Inverse Methods in Heat Transfer

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Lecture – 18
The General Inverse Methods Process

Welcome back. This is Week 4 of the course on Inverse Methods in Heat Transfer and what we will look at this video something that I alluded to and even discussed in mild amount of detail in the first week, which is the general inverse methods process. So, far in Week 3 and Week 2 we saw a lot of techniques, primarily we looked at different ways of looking at linear regression.

Now starting this week later on we are going to start doing non-linear regression also and later on machine learning methods as well. but all of them come broadly within this process that I call the inverse methods process. In fact, you can see when you come to the machine learning portion that the machine learning method itself is in some sense a subset of the general inverse methods process.

Now what I would like you to pay attention to within this video is to start thinking about, how what I am saying within this video make sense in the context of linear regression, what you learnt about linear regression in the last two weeks and as you go to the nonlinear portions later on this week also, see how it applies there. So, once you see the general process you should be able to fit in practically every method.

And I feel that this gives a lot of clarity to what generally happens within inverse methods. otherwise, we get the loss within the details of the methods.

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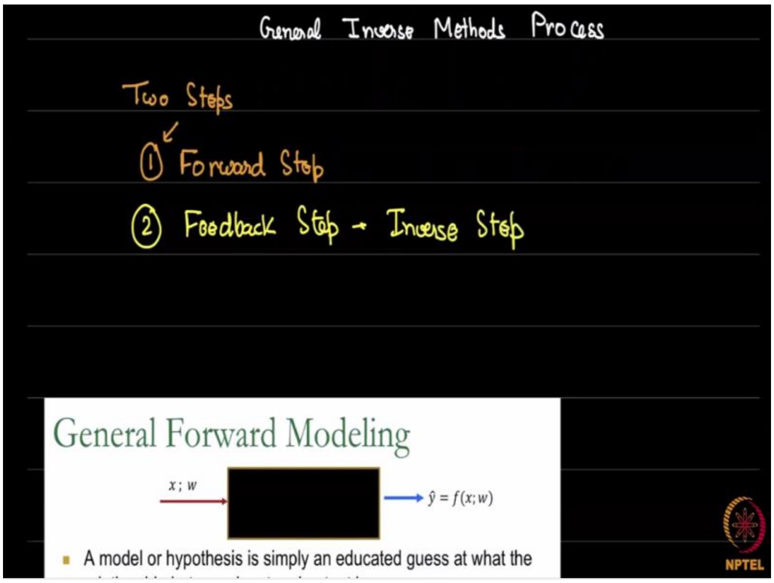

So, coming back to this, the inverse methods process generally involves two steps which we have seen so far really speaking. So, there is what we will call the forward step and I discussed this earlier as well in the first week and then there is the second step which we can call the feedback step for the inverse step. So, let us see details of this right now.

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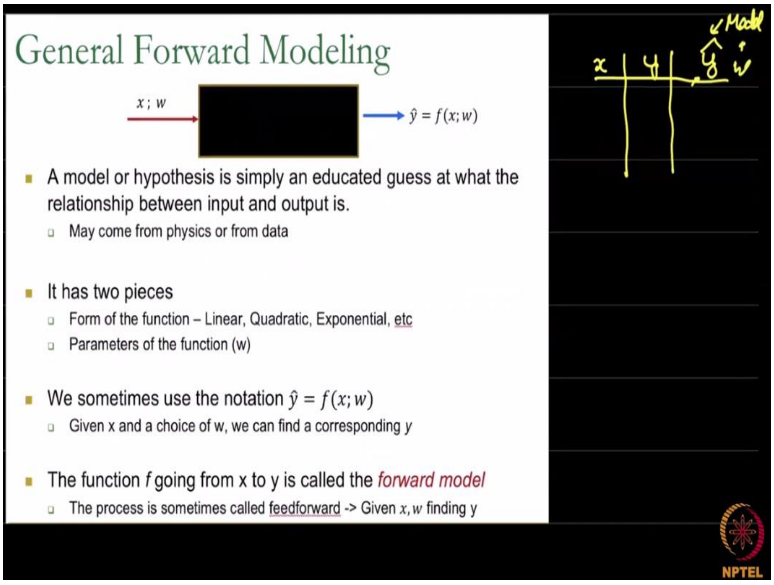

So, the first thing is the forward portion we call the modeling portion or the forward modeling portion. So, you would have seen within the linear regression portion, what it typically has is some x, some y and then of course, you have your model. This model involves some parameters w. so that is what I have written here.

