

**Oil Hydraulics and Pneumatics**  
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**Part - 1**  
**Estimation of leakage through spool and housing bore-concentric and eccentric leakage path**  
**Lecture - 43**  
**Estimation of leakage through spool and housing bore and Numericals on DCV, PCV and FCV**

My name is Somashekhar, course faculty for this course.

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**Oil Hydraulics and Pneumatics**

- Hello friends ..., Very good morning to one and all
- Hope you have enjoyed the [Lecture 13](#)
- Please note you have studied in the last lecture the followings:
  - Different types of FCV of interest are...
    - Needle Valve (or Metering Valve)
    - Needle Valve with Check Valve → Choke Valve or FCV
    - Pressure-compensated Flow Control Valve
    - Applications: How to Control the Strokes → Meter-in, Meter-out and Bleed Valves
    - Flow Dividers and its Applications
- In today's lecture we will discuss mainly on leakage flow prediction in concentric and eccentric spool valves, and also simple numerical calculations on important parameters of control element



Hello friends, very good morning to one and all. Hope you have enjoyed the lecture 13. Please note you have studied in the last lecture the followings; different types of Flow Control Valve of interest are Needle valve, also known as a Metering valve; Needle valve with Check valve, it is also known as Choke valve or a Flow Control Valve. Pressure-compensated flow control valve; also, we have seen the application of flow control valves in meter-in circuit, meter-out circuit and a bleed valves.

As we know in some of the applications like a shaping operations, planning operation; shaping operation you will see friends, the forward stroke is a cutting stroke. Return stroke is a very fast. In such cases, we are using these meter-in circuit and meter-out

circuit, whatever you want forward stroke should be controlled or a return stroke should be controlled.

Also, we have seen the flow dividers and its applications in multi cylinder operations, dividing the pump flow into equal halves or based on your requirement; sometimes 75 percent, 25 percent of the pump flow will divide irrespective of the pressure changes in the brand circuit. In today's lecture, we will discuss mainly on oil leakage flow prediction on concentric and a eccentric spool valves.

Already you have seen in previous class, the empirical relations to determine the oil leakage. In today's class, I will derive the simple equations to find out the leakage flow through the concentric and a eccentric spool valves and also, we will discuss in today's class simple numerical calculation on a different aspects of direction control valve, pressure control valves and a flow control valves.

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#### Lecture 14

#### Organization of Presentation



- Recap
- Leakage flow prediction in concentric and eccentric spool valves
- Simple numerical calculations on important parameters of control element
- Concluding remarks



To today's lecture, I will recap what we have studied as of now. Leakage flow prediction in concentric and eccentric spool valves, then simple numerical calculations on important parameters of control element, concluding remarks of the today's class.

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### Recap

### Course Outline



Sl. No.	Particulars	Lecture Hours
1.	Introduction to Oil Hydraulics and Pneumatics: Power Transmission Methods, Scopes, Application areas, Components and Subsystems, Merits and Demerits, Research Challenges	2
2.	Basic Laws and Symbols	2
3.	Pumps: Types, Characteristics, Operations, Efficiencies, Torque and Power, Numerical	3
4.	Compressed Air Generation, Preparation and Distribution: Compressors- Types, Characteristics, Operations, Efficiencies, Torque and Power, Pressure Drop and its Calculations	2
5.	Air Driers: Types, Characteristics, and Applications	1
6.	Valves: Constructional Details, Operations and Application Areas of Various Types of Directional Control Valves, Pressure Control Valves, Flow Control Valve, Numerical	4
7.	Actuators: Rotary and Linear Actuators - Types, Characteristics, Operations, Efficiencies, Torque and Power, Numerical	3
8.	Subsystems: Reservoirs, Hydraulic Fluids, Seals, Filters, Accumulators, Maintenance	3
9.	Circuit Design and Analysis: Development of Single Actuator Circuits, Development of Multiple Actuator Circuits, Cascade Method for Sequencing	4
10.	Hydrostatic Transmission and Control: Different Configurations and Analysis, Pump and Motor Characteristics	2
11.	Servo and Proportional Valves: Constructional Details, Operations, and Applications	3
12.	Role of Modeling and Simulation in Hydraulic Components- Case Studies	1



