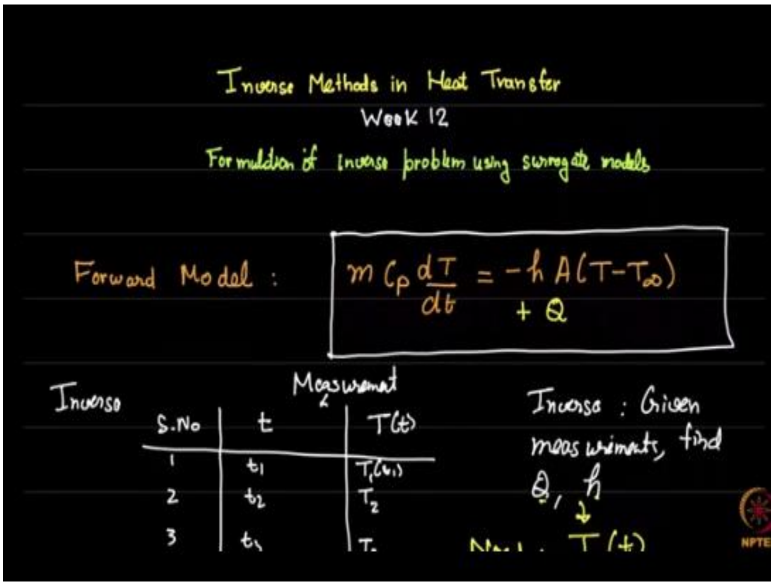
Inverse Methods in Heat Transfer
Prof. Balaji Srinivasan
Department of Mechanical Engineering
Indian Institute of Technology-Madras

Lecture - 66
Formulation of a Surrogate Model Based Inverse Solution in Unsteady Conduction

(Refer Slide Time: 00:19)

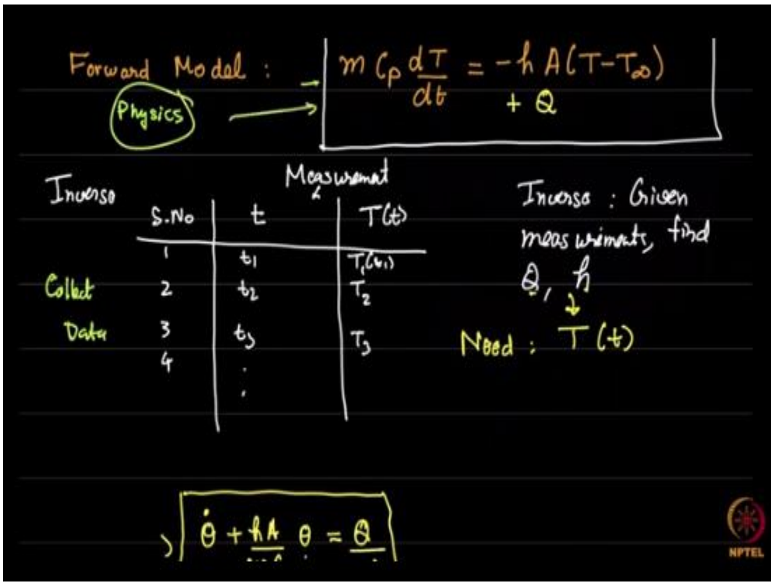


Welcome back, we continue week 12 of inverse methods in heat transfer. This is sort of the final video before the summary, and the way ahead video, which will be the next one. And I am going to continue this idea of looking at the inverse heat conduction problems that we looked at in the last video and also in week 4, the nonlinear problem. And formulate this problem for surrogate models, just like we formulated the problem for the PINN case.

Again, solving this surrogate model problem by hand would be impossible. And the computation part I was going to demonstrate it actually in conjunction with Metropolis-Hastings. But I would just briefly talk about how we can do this rather than do it. So, I will just show the formulation of the problem now and distinguish it with the other methods that we have seen so far.

