**Inverse Methods in Heat Transfer**
**Prof. Balaji Srinivasan**
**Department of Mechanical Engineering**
**Indian Institute of Technology, Madras**

**Lecture No # 04**
**Module No # 01**
**Review of Basic Heat Transfer for this Course**

Welcome once again to the inverse methods and heat transfer course offered on NPTEL this is week 1. What we are going to do in this short video is to review some basic heat transfer from undergraduate heat transfer. It is not going to be very long. Obviously, that undergraduate course is a full semester-long course. So, I cannot review that in a few minutes or even a few hours strictly speaking.

But a little bit of knowledge just to jog your memory, will also have some exercise problems within the week again to sort of refresh your memory about basic heat transfer. As I showed you in the last video you require the forward model in order to solve any inverse problem.

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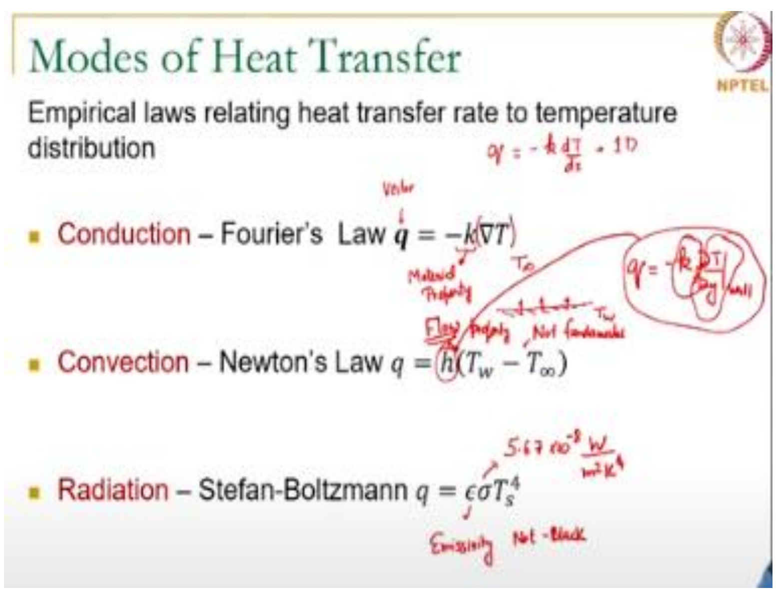


Recall that we have three modes of heat transfer with their corresponding usually empirical laws which relate heat transfer to the temperature distribution. So, the first one that all of you would have studied is Fourier's law of conduction or Fourier’s law which relates, so remember that all these rate laws relate the heat transfer rate to the temperature. So, only with the temperature gradient do you get conduction.

So, what Fourier law tells you is q, notice here this is in bold q is a vector as you know the gradient of T also is a vector. You would have usually seen it in the form,

$$q = -k \frac{dT}{dx}$$

This is the one-dimensional version. That is because heat transfer is a vector quantity, it has a direction as well as a magnitude. So, the direction in which heat flows is also, it is found out by the direction of the gradient, of which is which happens to be if you remember your vector calculus, the change the direction in which temperature changes the most rapidly.

So, conduction is determined by Fourier's law, this is an empirical law. Convection is not even an empirical law, it is sort of a forcing to this kind of form newton did Newton's law of cooling, q is in one dimension in some sense,

$$q = h(T_w - T_\infty)$$

If you have a plate the wall gets kept at some temperature. And let us say this is a hot wall and that is a colder fluid, the cooling that will take place, the heat transfer that will take place from the wall due to convection is typically written as $h(T_w - T_\infty)$.

Now there is a difference between k and h, k the conductivity is actually a thermo-physical property of the material. h on the other hand is not a property of the fluid or indeed of the solid. It is actually a property of the flow. please notices this you would know this again from heat transfer but I am re-emphasizing this. So, it depends not only on what the material is but on the speed with which water or air or whatever is flowing there. So, it depends on multiple conditions, on Reynold's number, on the parental number etcetera.

So, it is not just a property of the material, but it is also a property of the flow characteristics. That is because this is not a fundamental definition. $q = h(T_w - T_\infty)$ is not a fundamental definition at all. The actual definition of $q = -k_f \frac{\partial T}{\partial y}|_{wall}$, calculated at the wall. Now actually it is this $k_f$

of the fluid $\frac{\partial T}{\partial y}|_{wall}$, at the wall it is from here that we can actually calculate h. So, all the values of h that you see within textbooks are actually calculated knowing this,

$$q = -k_f \frac{\partial T}{\partial y}|_{wall}$$

So, you actually have to solve for the temperature of the fluid and then actually find out $\frac{\partial T}{\partial y}$ the gradient of the temperature at the solid or at the wall and that is really how you calculate h. So, all this is to say if you solve an inverse problem you have to be careful about what the known parameter is and what the unknown parameter is here. So this $k_f$ is a property of the fluid but $\frac{\partial T}{\partial y}$ obviously is not a property just of the fluid but of the entire flow and all those put together determine h.

So, you cannot just say h of water is something, you have to say h of water for a specific flow is a certain value. Finally, we have radiation. Radiation, of course, obeys Stefan Boltzmann's law of which is for a black body, I have also added an emissivity. This is in case the body is not black. So, unlike conduction and convection, radiation will work basically without a material itself. So, the second thing of course is σ is Stefan Boltzmann's constant, it is $5.67 \times\ 10^{-8}\ \frac{W}{m^2K^4}$.

