variation is triangular; that means the inner core is not actually participating in load sharing. The material used in the inner core is not participating. It has an advantage and that is well exploited by engineers. Not only by engineers, nature has already adopted them; we will see that. So, the message here is, why you developed truss in such a manner? You utilize the material very well because whatever the material they put in every part of the material contributes to load sharing. And you are also very clever, a transverse load is felt as axial loading.

(Refer Slide Time: 24:03)



Now you have advantage of this knowledge; if you had no knowledge of Strength of Materials, you have rail lines, several thousands of kilometers across the globe, you would have made it as a block like this; it would not have cavity on the sides. Because you want that to be solid, you know it should resist and nothing should happen. And in reality, all these are like T cross-section, you do not have material here.

So, tons and tons of steels have been saved. So, knowledge of Strength of Materials is essential and we have saved, we have seen, because of that knowledge; because the rails essentially transmit, it has to resist bending loads.

(Refer Slide Time: 24:53)

Introduction to Mechanics of Solids

Can one Visualize the Stresses Developed?

Copyright © 2022, Prof. K. Ramesh, Indian Institute of Technology Madras, INDIA

Now the idea is, you know, you and I have stress due to various reasons, you have to write examinations I have to come to the class and then make the lecture and lecture goes smoothly. There is no way that you can take an x -ray or some photograph and find out, what is the level of stress that you have. Fortunately, nature is so benevolent, you have a methodology called 'Photoelasticity' that gives you the stress pattern in colors. And as children, you would have had opportunity to tighten a nut by a spanner. So, that is what is shown here; and here, for me to show you colorful fringe patterns, these are not made of metal, but a suitable plastic amenable for Photoelastic analysis.

When I view it in the polariscope, I see, as the load is increased, you see more and more colors developed. Do you see that? So, you can infer that the colors are directly related to the load transmitted by the member and you can also see that in a nice manner.

(Refer Slide Time: 26:04)

Introduction to Mechanics of Solids

SWAYAM PRABHA

Can one Visualize the Stresses Developed?

Copyright © 2007 Prof. K. Ramesh, IIT Madras

Copyright © 2022, Prof. K. Ramesh, Indian Institute of Technology Madras, INDIA

So, you observe, you have the fringes developed here and you also see the cross-section. It is not constant, it is increasing. It is increasing in regions where more stresses are developed and if you look at, spanner is a well-designed machine component.

Even with this course or in the next course, you would not have solution for this; it is so complex a problem, but if you go to an experiment or a numerical method, it is easy for analysis. So, the first inference is I have beautiful colors, I could visualize they are related to the load applied.

(Refer Slide Time: 26:54)

Introduction to Mechanics of Solids

SWAYAM PRABHA

Visualization of Stresses due to Tension or Bending

- Photoelasticity can reveal the stress as fringe patterns (Smoother variation)

Copyright © 2007 Prof. K. Ramesh, IIT Madras

Copyright © 2022, Prof. K. Ramesh, Indian Institute of Technology Madras, INDIA

With this background, we will go and see what happens when I apply axial tension and bending to the same material. I have a simple beam made out of this, I get colors like this and what we have said? Over the cross-section, the response is uniform. What do you see in colors? You see every point, when it is axially loaded, it is a uniform color.

What happens when I go to bending? I have all the colors in this spectrum available in this when it is bent. So, the response by an axial load and response due to bending are different; that you can clearly see. Now, I will also repeat it with a smoother variation. Now, you observe how the bending loads are reflected as resistance in the colors. You actually find that the fringes are appearing and then, they move towards the center and if you finally go and look at; I have labeled this as 0, 1, 2 and 3. If you look at these fringes and find out the distance, the distance will be equal. So that, by inference, you can say the variation is triangular like this.

Now let me do; see I said that in this course, I am going to take a member which is slender without any step, without any hole; but in reality, you cannot have anything without a step or without some hole for you to join and so on. I can also conjecture, I mean, I can also get the idea, what is the way the stress problem becomes from the same photoelastic approach. I put a hole and then see, what happens? for you to make a new design, you get a flowery pattern, you can print it on your cloth and then sell it, but it is too difficult to analyze mathematically.

(Refer Slide Time: 29:00)

Introduction to Mechanics of Solids

Complexity of Stress Field Dramatically Changes

- The moment, a small hole is introduced – see what happens!
- If there is a step in the beam

Beyond this course!