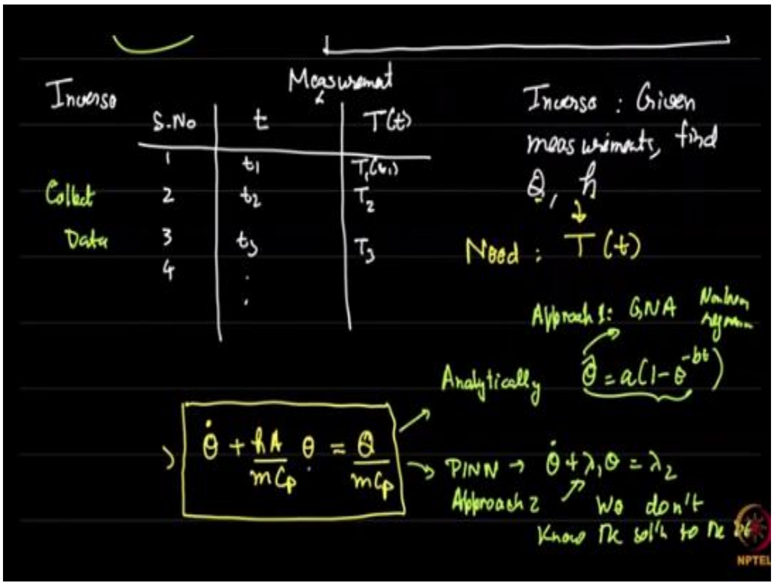
Once again, the forward model is something that we obtain from physics and we have the differential equation here, okay? So, we have the differential equation here. And now once this differential equation is available, we have several options.

(Refer Slide Time: 01:42)



One option is of course, after you collect the data, this of course is data collection.

(Refer Slide Time: 01:52)

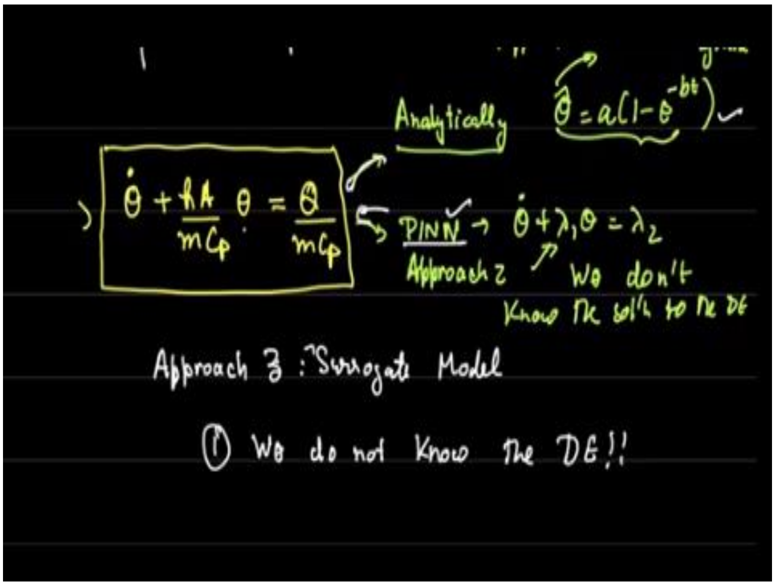


So once this forward model is obtained, you have two options. You come to the forward model and either solve it analytically which if you remember you had $\hat{\theta} = a(1 - e^{-bt})$. So, something of this sort or +bt whichever way you want to represent it. And another way is to start from here and apply PINN and simply use $\dot{\theta} + \lambda_1 \theta = \lambda_2$.

Now if we start with the analytical approach, we can use the Gauss-Newton algorithm and just do this as a simple nonlinear regression problem. So, this is approach one, okay? So, this is the first approach. The second approach is useful if we do not know the solution to the differential equation, okay?

So, notice the distinction, you use an analytical solution, if you know that the analytical, so you use the analytical solution if you know how to solve the differential equation. You use PINN or in many cases, you can sometimes use computation also in case you do not know the solution.

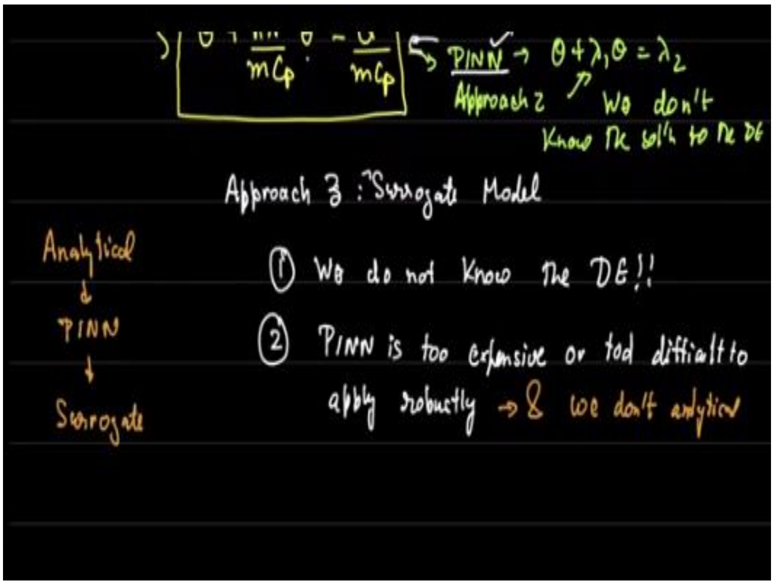
(Refer Slide Time: 03:24)