All this again will be very familiar to you from your heat transfer classes this is just to recap or review what we are doing.

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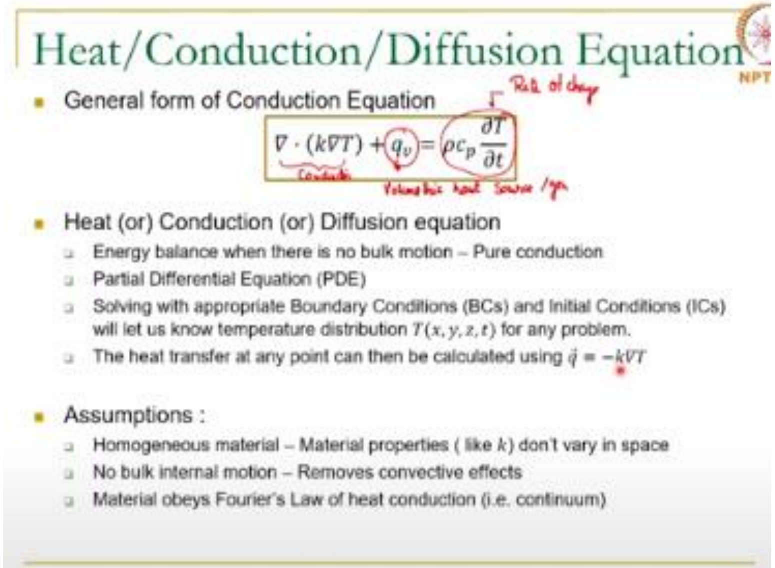


Once we know the modes of heat transfer, I want to look at 1 equation we will be using again and again which is sometimes called the conduction equation, called the diffusion equation, is called the heat equation, it has multiple names. But this is for the simple case, where you have like a quiescent fluid and there is just conduction here. So, the general form of the conduction equation looks like this.

So, this of course I have written in what is known as the coordinate-free form it is just the usual calculus form of this equation. So, the right-hand side measures the rate of change of enthalpy, but notice that rate of change in time. So, when a fluid is being heated there is some net effect that happens here. That might happen due to a couple of reasons. One of the reasons you might have a volumetric, v stands for volumetric here.

Volumetric heat source or heat generation. you have various names and this of course is the conduction term actual conduction inside the $k\nabla T$ comes due to Fourier's law. All these derivations you would have again seen in your undergraduate studies. But the important thing to remember is the net change in temperature happens due to 2 sources, either you have an internal source of heat generation or you have some conduction happening within the fluid. There is no convection here because we are assuming it is quiescent fluid.

So, as was written here, where does this equation come from? It comes from a simple energy balance. So, this comes from energy balance, there is no bulk motion, this is pure conduction, and this is a partial differential equation as you can see $\partial T$ with respect to time. How do you actually obtain temperatures? You obtain temperatures by solving using the appropriate boundary conditions and initial conditions. I will show you some common ones shortly within this video and you can obtain the temperature.

Once you know the temperature from Fourier's law, you can find out the heat transfer at whichever point you want. So, these are the 2 preeminent things that we want to solve for in heat transfer anyway, we want to solve for the temperature and we want to solve for the heat transfer. The assumptions that are made in this equation are that it is a homogeneous material and that k is constant throughout the material.

There is no bulk internal motion, that is there are no convective effects and the material obeys Fourier's law of heat conduction, which is typically true in case you have reasonably dense media and basically you have a continuum hypothesis is true.

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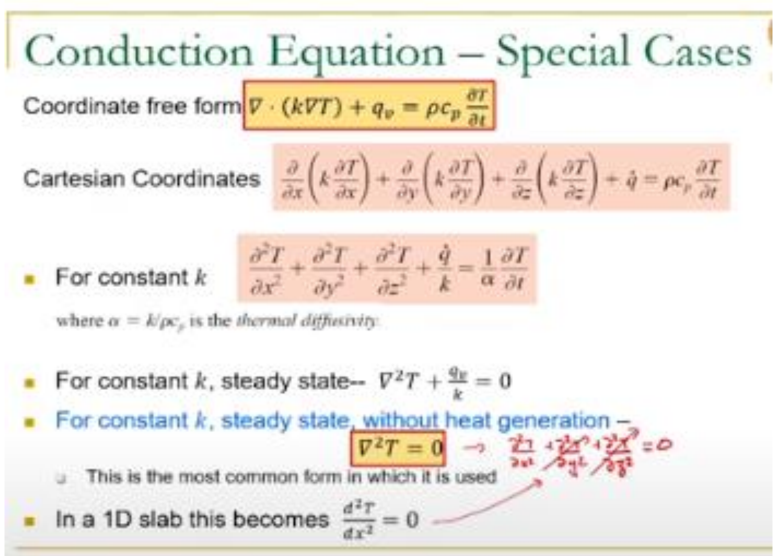


Let us look at now some special cases of this conduction equation, which is what we will be using, in fact, we practically always be just using the one-dimensional case. So as written above this is the general form of the equation in Cartesian coordinates in 3d. you can simply write this as

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = PC_p\frac{\partial T}{\partial t}$$

k still here because k could be a function of temperature and temperature itself is a function of x. So, that is a possibility that can happen often.