Yes friends, you have seen this. Again, and again, I am showing during my lecture. The course outline as you know since from the four classes, we are discussing the constructional details, operations and application areas of various types of directional control valves, pressure control valves, flow control valves. And in between we are also discussing some numerical correct. This is a very biggest chapter, also which is a heart of the fluid power system.

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### Recap

### Control Elements



- **Direction Control Valves** are required to start, stop and change the direction of an actuator by changing the direction of the fluid flow, so that the actuator will move either left or right in case of a linear actuator -Cylinders, clockwise rotation or anticlockwise rotation in case of motors
- **Pressure Control Valves** are required to control the Pressure level and hence decides the Force Output (F) of a Cylinder or Torque Output (T) of a Motor
- **Flow Control Valves** are required to control the Flow rate of the fluid and hence decides the Velocity of a Cylinder (V) or Speed of a Motor (N)



Direction Control Valves



Pressure Control Valves



Flow Control Valves



Now, as we know these control elements are playing a major role in controlling the direction, the pressure level and the speed of an actuator correct. As we know the direction control valves are required to start, stop and change the direction of an actuator by changing the direction of the fluid flow, so that the actuator will move either left or right in case of the linear actuator.



Clockwise rotation or anticlockwise rotation when you are controlling the motors; either hydraulic motors or air motors, only the medias are different. Also, we discussed in the previous class, pressure control valves which are very essential to control the pressure level. And hence, decides the force output in case of the cylinders and a torque output in case of the motors; meaning, the pressure acting over the area will matters.

Flow control valves are required to control the flow rate of the fluid and hence decides the velocity of the cylinder; left and right motion or the speed of the motors. We have discussed in last three classes rigorously on the different types of valves. These are some of the valves direction control valves, pressure control valves and a flow control valves.

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Recap

- ✓ Directional Control Valves discussed so far ...
  - Check valve, Shuttle valve, Twin pressure relief valve, Spool Valve, Fast response valve, Time delay valve and Pilot operated directional valve
  - Spool lap and flow characteristics
  - Leakage
  - Forces on spool valve
  - Valve material and
  - Valve specifications
  
- ✓ Pressure Control Valves
  - Pressure Relief Valve (PRV)
  - Unloading Valve
  - Pressure Reducing Valve
  - Sequence Valve
  - Counterbalance Valve
  - Brake Valve and
  - Valve Specifications
  
- Flow Control Valves basically..
  - Needle Valve or Metering Valve
  - Flow/Control Valves or Choke Valves
  - Pressure-compensated Flow Control Valve
  - Applications: How to control the Strokes → Meter-in, Meter-out and Bleed Valves
  - Flow Dividers and its Applications
  - Valve Specifications

Also, we have discussed in detail on directional control valves as check valves, shuttle valve, twin pressure relief valve, spool valve, fast response valve, time delay valve and a pilot operated directional control valves. Also, we have seen the various types of spool lap and flow characteristics, how the flow characteristics for the null cut valve, overlapped and underlapped spool valves.

Also, we have discussed the leakage through the gaps between the bore and the spool valves. Forces on spool valves, then what are the valve materials used for the manufacturing the hydraulic as well as a pneumatic valves and also, I have given the valve specification.

Under the pressure control valves friends, we have discussed many things. Pressure relief valves to control the system pressure, here we are setting the maximum pressure in the valve. If you are operating the double acting cylinder alone, the head side will whatever the pressure is there on the head side that is a system pressure what we are setting generally.

Unloading valves very essential to unload the system, correct? As we have discussed some of the applications of unloading valves how because they will monitor the pressure from the external branch circuits. Pressure reducing valves very very essential to limit the pressure in the different circuits. Even though, the main power pack is a same.

