Inverse Methods in Heat Transfer
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Module No # 01
Lecture No # 02
Inverse Problems - Definition, History, and Applications

Let us now come to forward problems versus inverse problems. So this is the first introduction I am going to give apart from the brief introductory video you would have seen on what inverse problems are? And it is best to see them in the context of problems that you are already familiar with.

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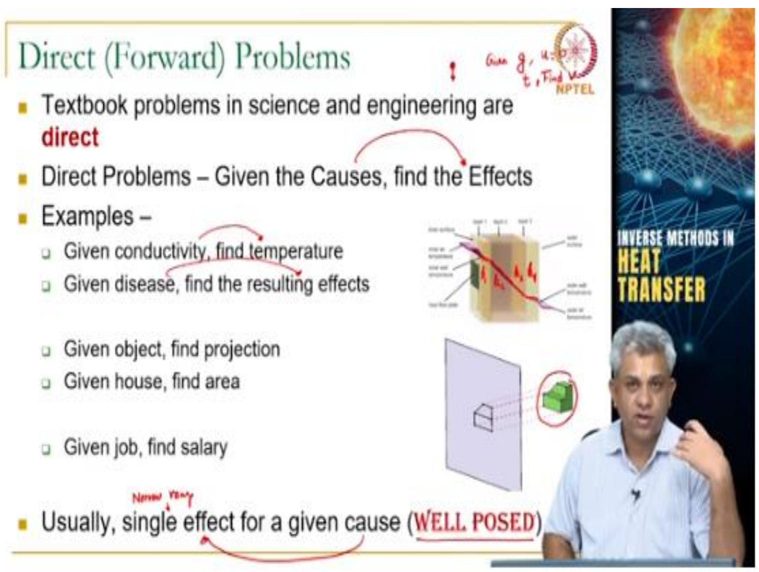

So, the problems that you are already familiar with are what are known as direct or forward problems. So, any textbook problem that you would have typically done in science and engineering is a direct problem. But as you will see shortly real problems in science and engineering are usually inverse problems. As you will see I will give you a diverse set of examples. these sets of examples are true not only in science and engineering but also true in general in many things in life.

So, the most interesting problems both in life and in science and in engineering turn out to be inverse problems but let us first look at direct problems. So, what is a direct problem? A direct problem is a straightforward problem that says given the cause finds the effect. So, for example, if I take a problem in Newtonian mechanics, I start with some ball at a particular place and I tell you that it starts from rest and you drop it. So, you drop it given the acceleration due to gravity, the initial velocity let us say it's 0 and I give you the time. I tell you to find the final velocity or find out how long will it take to reach the ground.

These kinds of questions are you have given the cause, the causes the ball started at a particular point and it had a particular speed and the causes of the acceleration due to gravity and you are asked to find out where it will reach at a particular given time. So given causes find effects. Let us see a few more examples. So let us see an example in heat transfer. Let us say so if I give you the conductivity.

So let us say I have a slab I have a composite slab I have a whole bunch of conductivities k_1, k_2. you would have done all these problems within your heat transfer course. So, suppose I give you a series of conductivities and I give you some boundary conditions also I tell you the left temperature is so much, the right temperature so much, or convection is so much on the right hand etc. I would always see this done these sorts of problems multiple times. The question is given the conductivity find the temperature profile. can you find out what this is?

So, what is the cause here? The cause here is whatever material property exists. Cause here is energy conservation. Cause here is the boundary conditions and you are asked to find out the effect which is the temperature profile. Very briefly very soon in the next slide, you will start seeing the inverse problem of this, but I would also like you to think about what the possible inverse problems could be as I am going over the direct problem.

Now if we look at the medical domain or a simple domain, let us say you have somebody like a trainee doctor and he would be asked stuff like suppose a patient has TB what would be the resulting symptoms? So that person could say something like there will be weight loss, there will be cough, there will be fever etc. So given the disease find what the effect on the patient's body is? so that would be an example of a direct problem, a forward problem. these are typically what are known as analysis problems.

Within engineering drawing, you could have something like given an object, find out its front view, top view, side view etc. So given an object find its projection given a body, given the light, find out what the shadow will look like these are again forward problems. So please keep on noticing some common characteristics between these problems. So again given this kind of body find out how much what will project onto a screen.

Again, suppose I give you the house which you are living in and ask you to find out the net floor area. That is an easy problem. That is a direct problem. you just go measure it you will get the house area. Suppose I tell you that there is a beginning software engineer and XYZ company what is their salary? you will at least get a range of salary. So usually, a single cause will typically have a single effect or a narrow range at worst like with the salary.

These problems are very typically forward problems and these are also called well-posed problems. What well-posed means? I will cover a little bit later this week. but we will call them well post problems. You can already intuitively feel this what this well-posed problem means. It has an answer and that answer is somewhat a little bit more definite than what? I will show you shortly now as a well-posed problem.

So, these are the characteristics of direct problems as I said that is what typically happens within the normal school kind of problems or normal science and engineering problems that we deal with within the academic setting typically.