Copyright © 2022, Prof. K. Ramesh, Indian Institute of Technology Madras, INDIA

And look at what happens when I have a step; and I already told you, when I have a shaft, you will have to have steps for you to put the pulley, you have to have a step for putting a bearing, you cannot avoid that. But the moment I put any one of these aspects, the problem becomes very complex; fortunately, beyond this course. And you can also make one more interesting observation. See in the case of bending, we have seen that as parallel lines essentially; and I have a line drawn here; in this zone, this is still parallel; in this zone, it is still parallel; some disturbance happens in the vicinity of a step.

That is how engineers handle it. They will handle it with a correction factor or a stress concentration factor and wriggle out of the complex problem. So even to understand that a beam will resist, which has a triangular variation, that was not a simple thing to visualize.

(Refer Slide Time: 30:25)

The slide is titled "Idealisations" and is part of a presentation on "Introduction to Mechanics of Solids". It lists four idealizations with their respective characteristics:

- Small deformation**
 - Apply the equilibrium requirements to the *undeformed* configuration.
- Material is Homogeneous**
 - Elastic property is same at every point
- Material is Isotropic**
 - Elastic property is same along any direction
- Material is an elastic continuum**
 - No defects!

Illustrations on the slide include a rectangular metal block labeled "Homogeneous" and a cylindrical concrete specimen labeled "Concrete cylinder – heterogeneous with pores etc.". A red starburst diagram labeled "Isotropic" is also shown. The slide includes the IIT Madras logo and a copyright notice for Prof. K. Ramesh, Indian Institute of Technology Madras, INDIA.

So now, let us look at the idealizations – small deformation. The reason why we go for small deformation is, you can make the problem much simpler to handle and the implication is, we work on undeformed configuration; but we will also make a violation when we want to go and do buckling analysis, I will have to write the governing equation only in deformed coordinates, fine.

And you also say the material is homogeneous. What is the meaning of homogeneous? When I consider any point on this, the properties remain same, elastic properties remain same. And you have a counter example, you take a concrete block; if you have seen how concrete blocks are made, you find they put pebbles, they put cement, sand and all that and it also has pores. Definitely not homogeneous, but you analyze it as homogeneous; that is a different story. And you also idealize the material behaves in an isotropic fashion. You take any direction, the elastic properties remain unchanged. These are all idealizations that make your life simple and another important one is, it is an elastic continuum, no defects. Even if there are defects, in this course, we will simply close our eyes. You have fracture mechanics which takes care of all the defects, fine.

(Refer Slide Time: 32:05)

Introduction to Mechanics of Solids

Historical Evolution of Stress Variation in a Beam

Da Vinci (1452-1519)

Galileo (1564-1642)

Mariotte (1564-1642)

Mariotte (Did both without assigning reasons)

Parent (1666-1716)

Coulomb (1736-1806)

Copyright © 2022, Prof. K. Ramesh, Indian Institute of Technology Madras, INDIA

And as I said, historical evolution of stress variation in a beam, it all started with Da Vinci and he had, in his notes, diagrams drawn like this and Galileo conjectured that the resistance is uniform and you have Mariott who has conjectured as a triangular variation in one fashion. He has also quoted another variation like this. Then you had Parent, he had also conjectured triangular variation, but the neutral axis, what we will call later, is shifted. So, it was finally, Coulomb who got the same triangular variation that I showed as the result that we are going to learn in this course. So, if you look at the time span, it has taken close to 400 years. So, it is not a child's play. If you do not understand some aspect of the course while developed, do not feel discouraged. So, we look at some of the historical developments.

(Refer Slide Time: 33:23)

SWAYAM PRABHA

■ First person to document a theory

Leonardo da Vinci (1452-1519)

This book was lost.
People could never use it in olden times!
It was later found in 1967.

Copyright © 2022, Prof. K. Ramesh, Indian Institute of Technology Madras, INDIA

So, Da Vinci, 1452 to 1519, he was the first person to document a theory. Until then, people were not documenting. He had also written a book, unfortunately, it was lost; they could not locate it. They could locate it only in 1967. So, whatever the knowledge that he gained; others had to struggle from scratch, but historians, you go back and then see and then reinterpret, what he could have understood, at that point in time? That is also interesting to see.

(Refer Slide Time: 33:57)

The slide features a blue header with the text 'Leonardo da Vinci (1452-1519)' and a portrait of Leonardo da Vinci. Below the header, there are three bullet points in blue text. The background is a colorful illustration of a man with glasses sitting on a bench under a large green apple tree with red apples. The scene is set in a green field with rolling hills in the background. The slide also includes the 'SWAYAM PRABHA' logo and a copyright notice at the bottom.

