Inverse Methods in Heat Transfer
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Lecture No # 03
Module No # 01
The Inverse Problem Solving Process

Hello and welcome back to the NPTEL course on inverse methods in heat transfer. in the previous portion of this week, We saw what the difference between a forward problem and an inverse problem was? In this video what I wish to do is to show you a very simple problem just as an illustrative example of how an inverse problem solution would actually work?

I am not actually going to put in any numbers and show you how this works I just want to talk about the overall process. So, if you pay attention in the process what you will notice is that for solving a single inverse problem, you actually need to solve multiple forward problems. This is what I call the inverse solution process and we will see this repeatedly throughout this course.

So, this video is purely meant to introduce you to what or how an inverse process could possibly work, and then we will see greater details in the coming weeks.

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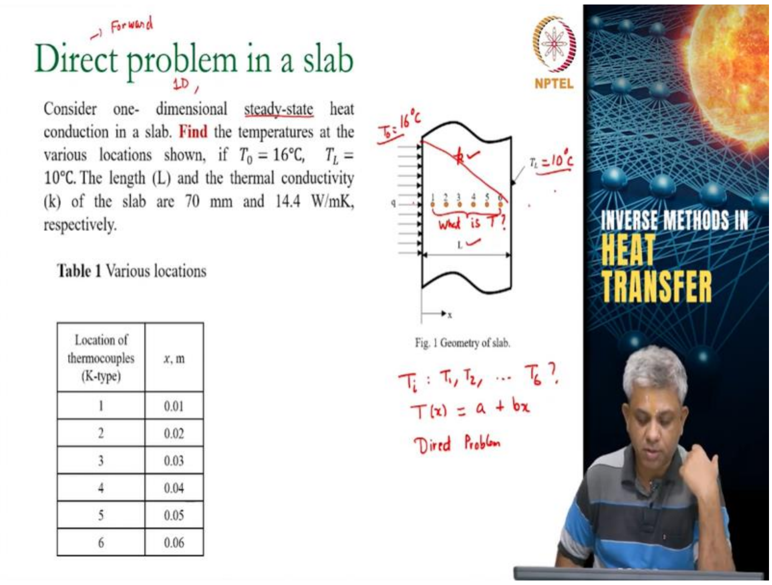

So here is the example of a direct or a forward problem. it is a very simple forward problem, you have already solved problems like this during your heat transfer class. So, the problem works in the following fashion or it is the following problem. You consider a 1D problem and its steady state and it is a conduction problem within a slab. this is a simple sort of problem which you have seen.

So let us say the left-hand side temperature is as given T_0 is 16°C and T at x = L is given to be 10°C. So, we are given the temperature, the left, and right end, and the length is given. You are also given conductivity. So, what you are asked to find out is what are the temperatures at these 6 points? Suppose we say T_i which is basically T_1, T_2 up till T_6, what are these 6 temperatures?

So, this is a simple enough problem you would have already seen this during your heat transfer class. You already know that basically say that the temperature is linear and given these 2 temperatures, you can basically find out that it is going to be a line going from 16 to 10 and you can easily solve for A and B here. So, this is a straightforward direct problem. given the left and right temperatures, find out the temperatures within. So, this is the forward problem so pay attention to this, because I will come back to this when we try to solve the inverse problem.

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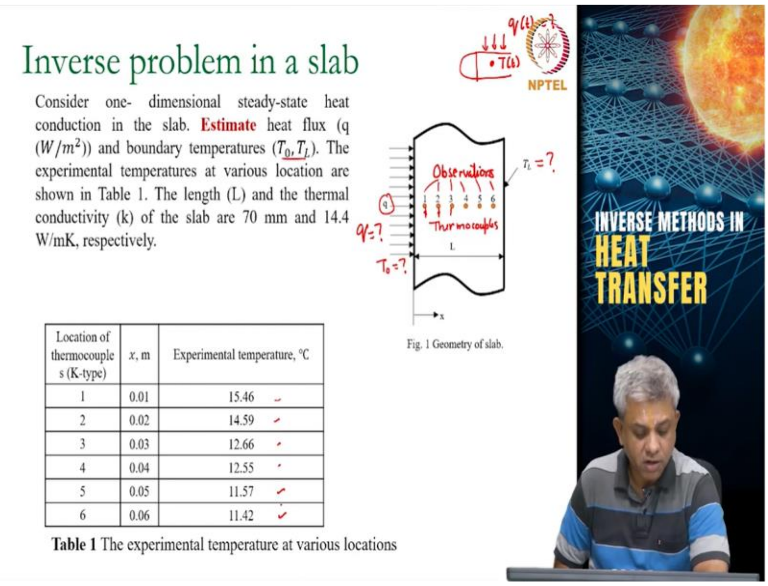