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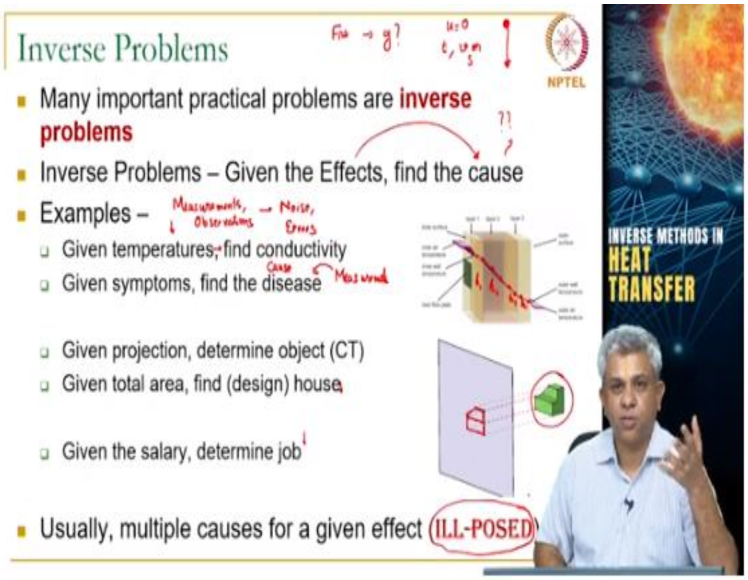

Now as against forward problems we have inverse problems and its best to see inverse problems in the context of forward problems. So now let us look at the same problems that we did in the last slide and now look at what their corresponding inverses would be? Now, as it turns out most interesting problems both in life as well as in science engineering are inverse problems and these represent the most practical problems that we can think of.

So here it is in the opposite direction, here it goes from effects to cause. if I know the effect, can you find out the cause? So, for example for thousands of years, people had been observing that balls or objects left in free fall, fall, but what is the cause? So, people had all sorts of hypotheses. till Newton came and sort of formalized it. so, what Newton did in some sense was also a sort of inverse problem.

So you can think of this like I am observing a ball and it is falling through the air, you know you drop it through the air and it reaches a particular point. so you are given the initial velocity, you are given the final time, and perhaps you can even measure the final velocity or the distance that it traveled. Now the question is can you find out g? Can you find out the acceleration due to gravity? now it might look like an obvious problem but please think about this in the realistic practical sense?

Now when it was Galileo or Aristotle or people within India so you have like all these historical examples, nobody knew that you have to now subtract out air resistance. Air resistance is a real problem that adds noise to this problem. they do not know whether in fact up until Galileo people thought that if you put larger objects they would fall faster which is not true. If you look at Newtonian mechanics that is simply not true. so if I put a larger object also it is going to fall at the same speed as a smaller object.

So that is the confusion that Crosses by the air resistance issue. so that is why inverse problems are many times a little bit harder to solve. So now the question is given these things can you find out g? So now let us look at the same problems we did in the last slide and go over those examples again. so now the same problem now I give you the opposite situation. When I said given temperatures, it is important to notice and I will come back to this point, I have not given the whole temperature. I have given you some measurements or observations of the temperature.

This is also a subtle difference again we will come back to this later on in the course. So when I tell you the temperature typically, I will give I have let us say some 10 thermocouples within the slab and I start measuring the temperature. Now notice that when you put a thermocouple there will be some noise, and there will be some errors. so we should account for that too, it will not fall perfectly within a straight line that is not going to happen. Because measurements have errors in them or measurements have some accuracy requirements built into them.

So now we ask the opposite question? We ask if I give you temperature measurements, can you find out the conductivity of the material or in this case can you find out the fact that there are multiple conductivities going on here and you can find out all of these out? So incidentally typically the forward problems that you solve again if you have done heat transfer, if you have done forward problems and heat transfer typically you will be given K. K is 10 W/mK etc.

Now the question that you are given is given the material constant find out the temperature. but in practice often the harder problem is finding out K of a material. When a material a new material is given you want to find out its conductivity how do people do it? They do it by making temperature measurements, putting in some theory, and then extracting this information out. So literally the way people find out K is through solving proper inverse problems.

Again, the real problem that a doctor faces is not a patient calling and say I have TB, you know what symptoms I will observe that even the patient knows. The patient usually comes with a symptom and asks the doctor for the disease so that is usually the direction in which we go. only then can we actually cure it. So typically, symptoms could be the doctor I am having cough, I am having fever then the doctor could say maybe Covid, maybe TB, maybe something else.

Notice the fact that your options actually multiply here unlike the forward problem. the forward problems are usually somewhat like a convergent problem. The inverse problem is somewhat like a divergent problem. so here you could have multiple diseases which correspond to the given symptoms. again, notice we are only making some finite measurements. So you might measure temperature, you might measure the frequency at which a person coughs, you might measure the weight loss of this person, you might do an RT-PCR test.

So all these measurements based on that, based on measurement you want to find out the cause. So that is what is again typical of an inverse problem. Again if we go to the engineering drawing problem, we give the projection or we give multiple projections, you might have in case anyone has done engineering drawing, you might have done this to find out the object. This typically happens within CT scans also will come back to that, later on, this week. I will just briefly discuss that because they are an important application of inverse problems.

But here the same thing here suppose I give you just this is the projection can you guess the object? Now you can see that it does not necessarily have to look like only this object. you need at least a few more projections for you to start guessing at what the object really is. so this is again typical of an inverse problem. Again suppose I give you that you have a 1500 square feet house if you are lucky and you want to find out a design, a floor pan for the house which satisfies certain constraints that is a design problem.