Leonardo da Vinci (1452-1519)

- He established all the essential features of the strain distribution in a beam while pondering the deformation of springs.
- Da Vinci did not have at that time Hooke's law and the calculus.
- Mathematical formulation had to wait till the time of Bernoulli and Euler

Copyright © 2022, Prof. K. Ramesh, Indian Institute of Technology Madras INDIA

And we sit in the Newton's garden, he is sitting with the apple tree and apples are falling and he is getting inspiration. So, we are also getting inspiration by looking at the history in the Newton's garden. Recently, there was a storm in the UK and this apple tree got uprooted. Seems they are going to replant it and so on and so forth. So, by digging into his notes, people have found that he had the essential features of the strain distribution in a beam, but he did not have Hooke's law at that time. There was no mathematical development of Calculus, so he had to wait; understanding could not be translated into mathematical equations, but he seemed to have understood some of it. Nuances, he could get it.

(Refer Slide Time: 34:54)

SWAYAM PRABHA Leonardo da Vinci (1452-1519)

- Despite Da Vinci's accurate appreciation of the stresses and strains in a beam subject to bending, he did not provide any way of assessing the strength of a beam, knowing its dimensions, and the tensile strength of the material it was made of.

Copyright © 2022, Prof. K. Ramesh, Indian Institute of Technology Madras INDIA

The slide features a portrait of Leonardo da Vinci in the top right corner. The background is a colorful illustration of a landscape with rolling hills, a large green tree with red apples, and a person sitting on a bench under the tree. A small inset image shows a man in a red and white checkered shirt, likely Prof. K. Ramesh, speaking.

So, even though he was able to appreciate, I mean, stresses what they had developed, but he was not able to give a recommendation, how to design a beam? What should be the cross-section, that kind of a solution was provided only by Galileo. Even though Galileo has given it, he had his own limitations.

(Refer Slide Time: 35:15)

SWAYAM PRABHA

1638

- First person to publish a solution to the problem

Galileo Galilei (1564 - 1642)

Galileo's demonstration of a cantilever beam

Copyright © 2022, Prof. K. Ramesh, Indian Institute of Technology Madras INDIA

The slide features a portrait of Galileo Galilei in the top left corner. A timeline shows the year 1638. The background is a colorful illustration of a landscape with rolling hills, a large green tree with red apples, and a person sitting on a bench under the tree. A small inset image shows a man in a red and white checkered shirt, likely Prof. K. Ramesh, speaking. Another inset image shows a diagram of Galileo's cantilever beam experiment, with a beam fixed to a wall and a weight hanging from it.

So, Galileo was 1564 to 1642; first person to publish a solution to a problem. So, you have a very famous picture of a cantilever beam and you look at this, what are the violations? We have said, in this course, we will have certain idealizations. First, we said it is a slender member; do you find the beam is slender? It is a short beam. So, that means he is going to

have problems because unless you simplify the problem, you will not be able to visualize, what way to model it, fine; and it is made of wood. Wood is not an isotropic material, it has fibers are aligned. So, it is a complex material.

(Refer Slide Time: 36:06)

The slide is titled "The Strength of Materials in 17th Century" and features the Swayam Prabha logo. It contains two bullet points: "The problem of beam strength was addressed by Galileo in 1638." and "Ruler offers more resistance when lying on the edge than when lying flat". The slide includes a portrait of Galileo Galilei, a diagram of a stone tower with a wooden beam labeled with points A, B, C, and D, and a diagram of a ruler being tested on a tree branch. A red 'X' is placed over the ruler lying flat, and a purple checkmark is placed over the ruler lying on its edge. A small illustration of a person sitting on a bench is also present. The slide footer includes the copyright notice: "Copyright © 2022, Prof. K. Ramesh, Indian Institute of Technology Madras INDIA" and the IIT Madras logo.

So, strength was addressed by Galileo in 1638, but he found a very interesting answer. The answer was, if you take a scale and then put it in this fashion and apply an end load, clamp it on one end and put an end load. I have a cross-section which cannot do this because it is not a scale. You can verify it; I get a deflection; you can take your scale in the room and then try holding it in this fashion and holding it horizontally. There is a stark difference between the moment of inertia that you have to calculate, he had got that intuitively, fine.

