Inverse Methods in Heat Transfer
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Lecture - 43
Why Machine Learning in Inverse Heat Transfer?

Welcome back this is week eight of inverse methods in heat transfer. Starting this week, we are going to discuss machine learning methods for generally for inverse methods in heat transfer, but I will also have about three four weeks so I think about weight eight week eight to week eleven will be just machine learning. I will probably teach a little bit more machine learning than you need for simply for inverse methods and heat transfer, but the reason for that as I will show by the end of this video, is that machine learning itself is a special case of inverse methods. So, there are actually two connections that we have here one is that machine learning is a type of inverse method or at least a certain set of machine learning methods are types of can be seen as a subset of inverse methods also.

Machine learning can be used for inverse methods in heat transfer okay. so, we will see both these connections a little bit today and this will become solid by the time we finish week eleven so let us start

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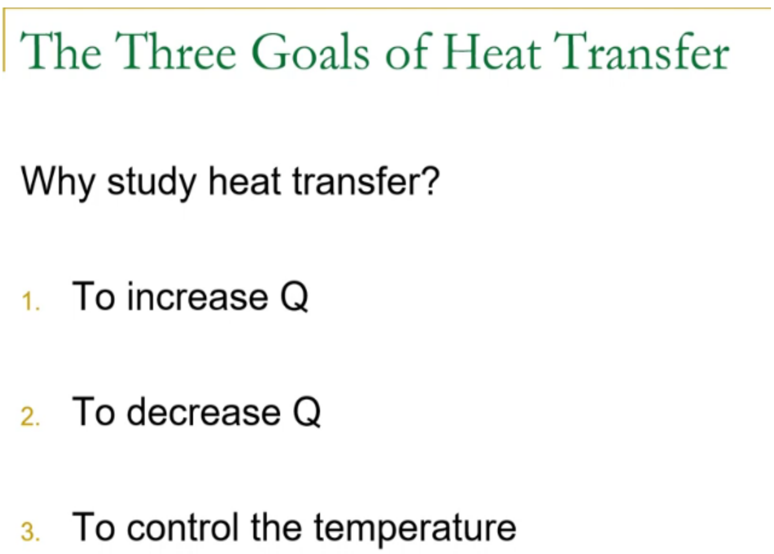

There are in general since we are looking at heat transfer why do we study heat transfer so typically we study heat transfer in practice so if you are looking at an engineering problem in order to increase the heat transfer in a specific problem let us say you want to increase you know the heat generation in a specific room or you want to heat specific process a little bit more or you want to decrease the amount of heat transfer you do not want heat loss okay you want to preserve the heat in a particular place or specifically you wish to control the temperature in some sense it comes as a subset of the other two.

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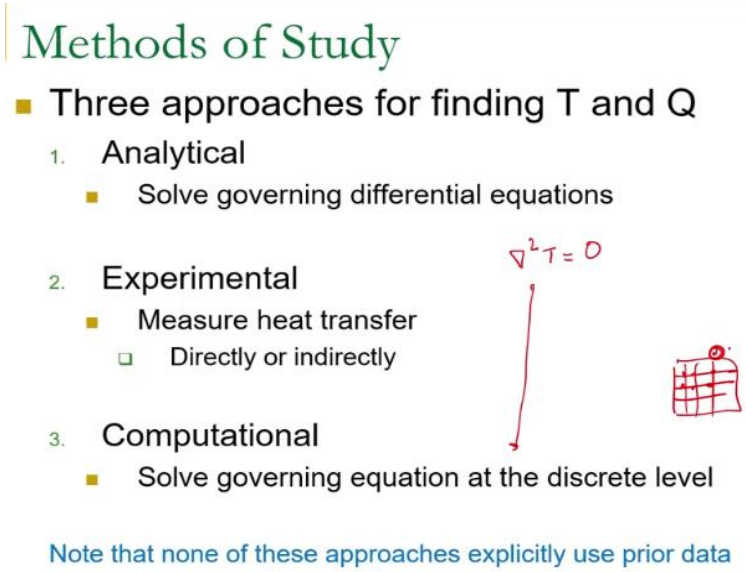

Now when we try to study heat transfer typically, we have three approaches that we use, one is of course you take the governing equations and you solve the differential equations or you take two experiments you measure the heat transfer in a specific situation directly or indirectly. what is meant by indirectly is you measure the temperature and from there you infer the heat transfer or finally computationally you take the governing equation so you could have let us say in 2D del square T equal to zero in case you have a steady state conduction problem with the constant with a steady state conduction problem with constant conductivity.