And for constant k you can just take out k from the equation and you will get this $\frac{\dot{q}}{k}$ and you will get a $\frac{1}{\alpha}$ on the right-hand side where alpha is the thermal diffusivity of the material. And if we have a steady state then this term here, the right-hand side term goes to 0. So, notice this term goes to 0 and k is constant so it comes to the bottom, so you get,

$$\nabla^2 T + \frac{q_v}{k} = 0$$

For constant k and steady state without heat generation so you can see that I am slowly relaxing assumptions as I go down from the top. First the most general form then you assume k is constant, then you assume k is constant and there is a steady state, then you assume k is constant, steady state and remove the heat generation, then you simply get $\nabla^2 T = 0$, this is a Laplace equation and this is the most common form in which you will usually form find the heat conduction equation.

Now even more commonly like we usually do in a slab, you really just have a very simple simplified form this is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

And since it is 1d, these 2 are not going to be there. so, you use T is simply a function of x. So, you get $\frac{d^2 T}{dx^2}$. This is usually the form in which you would have seen it.

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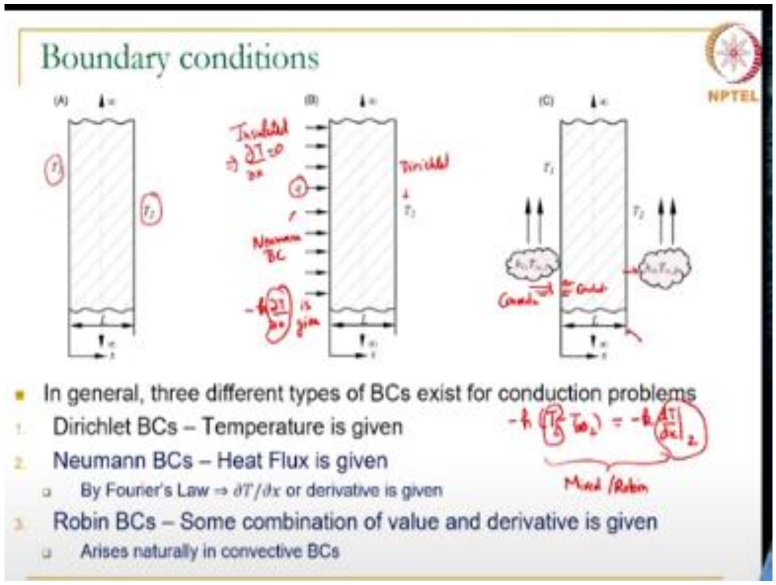


Now let us say we take a 1d case or in a general case also, usually we have to apply boundary conditions. So typically, there are 3 different types of boundary conditions that are applied for conduction problems. Usually, almost every problem will have either Dirichlet where the temperature is given. So, for example like we did with the direct problem earlier on this week. so you could have $T_1$ is provided and $T_2$ is provided and you basically have to solve for heat transfer.

In this case or you want to determine the temperature profile within this case. This is what is known as Dirichlet boundary conditions because temperatures are given. You have Neumann boundary conditions. Neumann boundary condition means that a heat flux is given. for example, this side here temperature is given so this side is Dirichlet. But on the other side, let us say some heater is given and you know that it is actually providing constant heat flux.

So, this side is a Neumann boundary condition because what is given is not T but what is given is $-k\,\frac{\partial T}{\partial x}$ is given, which is equivalent to providing us with $\frac{\partial T}{\partial x}$, because presumably we already know the conductivity of the material. So, therefore this is a Neumann condition a special case of this is, if something is insulated then what we would have is $\frac{\partial T}{\partial x} = 0,$ because it corresponds to the q = 0 case.

So, therefore this is a Neumann condition a special case of this is, if something is insulated then what we would have is $\frac{\partial T}{\partial x} = 0,$ because it corresponds to the q = 0 case. So, therefore it is the same as saying $\frac{\partial T}{\partial x}$ is 0. So, you can apply the Neumann boundary condition with 0 on the right-hand side and that will give you the insulated condition. The third which in some sense probably is in some sense most practical are Robin boundary conditions they are called robin. Dirichlet, Neumann, and Robin are the names of some scientists. So, what is given in such a case is a combination of the value as well as the derivative.

So, you do not have T and you do not necessarily have $\frac{\partial T}{\partial x}$, you have a combination of the 2 this naturally arises when you have convective boundary conditions. How does it arise? You will say that convection from this side would be equal to conduction from this side. so, you will have something like,

$$-h\left(T_2 - T_{\infty_2}\right) = -k\,\frac{dT}{dx}\,\Big|_2$$

So, you notice $T_2$ is participating $\frac{dT}{dx}$ at $x_2$ is also participating so this is what is known as a mixed or a Robin boundary condition. So, these are the possible boundary conditions, that you have in general a simple conduction case. So, what we saw in this video this far was that we just looked at quick conduction convection and radiation, we looked at the heat conduction equation which is the most common equation.

We will be using it throughout the rest of the course and we also looked at some of the popular boundary conditions we can use for such problems. In the next video, I will actually use some of the material that we saw in this video and go ahead and show you a few forward problems that will be using as example problems for the rest of the course.