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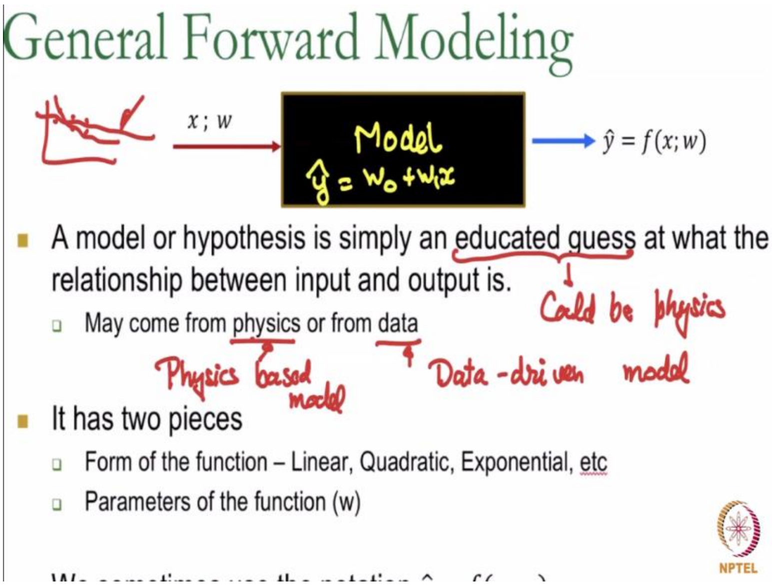

Imagine there is this box, this box essentially is our model. what you have going in is for example in the slab and thermocouples example this would be the slab location. This w would be our guess at the parameters, that relate the location to the temperature at that point. So, for example, this model is essentially an educated guess at whatever relationship we have between input and output. Notice that this is still a guess.

Now you might say that the model obtains from Physics, but Physics is also in some sense a guess, at least that is the view point that I take, but nonetheless it is an educated guess could be Physics. As I have written here this educated guess may come from Physics or it may be purely data driven approach and this is what we call the data driven model and we will see this distinction once again when we come to the end of the course or the machine learning course.

This will be a Physics based model, but either way whether we got the models from physics or whether we got the model from data. For example, you might remember last week, that I showed you that for the temperature data, we could even fit a quadratic and that would be a data driven approach. whereas Physics said that no it has to be a linear temperature dependence on the sensor data.

In fact, we saw that the quadratic fit, fits better, but nonetheless we went with the linear fit because we assume that the Physics actually governs this what is happening within this slab and not assumed I think it is a reasonable assumption to make, even the general long history that we have had with how temperature varies within a slab, but based on that we made a model which was basically $\hat{y} = w_0 + w_1 x$.

This is an example of the model of course you have other models. So, when we make a guess for what the model is. Now it is useful to split it into two parts.

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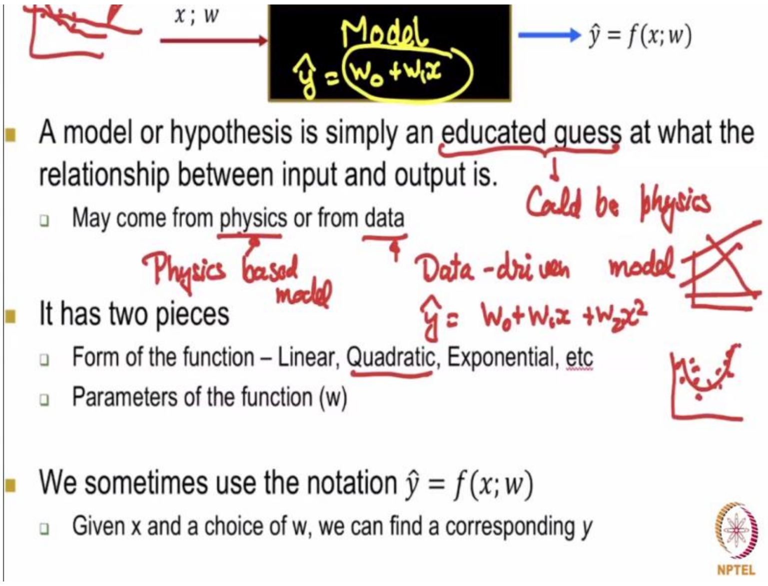

