Inverse Methods in Heat Transfer
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Lecture No # 10
Module No # 02
Summary of Week 2

Welcome back. In this video, we will be looking at a summary of what all we covered in week 2.

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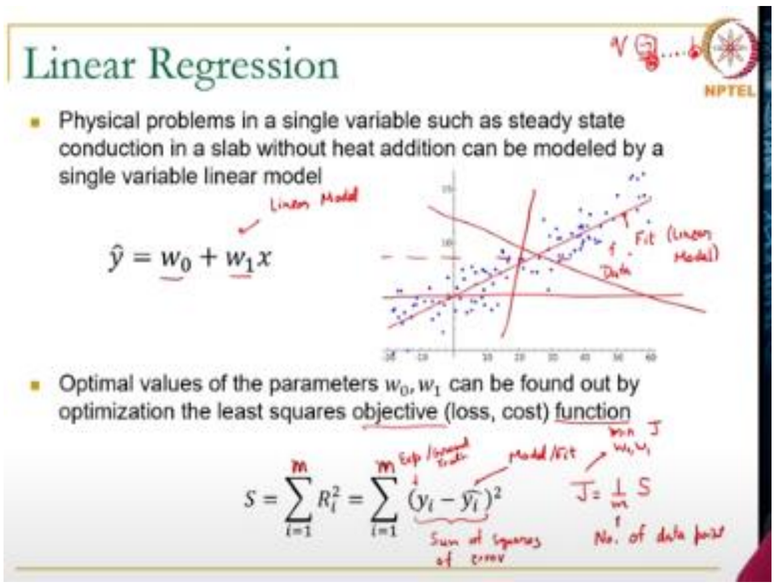

The primary contents of week 2 were linear regression, basically linear techniques for inverse problems in other words. So, when you have physical problems such as you know what we did again and again this week, which was steady state conduction in a slab without heat addition. so we took a slab, there was some heat transfer because of that gradient in temperature in the middle and we kept some thermocouples in the middle and made measurements and try to see if we could figure out the end temperatures as well as the heat flux within the slab, using just inverse techniques.

Now in this case, the general problem is that of fitting some scattered data. So, this is the original data set and we try to fit a line. So, this is the general example of how something of this sort could look like, as you can see the linear fit with a linear model cannot fit all data points simultaneously, if there is noise. We will come to characterization of this noise within week 4 and week 5, but right now we have some noisy data.

In our case this noisy data is because of the thermocouples. The sensor might be noisy or there might be other physical effects. Whatever be the case we use the linear model and the linear model's expression was fairly simple.

Since this is one variable, we have a linear model which is simply $\hat{y} = w_0 + w_1 x$. Now what we want of course is the best linear model, that is possible for the given data. You could have as I talked about later during the week a constant really bad fit of a line which is kept somewhere else. It could be at the mean or you could have bad models of this sort so you could have models which are linear but do not really fit the data.

But we want optimal values of the parameters w_0 and w_1 and for that purpose we derived or we defined more accurately an objective function. And this objective function was called the least square objective function, because what we are trying to do is to minimize the sum of squares and the sum of squares of the error. So if this is the experimental or ground truth and if this is the model or the fit, the error is given by sum of squares of these terms.

And I also use the term J which was just the mean value of S where m in this case I have written n really speaking you should treat this as m does not matter M or m; m is the number of data points. So we have defined these quantities and what we decided was for optimal values of w_0 we want the minimum value of J, over all possible values of w_0 and w_1, because that will give us the best fit curve.

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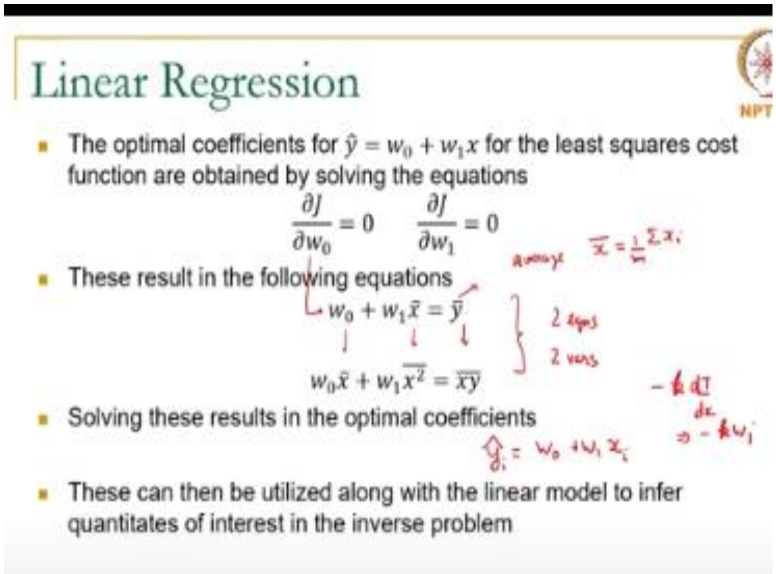

So how we obtained the best fit curve was to say well J has to be optimum with respect to w_0 and w_1. Therefore, gradient of J with respect to w or the partial of J with respect to w_0 and partial derivative of J with respect to w_1 should be both 0. So, once we applied this, we obtained the following equations. you might remember the physical meaning of the first equation. It simply says that the prediction at $\hat{x}$ or at $\bar{x}$ should match the average of the predicted value.