So design problems typically also are inverse problems. Finally, you might know the salary range you are suited for but you might have preferences you want to know which job will satisfy this and satisfy some other constraints like you want to live in the city etc. These are all inverse problems. obviously, all the techniques we are going to do is are not going to help you find a job or something of that sort. What they are supposed to do is. my point here is inverse techniques are more general than just things applicable to heat transfer.

It is mathematically a robust subject that applies itself to multiple Fields. I will come back to that you will have very surprising applications as I will show later on this week itself. So, what is characteristic of inverse problems is? there could be multiple causes that correspond to a given effect. so, these problems are known as ill-posed problems. So, remember well post problems forward problems ill post problems are typically what are known as or that is what typically makes inverse problems harder.

Otherwise, you know inverse problems are just like forward problems. you just solve on the opposite direction. but ill post problems is if typically an inverse problem is ill-posed, it makes the solution of an inverse problem a little bit difficult.

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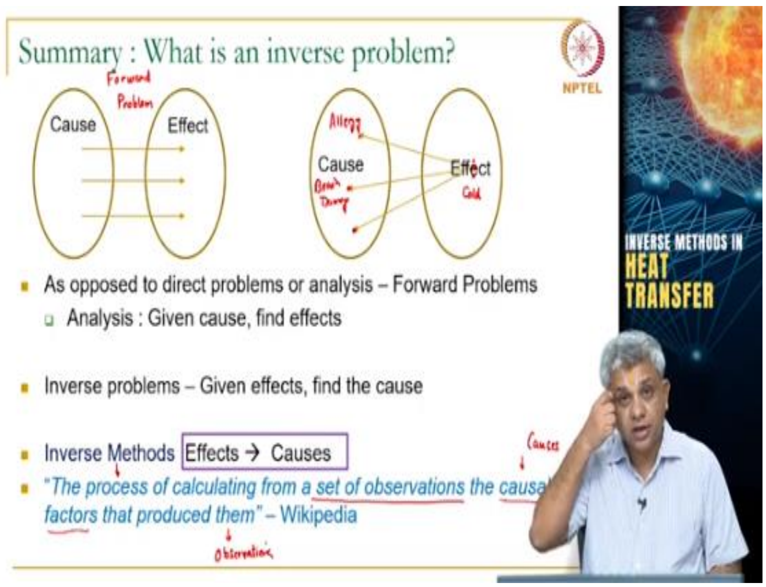

So just to summarize, what is an inverse problem? So typically direct problems or analysis problems or what we usually do they are forward problems. A forward problem goes from cause to effect. I will show you another characteristic of it shortly. But inverse problems go in the opposite direction, given the effect finds the cause. Now notice it seems like the same effect has multiple causes. this is possible just like the projection case or usually what tends to happen more often like I said that these are divergent nearby effects can actually have very far away causes. For example, 2 people could go to a doctor with very similar symptoms, in fact read a horrifying case very recently like last week of a lady who had gone to the doctor with just some runny nose, it turned out that she had some damage in her brain some cerebral spinal fluid was leaking out. And the doctor it was he the doctor figured it out based on some small indication I think maybe she had a mild headache or something of that sort.

She thought she had an allergy but it turned out you can have an effect that is just a symptom of a cold. but it could be something as far away as allergy versus brain damage. So this is a dramatic example of an inverse problem, where just small changes in the effect just some mild changes. which is why we go to you know medical professionals. Because they can figure out these subtle signs. if they are good, they can figure out these subtle signs that could be like vastly different causes for the same thing it could be harmless or it could be very harmful.

Same thing happens within the engineering domain too the numbers can be vastly different based on a small measurement change at one place, this is what is known as the stability problem, which I will come to once again later on in this week. So, we go in this direction from effects to causes so just to give you sort of an overall definition of what we have looked at over the last couple of slides.

It is that the inverse method or inverse method is a process of calculating from a set of observations. so please notice this word from a set of observations. So, the set of observations could be thermocouples which you have kept or the doctor could be making some measurements on your body or you could be taking some projections of the object etc. So, these are all measurements or observations from that you want to find out the causal factors.

"Causal factors" basically simply mean causes or numerical causes that produced them. "them" here means the observations. So, you are observing and you want to find out what possibly could have led to these observations. so that would be a definition of what an inverse problem is?

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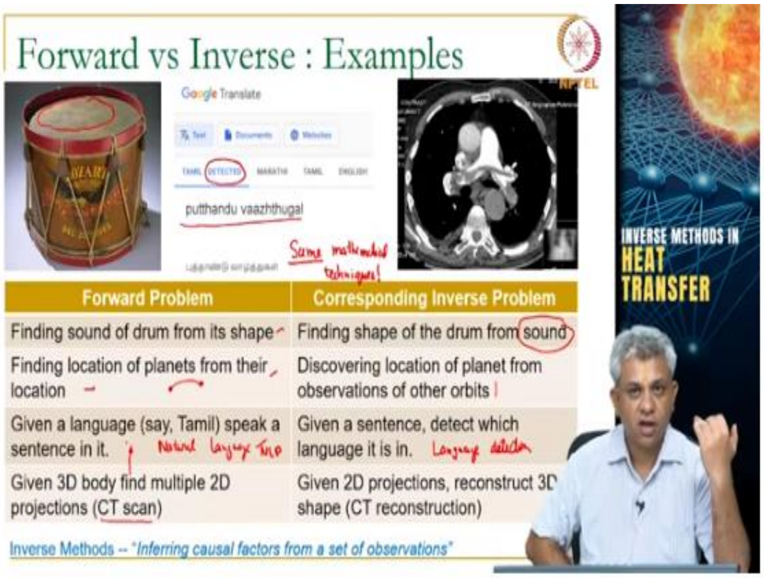

So let us now look at a few concrete examples, I gave you sort of toy examples. I want to give you more realistic examples and then I will show you some historic examples also in the next slide. So here are some examples of forward versus inverse problems in various fields. so, remember what inverse problems are, or inverse methods are? you want to infer the causal factors from a set of observations. So please keep this at the back of your mind as we go further.