In such a case you can solve this equation computationally at the discrete level so you can solve it and look for temperatures at specific points and then infer the heat transfer at the boundary. Now all these three approaches explicitly do not include prior data. As we saw in the last week during the probability week prior data has a strong effect on how we should evaluate post it so that is one angle that we can take in fact machine learning though I will not do it in the next three four weeks also strongly tends to use probability theory in fact some of those methods are exactly the same as what we did last week also, metropolis hastings general monte Carlo etc. but none of these approaches talk about prior data

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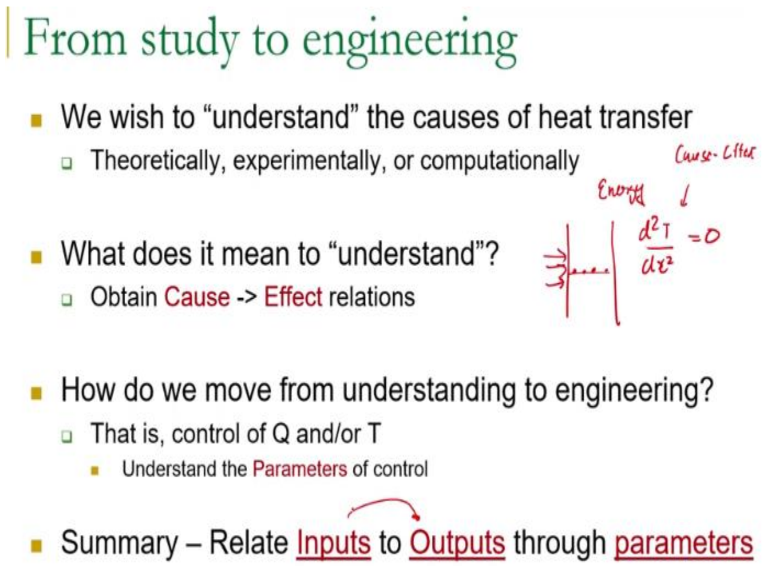

So simple study to engineering when we talk about a simple study, we typically say I want to understand the causes of heat transfer theoretically, experimentally or computationally. what is happening in this specific situation in a pipe etc. but what does it mean to understand what we want really when we say we wish to understand is to obtain a cause effect relationship okay. so now this will trigger memories of the first week of this course going from cause to effect or going from effect to cause. so how do we move typically simply from understanding, once we understand the cause effect relationship, so for example, I could say something like here is a slab the cause is the heat flux that was given to it and the effect is the temperature that I see here and this effect comes from the fact that energy is conserved and we can encapsulate our cause effect relationship as a differential equation.

But engineering is a little bit more subtle we are not satisfied with just this. I simply do not just want to predict what happens here, but I wish to control it as we saw earlier, we want to control q and temperature. Now what you need to do in order to control is to understand the parameters that control it okay. so overall when you look at engineering you want in some sense to parameterize the input output relationship through some parameter. so, this will seem like it is a little bit obvious but as you come through the machine learning chapters or as we have seen through even the inverse portions so far You will know that parameters are not as straightforward as they appear.

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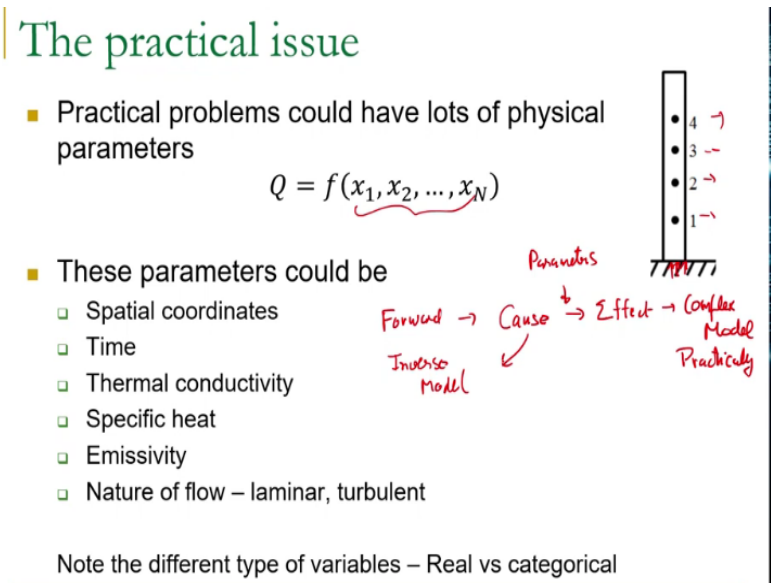