So, he was able to say, you should not use the beam in this fashion; you should use the beam in this fashion. The ruler offers more resistance lying on the edge than when the lying flat. It is a very important observation; valid observation. He did not have a mathematical explanation for it, we can give a mathematical explanation once you learn bending.

(Refer Slide Time: 37:12)

The Strength of Materials in 17th Century

- All Galileo's work on the mechanics of materials is included in the first two dialogues of his book "Two New Sciences"
- "... under simple tension, the strength of a bar is proportional to its cross-sectional area and is independent of its length." This strength Galileo calls the "absolute resistance to fracture".

Copyright © 2022, Prof. K. Ramesh, Indian Institute of Technology Madras INDIA

And you know, they have all worked on tension, you know; Under simple tension, the strength of a bar is proportional to its cross-sectional area and is independent of its length.

It is a very correct observation; length does not contribute to it, but it is only the cross-sectional area. See, translating this understanding on a tension specimen to bending was not simple. They have all understood what happens under tension. Some observation that they could do on bending.

(Refer Slide Time: 37:51)

The Strength of Materials in 17th Century

Galileo assumed that the beam rotated about the base at its point of support, and that there was a uniform tensile stress across the beam section equal to the tensile strength of the material.

Copyright © 2022, Prof. K. Ramesh, Indian Institute of Technology Madras INDIA

And you know, he got this distribution like this; he said that this acts like a fulcrum and then you had resistance developed like this; all that is not supported by theory now.

He could not proceed. So, you should understand the difficulty they had at that time.

(Refer Slide Time: 38:08)

Choice of Problem

Problem is complex
Beam is not slender
Material is wood - orthotropic
Studied Hollow beams
Observed examples from nature.
Understood they are stronger

Copyright © 2022, Prof. K. Ramesh, Indian Institute of Technology Madras, INDIA

And the reason I have said; certain aspects I said earlier; even the problem is complex because I am having shear force and the bending moment varies across the length. While we develop the beam theory, we are very clever; we will take a beam under pure bending. We will avoid shear. So, why Galileo could not succeed? He took a complex problem; he took a beam that is not slender; material is wood, it is orthotropic, but they also look at what happens; see, when you want to have something to support load you want that to be solid, is not it? Solid means something completely filled, but he found in nature, when you have wheat fully grown, you find the wheat stacks are hollow and they carry at the end, heavy grains.

So, he wondered that hollow beams can be stronger. Nature has taught us, but no explanation at that time, but when you say hollow beam, what you have to compare is, what is the cross-sectional area of a hollow beam and what is the cross-section that you make out of the same material as solid. If you have the hollow beam like this. So, imagine that this is hollow, if I compare with the same size of the solid shaft, you cannot compare; that is not a valid comparison. In Strength of Materials, we would like to use minimum amount of material.

He found many materials, when they are loaded, they behave like a spring; in what aspect? The spring, when you elongate, leave it, it comes back to its original position. And he also found that this varies linearly. But let me ask a question, you go back and then find out you all have a pen like this which has a jotter type of arrangement; you will have a spring when you put an axial load on the spring, with your background, it is possible for you to find out, what is the force transmitted at a general cross-section? It is not axial load. Please go back and then see.

(Refer Slide Time: 41:11)

The Strength of Materials in 17th Century

- Established the relation between the magnitude of forces and the deformations that they produced.
- Discovered in 1660 but published only in 1678.
- The linear relation between the force and the deformation is the basis for Hooke's law.

Copyright © 2022, Prof. K. Ramesh, Indian Institute of Technology Madras, INDIA

So, when you say Robert Hooke looked at the spring, when you say axial load; when you pull the spring, it is a representation here. I have just put zig-zag lines, but actually spring is coiled like this. So, if you look at what kind of forces transmitted at a generic cross-section, it is not axial load. But this example is good to show that there is elastic recovery and the force deformation relationship is linear. The resistance developed in an axial pull or what happens in a spring subject to axial pull are different because you have to understand; you have the background, let me see. I want to know in the next class, what is it that you can do. And this linear relation formed the basis of Hooke's law.

See, even though he determined this in 1660, he published it only in 1678. See those days, though my slides are very poetical nicely colorful richly decorated, Science development has not been poetical; there were fist fights there were fights that people want to say, I have done it first, fine; and they have also camouflaged these results as anagrams and then; see, knowledge is power, so they were not willing to disclose. Imagine, 18 years, he has not published the result.

(Refer Slide Time: 42:43)

1666

- Distribution of tension
- Location of Neutral plane.