For example, the main power pack, when a system pressure we are setting 150 bar, using pressure reducing valve we are able to control the 1 circuit for 50 bar, another circuit for 100 bar and main one is for 150 bar. Pressure reducing valves are very very essential. Always they are open type.

Sequence valve, when we are operating the two or more cylinders; time-based sequence, when we are doing the sequencing the cylinder, for example one cylinder will move and holds the work piece; other cylinder will come and heads and complete the operation either a punching or a bending anything. Then, this will go, then other will go or both will move at a time. Meaning, when you are sequencing the multi cylinders, this sequence valves are used.

Also, we have seen the counterbalance valve, very essential when the cylinders are mounted in the vertical directions. Also, we have seen the brake valve which are very essential to control the motors; hydraulic motors or a pneumatic motors which are rotating very fast. Counterbalance and a brake valves are very very very essential which will create the back pressure to load the load very smoothly.

Also, we have seen the valve specification. In all you will see friends; valve specification for the direction valve and the valve specification for the pressure control valves were

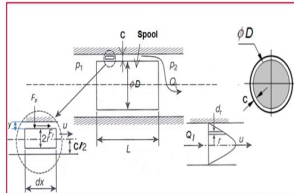
discussed in detail how to select the valves for a specific application. In flow control valve, we have discussed needle valve or a metering valve. They are throttling like a taps. If you will open the tap, more flow will go; then, actuator will move very fast.

If you will restrict the flow, actuator speed will be controlled. Simply they will the acts as a faucet in the main flow paths. Flow control valves or a choke valves, here metering valve integrated with the check valve; meaning, this is very essential to control the flow when which direction you want, whether forward direction or a return direction. This will calls for your meter-in, meter-out and various applications in industries.

Pressure compensated flow control valves, to control the orifice size based on the upstream pressure; it will adjust automatically the orifice. Also, we have seen applications how to control the strokes of the piston, what they will call meter-in, meter-out and bleed circuits, flow dividers and its applications. See friends, now these are all what I am showing you in this slide, all the recap what we have studied in directional control valves, pressure control valves and a flow control valves. Please brush up all these things.

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- Derive an expression to estimate the leakage through the spool and housing bore for concentric leakage path



$D$	Spool diameter (m)
$c$	Radial clearance or clearance height (m)
$\Delta p = p_1 - p_2$	Pressure drop over the clearance (Pa)
$L$	Length of the leakage path (m)
$u$	Velocity (m/s)
$r$	$c/2 - y = 0.5c - y$
$y$	Distance between the element side surface and solid boundary (m)
$Q_l$	Leakage flow rate ( $\text{m}^3/\text{s}$ )
$F_p$	Pressure force acting on the fluid element (N)
$F_r$	Shear force (or frictional force) acting on the fluid element (N)



Now, quickly, we will go to the today's lecture, mainly as I have told you a derive an expression to estimate the leakage through the spool and the housing bore for a concentric leakage path. What is the meaning here? You will see here, the spool is concentrically mounted in the bore; meaning, flow will takes place through the gap.

Generally, this is treated as a flow through the parallel plates. In hydrostatic element, they will derive the flow through the gap; meaning, this is the radial clearance.

Now, we are deriving this using the very simple figure. You will see friends here, here in the middle one is a spool the mounted concentric; meaning, axis of the bore and the spool axis are coincides each other. You will see in the side view also. Always there is a clearance because spool is moving inside the bore, it requires the small clearance. If clearance increases, leakage increases.

Please understand friends, the clearance is very very critical in case of the spool valves. If clearance increases, leakage flow increases. Then, if clearance decreases, manufacturing cost increases. Take care for this also. Now, I am deriving the flow through this passage.  $Q_l$  is a the leakage flow, correct? Here, I am using some of the notations. Here, notations are  $D$  is a diameter of the spool. Correct,  $c$  is a the radial clearance or a clearance height.