The practical issue is this physical problems have lots of physical parameters so if you want heat transfer heat transfer could depend on a whole bunch of parameters let us just call them x1 through xn as we have been doing so far we have been calling them w1 through wn let us call them x1 through xn for now so these parameters could be spatial coordinates for example I have this fin heat transfer problem and I wish to know what is the heat transfer at the base it could depend on number of things, it could depend on the temperature measurement here it could depend on what time it is, in case it is an unsteady problem it could depend on the thermal conductivity, it could depend on the specific heat of the material emissivity in case there is radiation, whether it is laminar flow, whether it is turbulent flow around it all sorts of things.

So, there are a whole bunch of variables that are setting here like I said x1 through xn every single thing is actually a parameter of the problem now when we map so remember we have two things. we have the forward model and we have the inverse model. The forward model says cause to effect but this cause to effect depends on a number of parameters. so, you actually have in this case a fairly complex model okay it is not a straightforward simple model in most engineering problems that is the practical issue okay so this is a complex model practically.

Now the inverse model can be affected only if you have the forward model. Really, I mean without the forward model what are you going to do you cannot do in any of the inverse techniques that we looked so far.

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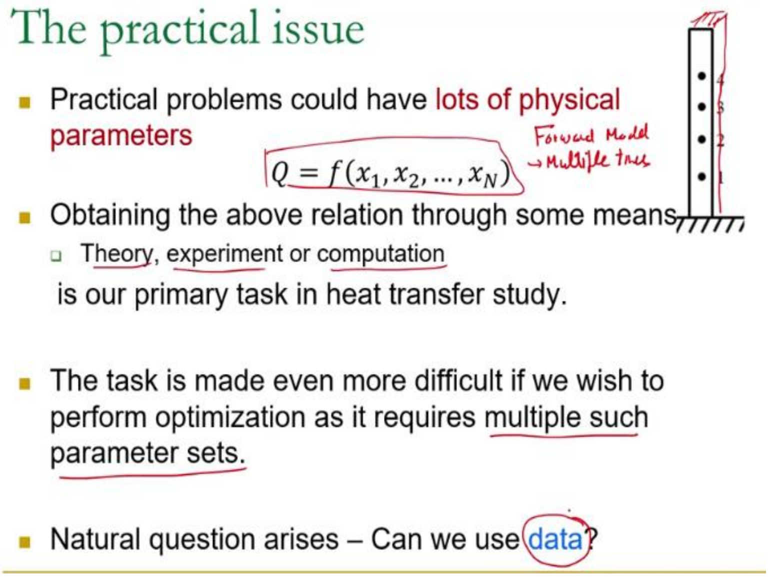

Now what we need to do is to try to obtain this relationship, this forward model in some way. Typically, in a usual heat transfer course in an undergraduate or a graduate heat transfer course we are building these models either through theory or through experiment or through computation okay. Now this becomes even more difficult when in practice you want to do optimization okay what is meant by optimization can you find the best cross section of the fin okay so we have assumed a rectangular fin here, but can you tell me what sort of Cross section of the fin will maximize heat transfer. That kind of problem requires you to build this forward model multiple times. why is that?

When you do one forward propagation? so typically when people do engineering optimizations, which is another type of inverse problem incidentally okay. so, it is a type of inverse problem because what you are saying is, I know the performance of the fin, I want so much heat transfer what is the shape. so, the forward problem will be given the shape find the heat transfer, the inverse problem will be given the heat transfer find the shape so when you are trying to find the shape, you have to do a lot of forward propagations remember our gauss newton algorithm, so each time you take a shape choice you make a forward propagation okay

So, once you do a forward model this becomes very difficult, because it requires multiple such parameter sets. you cannot just fix the shape also; you have to keep on giving multiple shape parameters and moving through this this is what makes practical inverse problems or practical engineering difficult okay. so, the question here is instead of simply using only theory computation or prior experiments or just experiments that you are doing right now can you use data? why do we need to use data?

