Inverse Methods in Heat Transfer
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Lecture No # 07
Module No # 02
Example Application of Linear Regression for an Inverse Conduction Problem

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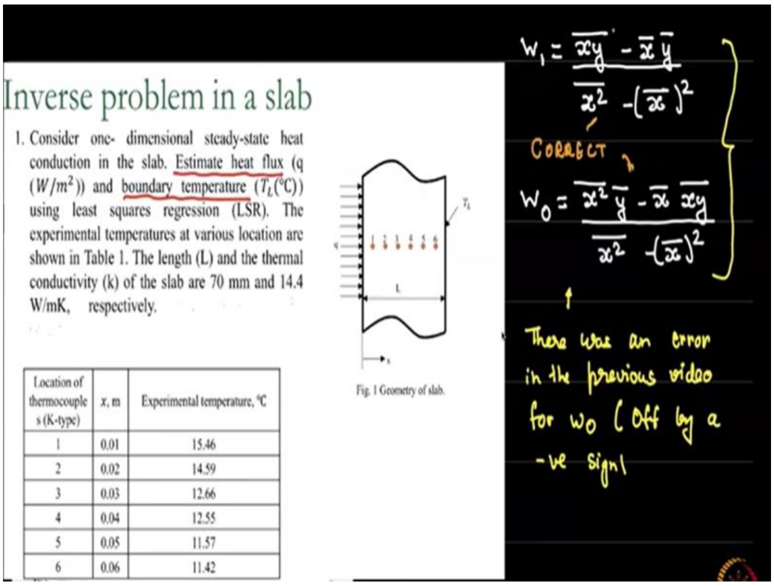

Welcome back. In the previous video we derived some simple expressions for linear model, recall that we were looking at the inverse problem in a slab. We have given thermocouple measurements at a few locations and we are asked to find out the actual temperature. So, what we are asked to estimate is that heat flux and the boundary temperature. So, we will try to answer this question, using the expressions that we derived in the last time.

Please notice that there was an error in the previous video for w_0. I had actually this is was off by a negative sign. So, I think I have put a minus sign up here and a plus sign here it is the other way around. So, what is written here is actually accurate the previous video was wrong. So, these are the correct expression so you can note that down so these 2 are correct now.

$$w_0 = \frac{\overline{x^2}\,\bar{y} - \bar{x}\,\overline{xy}}{\overline{x^2} - \bar{x}^2}$$

So that is a small note I had also told you in the last video that there are slightly differing expressions that you will see usually you would have already seen this in textbooks, like I said many people in school do statistics and you might have an expression there and this is very similar to that, except I am using averages here and the school expressions are you might see in other places also beginning statistics textbooks tend to use some the summation instead of the average and I will write those expressions.

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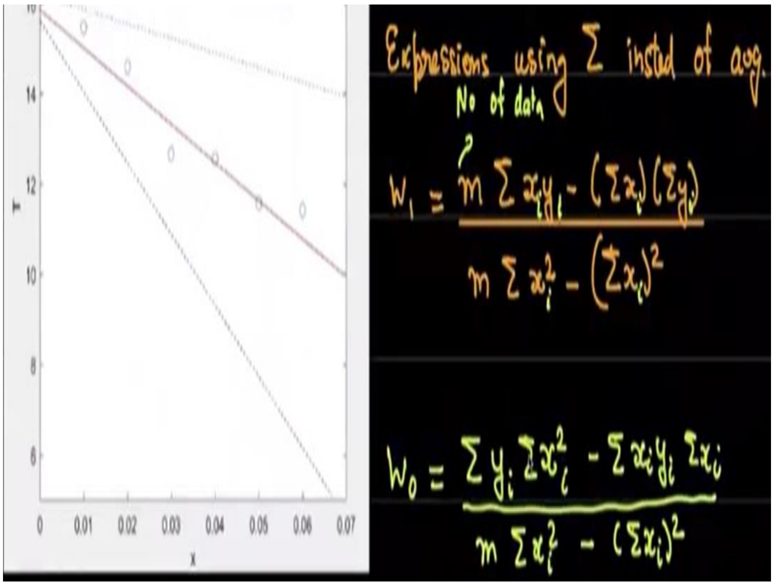