Edme Mariotte (1620 - 1684)

Copyright © 2022, Prof. K. Ramesh, Indian Institute of Technology Madras INDIA

Then you have Mariotte, 1620 to 1684, and as we have already seen that he had proposed two results of variation in the beam without a reason. So, we will have to say that he had right thinking in that direction, but not completely he could succeed. He had postulated this variation as well as a variation like this. Though this matches with current understanding, there is no clarity.

(Refer Slide Time: 43:18)

1687

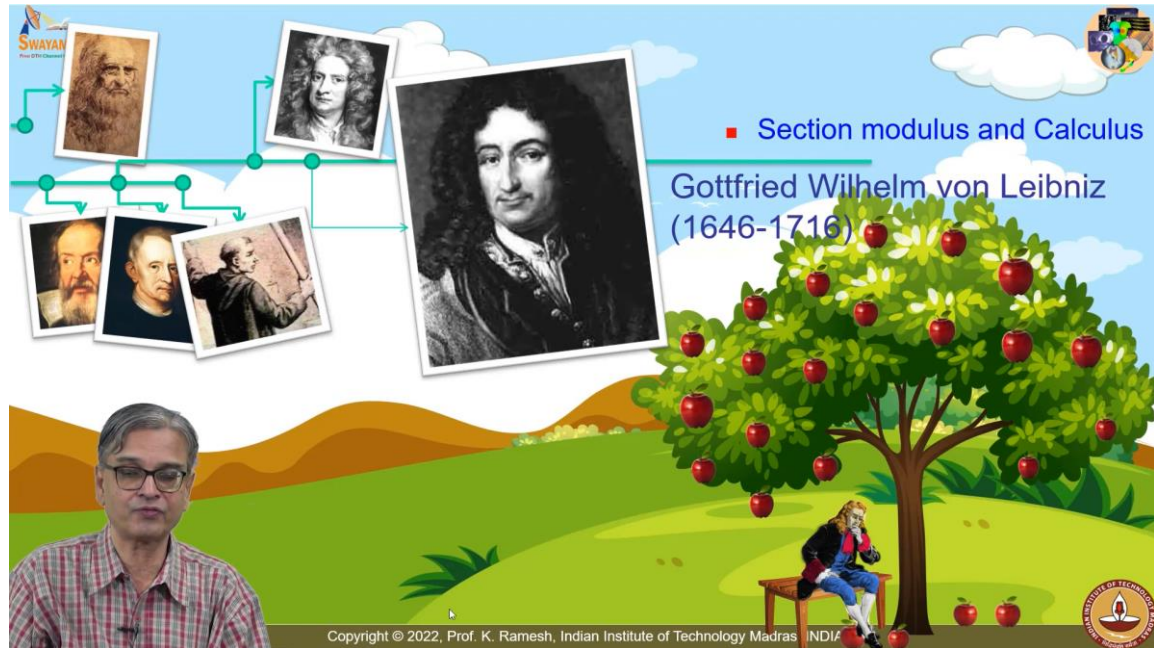
- Equilibrium of Forces and Calculus

Sir Issac Newton (1642 - 1727)

Copyright © 2022, Prof. K. Ramesh, Indian Institute of Technology Madras INDIA

Then you had Issac Newton; made very significant contributions and the modern world gives credit for Calculus to Newton. We will see some information from India in the next class.

(Refer Slide Time: 43:33)



We also had Leibniz, And I said, the moment you go from simple tension to bending, shape of the cross-section matters. The shape of the cross-section does not matter in tension or compression; only the cross-sectional area matters. Even Galileo found it in his experiment, shape matters. And this was brought out when you do the bending, people also developed something called as section modulus and that is credited to Leibniz. He has also been credited for development of Calculus.

So, in this first lecture, we had a brief overview of what is the type of mathematical step that we are going to make. We are not going to live in rigid body idealization anymore. We would take a baby step for deformation; we will make that as small deformation. And we have also looked at, *a priori*, what is the result, how resistance develops in simple tension, how the resistance develops in bending, how the resistance develops in torsion, but we could see, experimentally by Photoelasticity, the play of colors in tension and bending that has given an inference, what I said that the cross-section equally participates in axial tension whereas, in the case of bending, the inner core does not contribute to load sharing. And we have also said, material is elastic continuum, we will hang on to it because all our mathematical development depends on that idealization. And I said, you have a spanner and spanner is made by forging.

When you do forging, the material is no longer isotropic you have grains aligned, but we will still say, material is isotropic and live in a comfortable domain. Thank you.