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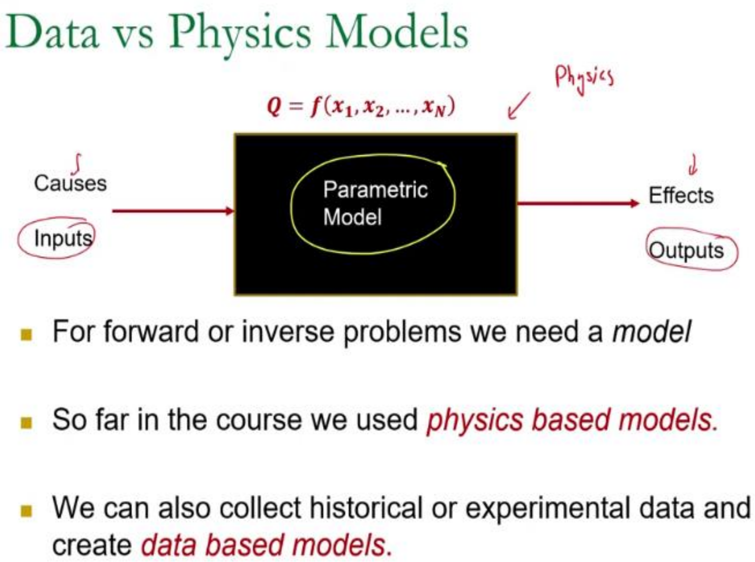

You have two types of models so as we have discussed you have causes which we are going to call inputs and you have effects which we are going to call outputs. In the middle is some parametric model okay so this parametric model which sits here could have come from anywhere. Now typically in all the cases that, we did so far especially since we repeated the slab case multiple times this model came from physics okay, we said that energy is conserved and we basically made we built this model so we have used physics-based models. But this physics-based model can be replaced or augmented by data-based models.

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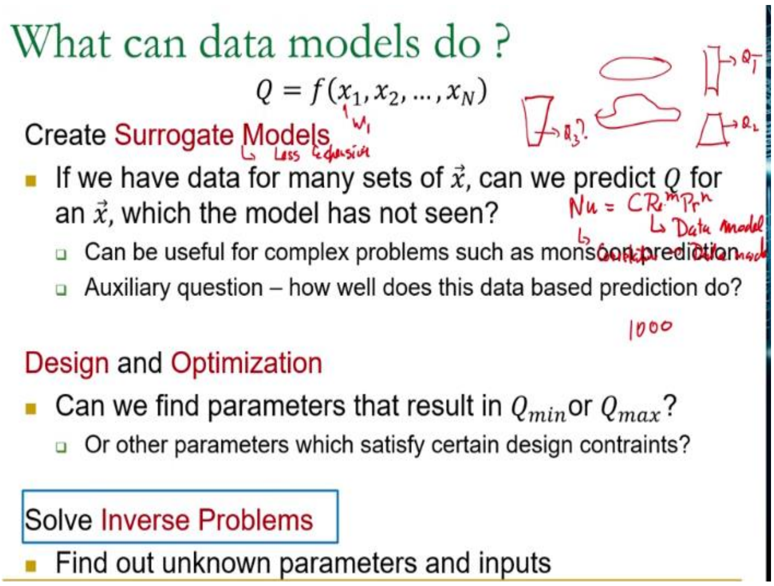

So what can a data-based model do so one big term that you will see typically in the industry is this thing called a surrogate model. so if you have data for many sets of these parameters imagine this x is w, can you predict q for a parameter which you have not seen? so a practical example is suppose I know for multiple shapes, heat transfer for multiple types of shapes or let us take fins, this shaped fin a shaped fin like this, if I give you multiple shapes, can you now give me a new if I give you a new shape something like this can you predict the heat transfer without actually going through physics.

If I tell you shape one gives me heat transfer one, shape two gives me heat transfer two, if I give you shape three can you give me heat transfer 3. This case is known as a surrogate model in that we try to build it these based on prior computations and prior experiments but we based kind of throughout the physics completely out of it for example you would have seen a model such for example Nusselt number is some constant times Reynolds number power m Prandtl power number power n this can usually be a full surrogate model or a full data model.