Welcome back to the inverse methods in heat transfer course on NPTEL. So in this video, I would like to talk about some forward problems which are common in heat transfer or actually in the course that we are going to do. There are of course an infinite number of forward problems in heat transfer. But I want to use a few examples just to give them as corresponding inverse problems when we go through the rest of the course.

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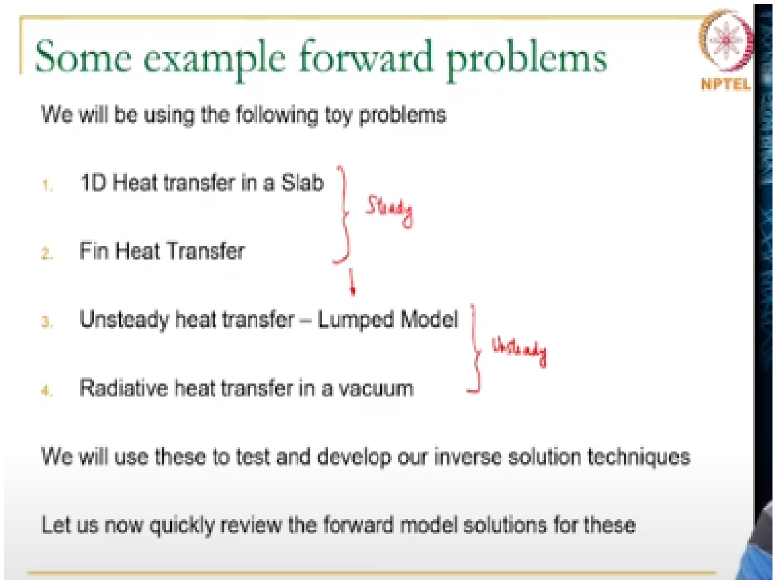


So, some possible examples forward problems that we could use. So, some toy problems will be used. so, one is the very first one that I showed you is the 1d heat transfer in a slab case. A very common case you can actually build your intuition of the simple case really well because it has multiple characteristics that are required for inverse problems. The second is a case of a fin and heat transfer in a fin, we will use that as well.

The first 2 are basically steady-state cases, and the next is the general unsteady heat transfer a lumped model case. We will also look at later on in the course, a lumped model with heat generation within it. And then we are also going to look at a very simple case it is radiative heat transfer in a vacuum, again we are going to do answer the unsteady case. Now as we saw earlier on we saw that when we looked at any inverse problem you have to solve multiple forward problems and you need a forward model in order to be able to solve an inverse problem.

This forward model and we are going to use these 4 forward models and a few more through the rest of the curves but these 4, I will repeat. So, this is just so that we have some simple case, that we can go back to and compare the various ideas that we are looking at. So, we will use this to test and develop our inverse solution techniques. So for example we may try some techniques for the steady state cases and when we try to apply them to the unsteady case, then suddenly it will stop working.

So, then we know that we have to develop our technique a little bit further and we have to do something more in order to make this thing work. So, what I want to do in the rest of this video is to review the forward model solutions for these. Again, these are things that you should already be familiar with please feel free to skip through this. You might also see some exercise problems related to this and a few similar problems within the homework assignments for this week.

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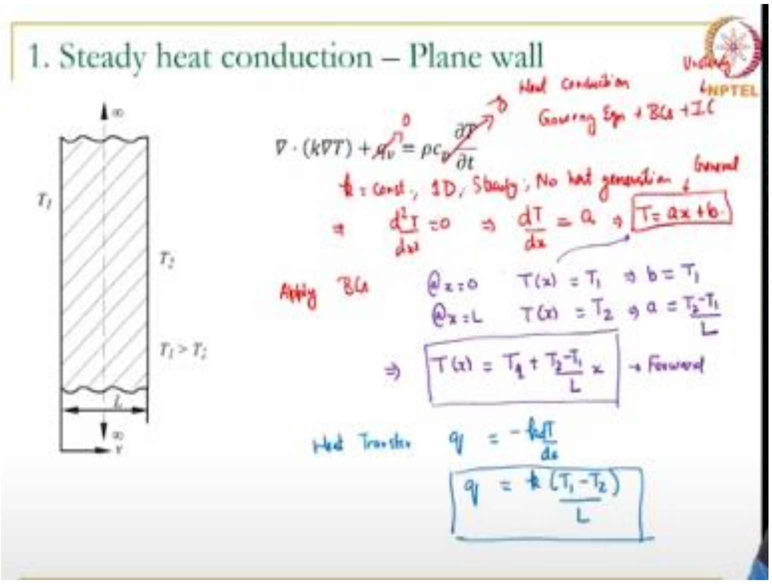


The first problem we will be looking at is steady-state heat conduction in a plane wall. so, this is the first direct problem that we looked at also. So, if we start with the biggest possible equation or the most general equation. This of course was the heat conduction equation. Please remember that solving any problem of this sort or developing a forward model involves just the governing equation plus the boundary conditions plus initial conditions in case it is unsteady.

And solving for that by applying the boundary conditions applying the initial conditions and getting the full solution is the general process again you will be familiar with this not only from heat transfer but other courses also. So, if we take this case and we simplify this further, we assume k is constant, we assume we are dealing with a 1d or a quasi 1d case, so such as a slab and this dimension is really high in comparison to the width of the slab sorry depth of the slab.

