

**Inverse Methods in Heat Transfer**  
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**Lecture - 67**  
**Summary of Course**

Welcome back. This is the final video of the course. I will offer a very short and very rapid summary of what we covered in the course.

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- Inverse Problems – Definition
  - ① Algebraic Techniques ✓
    - Linear Models -- Linear, Polynomial, Weighted Regression
    - Nonlinear Models – GNA, Levenberg-Marquardt, Tikhonov ← Regularization
  - ② Probabilistic Techniques → Saflaji
    - Bayesian, MH MCMC
  - ③ Machine Learning
    - Neural Networks – PINNs, Surrogate models
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First of course, we looked at the introduction and looked at what inverse problems are. In that we tried to figure out the cause from effect. And that makes inverse problems a really general topic for all of science and engineering. Specifically, within heat transfer, we saw a few examples. We just looked at a couple of examples, illustrative examples within the class.

In case this were a live class, we would have done a whole lot more. But in video recordings only so much could be done. But we keeping with the name of the course, we looked at several methods. So, the methods that we looked at were first of course, linear models, which could be used for inverse and within that we saw linear regression, polynomial regression, weighted linear regression, etc.

Then we looked at nonlinear regression, specifically Gauss-Newton algorithm. Then we looked at variants of Gauss-Newton algorithm, which were Levenberg-Marquardt

and Tikhonov, which were used as regularization techniques for these. So, these two were regularization techniques. Then we switched our attention. This was our first set of techniques.

The second set of techniques were probabilistic techniques, where instead of interpreting the problem as one of simply finding a function, we find, we interpret the problem as trying to find out a probability density function. So, after introduction to probability and all the other terms, we looked at the Bayesian approach, specifically using the Metropolis- Hastings, Markov Chain, Monte Carlo.

The Metropolis-Hastings is basically a sampling technique where if you cannot do or if you do not wish to do offline Bayesian because it is really expensive, you start sampling so that every sample tells you where next to look. So, Markov Chain Monte Carlo is simply a probabilistic technique or a technique to reconstruct the probability density function.

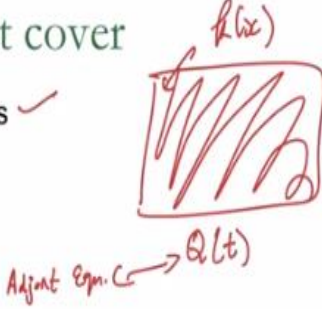
And we saw how once you know the probability density function, you can actually find out a whole lot more not just the parameter, but also the uncertainty in the parameter, okay? Whereas whether it is algebraic techniques or machine learning techniques, we look only at the final answer rather than the uncertainty involved in our determination. Finally, we looked at machine learning.

The last four to five weeks were primarily on machine learning, where we looked at an introduction to neural networks. We looked at physics informed neural networks and we also saw how surrogate models could be used.

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## What we did not cover

- Design of Experiments ✓
  - Function estimation
  - Adjoint techniques 
  - Practical techniques to solve inverse problems with CFD
  - Uncertainty analysis ✓
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Now what is it that within this course we did not cover? As I had said earlier, this is basically an undergraduate course and that too a really introductory one. What we did not cover is a vast set of topics that make it for a graduate course. So, we did not cover even though I mentioned briefly about how a sensitivity matrix could be used for using design of experiments, you know designing where to put thermocouples you know how exactly to find out uncertainty.

All these topics were not covered within this course. Secondly, we only looked at parameter estimation. That is, we just assumed that we know only, we want to find out thermal conductivity or some such parameter. Whereas usually you would, not usually, but in some cases, you have an entire function to be estimated. For example, let us say the thermal conductivity varies across an entire body.

So,  $k(x)$ . In such a case, you would need a function estimation approach. There are other cases such as in turbulence where you need a function estimation approach. Of course, finding out the entire  $Q(t)$  also is a function approach, function estimation approach. Adjoint techniques, I had briefly referred to this.

These come in two different ways even though the meaning actually comes in the same way. Adjoint techniques can be direct techniques to write what is known as an adjoint equation to the governing differential equation, which lets you find out things like derivatives such as sensitivity coefficients directly. It turns out it is also very useful whenever you have function estimation problems.

One other thing that I did not discuss which actually I was planning, these other techniques that I told you these, the other four topics that I told you, I was not planning or cover in any way. But some practical techniques to solve inverse problems with CFD would have been an important topic. Unfortunately, time did not permit me to discuss this.

The idea is in cases where we do not know analytical solutions, even though I showed neural networks, traditionally people tend to use normal CFD computation so out of which you build surrogate models. So, to walk you through a full case study would have been nice. But unfortunately, we did not get time to cover them.

Nonetheless, I hope you found this course covering a wide variety of techniques useful and I hope you continue your journey in inverse methods because it is a vast topic. And as I said in the beginning covers in fact many topics can be thought of as special cases of inverse method. So, I hope to continue your journey here and I was a little bit useful in encouraging you to learn a little bit more about the subject. Thank you very much.