This is a simple example of that, because really speaking you cannot derive this directly out of physics. If you have seen this dittus Boelter and all these other types of heat transfer correlations so Nusselt number correlations are a simple example of a data model. so, correlations are a simple example of a data model I do not want to call it data-based model because it reminds people of databases which is another topic entirely in computer science. so, when you want to predict monsoons when you want to predict weather such kind of models in fact, we make data models based on ods some mix of physics and some mix of data so design and optimization once again can you use these parameters that will result in finding out the minimum heat transfer curve configuration or maximum heat transfer curve configuration.

So, all these are problems where people use surrogate models. It is used quite often in the industry. for example, you want to let us say find out you are designing an automobile and you want to design let us say the HVAC the ac system within the automobile or you want to design the shape of the automobile itself, what you could do is, you do a few simulations let us say you do a thousand simulations and each simulation is expensive and based on that you find a correlation between some shape parameters and the amount of drag or the amount of heat transfer that is coming here and then you give a new simulation entirely and this model is less expensive.

So typically, surrogate models are used, because they are less expensive. we will see this towards week ten or week eleven of this course. we will see some simple examples again we are going to do just classroom examples, but you will see through the examples that this is generalizable to any case. Now we are interested primarily in solving inverse problems so data models can be used as surrogate models or they can be used to solve inverse problems that is to infer parameters also.

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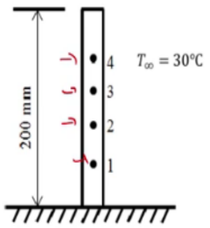
Back to our example



S.N	x (mm)	T_{exp}
1	25	90.90
2	50	84.47
3	75	78.13
4	125	65.35

$\frac{hP}{kA}$

Inverse Problem : Find the fin parameter m

- Physics based forward model: Use $\theta = \theta_b e^{-mx}$ Forward model Infinite Fin
  - Then, use some regression technique
- However, what if we did not know what model to use?

So, let us go back to our old example okay so the old example that I showed just now or you can think of the slab example also. but this is a little bit neater because it is a nonlinear solution. so, suppose somebody made these measurements just like in the slab case, we put a fin and we made these four measurements of experimental temperatures at these four thermocouples.

Now for some reason either you do not know the physics of the problem we are we are going to pretend as if we do not know the physics of the problem we do not know the fin equation and we want to find out the fin parameter m. so you might recall what m is this hp by k a, I had derived this in the first week so suppose I want to find out this parameter and the only four pieces of information I have are this then how do I go about doing that now the forward model based on physics assuming an infinite fin or sorry infinite fin is this this is the forward model. but suppose as could have as could happen with multiple of you do not even remember that this is the forward model or you do not know how to derive it or you do not even know the physics of the problem which would happen in the case of like it is an entirely new problem within an industry.

You do have some data but you do not know the physics, so in this case if you know the forward model we can use some regression technique, you can use a linear regression technique by taking log on both sides or you can simply use the gauss newton or Levenberg Marquardt technique by not even taking the log and just continuing with it now what if you do not know what model to use?

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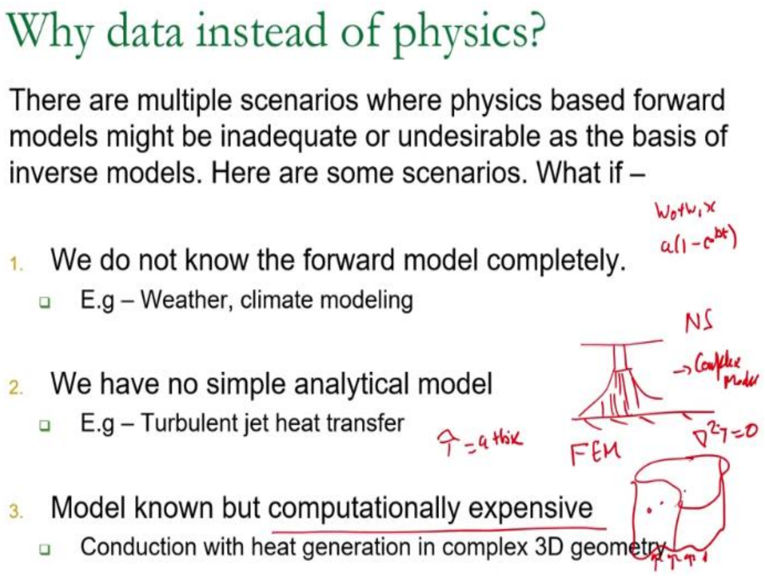

So, there are multiple such scenarios okay we do not know the forward model completely so for example weather we know some of the model we know what the fluid dynamics is, we know what the atmospheric dynamics is, but we do not know all the parameters or we have no simple analytical model.