So, in our case this would say that at the average location of the thermocouples you should get the average temperature measured by the thermocouples and w_0 and w_1 should satisfy this equation this comes from $\frac{\partial J}{\partial w_0} = 0$. Similarly, if you set $\frac{\partial J}{\partial w_1} = 0$. You get an equation which is just the weighted version of the previous equation. You can notice that each term is roughly like multiplying the term in the previous equation by x, except of course we have this bar, bar meaning average.

So, for example, $\bar{x}$ would mean $\sum_i x_i$ divided by number of data points (m).

$$\bar{x} = \frac{1}{m} \sum_i x_i$$

so, we had covered this in detail the last week. Now since this is 2 equations in 2 variables you can solve this to obtain the optimal coefficients. I will not write the optimal coefficients again; they were sitting in the first video of this week. And then along with this, so once you have the model now the model is $\hat{y} = w_0 + w_1 x$ at any point.

And you can use this to infer all sorts of things if we wanted the left temperature for example, we set x = 0. You want it at some specific other location you can plug in the specific value of x and get that or let us say you want the heat transfer. Heat transfer would be defined as $-k\,\frac{dT}{dx}$, which in this case $\hat{y}$ being the temperature this would give us - k times w_1, as we saw during this week. So, the point is once you know the temperature model within the slab, you can re-derive the heat transfer or the temperatures at various locations.

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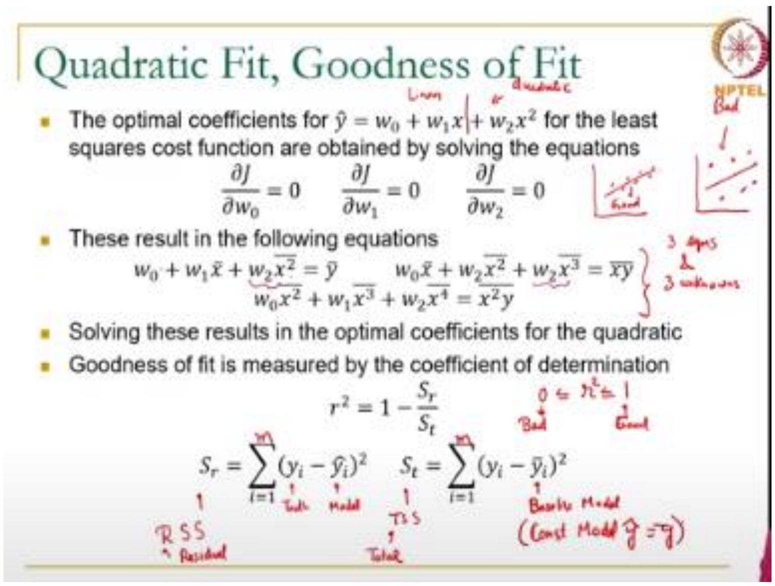

We also extended this to quadratic fits and now instead of the model simply stopping here, where this was the linear fit, you have the extra quadratic term. And as I said you can apply this to the case where you have for a simple case where the slab has a heat addition in the middle. And we are giving that example as an exercise within this week's homework. So, in this case instead of 2 parameters, you have three parameters w_0, w_1 and w_2 and now you can solve all 3 of these equations simultaneously.

We wrote down the equations as you saw the first 2 equations look almost identical to the linear case, except, now of course you have this additional term but notice the progression constant $\bar{x}$, $\overline{x^2}$, $\overline{x^3}$ so on and so forth. Same idea as the last time and the last equation is $\overline{x^2}$, $\overline{x^3}$ and $\overline{x^4}$. And you have 3 equations in 3 unknowns now. you can now solve this to obtain the optimal coefficients of the quadratic and the rest of it remains the same as before.

Now apart from this whenever you have data and you have a curve fit, you can see that even though you may get the same curve for 2 completely different data. In this case this is a good fit or at least it is a better fit than this one. so, this is not such a good fit so at least this is worse. So, we try to characterize one single number that gives us this idea. we will use this more during the rest of the course also.

So, this goodness of fit is determined by something called the coefficient of determination and the coefficient of determination is given by this expression

$$r^2 = 1 - \frac{S_r}{S_t}$$

We must remember that this will always come out to be between 0 and 1 and 0 means a bad fit and the best possible fit that you can get is r^2 = 1, this would happen if the net error is 0. The definitions of these 2 quantities S_r and S_t, are S_r is given by the summation from 1 to m the number of data points of the truth - our model.

So, S_r basically it is also called RSS, residual sum of squares and S_t is sometimes called TSS, total sum of squares and what this indicates in this case is sort of a baseline model. A baseline model is simply a constant model with every prediction simply being just taking the average. So, imagine you have a set of thermocouples and the average temperature measured by those thermocouples is let us says 30℃.

And then you say that that means that the temperature in the entire slab is 30℃. Obviously, this is a bad model, but your any model that we give should improve upon at least that and if it does not. Or if it does only mildly so then that is obviously a very bad model, because we have taken a very bad and sort of baseline assumption here. So, another way of saying this as we had discussed in the previous video, was to say how much of the variance in the data is accounted for by the model.

So, if my model accounts for a lot of variances of the data, then r^2 will be approximately 1 and that means it is a decent fit. One note that I have to point out here and I will re-emphasize in the later weeks is the coefficient of determination is good only particularly good for linear models, the models that will be dealing with in this week as well as the next week. And for other models we use something a little bit more sophisticated. so that is it for this week I hope you are able to do the assignments and I will see you in the next week. Thank you.