Here, I am showing you in the units meter-meter and  $\Delta p$  is a  $p_1$  minus  $p_2$ . What is this? Pressure drop over the clearance and  $L$ ;  $L$  is a length of the leakage path and  $u$  is a velocity meters per second and  $r$  is a you will see here this arc, this  $r$  is how to get it this  $c$  by  $2$  minus  $y$ ;  $y$  is the distance between, here I marked here the  $y$  is the distance between the element side surface and the solid boundary.  $Q_l$  is a leakage flow rate.

If will provide all these in the in this unit, I am getting meter cube per second.  $F_p$  is a you will see here,  $F_p$  is a the pressure force acting on the fluid element. You will see here friends, I have shown you the fluid element here in the bore, you are all see here I am taking this, this I enlarged here. For the derivation purpose, I am doing here the  $F_p$ , the  $F_r$   $F_r$   $F_\tau$  or whatever;  $F_\tau$  is nothing but the shear force or a frictional force acting on the fluid element, what I have considered here. Correct?

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- Assuming the steady-state flow and forces at equilibrium, we can write the following equations as



- The pressure force  $F_p$  is given by :

$$F_p = 2 \pi r D dp$$

- The shear force  $F_s$  is given by :

$$F_s = 2 \pi r D dx$$

- Also:

$$r = 0.5 c - y$$

$$\frac{du}{dy} = -\frac{du}{dr}$$

- From newton's law of viscosity, "shear stress  $\tau$ " is given by :

$$\tau = \mu \frac{du}{dy} = -\mu \frac{du}{dr}$$

$$F_p = F_s$$

$$\Rightarrow \text{from above equation } \mu \frac{du}{dr} = r \frac{dp}{dx}$$

$$\text{so above equation can be written as } \mu \frac{du}{dy} = -r \frac{dp}{dx}$$

$$\Rightarrow du = -\left(\frac{r}{\mu}\right) \frac{dp}{dx} dr$$

- The pressure gradient " $d_p / d_x =$  constant" and is given by :

$$\frac{dp}{dx} = \frac{\Delta p}{L}, \text{ where } \Delta p = (p_1 - p_2)$$

- So the above equation  $du$  becomes :

$$du = -\left(\frac{r}{\mu}\right) \left(\frac{\Delta p}{L}\right) dr$$



Now, we will see while deriving the flow through the leakage path, meaning parallel plates, meaning the spool and the bore parallel plates flow through this to arrive this, what I am doing friends? Assuming the steady state flow and a forces at equilibrium. We can write the following equations as the pressure force  $F_p$  is given by  $2 \pi r D$  into  $d p$ .

The shear force  $f \tau$  is given by  $2 \pi \tau D d x$ . Also,  $r$  as I told you  $1$  by  $2 c$ ; meaning  $0.5 c$  minus  $y$ . Also, we know that  $d u$  by  $d y$  equal to minus  $d u$  by  $d r$ . So, from the Newton's law of viscosity, shear stress  $\tau$  is given by  $\tau$  equal to  $\mu$  into  $d u$  by  $d y$ .  $\mu$  is a viscosity; dynamic viscosity is equal to what? Minus  $\mu$  into  $d u$  by  $d r$ . Now,  $F_p$  equal to  $F \tau$ .

Here, from the above equation, what we will get?  $\mu$  into  $d u$  by  $d r$ , this;  $\mu$  into  $d u$  by  $d r$  equal to  $r$  into  $d p$  by  $d x$ . So, the above equation, this can be written as  $\mu$  into  $d u$  by  $d y$  minus this is I am written as  $r$  into  $d p$  by  $d x$ . So, the  $d u$  term; you will get the  $du$  term, how it is? Minus  $r$  by  $\mu$ , it will go here;  $d p$  by  $d x$ , this  $d r$  will go this side. Correct?