So let us look at some forward problems versus their corresponding inverse problem. so here is a famous problem. this is called finding out the shape of a drum by hearing it. So, for example a forward problem well defined problem in acoustics in case you have done. this is supposed I gave you the shape of the diaphragm, suppose I say it is circular. And I give you properties also just like conductivity in thermal, you have to give the properties the elastic properties of this diaphragm and you want to find out, what sound will it make if I hit it.

That is a forward problem given the shape of the drum find out the sound that it will make. What is the inverse problem? The inverse problem is from the sound you are only hearing the sound, can you find out what the drum itself looks like? So this looks like an impossible problem and in fact, there was a very famous article very recently about this recently meaning about 20, 30 years ago. mathematicians showed that this is actually not a well-defined problem.

For the same sound, you can have different sort of shapes, but you can get remarkably close to the original shape just by listening to the sound. So that set of analyses is possible now. here is another example I will spend a little bit more time on this in the next slide, because it is of historical importance and inverse methods. If you know its current location this is very much like the ball problem with Newtonian mechanics. given the current location can you find out the future location based on orbital mechanics?

Now the inverse problem is you are given some neighboring orbits and you want to find out a missing planet. I will show you that this actually happened historically. so that would be an example of an inverse problem. again, I will skip this for now and we will discuss this in Greater detail in the next slide. But here is a more interesting problem. so suppose I give you a language I say Tamil and say happy New Year or New Year wishes, New Year greetings to you.

So in Tamil, you would say “Putthaandu Vaazhthukkal”. so, which would is a forward problem given a language to find a sentence and find an equivalent translation set. Even though there are multiple ways of translating I said the range is narrow. Now here is an inverse problem. the inverse problem is and this is what Google has done here incidentally Google translate has done this I just wrote this word called “Putthaandu Vaazhthukkal”.

And you notice this word it says detected it found out the language. so whenever from the language you want to find out what language it is that is a language detection problem. That is an inverse problem given the language finding out what it is? So that is the cause. the cause of the sentence is the original language now surprisingly enough it might seem like these 3 are very different problems. how is language in any way shape or form connected to the sound of a drum or the motion of a planet?

Let us come to one more example, again I will let me spend some time here. so when you go for a CT scan. So what happens is like you have these detectors and you know you have essentially mathematically speaking multiple projections of your body being thrown off and being detected by multiple detectors. So this is what we would call a CT scan. so there are multiple 2D projections. Now if you want CT reconstruction, you want to find out the 3D body that corresponds to all these 2D projections. so that is what a CT reconstruction problem is.

Now if you look at it very diverse fields from acoustics to orbital mechanics to natural, what is known as natural language processing, this is known as NLP Natural language processing, to CT reconstruction, to biomedical applications, as it turns out all of them use the same mathematical techniques. This is where the power of mathematics lies, in unifying if many of you would have done some courses in solid mechanics, some courses in fluid mechanics, some courses in heat transfer if you notice many of the differential equations will be the same.

Now as far as the process is concerned so the stretching of a bar is entirely different from what happens in you know really viscous fluid is different from what happens within a slab. But the equations will look very similar and in fact they are identical except you might want to switch t for velocity or temperature for velocity or something else. Now here is the point mathematics unifies this seemingly completely different application. now notice again I will tell you there is nothing different than having a biomedical application than let us say language speaking.

But nonetheless, it is identical the mathematical techniques that we use to detect a language will turn out to be very similar at least the sequence of series of techniques that will cover within this course are capable of handling both. When we come to the end of the course I will discuss how you know something like language detection could happen even using exactly the kind of inverse techniques that we have used for heat transfer.

So that is in some sense the exciting part. the part that all of these subjects come together and that is really what we are also seeing in the modern machine learning revolution because all of it is essentially a subset of inverse techniques. So my point of showing these diverse examples is not just to show you examples but to tell you that there is the mathematics that unites all these fields and that is why we will also be covering some mathematical techniques.

The course is a mathematical course incidentally. so, it is a numerical course, it is also a mathematical course because it is mathematics that unites these diverse applications.