There is a third approach which is what we call the surrogate model approach. Now how does this work? The surrogate model approach is useful in the following cases. We do not know the differential equation itself, okay? So, notice even PINN requires you to know the differential equation.

The Gauss-Newton approach requires you not only to know the differential equation, but actually the all also the actual solution. In that case, this is really effective. If you do not know the actual solution, but you know only the differential equation you can use PINN. But what happens if you do not know the differential equation at all?

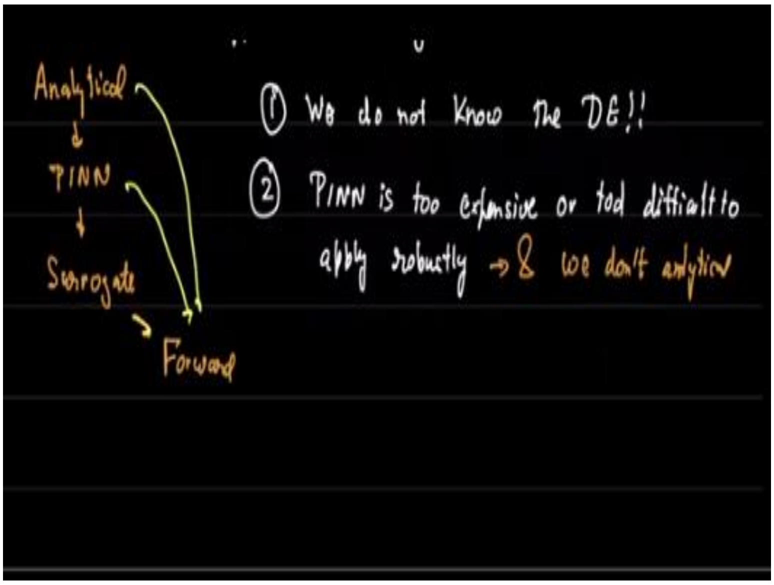
(Refer Slide Time: 04:18)



Then you use the surrogate model approach or PINN and PINN is too expensive, let us say which can happen. So, I did not talk about numerical issues with PINN. PINN can actually be too expensive or too difficult to apply, which is the same thing. There are robustness issues with PINN. Again, this course was too short for me to discuss those issues. So, in case these two are the case, this happens and of course we do not know analytical solution.

The assumption is if you knew analytical, you should immediately jump there. If you do not know that you jump to PINN. So analytical if you do not know that jump to PINN. If for some reason PINN is not applicable that is when we move for the surrogate approach. Now how is surrogate approach done? So surrogate approach is, we still need a forward model.

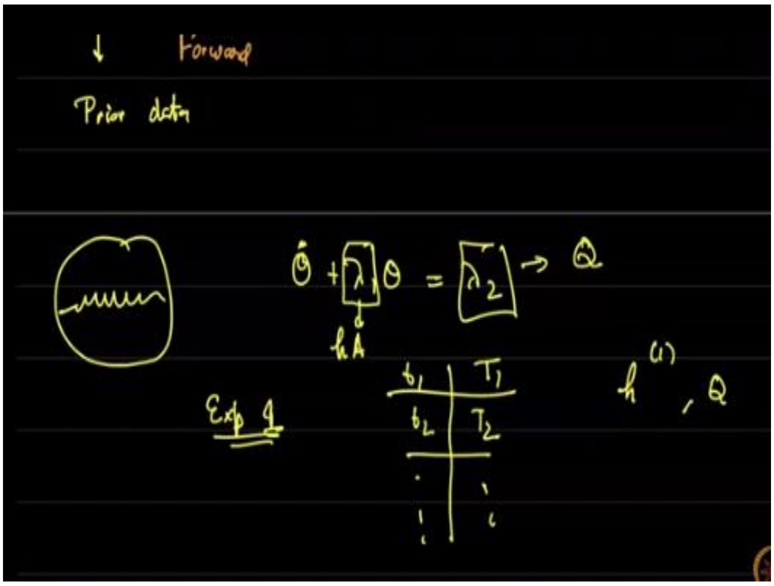
(Refer Slide Time: 05:32)



Remember, the moment you have a forward model, all you need is some iteration in order to get stuff back. You just need some optimization scheme in order to solve the problem. In fact, I had given you an example of a surrogate model in the last week also. But here is a little bit more about how you can formulate.