This is  $d u$  by  $d r$  correct? Now, we will see friends, the pressure gradient  $d p$  by  $d x$ , this  $d p$  by  $d x$  is a constant and given by  $d p$  by  $d x$  equal to  $\Delta p$  by  $L$ . This  $\Delta p$  is a pressure difference across the leakage gap;  $p_1$  minus  $p_2$ . As I have told you  $p_1$  minus  $p_2$ . So, the above equation  $d u$  what happens minus  $r$  by  $\mu$ , as I have told you here  $d p$  by  $d x$  equal to  $\Delta p$  by  $L$ ; meaning, you will substitute here  $\Delta p$  by  $L$  in to  $d r$ .



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- Rearrange the terms, so the above equation  $d_u$  becomes :

$$du = - (r dr) \left( \frac{1}{\mu} \right) \left( \frac{\Delta p}{L} \right)$$



- Now the velocity distribution in the radial clearance is found by integrating the above equation as:

$$\int du = - \int (r dr) \left( \frac{1}{\mu} \right) \left( \frac{\Delta p}{L} \right)$$

- This gives equation as:

$$u = - \left( \frac{r^2}{2} \right) \left( \frac{1}{\mu} \right) \left( \frac{\Delta p}{L} \right) + a \text{ where } a = \text{constant}$$

$$u = - \left[ \left( \frac{1}{2\mu} \right) \left( \frac{\Delta p}{L} \right) \right] (r^2) + a$$

- Now find out constant "a" by applying the boundary conditions in the above equation as :

$$u = 0 \text{ for } r = \pm c/2$$

- After substituting obtained a in u, we may get, the following expression for the velocity distribution as follows

$$u = - \left[ \left( \frac{1}{2\mu} \right) \left( \frac{\Delta p}{L} \right) \right] \left[ \frac{c^2}{4} - r^2 \right]$$



So, rearranging the term. So, the above equation  $du$  equal to minus  $r$  into  $dr$  because I am integrating this, that is why minus  $r$  into  $dr$   $1$  by  $\mu$  delta  $p$  by  $L$ . These are the constant terms; variable term is only  $r$ .

Now, what I will do? Now, the velocity distribution in the radial clearance is found by integrating the above equations; meaning, if we will integrate this, what happens friends? Here minus  $r$  square by  $2$ , this is a constant.  $1$  by  $\mu$  delta  $p$  by  $L$  plus  $c$  is a constant. To get the  $c$ , what I will do? I will apply the some boundary conditions, correct? Same thing, I am doing here.

Now, what I will do? Now, to find out the constant "a" by applying the boundary conditions in the above equation as  $u$  equal to  $0$ , when it is for  $r$  equal to plus or minus  $c$  by  $2$ . Apply the here, upper limit and lower limit. After substituting, after getting the  $a$ , you will substitute here to get the  $u$ ; meaning, velocity distribution as  $u$  equal to  $1$  by  $2$   $\mu$  delta  $p$  by  $L$ . Here after substituting this and this, I will get  $c$  square by  $4$  minus  $r$  square.

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- The leakage flow rate is given by :
 
$$Q_l = \int_{-c/2}^{+c/2} u(\pi D) dr$$

$$Q_l = \frac{\pi D c^3 \Delta p}{12 \mu L}$$
- It is important to note that leakage is **inversely proportional to the viscosity ( $\mu$ )** and **directly proportional to the cube of the radial clearance ( $c$ )**.
- If the radial clearance is doubled due to wear, the internal leakage increases eight times
- The power loss due to leakage is given by...
 