And this is steady. so, because it is steady this term disappears, because k is constant you can take it out we will assume no heat generation. We will also do some examples of heat generation later on in the course. But let us, for now, assume that this is 0 as well and then because it is 1 d we finally get $\frac{d^2T}{dx^2} = 0$. So, of course, if you integrate it once you get $\frac{dT}{dx} = a$. So, a is a constant if you integrate it once more, this gives you, $T = ax + b$. This is the most general solution.

You apply the boundary conditions. The boundary conditions are at x = 0 temperature T(x) is $T_1$, at x = L temperature T(x) is $T_2$. So, if you apply this condition here. This will give you b is $T_1$, when x = 0 temperature has to be $T_1$, so b is $T_1$. When you apply you will get,

$$a = \frac{T_2 - T_1}{L}$$

So this gives us the solution as

$$T(x) = T_1 + \frac{T_2 - T_1}{L} x$$

So this then serves as our forward model. Whenever, we want to actually guess for the inverse solution, as I showed you in the previous video.

You know you can also find out the heat transfer in this case. so suppose you just want q the heat flux, which is $-k\ \frac{dT}{dx}$. In this case, q is a constant in the entire slab, you can argue physically about why this is also so. This gives you q is $-k\ \frac{(T_1 - T_2)}{L}$. so both these in some sense are part of the forward mode. So, this is the really simple case of, in fact, the first case you typically do in an undergraduate heat transfer course of steady-state heat conduction. Next, we will see a slightly more complicated case.

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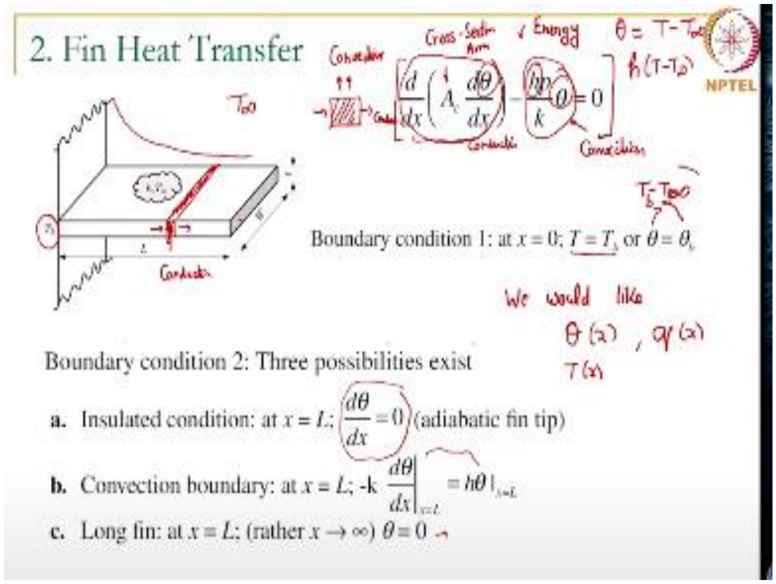


Let us come to the next case which is also a steady case but it is in a slightly different geometry. It is a fin heat transfer case. You would have seen this a typical fin need not always have a uniform cross-section. In this case, we have chosen a fin-width cross-section which is a rectangle. But let us say you have a general fin. you would be familiar with the equation based on balancing energy.

So let us say, you would have seen this kind of derivation once again. so, if I take the small element notice what happens. so, there is conduction within here. So let us say the temperature is maintained at $T_b$. At the base of the fin you have kept like a temperature of $T_b$, the outside temperature is $T_\infty$. Let us say for the sake of argument that $T_b$ is hot and outside fluid is a little bit cold. When you do that the fluid will get cooled. so, the temperature will actually decrease from the base to the tip.

Why would it decrease? It decreases because of the cooling on the side. so if I look at the small element you have conduction going on this side, but you have convective losses here and energy balance gives you this equation.

$$\frac{d}{dx}\left(A_c \frac{d\theta}{dx}\right) - \frac{hp}{k}\theta = 0$$

Now what you see here is $A_c$ is the cross-sectional area and you can see this term, I will explain what theta is shortly. Theta is basically the temperature deficit with respect to $T_\infty$.

So what this is saying is if you multiply by k on both sides, this then are the conduction term and this here is the convection term. How is this the convection term? Again, you should remember this, but in case you do not this is just as a refresher you have $h\theta$, $\theta$ is $(T - T_\infty)$. So, what we are saying here is simply a simple balance equation, whatever is conducted here the conductive loss is because of convection from the side.

So, you have an element this sort this temperature difference is purely because I am losing. if this side was convectively insulated this temperature and this temperature would be the same. But as it happens the right-hand side temperatures a little bit lower, because I am losing some heat from the side. so that is what this simple differential equation maintains. Now once you have the differential equation you obviously need the boundary conditions also.

What are the boundary conditions? At the left end let us say I am maintaining the temperature at T = $T_b$ or we can say that $\theta_b$ is $T_b - T_\infty$. So either way since right now this equation has been written in terms of $\theta$, rather than in terms of temperature. So $T_b - T_\infty$ so the boundary condition at the left is fixed. On the right-hand side usually, we have 3 possible boundary conditions that are usually usual for fin conditions.