So how you formulate with the forward problem is where you either use PINN or you use the analytical technique or the surrogate technique.

(Refer Slide Time: 06:05)

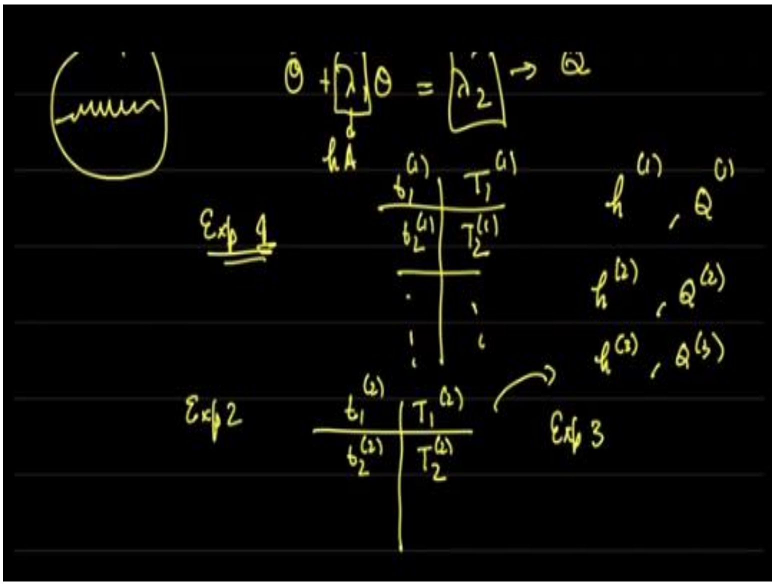


Surrogate technique can work, if you do not even know this, you need something else, you need prior data. What is meant by prior data? So now notice, our differential equation is θ_1 plus this equal to λ_2. Now I just said I do not know the differential equation. I might not know that, but I might know that θ_1, θ_2 are parameters.

For example, θ_2 is some variant Q by MCP, but let us say some normalized heat that you added to the system. So, there was the system. We added heat to it and we know the heat that we added. So, we just measured what happened and this is the convective heat transfer coefficient. If you go back to the notes, you will find that.

So, suppose for some experimental system, so someone somewhere, let us say in the United States did some experiment and they measured t_1 and T_1, t_2, T_2, but this was for some h and some Q, okay? So, they made these measurements. So let me not call it subscript just not to confuse you. So, $h^{(1)}$ and $Q^{(1)}$.

(Refer Slide Time: 07:26)

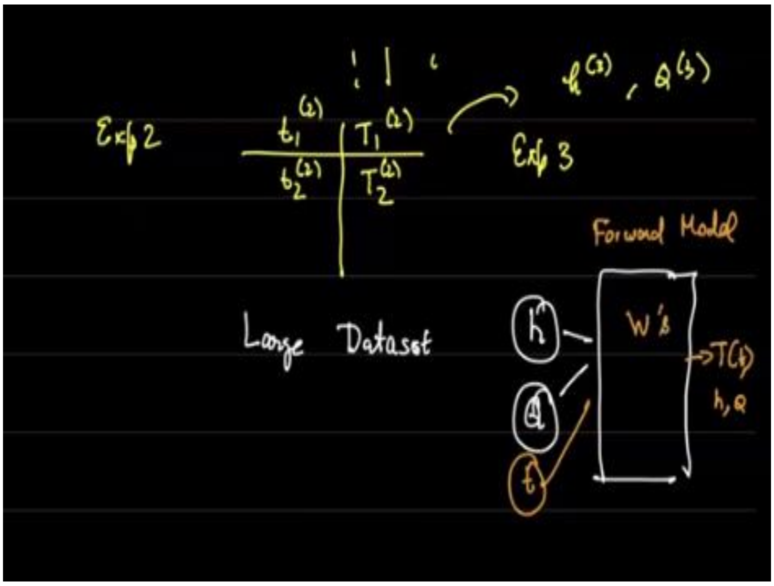


Then someone somewhere else did some other experiment. Again, they said T_1, t_2, T_2. But we can put a superscript from this was experiment 1. This was experiment 2. So, $h^{(2)}$ and $Q^{(2)}$, okay? So, this goes on. Then of course, someone else did experiment 3, which is, $h^{(3)}$ and $Q^{(3)}$ and then you have this corresponding data set for all three, okay? You keep on doing these and over time, now where would this happen practically?