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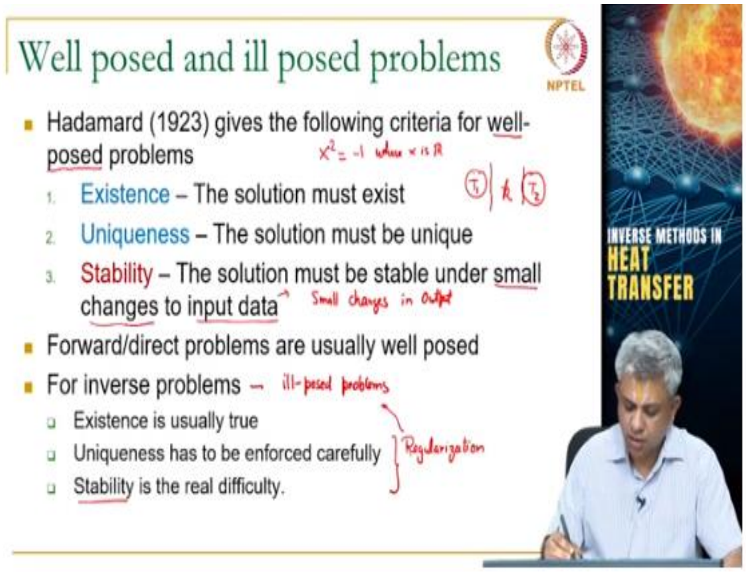

I had talked earlier about well-posed and Ill-posed problems. so let me just show you some formal definitions of what these are? I had casually introduced you these. so these categorizations of what constitutes a well-posed problem was first given by this person called Hadamard, the scientist called Hadamard in 1923. He gave the following criteria for what constitutes a well-posed problem.

First is when you give a problem there should be a solution that exists. So, you cannot give a problem for which there is no solution at all. So, an example of a bad problem or an ill-posed, you cannot really call it an ill-posed problem, but you can call it a not a well-posed problem is. suppose I say solve $x^2 = -1$, where x is real. This is technically speaking mathematics genes will frown at this kind of example for me saying that this is not a well-posed problem.

But anyway, I mean it is not a proper problem because the solution does not exist at all within a real number. So, a well-posed problem must have a solution that exists. So you must give adequate boundary conditions. for example, if I give you only one boundary condition then it is not really a well-posed problem. Let us say I have a slab I give you one side, I do not give you the other side you cannot really find out the temperature distribution.

The second thing is the solution must be unique so given a certain set of boundary conditions and a particular thermal conductivity and given that its steady-state etc. You know that there is only one possible temperature profile possible, so the solution is unique means it is a well-posed problem. The third and a very important criterion is where inverse problems typically run into trouble is stability. so the idea is this, if you make small changes to the input data. the input data could be.

Let us say again in the slab example this could be some small perturbation to the left temperature, some small perturbation to the right temperature, and some small perturbation to the conductivity. If you do that you do not expect that the temperature profile will finally jump to something else. So you expect that small changes in input will cause small changes in output. So that is small changes in the cause, cause small changes in effect. we are not accounting for turbulence etc.

But other than that typically looking at a well-posed problem means small changes here lead to small changes in the output and that is what is defined as a well-posed problem. There are more technical requirements here but I will not get into those. but this is just to give you a little bit more detail about why we call inverse problems ill-posed. So when we look at a forward or a direct problem these are usually well-posed problems typically these are well-posed problems.

The inverse problem's existence usually is true I mean there is a temperature profile. if I give you some thermocouple measurements and I ask you what is the temperature profile you know that something does exist there. Does uniqueness mean you have to be a little bit careful how do you ensure? We will do this later on in the course. most important is stability. Like I said with the doctor example, in case just one small change is made to the symptoms it can lead to a large change in what the possible diseases could be.

So stability is a real difficulty with inverse problems. so please remember these inverse problems are typically ill-posed problems. Usually I mean not all inverse problems but usually ill-posed problems and primarily because there are stability issues. And what we tend to do is we use a technique called regularization to reduce this ill-posedness and to increase stability. so to reduce ill-posedness of the solution we use a technique called regularization.

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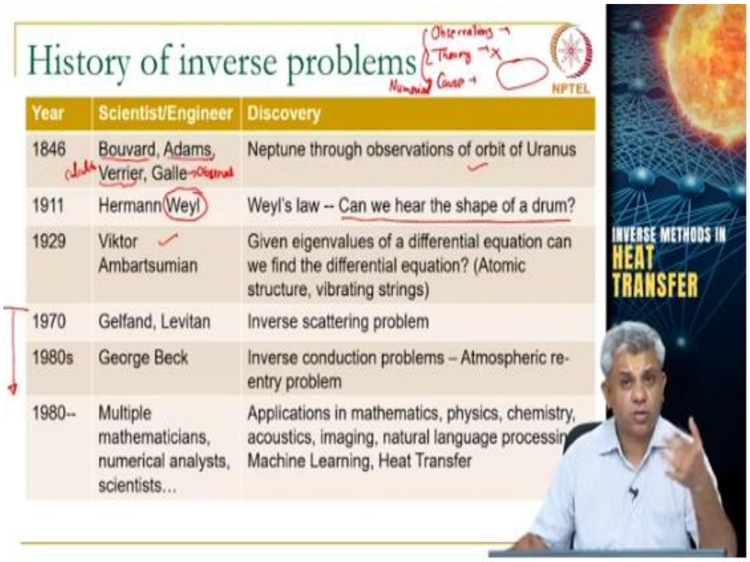

So we will see that later on in the course so let us now come to a brief history of what inverse problems are. So really speaking the formal history of inverse problems starts somewhere near the nineteen seventies where you can say is the cutoff of really calling something an inverse problem. I do not know the precise history which might be the nineteen sixties too but somewhere here is the cutoff point somewhere here 1970.

But before that people have been solving inverse problems implicitly like I said most interesting problems even if you can call a design problem inverse problem. But within science, the use of some proper techniques to get here, starts with gauze or even with the first example that I have shown here not a collaborative effort of these 4, but historically there were 4 people involved in this beautiful discovery sort of dream discovery of Neptune through the observations of the orbit of Uranus.