So, expressions using summation instead of average these are perfectly equivalent expressions. So, I am going to write the same thing involving sums, rather than averages here. So let me write the expression now so w_1 is the number of data points m, multiplying $\sum xy$, summation is all over the number of examples multiplied by $\sum x \sum y$, the whole thing divided by $m\sum x^2 - (\sum x)^2$.

$$w_1 = \frac{m\sum xy - (\sum x)(\sum y)}{m\sum x^2 - (\sum x)^2}$$

So, you will see that it is more or less identical to the expression here, wherever there was $\overline{xy}$, we have summation and basically you can actually quickly find out that at a few places we are going to get m, which is the number of data points. So, we collect that now similarly we can write an expression for w_0 also. w_0 is given by,

$$w_0 = \frac{\sum y_i \sum x_i^2 - \sum x_i y_i \sum x_i}{m\sum x_i^2 - (\sum x_i)^2}$$

So, the expressions that we have. now what we want to find out are the heat flux and the boundary temperature. So, what we need to do with this is?

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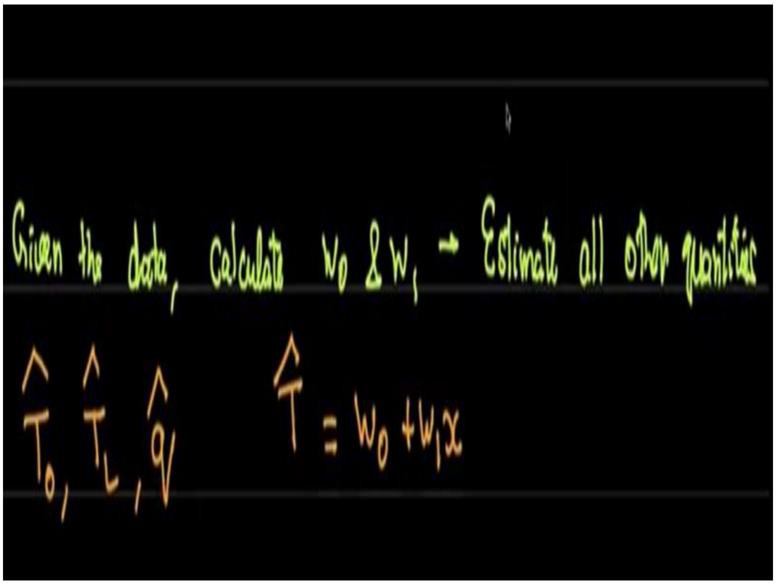

So given the data calculate w_0 and w_1 and from here, estimate all other quantities. Now what are the quantities we want to estimate? We want to estimate the temperatures, let us say I have only asked for T_L which is the right-hand side temperature. But let us say we want T_0, T_L and we actually want to estimates for this and we also wanted estimate of the heat flux.

Now how would we calculate these? Let us see that just shortly. but once we have the model of $\hat{T}$ being,

$$\hat{T} = w_0 + w_1 x$$

obviously we can find out $\hat{T}$ at x = 0, $\hat{T}$ at x equal to length, which is 0.07 as given in the question. And then you can find out q as,

$$q = -k \frac{dT}{dx}$$

we will do that within this video in the next slide.

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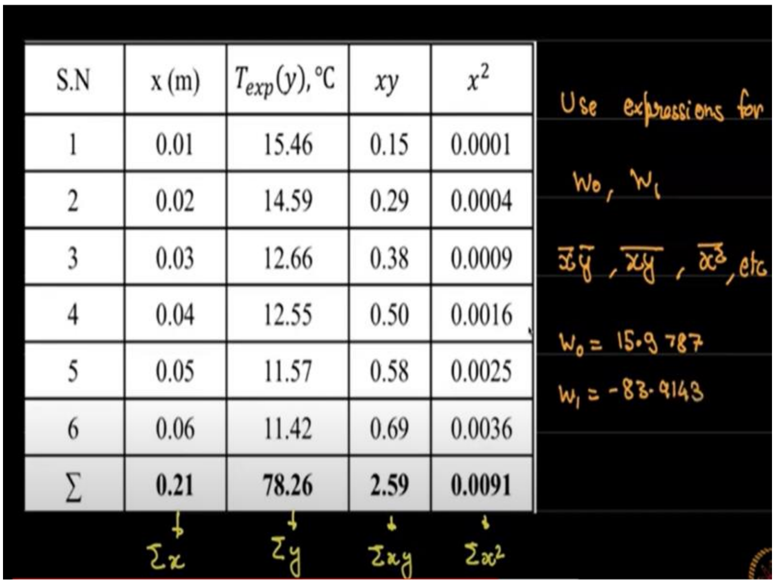