One is the form of the function what do I mean by form? You can clearly see that the form of the function that looks like $w_0 + w_1x$ is different from $\hat{y} = w_0 + w_1x + w_2x^2$ which would be a quadratic function. How are these two different if you look at geometrically whatever linear function you give regardless of what w_0 and w_1x you give it will always only look like, it can only look like straight lines, whereas a quadratic can look like this so it can capture more complex data relationships.

So, this is what is called the form. Now quadratic a specific case of a quadratic function is also a linear function because you can simply set $w_2 = 0$. So, a quadratic covers linear, but a linear does not cover quadratic. Similarly, if I have a sinusoid even a quadratic cannot be covering sinusoid.

So, each function has its own characteristics, which we will call very roughly the form, all of you intuitively understand what that is. Parameters of the function are the knobs that we are turning. So, w_0, w_1, w_2 decide the details of the function. Now, all these are still lines, but the slopes of the line how flat they are, how steep they are all that are the parameters of the function.

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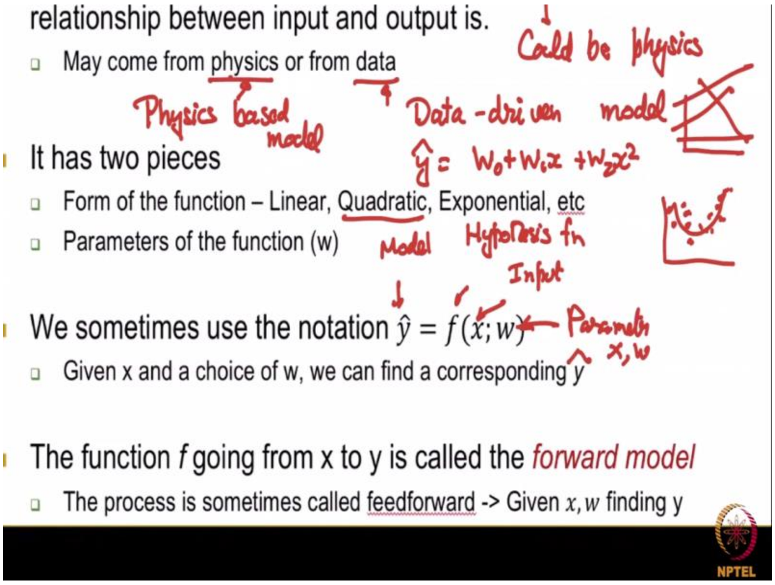

So, sometimes use the notation notice this, $\hat{y}$ which is of course our model is a function. This is our model function or called the hypothesis function. x is the input, w is a parameter and in order to distinguish the fact that the form and parameters are different we do not call it x, w. For example, if I look at this function you can technically say, this is a function of w_0, w_1, w_2, x and x^2 or w_0, w_1, w_2 and x, but that would be x, w.

But mathematically it is a little bit clearer, in order to separate these different roles that are form in a parameter how to say (x; w). So, this is a notation that you might file within the literature especially within machine learning literature. Now, what it indicates is this if you give me x for example, again please go back to the physical example if I give you location.

And I give you the two values w_0 and w_1 you can obviously find out the corresponding prediction for the temperature. This thing that goes from the input to the output or from the independent variable to the dependent variable is called the forward model. This process is sometimes called feedforward, you will see this (()) (08:06) what about the feedforward step people might ask you stuff like that.