So what happened was the observations that people were making about how Uranus orbit was moving were different from what Newton's loss of gravitation would have predicted. so this is what our Kepler's Law of Motion whichever way you want to see it would have predicted. So both these guys' Bouvard and Adams independently speculated that the reason why this is not quite matching might be because there is an extra planet.

So that there is a planet after Uranus that is causing these perturbations and if we account for the gravitational effects of that we will be able to see that this accounts for Uranus. So if I remember right its barrier who did the calculations and said something like if what we have speculated is right and that there is an extra planet somewhere here. Now observing Uranus, so now notice this make a set of observations and make a guess about the causal factor. So notice this you made a set of observations about Uranus you have a theory.

So typically the typical mixes are you have observations, you have a theory and based on these you guess at a cause, you try to in fact give a numerical cause as much as possible. So your observations are in the language example the observation would be 2 words “Putthaandu Vaazhthukkal”. You have a theory that each language works a particular way I will show you how machines do it maybe later on in the course, but you actually try to guess at the cause. the cause is the person must be speaking Tamil. that is why he made those strange noises.

Similarly here the observations were observations of the orbit of Uranus they had a theory. the theory here was Newton's theory of gravitation. They said we can account for this if I add an extra planet remarkably enough, Verrier did the calculations and said if these calculations on this theory are correct we should find Neptune's roundabout route above around approximately here.

And this person did the observation and magically enough, they found it almost exactly where the theory predicted. So this is a remarkable success of the scientific method of Newton's law of gravitation etc. Of course, there is a counter example when Mercury's motion was not orbital motion but what is called as a Precision of the perihelion of mercury once that was there. So people made the same thing like maybe there is like another twin planet somewhere there and they did that calculation completely went off.

People did not observe any new planet and it took Einstein the theory of general relativity in order to show that that accounts for the extra motion of mercury. So it is possible that your theory is right and your observations are right, then your cause would be right. but sometimes it is possible that your theory itself could be wrong and you have to modify your theory. So that we are obviously not going to deal with within this, though I will touch upon this when we come at the end to neural networks.

So but it could be either possibility. so historically this neat problem, even though it is not if you look at discussions of history of inverse problems you will find this. but it is a good example of how it could have gone both ways. With Mercury and Neptune one to the confirmation of the theory and an actual observation of the causal factor and one which shows the theory itself had to be modified. Now preceding further the first real mathematical attempt at a strong inverse problem was by Hermann Weyl.

This is a mathematician 1911. I showed you the example of a drum so the problem that he solved was much related to that. So this title I have given here can be here the shape of a drum came later in the nineteen sixties or seventies was a popular article. I think in the American mathematical society or physical society I forget which publication this was in. But is the same problem that I showed you, so he solved the mathematical equivalent of this problem and in some sense you can say that the first strong application of inverse kind of techniques here.

Then in 1929 I think this was Viktor Ambartsumian I am not sure I am promising the name right. In 1929 there was an application in a mixture of astrophysics atomic structure etc. Now look at this problem you might not understand it in the version given here but you can think of it as a version of. If I give you the matrix can you find out its Eigen values that would be a forward problem. But if I give you a set of Eigen values kind of, you find out the matrix which will cause it you can see automatically that there could be multiple matrices.

So how to solve it in a physical setting like i said in an atomic structure or a vibrating string stretching was something that did I think as part of his thesis maybe I am not sure I do not remember perfectly well. So nineteen seventies like I said roughly is where you can start seeing this approach being formalized. so Gelfand and Levitan both Russian scientists started solving the inverse scattering problem. scattering problem is something that occurs for light rockers in general with radiation so it has connection with heat transfer also.

So these people used a series of techniques, we won't be getting into those techniques we are going to look at some simpler techniques within this course. Now within our field within our field meaning the field of this course one of the first people or probably the first person was professor

George Beck nineteen eighties there is a famous couple of books by him too in inverse heat transfer. he was commissioned I think by NASA to solve, I will show you this example shortly.

Solve the re-entry problem like a vehicle is re-entering the atmosphere and they want to make measurements on the vehicle. But they are not able to do it because of the high heat transfer there. So they had to solve an inverse conduction problem there and that in some sense is the first formal application and I think probably because of a space application far more than nineteen seventies by girlfriend and leave it and you can in some sense also think of this as the time that inverse problem suddenly exploded.

So after that there have been a huge number of applications in mathematicians who are interested in the theory, physicists of course in a large number of fields, I have already talked a few here are acoustics atomic structure, Then geophysics finding out the constituents of a planet based on you knows measurements that you are making from on the surface etcetera. Then imaging like I showed you CT scan etc. natural language processing is the example I showed you of Tamil.

Machine learning again these 2, I will cover at least a little bit of this later on in the course and then of course the reason why we are doing this course in heat transfer. so a large number of applications in all these problems and that are a brief introduction of the history of how inverse problems are developed.

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Now having looked at how inverse problems developed across various Fields let us look at some possible applications of inverse heat transfer. I will show you the broad applications in this slide and I will show you one specific application in fact historically the first application in the next slide. So you want to estimate so that is a keyword we are estimating why are we estimating? We are not directly calculating because we already know the solution is non-unique. so when the solution is non-unique you are only going to estimate.