You can also have a fourth one which is a fixed temperature that is not very common. so you can have an insulated usual Neumann type of condition. Let us say the step is insulated then you will have $\frac{d\theta}{dx} = 0$. If you have a convection boundary or a convective boundary condition, remember once again you have this robin sort of condition,

$$-k\frac{d\theta}{dx}|_{x=L} = h\theta|_{x=L}$$

Or this is usually for a fin we can think of a really long fin.

So long in comparison to the thickness t, so if it is really long then you can as well assume that this gap almost goes to 0. so, $\theta$ goes to 0 at this end. So, these 3 are the usual boundary conditions that you will see within textbooks within some other textbooks for example, Incropera, etcetera. you will also see the fixed temperature case. But above and beyond this just like with the slab case, we would like to get the temperature as a function of x. And you can also ask for heat transfer as a function of x as temperature, heat transfer 2 quantities that we are interested in.

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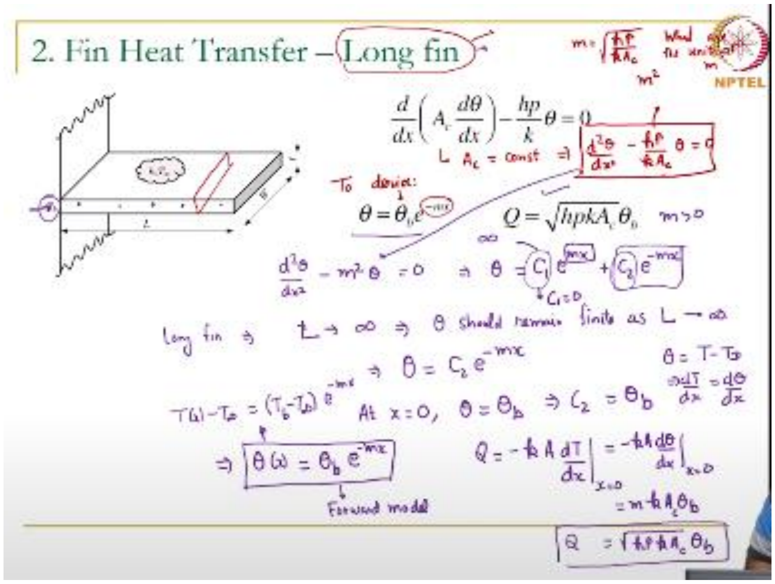


So if we go ahead and do that what we would find is for the specific case of a long fin, that is what I am going to solve for. We will obtain a simple expression that p is or $\theta = \theta_b\, e^{-mx}$. So let us now derive that. We will also obtain the net heat transfer, in this case,

$$Q = \sqrt{hPkA_c\theta_b}$$

So let us derive that as well. So, I will just go ahead and do this again. you should be familiar with this from your heat transfer class but let us just go head.

So, in this case, $A_c$ is a constant, because it is a rectangular fin, so this tells us $\frac{d^2\theta}{dx^2} - \frac{hP}{kA_c}\theta = 0$. So, this is our governing equation. This quantity $\frac{hP}{kA_c} = m^2$; where m is the fin parameter. it is called the fin parameter; p is the perimeter around the fin. If you have sort of forgotten it, I request you to look up a simple heat transfer textbook undergraduate textbook and just refresh your memory.

None of this is very deeply involved in the rest of the inverse heat transfer course but it is good to always remember this. So, $m = \sqrt{\frac{hP}{kA_c}}$. The unit is of m or something actually let me ask you to think about it. what are the unit? The clue lies here, you can figure out what the unit is of m are? I will give it to you later on during both probably the assignment for this week as well as later on in the course.

So, we want to find or we want to derive these 2 expressions from this equation. The specific case we are looking for is the long fin case. So now if this is the differential equation, you can write this in the following form,

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

So, this tells you, that theta you would know this kind of equation $\frac{d^2\theta}{dx^2} = m^2\theta$. So, this means $\theta = C_1 e^{mx} + C_2 e^{-mx}$; why is that you can see if you differentiate this twice you will get a $m^2$ up front. So $\frac{d^2\theta}{dx^2}$ is simply going to be $m^2\theta$. Now we have 2 constants there are several ways of finding it out. The easiest way I know is, since this is a long fin, it means, at the right end the length tends to infinity. This means theta should remain finite as L tends to infinity.

This means $C_1$ should be 0, why because if m is positive; as L tends to infinity, this term will otherwise go to infinity. We do not want that so we will have only this. so, this tells us $C_1$ is 0; so, we have $\theta(x) = \theta_b e^{-mx}$. So, one of these constants is eliminated, one constant still remains. How do we find that out? We know that at x = 0, $\theta = \theta_b$, which means $C_2$ should be $\theta_b$. So this tells us the temperature or the temperature deficit is $\theta_b e^{-mx}$. This is just like I said here all right.