S.N	x (m)	$T_{exp}(y)$, °C	xy	x^2
1	0.01	15.46	0.15	0.0001
2	0.02	14.59	0.29	0.0004
3	0.03	12.66	0.38	0.0009
4	0.04	12.55	0.50	0.0016
5	0.05	11.57	0.58	0.0025
6	0.06	11.42	0.69	0.0036
∑	**0.21**	**78.26**	**2.59**	**0.0091**

So now suppose we come here and we use the summation expression. So, we use expressions for w_0 and w_1, which involve either average of xy, x^2 average etc., we have all these within here. So, if we involve all that we basically look at this table. So, I have made a table again you would have been used to some such thing within your school. Please notice the parameters of this table. Here is the serial number 1, 2, 3, 4, 5, 6. This is how we are going to simply write down data.

Here is the single variable over which the data depends the location. Here is the T experimental temperature or what we are calling y here. This is the noted down temperature here and then the auxiliary quantities which we require for the expressions shown here, which is $\sum xy$, $\sum x^2$ etc. So, $\sum x$ and $\sum y$ are already available we also want xy and $\sum x^2$.

So, if we look at these expressions, we see sigma notice this, this term then becomes $\sum x$, this term here $\sum y$, this term here is $\sum xy$ and this term here is $\sum x^2$. Now using these expressions, I will not actually put these in here you can calculate what w_0 and w_1 are. If you calculate these quantities, they come out to $\sum$, so w_0 comes to 15.9787, if you calculate it and w_1 you can calculate this to -83.9143.

Now you can use an excel sheet as I have shown here or you can use a MATLAB program, which I will show you in a next week. How to write a simple MATLAB program to calculate this but we will use this technique as well as couple of other techniques to in order to come to the same conclusion. So, I want to use MATLAB during that time, but for now let us assume you are going these calculations either by hand or by using an excel sheet.

If you are doing the assignments and taking this course for credit, you can do the assignments using any technique that you want. So, I do not really care. In the final exam if you take this course this for credit, then we will figure out how to give you calculator to in order to compute these reasonably fast. So, once you come here, we get these values as I write down in the previous slide w_0 is 15.9787 and w_1 is -83.9143. Now once you have these values we can now determine as I said the temperatures

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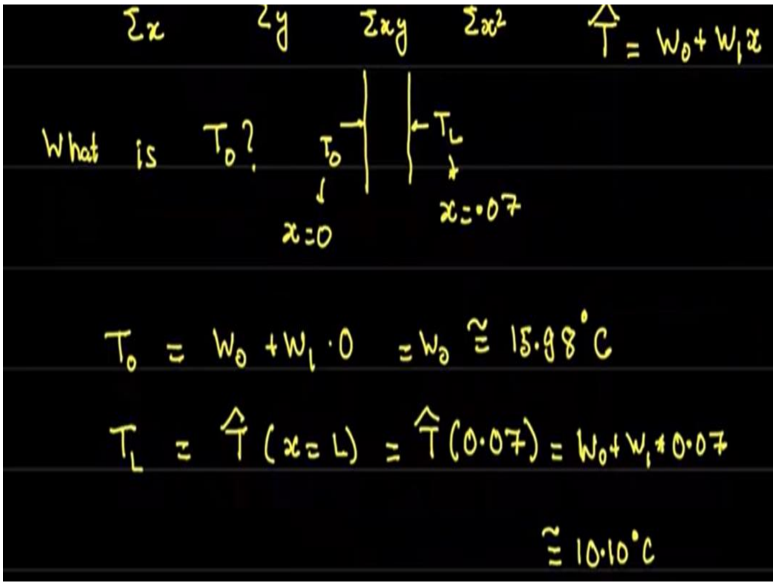

So, suppose I ask the question what is T_0 that is the temperature on the left boundary? So, temperature at the right boundary now notice as far as the measurements are concerned. We do not have that value we only have thermocouple values in the middle of the slab 0.01 up to 0.06. we do not have a value at the left or the value at the right.