So, if you have something like a turbulent jet heat transfer, so which is a case like you have a nozzle here, you have some plate here, so for example you can imagine some in the food industry somebody is just heating food through a jet of this sort. when you do that, you have no simple analytical model here you have to solve the full Navier stokes equations and this is an expensive proposition, unlike our simple things like w zero plus w one x or a into one minus e power b t, all these are simple models. this is a really complex model. we will come to more details of this in the final week of this course.

But in general, you have no simple analytical model means you have to rely on a simpler model suppose you do know the model. for example, I am doing something like heat conduction in a somewhat complicated shaped body. so, I am doing heat conduction in this body or something like a laptop you have heating of the chip something of that sort. In such a case we know the equation let us say we know we know the steady state equation here, but it is computationally very expensive it is computationally expensive because you will have to put a fem model or cfd model or a finite volume model here and compute this and for each computation each time you say, if I if the conductivity is such and such then the temperature will be or if I am adding heat to it from one direction just like the slab, each time I want to do the simulation again, I do not have a simple formula like t equal to a plus b x.

It is a very complicated formula and the only way you can calculate it is by doing a full computation. in all these cases we can use prior data that is we can do some simulations some ten thousand simulations or thousand simulations say and collect the data and make what is known as a surrogate model.

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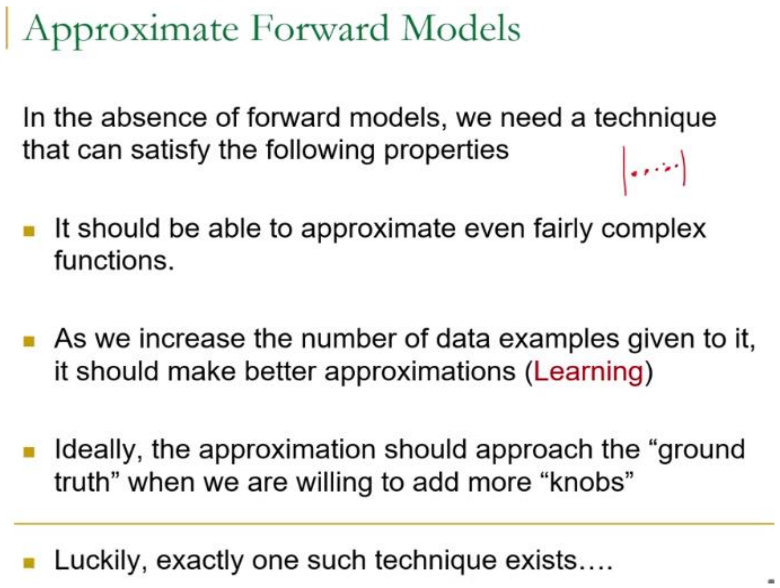

So, what we have in such cases are approximate forward models. so, in such cases we should be able to approximate even fairly complex functions, this is our desirable. as we increase the number of examples it should be able to make better example approximations. This is a key thing which leads to machine learning.

This is somewhat similar to what happened in the slab if I give you two points you will make one approximation if I give more and more points, I will make better and better approximations. but do note this that as we add more and more data, we should get better and better.so ideally you should approach the exact values as you add more and more parameters.

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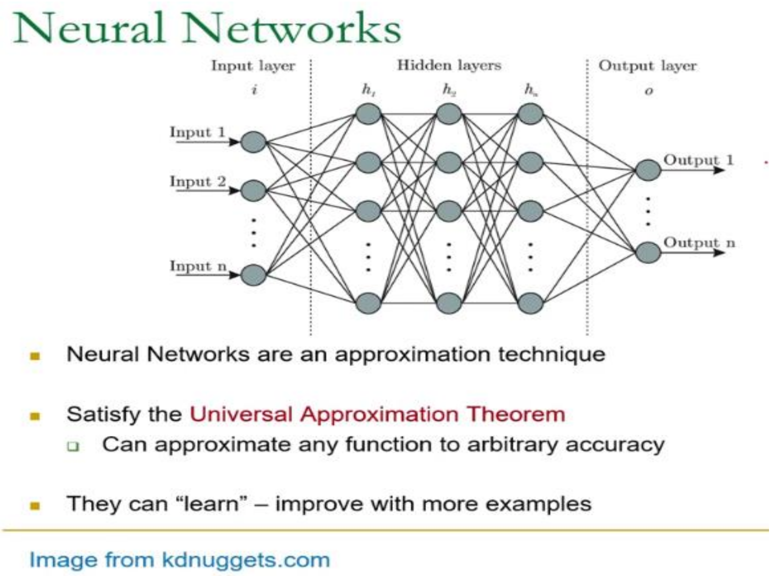