So here is the inverse problem, the inverse problem is like this. suppose you are given something else instead of giving you, the left and right temperatures. Suppose I give you measurements used based on thermocouples. So, we are making now remember what we talked about earlier about making a series of observations. So, we now observe the temperatures at these 6 points. And we ask 2 opposite questions or 3 opposite questions but essentially 2, which is we do not know what the left and right temperatures are?

Can we find those out? if we cannot find them, we can at least estimate them within a certain range. Similarly, the question is what is the heat flux? Can now remind yourself of this the actual practical case that I showed you about the atmospheric re-entry vehicle? you can see that it looks more or less like this except it is kind of turned on its side. So, if you remember we had a very similar case where we measured the temperature inside and we wanted to find out what q is given T?

So, the same thing here sees, given all these temperatures, find out what T is? So, this is in some sense related to the practical problem also. but you can also see that it is very close to the forward problem that I just showed you. Now you might think that this is straightforward, but there is a little bit of a problem when I show you the plot of how these measurements look like in practice. So, these are actually from something like a practical case not quite. We did not measure this actually in the lab but this is somewhat indicative of what you would see in the actual lab.

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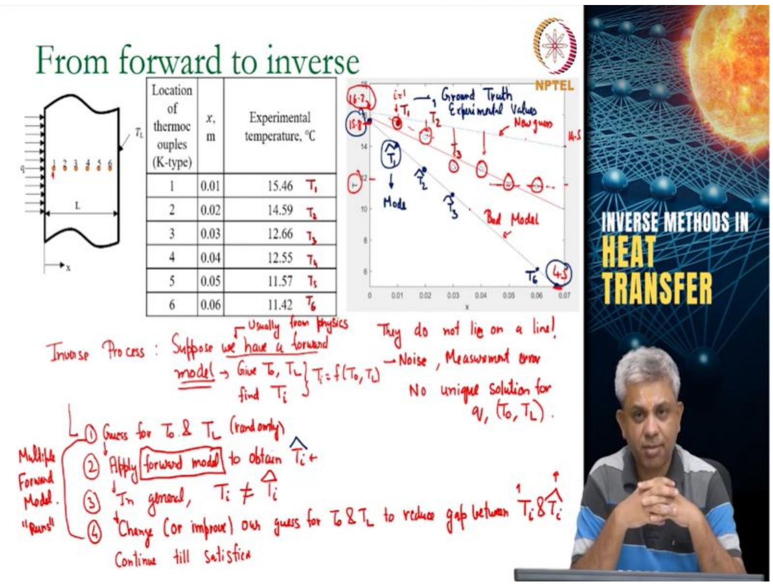

Now, here are the actual temperature measurements plotted. these are the x locations so this is i = 1, i = 2, etc., you can see that these are the temperature measurements. Notice that they do not lie on a line. as I told you with the ball falling through the air, in practice what you are going to see are you may call it noise, may be due to noise within the thermocouple, may be due to measurement error or maybe due to material in homogeneity.

So, it might be due to other genuinely physical effects that we are not accounting for. but what you notice is these points which indicate the temperatures are actually not perfectly within a straight line. Suppose you try to put a line through these 2 points and try to predict what T_0 and T_L are if that is what we are trying to predict that will give you one prediction or if you try to predict the heat flux. This line will give you one prediction. if you draw this line that will give you some other predictions.

So, you see now you have a problem of uniqueness, as is the case with inverse problems. So, you do not have any unique solution, you do have solutions, but you do not have a unique solution for either Q or for T_0 and T_L alright. So, in such a case how do we actually go about solving a problem? So, you see 3 lines here the general solution process is like this. So, this is what as I said I will call the inverse process.