So, feedforward is going from x and we to finding out y. Now, the exact opposite of this; so please remember this; this is what is called the forward model. Now, the exact opposite direction of this is the inverse process.

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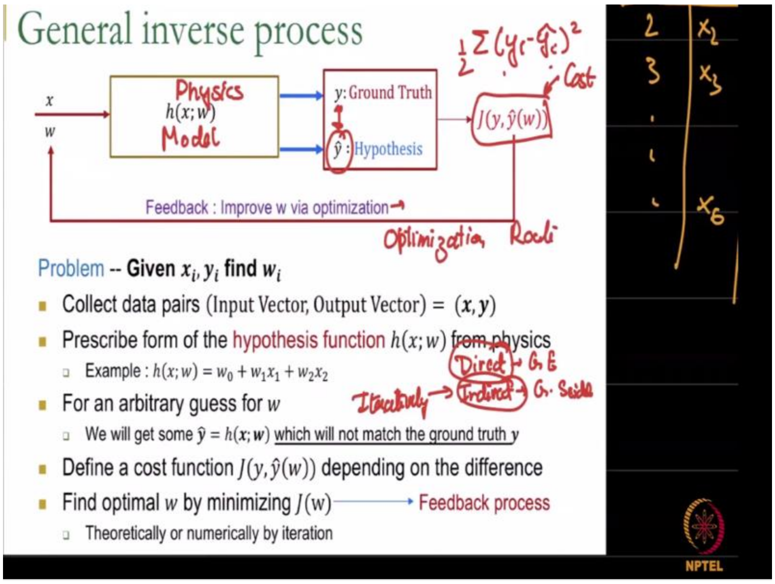

Now, how is the inverse process? I am going to actually combine both the forward process and technically speaking the inverse process is just this or the feedback process. So, I am going to combine feedforward and the feedback. we will make up this entire thing which is the inverse process of the solution. Now again go back to your linear example that you have seen so far within the course; we actually had a dataset.

The dataset looks like this 1, 2, 3 etcetera, x was x_1, x_2, x_3 these are the various locations we went until x_6 in our slab example then you have your model prediction then that is $\hat{y}_1$, $\hat{y}_2$ etcetera up till $\hat{y}_6$ and you had the actual measurement which is $y_1, y_2, \ldots\ y_6$. Now the point is this suppose I go to location 1 within the slab, I know x_1, I already let us say for in some magical way.

I have this guess, then given a guess and given x_1, I can find out y-hat so that is what is written here. First, we collect data pairs we collect an x, we collect y for that xy for that. Again, if time permits later on this course, I will discuss what happens if you do not have some of these data points are missing then what can we do. I am not sure whether we will have the time.

If time permits, we will look at it within the advance portions of this course, but let us say the general problem the inverse problem is given some locations and the temperature measurements on that find out the parameters. So, what we do for that is the first step of course invariably is to collect all these data pairs, location temperature, location temperature location then prescribe this word is very important you prescribe a form of the function.

Whether you are doing machine learning later on in this course you are doing non-linear or you are doing linear, you always prescribe a function from Physics is what we are going to do within the first portion of this course. For example, if you have two variables you could check $w_0 + w_1x_1 + w_2x_2$ in case it is a linear dependence on let us say both the x and y locations of this point or x_1 and x_2 locations of this point.

Now note this point for an arbitrary guess for w. Now it will look like in all our linear regression, we really never guessed for w. We just wrote our normal equations and derived it. We just found out a single w, but just imagine that you are not doing that, just like linear equations have, I hope some of you have seen these linear equations have both direct methods like Gauss elimination and indirect methods, like Jacobi, Gauss-Seidel some of you might have seen these methods.

So, if you have a set of equations you can always solve them either directly that is one short solution or indirectly or what we would usually call iteratively, that is you take a guess and improve that. Now assume we are going to use some such method and in the next few videos we will start discussing some indirect methods.