So, one such technique that does this is what is known as neural networks. so neural networks effectively which we will use over the next two three weeks. I will introduce you to neural networks really speaking in the next week. this week is just sort of a soft introduction to machine learning. these are an approximation technique, they satisfy something called the universal approximation theorem, which says that a neural network can approximate any function like really almost any engineering function not almost any engineering function can be approximated to arbitrary accuracy

And more importantly they can learn that is the more the number of examples you give the more the number of simulations more the number of experiments you give it will generally improve okay it will improve the prediction.

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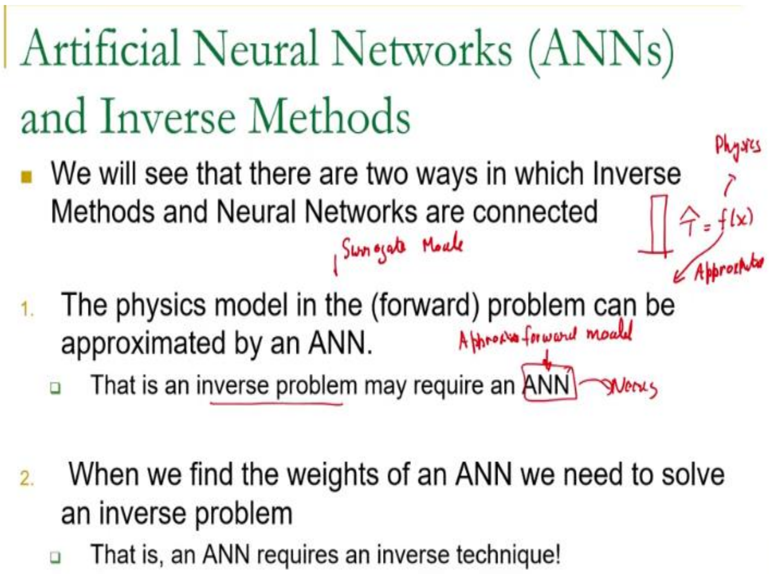

We will see that there are two ways in which inverse methods that we are going to do and neural networks are connected. The first is that the physics problem or the surrogate model, this is what is called a surrogate model. so, when we have such some things such as a fin or a more complicated problem the relationship, we want which is we want T-hat let us say as a function of x.

This function can be approximated by a neural network so this function had an exact value from physics and what we are going to do is, suppose I do not know this function f, I am going to replace it by another function let us say f tilde, that is going to be a neural network.

That is the first use of how a neural network works okay so another way to say it is to solve an inverse problem the forward model requires an ANN okay so the forward model is the ANN and this if it looks abstract this will become a little bit clearer when we actually apply it in the next couple of weeks however there is another connection the deeper connection that the ANN itself requires the solution of an inverse problem.

I am solving an inverse problem; the inverse problem needs a model and that model itself needs another inverse problem. so that is the strange thing, so which we will see this week itself by the time we end this week you will understand why this kind of looping or fractal nature of an inverse problem comes up. you wanted to solve an inverse problem needed a forward model, that forward model is an approximate forward model and that approximate forward model turns out to be a neural network, which in turn requires you to solve an inverse problem

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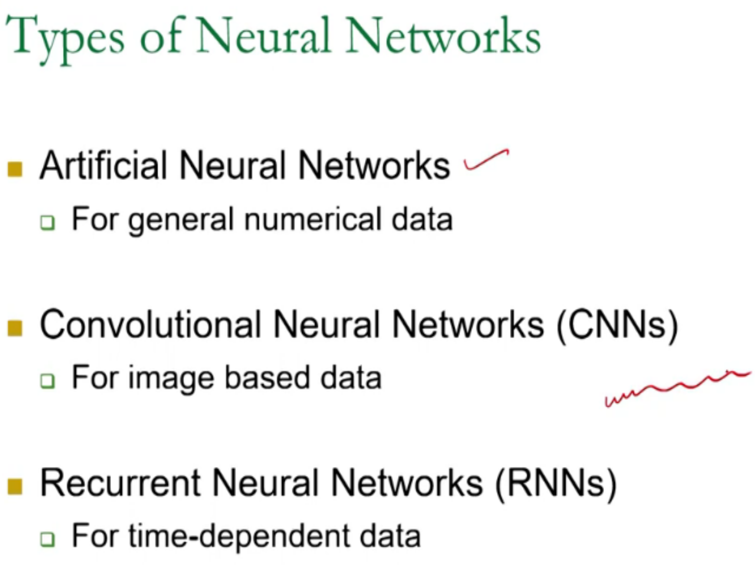