What about this second relationship? The second relationship talks about heat transfer. so the heat transfer in this case is, heat transfer at the base is - k times the area since this is the net heat transfer not just the flux - $\frac{dT}{dx}$ calculated at the base.

$$Q = -kA\frac{dT}{dx}|_{x=0}$$

Now since $\theta = T - T_\infty$, this means $\frac{dT}{dx} = \frac{d\theta}{dx}$, If you differentiate both sides,

$$Q = -kA\frac{d\theta}{dx}|_{x=0}$$

Now you apply this here, this basically becomes the minus and the - m cancel out this becomes $mkA\theta_b$. If you multiply outcomes to exactly,

$$Q = \sqrt{hPkA_c\theta_b}$$

Once again for the long fill this then becomes the forward model, so just to tell you or to connect this with the inverse problem suppose I put a whole bunch of thermocouples here and I ask you to find out what is $T_b$?

You can do the same thing guess for a $T_b$, put this forward model find out how well it matches that and start iterating. we will do a slightly more complex problem than this one but I am just giving an example of how this can be used. Incidentally, this is just theta, remember always that the temperature is it itself given by $T(x) - T_\infty = (T_b - T_\infty)e^{-mx}$. So this is the temperature expression because $\theta$ is given by $T - T_\infty$.

So that is all that we need typically for the fin heat transfer problem. you can also try other boundary conditions and they will give you corresponding expressions. Especially the insulated case as you might remember from a heat transfer class is an important subcase in fin heat transfer.

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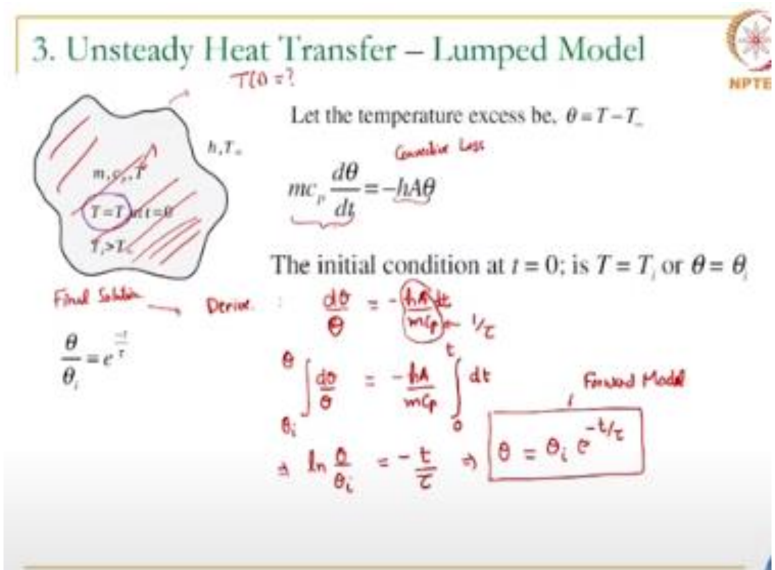


So, we looked at 2 steady cases. Now we can start looking at some unsteady cases just as an example. So, remember the very first problem I told you in solved by professor Beck was unsteady problem, this we will also look at some simplified unsteady problems within this course. So, a simple example that you might remember or some simple ideas that we use with the non-steady heat transfer is that of a lumped capacitance model or a lumped model.

So, let us take you know some arbitrary body, let us say it is hot and it is suddenly quenched in a fluid and assuming certain things that the Biot number is of a certain sort. You can now start finding out the temperature variation of this body over time. So, we let us take this case let us say the temperature is roughly uniform within the body and it is kept at an initial temperature of T = $T_i$. So, suppose the temperature is T = $T_i$ initially, we want to find out how does temperature vary with time.

In such a case, once again we do the same trick of finding out temperature excess or temperature deficit whichever way you want to define it. $\theta = T - T_\infty$ if you look at this entire body, if we are just assuming convective losses and no major comparative conduction within because of the Biot number, you can write down this equation. Why does the temperature change you go back to the original energy equation and you say,

$$mC_p \frac{d\theta}{dt} = -hA\theta$$

So, this is the change in enthalpy of the object, rate of change of enthalpy and this basically is the convective loss. Another way to say it is the reason the temperature of the object is decreasing is because, you have kept it in a fluid which is causing convective loss from the side. So again notice this $hA(T - T_\infty)$ is the convection and this is what accounts for rate of change of temperature.

Once you have this you can now go head and find out the how the temperature varies over time. Now notice this is a unsteady problem so we have to give initial conditions. Initial condition is that at T = 0 temperature is $T_i$ again I have ignored all temperature variation within the body because of the Biot number. The final solution turns out to be,

$$\theta = \theta_i \, e^{\frac{-t}{\tau}},$$

where $\tau$ is a corresponding time constant, again it depends only on the parameters here. It will look remarkably similar to the fin heat transfer case.

So we can now try to derive this we proceed just like we did earlier, so let us take this equation so you say then,

$$\frac{d\theta}{\theta} = \frac{-hA}{mC_p} \, t$$

Let us call this parameter as $\frac{1}{\tau}$ the time constant. I will not get into the physical significance of this you should have seen this in a heat transfer class anyway. So now we integrate this from the initial condition, which was theta i to the current condition where let us say the temperature is theta.

$$\int_{\theta_i}^{\theta} \frac{d\theta}{\theta} = \frac{-hA}{mC_p} \int_0^t dt$$

So, if you do that you will see $ln\ ln\ \frac{\theta}{\theta_i} = \frac{-t}{\tau}$. So this means $\theta = \theta_i\, e^{\frac{-t}{\tau}}$. so, this then is the simple forward model for the lumped model. You can also think of a case where you actually have heat addition which we will do later and this model changes slightly in that case we will see examples of that as well during the course.