So, this however is at the location x = 0, this is at the location x = 0.07. so, if we plug in these values, we now notice T_0 is so remember the temperature, this is our model, the temperature is $w_0 + w_1$ x. So T0 will be,

$$T_0 = w_0 + w_1 . 0$$

so this is simply w_0 and that gives us the temperature, which is 15 point, approximately 15.98°C. The temperature at the right-hand T_L can be calculated similarly, this is simply the temperature predicted temperature at x = L which is $\hat{T}$ at x = 0.07.

Since the length of the slab is 70 mm. so this then becomes,

$$T_L = \hat{T}(x = L) = \hat{T}(0.07)$$

$$= w_0 + w_1 \times 0.07$$

$$= 10.10\ ^{o}\mathrm{C}$$

If you calculate this comes to 10.10 degree Celsius approximately. You can see both these reflected in the graph, that I have shown here. you see around 15.97. So, this happens to be the best fit which I have drawn. So, this happens to be the best fit, I had written this was the visual best fit.

But actually, speaking I calculated this explicitly with the coefficients here with w_0 = 15.97 and w_1 = - 83.91. So how these lines were drawn. you can notice that the temperature at the right end is 10 point something, on the left temperature is some 15 point something. That's what we have now found out. Now finally the all-important question really speaking for heat transfer in JRS.

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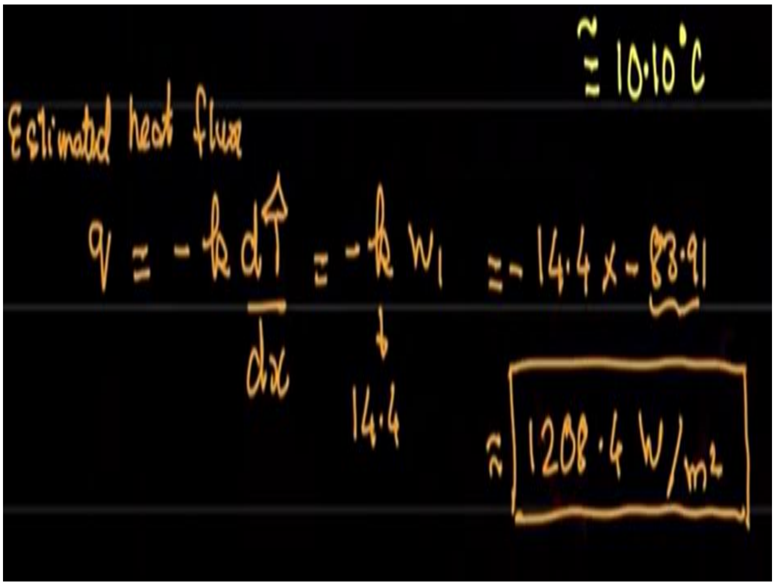

what is the estimated heat flux? So, the estimated heat flux q is,

$$q = -k\,\frac{d\hat{T}}{\mathrm{dx}}$$

Now $\hat{T}$ is $\mathrm{w}_0 + \mathrm{w}_1\mathrm{x}$. This tells us that this $\frac{d\hat{T}}{\mathrm{dx}} = \mathrm{w}_1$. So, this is -k $\times$ w_1, the value of k is given within the question to be 14.4. So, if you actually plug this in -14.4 $\times$ -83.91 which is the value of w_1, you will get approximately $1208.4\frac{\mathrm{W}}{\mathrm{m}^2}$.

So, this is the estimated heat flux. So, this is really a very simple of how an inverse problem is solved. So just to recapitulate, you are given a series of locations and series of thermocouples. you basically built a model inside it was linear, based on that linear model we found out the best possible coefficients w_0 and w_1 by solving a linear regression problem. And we actually derive the analytical expressions in the last video.

Once we derive that we can now find out these w_0 and w_1 the numerical values. once the numerical values were found we can now derive the quantities which correspond to the left temperature right temperature and the estimated heat flux. This is in a nutshell very similar to how the inverse process will take place at least for the sequence of problems we will be doing, like I said within the last week, we will also deal with slightly more sophisticated inverse problems.

But the preliminary aim of this course is to introduce you to this wide variety of technique which already exist and though this problem is simple, it is a problem we will return to multiple times. Because it is very simplified thing that we can use and solve things by hand to understand various techniques. Now as I said in the previous video, this does not tell us how to go ahead and do quadratic models, how to do cubic model or other polynomial models.

And another question that I have not yet addressed is well we can see that this line somehow fits the data. But how good is this line? So, these questions will be handled in the coming few videos within this week thank you.