So as it turns out an ANN or a neural network. Artificial neural network also requires an inverse technique. as it turns out we have multiple types of neural networks we will be concentrating primarily on these artificial neural networks within these three weeks, because it is a short course. I think within the syllabus I had included a few other types of neural networks too, but we would not have the time to do that so general numerical data of the type that we have been using this is good.

If time permits, I will briefly introduce you to CNNs in the final week in advanced techniques for inverse problems now sometimes the experimental data is not a number but it is an image. so, you have some thermography whichever method that you have for images, in case it is an image-based data then you can use something called CNNs convolutional neural networks and if it is time dependent data its data that comes over time okay sort of like what we did with the unsteady case but a little bit more complex. so let us say you have weather data over a long period in such cases its recommended that you use something called recurrent neural networks.

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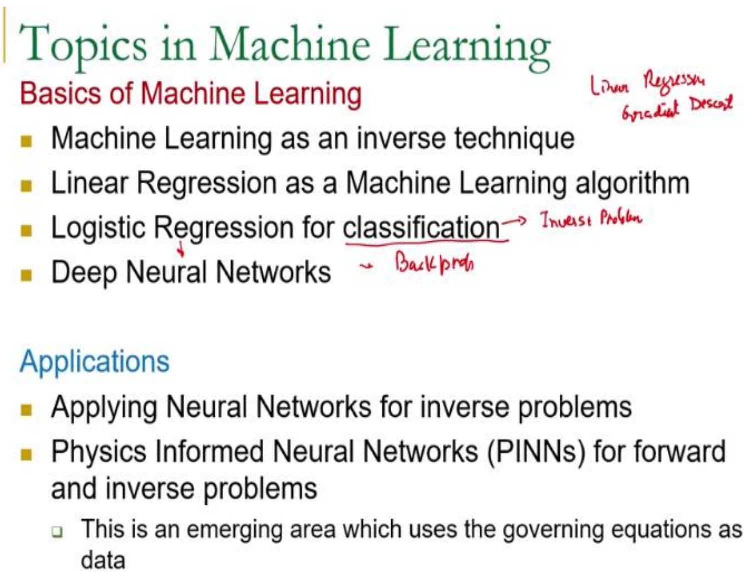

So, the topics that we will be covering in machine learning within these three four weeks are the first couple of weeks, we will be looking at the basics of machine learning. this week I will just talk about why machine learning is primarily an inverse technique and I will also show you I will go back to linear regression which we did with gradient descent and I will talk about a couple of details there that make it effectively a machine learning technique. then I will come in the next week we will do classification which is not directly related to let us say inverse problems in Heat transfer.

Even though I will give you a couple of motivations it is kind of a constructed example you know truly speaking it is not that common but however logistic regression kind of naturally leads to deep neural networks and as I said classification by itself is an inverse problem. so, within the broad idea of looking at inverse techniques not necessarily only in heat transfer, it is important to look at logistic aggression so we will do that. so, the third week I will come to deep neural networks and we will discuss this algorithm called the back propagation, which is what makes it possible for la deep networks and large networks and in the fourth week we will talk about applications. we will apply neural networks for inverse problems some of the problems that We did earlier pretending as if we did not know the physics of the problem.

Of course, this helps us in applying it to very general problems also finally we will come to physics informed neural networks they are an emerging technique very powerful technique in fact especially for inverse problems. so, this is the flow of topics that we will be looking at in machine learning over the next few weeks. so, I hope you find these set of topics also useful, typically these are not taught within a general inverse course, but we decided to sort of cut down on some advanced techniques such as the adjoint technique etc. within inverse heat transfer and come to machine learning because this is an emerging area. so, I hope you enjoy the and learn a little bit from the next few videos thank you.