So let us say you have multiple cars, multiple new models of cars and people are doing experiments there and you want to figure out how does the air conditioning work within the car and how does the temperature change at the place where the driver is sitting. So, you can take a temperature measurement between the time that the AC is started and till the time it reaches full cooling.

So, you can make these measurements and different cars. So, lot of people depending on various locations, let us say in India and Ahmedabad, in Mumbai and in Chennai, in Delhi etc., people do all these measurements and you have a large data set. So, this is the key.

(Refer Slide Time: 08:48)

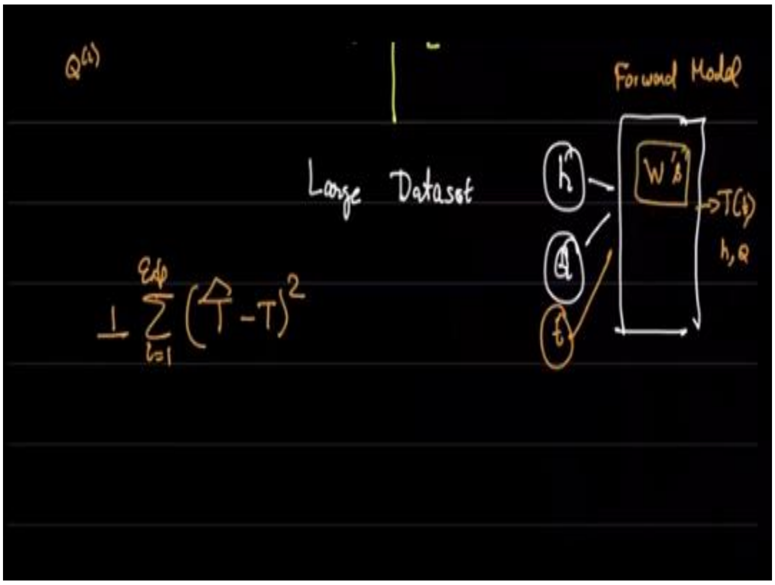


You have a very large data set. Now once you have this very large data set, you again make a neural network, but this neural network is like this. You say given h and Q here is a neural network. Notice the difference between this neural network and the neural network that I showed for PINN. So, in PINN the input was x or t and the output was let us say the time the temperature here.

Here it is different. So, you give h, Q. You also give the time, okay? So, this is what is new. Given these three find out temperature, okay? So, this temperature is now a function of h, Q and time. So, this now becomes our forward model. But the forward model has other parameters, W, okay. So, what does gradient descent look like here? So first the data set.

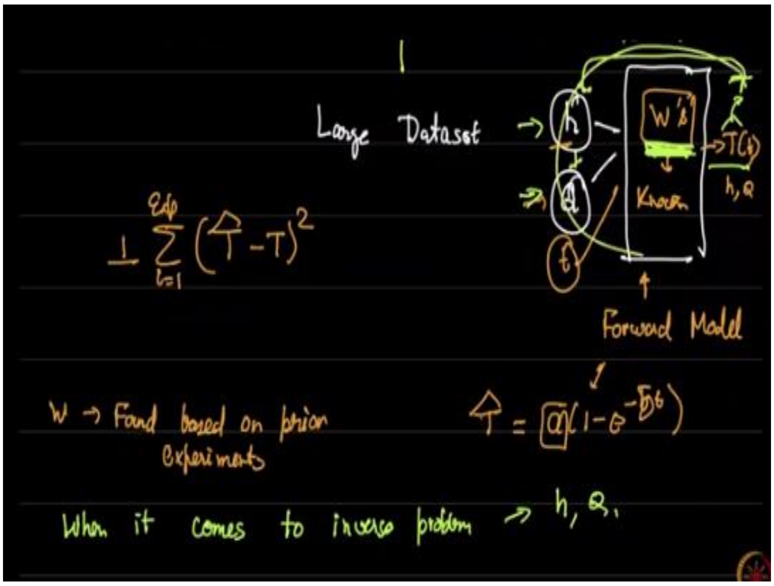
What you do is take all these data. Notice t_1, t_2, $h^{(2)}$. So, in this case you would have $h^{(2)}$ and $Q^{(2)}$, etc., going and this temperature this time going and that will give you a temperature T. Your model will say some other T so you will find the difference.