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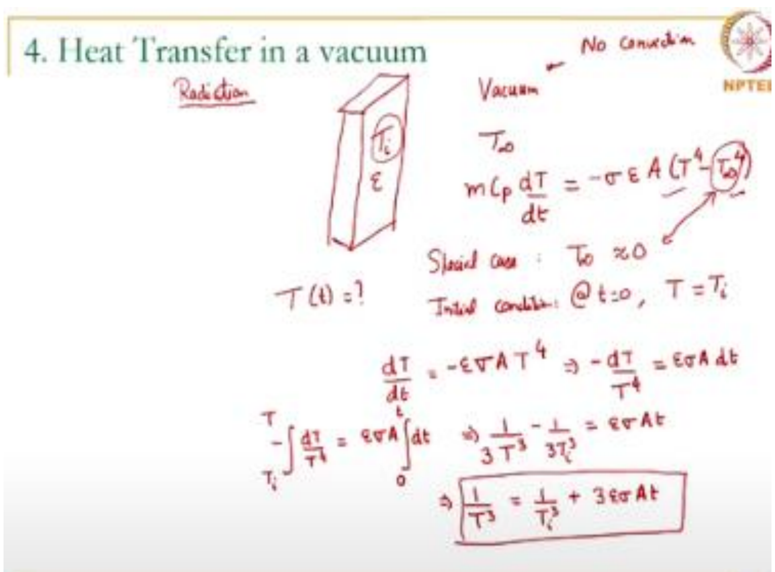


So, we just saw an example of a simple lumped model case, which was an unsteady case, also a forward model, the first 2 examples we saw were study models. So, the final forward model that I wish to within the week. Like I said we do small variations of this throughout the entire course is that of a heat transfer in a vacuum. This is just to add some kind of variance to what we are doing.

We are going to add a case with some radiation in it. So, we will sort of think of a simple let us say heat shield or something of that sort some idealization, which also let us say looks like a slab. It is going to look very much like a lumped model. let us say we have some the sheet kept at an initial temperature of $T_i$ and it is now in a vacuum. So, because this is, vacuum you are not going to have any convection.

So the only, heat transfer the only reason if you keep something in a vacuum, And let us say the ambient temperature is lower than the temperature of this body, the only reason why there is going to be any heat transfer at all, is only due to radiation outside of this body. Within the body, you could have conduction but again we will keep the Biot number so that there is not much conductive effect, so or the equivalent of Biot number in this case.

So, vacuum we have let us say the outside temperature is $T_\infty$. In this case, what is going to be the differential equation we start with? Once again, we can start with $mC_p\frac{dT}{dt}$, I am not going to define a $\theta$ here you will see shortly why. This is there is going to be loss and this loss is going to be $-\sigma\varepsilon$. Let us say this is not a black body and you have some amount of emissivity between 0 and 1. so this times $A(T^4 - T_\infty^4)$.

So, whatever is the wall temperature $T^4 - T_\infty^4$, from by Stefan Boltzmann's law you can actually derive this. Now this would be the general case. let us take a even more special case which makes the integration a little bit easy may might actually look at this full case also later on in the course. But the special case let us say that it is vacuum with approximately you know outer space, so let us say $T_\infty$ is really low approximately 0 in comparison to the heat shield temperature.

So, $T = T_i$, initial condition is at t = 0 temperature of the body is $T_i$. so again, we ask the question how does the temperature vary with time? The inverse problem would be supposed I make some measurements of the temperature with time can you find out the emissivity of the body. So, something of that sort would be the inverse problem, but the forward problem here is given the emissivity, can you find out how temperature will vary with time.

So again, we can take this equation so now the equation becomes,

$$\frac{dT}{dt} = -\varepsilon\sigma A T^4$$

Since $T_\infty$ is 0, so i am just saying $T^4$, so this gives us,

$$-\frac{dT}{T^4} = \varepsilon\sigma A dt$$

Once again, we integrate, we integrate from the initial temperature where we started to the current temperature,

$$\int_{T_i}^{T} -\frac{dT}{T^4} = \varepsilon\sigma A \int_0^t dt$$

So, if you do the algebra this gives you, $\frac{1}{3T^3} - \frac{1}{3T_i^3} = \varepsilon\sigma A t$. So, this gives you, $\frac{1}{T^3} = \frac{1}{T_i^3} + 3\varepsilon\sigma A t$. So, this is how the temperature varies with time and this would be the appropriate forward model for the heat transfer in a vacuum with pure radiative loss problem. So now we have looked at 4 problems. One problem with pure simple conduction in a slab, then with a fin, then the lump's capacity model, and with heat transfer in a vacuum. the last 2 problems were unsteady.

What we will do now starting from next week this is the end of the first week we have looked at what inverse problems are? Why inverse problems involve multiple forward problem solutions and some examples forward problem solutions which will be using on later on in the course. Starting from the next week we will start dealing with linear techniques for solving inverse problems and that would be the first place where we do this and you will see at least some of these problems reappearing there. So, I hope to see you in the next week please do try the exercise problems. Thank you.