So non-numerically you are going to estimate without any kind of stuff. But you can actually give some estimate with some error bounds, that is what we would like to do. so estimation of thermos-physical properties of a material like conductivity etc. So that you can try to do as I talked earlier in fact a straightforward important industrial use of inverse methods is there. You want to find out the boundary heat flux in force convection that is an example of how do you find out the heat flux even all the other details.

You want to find out spatially varying heat transfer coefficients in different geometries that are an example of inverse heat transfer. If you want contact conductance sometimes it is very hard to directly measure this in a conjugate heat transfer problem that is an example of inverse heat transfer. Of course, you can look at radiation also so you want to find out bulk radiation properties you want to find out emissivity etc. And in various kinds of environments all these are examples of inverse heat transfer problems.

We will primarily be looking at toy problems within this course, that is just to there is a reason why we just did not call it inverse problems in any transfer but inverse methods in heat transfer. Because the emphasis here is going to be on methods. so that you can apply it across different fields and not just heat transfer. But the context is heat transfer otherwise it becomes too difficult for people to sort of pin down what they wish to do.

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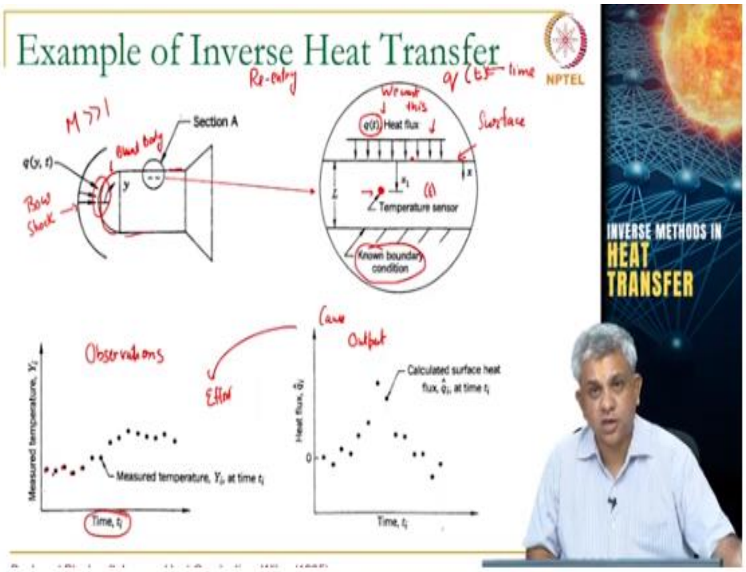

Here is an example of an inverse technique applied to heat transfer problem as I said the historically first example we have of such a thing. So this was by Professor George Beck and he was like I said I think this was NASA which called him to figure this out. So here is the problem you have an atmospheric re-entry vehicle some of you would have seen this kind of shape I am not sure if the movie margin shows that. But I think the movie mangalyan if any of you have seen this Hindi movie.

It shows what happens when you enter into the atmosphere you will have this typical blunt body shape this is a high mark number. So typically 3, 4, 5, 6 so typically there will be this what is known as a bow shock up front. And because of that there will be huge heat transfer here as you can imagine something is coming back rapidly into the atmosphere that is why it is a re-entry problem. Now all this updates so this this material here you it is kept as protective material to protect whatever components exist here.

So it rapidly sort of decays and you want to find out whether it will be able to take the heat transfer load so you want to also make live measurements of this. So suppose I zoom in on this portion what you will typically see is something of this sort so what you want to measure is this, we want this. it is not a single number notice this is q(t), which means it is time you want heat transfer as a function of time at the surface.

So this is the surface here now they want to measure this let us say NASA wanted to measure this. now the point is you cannot keep a thermocouple here. Obviously not you cannot keep it there it is just going to I mean the heat transfer code is just 2 way nothing is going to no material is going to withstand that so what you do is you bury it within. you keep it somewhere inside. So now here you have this interesting inverse problem. why it is an inverse problem? the forward problem is like this.

Suppose I give you q(t), can you find out how the temperature varies with time that is a straightforward problem these are known boundary conditions on the other end it is more or less like a slab. At least if we want to model it simply here is a slab one end, I know the boundary condition, I know the heat flux, other end I know the boundary condition. Let us say it is either heat flux or the temperature and find out a temperature at a point in the middle that would be the forward problem.

But here we want the inverse problem what is the problem that we want? What we have measured here is the set of observations at various times you make measurements. So you keep on making measurements at here and you know this since it is buried within it is not going to burn away. So it makes good reliable measurements let us say approximately reliable measurements and it tells you here is my time history of temperature.

Can you please tell me what would be the heat flux at these corresponding regions? So this was the problem as professor Beck talks about in his book which I have written here which is where I got the pictures from, this book is called inverse heat conduction written in as you can see 1985. it is called inverse heat conduction ill-posed problems.

So these ill posed problems are because it is there is no unique q that corresponds to this temperature so how do you find out what actually happens in practice is the interesting problem. So you can see here this is the output, even though this is the cause and this is the effect. the cause is the heat flux which is being you know generated due to the re-entry vehicle entering rapidly into the atmosphere and the effect is the temperature by looking at the temperature we want to find out what the cause was?

So it is like the sun heats us based on what we whatever we are getting we want to guess you know various things about this and what is its temperature? What is its composition? Again that would be an inverse problem typically in a heat transfer prose you will be doing the other way around given the emissivity etc, Find out the temperature on the earth. we want to do exactly the opposite you want to guess at the opposite thing.