(Refer Slide Time: 10:19)



So overall, you will simply still have $(\hat{T} - T)^2$ summed over all the experiments and all the individual data over that experiment, you can basically just collect all these together and just mix and match them into a single bag, okay. So, once you put all that together, you get this model, you get W's of this model. Once you get W's of this model, then you can do the standard thing.

(Refer Slide Time: 10:51)



Then this now is the forward model. How is this the forward model? Now notice, compare this with the actual analytical forward model which was $\hat{T} = a(1 - e^{-bt})$. Here notice whatever role a and b were playing is the same role that h and Q will be playing in our forward model. These are now known. After you have fit, so w is found based on prior experiments.

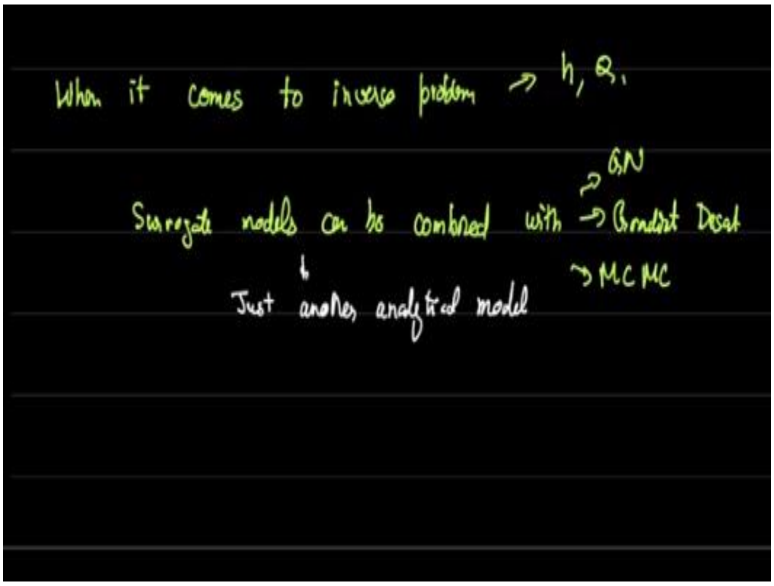
By the time it comes to solve the inverse problem, which is for a new given set of data, the only unknowns are h and Q. So, the way this works is this. You guess a value of h and Q. Based on prior data W has been fixed to be optimal, it predicts a T. This obviously will not match the new experiment. So, you predict a $\hat{T}$. This will not match the new experiment; go back direct h and Q and you keep on doing this.

So, notice the differences between these three approaches. In the first approach, you really have no new parameter to discover. You just know that these two were the unknown parameters or the free parameters left within the analytical model. You have only as many parameters as a unknown physically at that specific inverse problem, okay.

In PINN, once again you have these two parameters, but theta is expressed as a neural network. So neural network has some whole bunch of parameters W. Those W as well as λ_1 and λ_2 are solved simultaneously during solving the inverse problem. Finally, when it comes to the surrogate model, you first solve for W separately, when based on past data.

And then when you want these new parameters, let us call them λ_1, λ_2 or h and Q or whatever we want to call them, whatever new pair or a and b, the new parameters that you wish to solve a and b are solved during the inverse problems solution just like the conventional Gauss-Newton algorithm technique.

(Refer Slide Time: 13:28)



Now finally, I want to point out that surrogate models can be combined with all the models we found earlier. So, they can be combined with Gauss-Newton, they can be combined with gradient descent. And they can be combined with Markov Chain Monte Carlo or MH MCMC because basically at the end of the day, the surrogate model is just another analytical model.

In one sense, all three of the approaches that we have discussed, whether it is the conventional analytical solution, whether it is physics informed neural networks, or whether it is normal neural networks with surrogate models, in all three cases, we are assuming some analytical form, okay. How we apply it depends on our specific cleverness.

In the case of neural networks with our surrogate models or PINNs, you always have to still determine the parameters W. So that is how you tend to do surrogate models. I apologize for not showing the code example here, but it became a little bit too complicated to demonstrate in this online class. So hopefully, you can try some surrogate models yourself in your specific application.

And I will end the course as far as techniques are concerned here. In the next video I will simply give you a short summary of the techniques that we discussed and what techniques we did not discuss. So, I hope to see you in the next video. Thank you.