So, let us say we take an arbitrary guess for w. So, for some guess of w, we give let us say 0, 0 and you can find out what happens to $\hat{y}$ for that guess. Now, obviously for an arbitrary guess of w obviously $\hat{y}$ is not going to match y so they are going to be different. So, we define a cost function which we did, last time we have an objective function saying only that w is good which minimizes something like the Least square value and in the machine learning portions we will be sticking with this kind of fast functions.

So, you just sum up the gap between y and y-hat. for an arbitrary guess obviously these two are not going to be closed. Now, finally what we did was we find out the optimal w by minimizing J and this is what is the feedback. So, you give x, you give w, you use your model, this model usually comes from Physics. You say if this was the temperature and if these were the parameters then what would be my model prediction.

And then you check it against the actual ground truth and that gives you the differences measured by the cost. Now if this difference is too high, I am speaking colloquially here we will see the actual process in the next few videos. The difference is too high in truth. So, you back improve this by some optimization method. So, this will require an optimization routine.

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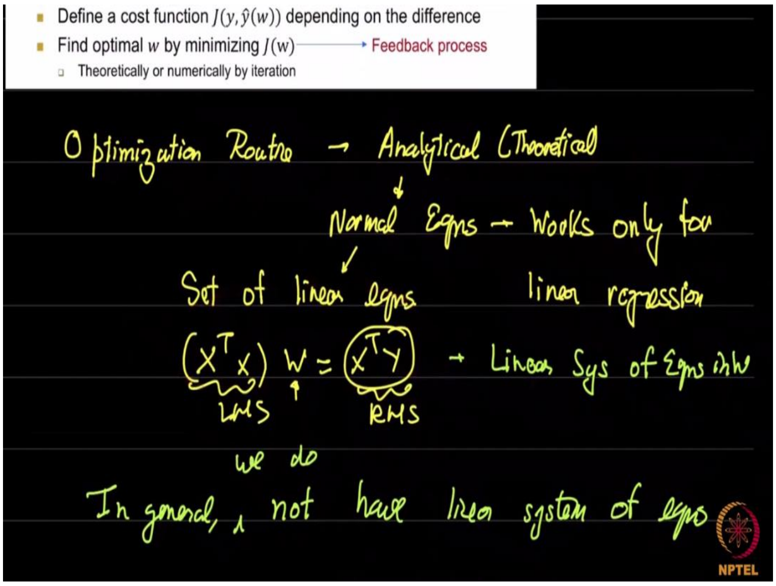

Now what we did in the previous weeks were our optimization routine was essentially analytics. So, basically, we did a theoretical calculation and we came up with the normal equations. Now as we will see this normal equation approach works practically only for linear regression. It is still powerful as we saw last time, but this approach works only for linear regression and even this case this was a set of linear equations.

So, we had our set of linear equations $X^T XW = X^T Y$. This was the right-hand side, this was the left hand side all this coding we did and we solved for w here. but in general, as we will see later on this week, we do not have a linear system of equations. You can probably intuitively see this if you do not that is okay, we will make this explicitly clear later on this week.

If you do not have a linear model, if the model itself is not linear and w if this model is not linear in w. If this model is not linear in w, then this equation will not be a linear equation for w this might seem obvious, if not like I said we will discuss this. So, this is linear equation in w or a linear system of equations in w. What we will see in general is we will get a non-linear system of equations in w.

And we do not have a simple process of solving it analytically, which is why we will have to solve it numerically by iteration. So, the way we are going to build this numerical solution by iteration is as follows. I will first show you a non-linear problem in the next video and after that we will systematically first solve this linear system or actually speaking the linear model, we will try to solve it by iteration pretending as if we do not actually have an analytical solution.

And then after that slowly we will build up how to solve the non-linear equations. So, finally we will end up with something called Gauss-Newton algorithm. You will see the name of Gauss coming everywhere, you see Gauss elimination, Gauss-Seidel and the algorithm that we will finally discuss at the end of this week we will be called Gauss-Newton algorithm that is the general algorithm that we can use to solve a non-linear inverse problem.

So, I will see you in the next video where we will discuss why non-linear models arise in heat transfer. Thank you.