$$\Delta N = Q_l \times \Delta p$$

$$\Delta N = \left( \frac{\pi D c^3 \Delta p}{12 \mu L} \right) \times \Delta p = \frac{\pi D c^3 \Delta p^2}{12 \mu L}$$
- Now from the leakage flow rate equation:
 
$$Q_l = \frac{\pi D c^3 \Delta p}{12 \mu L}$$
- The difference pressure is given by re-arranging the term:
 
$$\Delta p = \left( \frac{12 \mu L}{\pi D c^3} \right) Q_l = R_l \times Q_l$$
- Where  $R_l$  is the resistance to leakage and is given by :
 
$$R_l = \frac{12 \mu L}{\pi D c^3} \text{ (Ns / m}^5\text{)}$$



So, the leakage flow rate is given by  $Q_l$  equal to integrate minus  $c$  by  $2$  to plus  $c$  by  $2$   $u$  into  $\pi D dr$ . Substitute all the values here friends, after substituting this and applying the boundary condition, I will get the leakage flow through the gap is  $\pi D c^3$  by  $12 \mu L$   $\Delta p$ .

Please understand, this is very important equations. It is important to note that the leakage flow  $Q_l$  is inversely proportional to the dynamic viscosity  $\mu$  and directly proportional to the cube of radial clearance,  $c^3$ . If the radial clearance is doubled due to the wear, what happens to internal leakage?  $Q_l$  increases 8 times. We got doubled 2, 2 to the power of 3, 8 times it will increase.

The power loss due to leakage is given by  $\Delta N$  equal to  $Q_l$  into  $\Delta p$ . So,  $Q_l$  we know  $\pi D c^3 \Delta p$  by  $12 \mu L$ . This is  $\Delta p$ ;  $\Delta p$  into  $\Delta p$ ,  $\Delta p$  square. After simplifying, this is a the power loss. Now, from the leakage flow rate equation, the same equation I am writing here, the differential pressure is given by rearranging the term. Here, I want  $\Delta p$  friends.  $\Delta p$  is a pressure drop across the leakage path;  $\Delta p$  equal to what it is? Here  $12 \mu L$  will go this side;  $12 \mu L$  divided by  $\pi D c^3$  into  $Q_l$ .

Here this is  $12 \mu L$   $\pi D c^3$  is a constant term for the particular combination of spool and the bore. That is why I am taking  $R_l$  into  $Q_l$ . Here,  $R_l$  is the resistance to

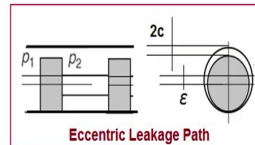
leakage and is given by  $12 \mu L \pi D c^3$  Newton's second per m to the power of 5, if we will substitute all the values taken in the same units.

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### Eccentric leakage path



- For the eccentric mounting spool in the housing bore as shown in Figure below, the radial clearance is not constant ...



- Hence the flow rate in the case of eccentric mounted spool is given by :

$$Q_l = \frac{\pi D c^3 \Delta p}{12 \mu L} \left[ 1 + \frac{3}{2} \left( \frac{\epsilon}{c} \right)^2 \right]$$

- So if the inner cylinder just touches the outer cylinder, the flow rate is increased by 2.5 times the value with the concentric cylinders assuming the same pressure drop



Similarly, as we have seen the leakage flow through the concentric spool valve because clearance is same, diametrical clearance or whatever the radial clearance is same throughout; but in eccentric leakage not like this because spool is going to sit on the one side of the valve, like this. Then, what happens here? You will see here spool is here. The axis of the bore and the axis of the spool are offset eccentricity epsilon, what I have marked here epsilon and this clearance also you will see, this clearance is not constant. Previously, c is constant.

Now, for the eccentric mounting spool in the housing bore as shown in the figure below. The radial clearance is not at all constant, it is varying. Hence, the flow rate in the case of the eccentric mounted spool is given by same it is as you will see friends here,  $Q_l$  equal to  $\pi D c^3 \Delta p$  by  $12 \mu L$ , this term is adding. This is to take care for  $1 + \frac{3}{2} \left( \frac{\epsilon}{c} \right)^2$ . So, if the inner cylinder just touches the outer cylinder, the flow rate is increased by 2.5 times the value with the concentric cylinders assuming the same pressure drop.