So this is an example or historically first example of inverse problems being applied in heat transfer. And like I said after this the entire field of both inverse heat transfer and inverse methods in other places took off including CT scans etc.

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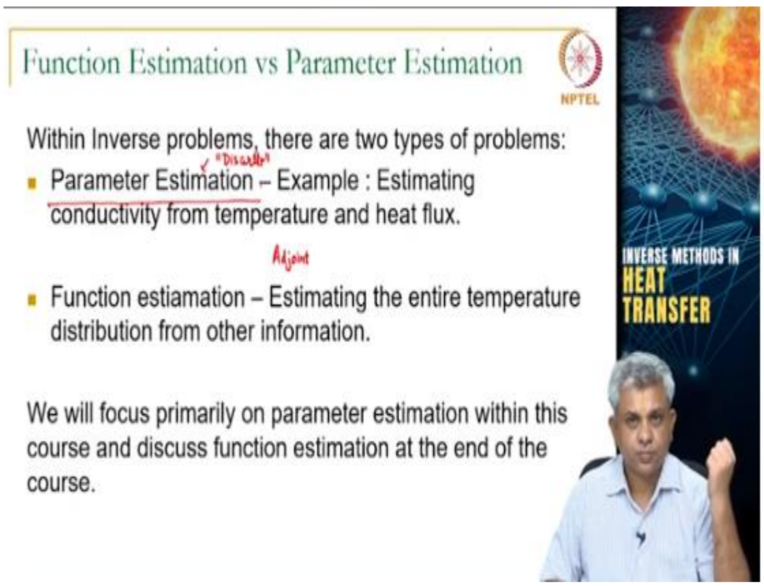

Within inverse problems whether it is in heat transfer or in other fields, there are 2 types of problems. this is called a function estimation problem versus a parameter estimation problem. So a parameter estimation problem is you want to estimate the conductivity from the temperature or the heat flux. So parameter estimation problem is finding out one property or one number or a couple of numbers. it is a limited number of numbers. this is in some sense a discrete problem.

So like I said 1 or 2 things that you wish to find out, it could be a few things also like, I said if there is a slab with multiple you know multiple different materials it could be k_1, k_2, k_3, k_4 that is a parameter estimation problem it's an easy problem. The function estimation problem is a harder problem. it is sort of the problem that I showed you in the last slide when I showed you the atmospheric re-entry problem. So the function estimation problem is find an entire function. so it could be like I have said the entire temperature distribution within a slab from other information.

So that could be one such example. another such example is like we did find out temperature as a function of or q as a function of time. So this is an entire function you are trying to find out not only at 1 or 2 points or 2 or 3 parameters but an entire function is what you are trying to estimate. this typically involves usually what is known as solving an adjoint problem. So I might or might not cover this but I will give you a brief idea about how to do this towards the end of the course when we looked at advanced techniques.

So primarily within this course this is what we will be concentrating on parameter estimation. So I want to point that out more advanced courses will also cover function estimation. But this is since its being taught primarily at the undergraduate level and parameter estimation itself as very interesting applications and inverse in interesting techniques that is what will primarily look at. As I have mentioned here function distribution will be looked at towards the end of the course probably the last week.

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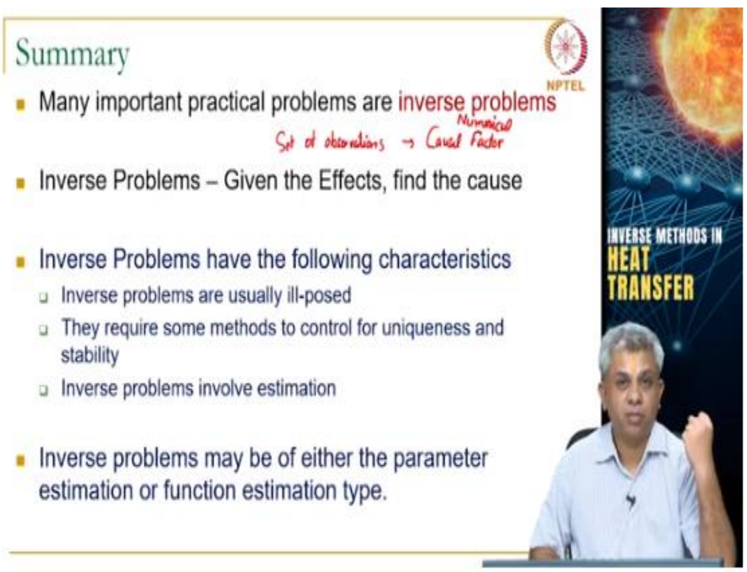

So here is a summary of what we have seen so far about inverse problems. Many important practical problems are inverse problems. An inverse problem is something which says given the effects find the cause or given a set of observations, find out the causal factor. Within this course, this causal factor will often be a numerical causal factor there is actually a number associated with it rather than something like a disease.

But I will come back to this later. They have the following characteristics. They are usually ill-posed, remember ill posed means that it will be non-unique or it could be unstable. These involve estimation as I said you cannot just calculate it you actually have to estimate within a certain region or a range as we will see within the probability and statistics things. They can be either of the parameter estimation type or the function estimation type.

So numerically that is what we are trying to estimate, we are trying to estimate the parameter or we are trying to estimate the function. So thank you in the next slides we will start looking at why the general procedure of solving an inverse problem is again I will do it primarily non-technically and why we involve forward problems while trying to solve the inverse problem. Thank you.