Now, this is supposed to be something. suppose we have a, I am going to call it forward model. What is the forward model? The forward model says if you give me T_0 and T_L, I can find T_i for you just to show you here is the forward problem or the direct problem. What did we give here? We gave you T_0, gave you T_L, these 2 values were given and we were trying to find out the T_i temperature at these 6 points. So that is T_0, T_L finds T_i. So, this means that T_i all the temperatures are a function of T_0 and T_L.

So this is what we have is this model. how do we have this model? Usually from physics, which we will see a little bit later today. So, from the physics of the problem, you actually know that given the left and right temperatures, I can find out what the temperature distribution is going to look like. So, what is the process? The process starts as follows step 1 guess for T_0 and T_L. What do I mean by that guess for T_0 and T_L? Suppose I guessed T_0 is, let us say 15.8 and T_L is approximately let us say somewhere around 4.5.

So, this is my guess. how I got this guess? I got them randomly. I just got lucky that I guess somewhere close to the actual value right now. But so I can just make some guesses 15.8 and 4.5 or 0 and 0 whatever you have? Now once you guess that, you say to apply the forward model to obtain T_i what does this mean? What this means is suppose I guessed these 2 temperatures I immediately know from physics from my forward model that the temperature at this location.

This is the new temperature so and so forth so this is T_6. Now just in order to ensure that we do not confuse notation. we have the actual temperatures. So, these temperatures are the actual temperatures this, we will call T_1, T_2, T_3, T_4, T_5, T_6. So, this, I will call T_1, T_2, T_3, etc. but what about the values that I just obtained? This I am going to call $\widehat{T_1}$. these are from my model and these are what are in machine learning this is called Ground truth. these are the actual experimental values so notice this.

So, this is the experimental temperature 15.46 for example here, whereas your model based on these bad random cases of 15.8 and 4.5 gave you some other value which we are going to call $\hat{T}$. So, I am going to call this $\hat{T}$. Now notice that in general T_i will not be equal to $\widehat{T_\iota}$. that is whatever we guess the temperature to be that is not generally going to match what reality is. In fact, you can see that reality can never or your model guess is never going to perfectly match reality because you are only guessing. this is a model with some values of the temperature.

And that is not going to match the real value of temperature ever. Notice these are very different so the values here. So, what do we do? So, the next step is to change or more importantly improve our guess for T_0 and T_L while reducing the gap between T_i and $\widehat{T_\iota}$. So, to reduce the gap between reality and the model we improve our guess for T_0 and T_L. So let us say this was a bad model and you made a new guess. I am going to guess T_0 is like 16.2 and T_L is something like 14.5.

So, suppose you make that guess, you will land up here. So, this is the newer guess. it started here it ended up here, and you still have a gap. Now suppose you keep on iterating you know you go here you come back here after some time you come to some line that looks like it is kind of ok. How do we decide it's ok and how is it not ok and how do we actually make sure that we always come towards the other answer rather than going further and further away this we will discuss next week.

But for now, roughly the processes make a guess, improve the guess, and keep improving till it starts looking like you are fitting your original data fairly well with your model. So, you continue this process till you are satisfied. Effectively you go here and you improve. now notice that in this whole process as you are going back and forth, what are you doing? Each time you are applying a forward model. I made a guess and applied a forward model to get some values of $\hat{T}$.

Now I make another guess with another 2 values and that gives me a new guess for the intermediate temperatures, keep on making it. So, as you can see that the entire process involves multiple forward models. so multiple forward models I am going to call it runs. As if it is a computer program so you take a forward model. Let us say you make a guess and a computer program gives you a forward model, get a $\widehat{T_i}$ to improve, and keep on running it multiple times.

So, the catch here is each inverse problem solution involves multiple forward problem solutions. So, without having a forward model, really speaking you cannot have or you cannot solve an inverse problem. So, the inverse problem is the inverse of the forward problem. So, you actually need a forward model in order to run the inverse problem just like with the Neptune orbit case that I discussed.

We needed to know Newton's laws of gravity in order to figure out that Neptune was there. you cannot randomly guess. it is that model which helps you iterate and come to the final solution. So, what this tells you is in order to solve inverse heat transfer. you actually need a forward heat transfer model. So that knowledge has to be brought to bear in solving in fact multiple times during the solution of a single inverse problem.

What we will see in the next couple of videos is a very quick recap of some simple forward heat transfer cases which will be used as example-type problem cases throughout the rest of the course. Thank you.