Fundamentals of Combustion

Prof. V. Raghavan

Department of Mechanical Engineering

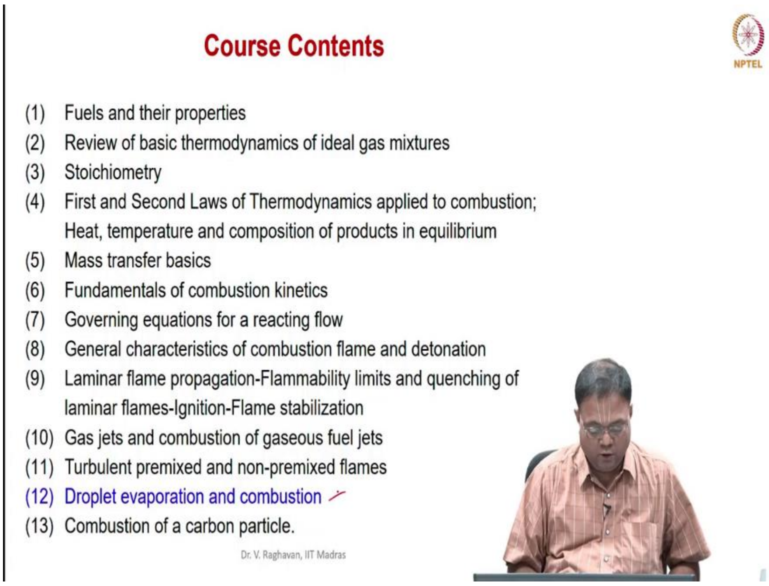
Indian Institute of Technology, Madras

Lecture - 54

Droplet evaporation and combustion - Part 1

Steady evaporation of liquid droplet

(Refer Slide Time: 00:14)

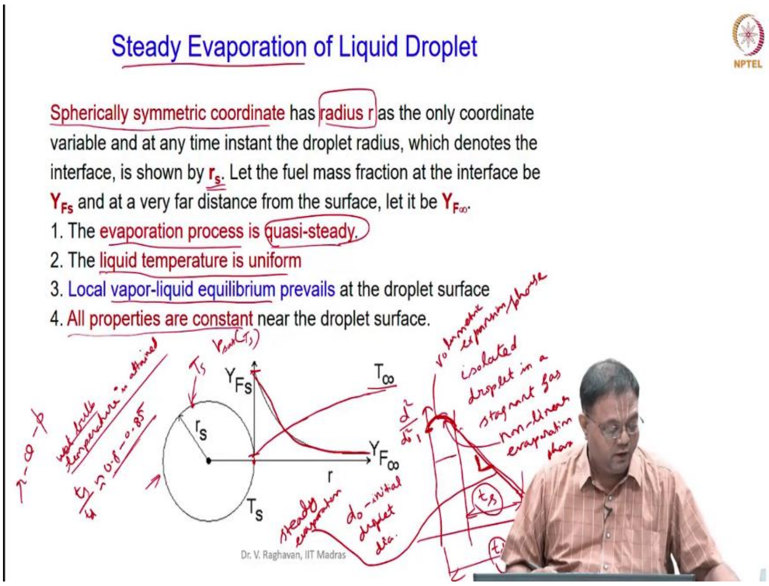


Course Contents

(1) Fuels and their properties
(2) Review of basic thermodynamics of ideal gas mixtures
(3) Stoichiometry
(4) First and Second Laws of Thermodynamics applied to combustion; Heat, temperature and composition of products in equilibrium
(5) Mass transfer basics
(6) Fundamentals of combustion kinetics
(7) Governing equations for a reacting flow
(8) General characteristics of combustion flame and detonation
(9) Laminar flame propagation-Flammability limits and quenching of laminar flames-Ignition-Flame stabilization
(10) Gas jets and combustion of gaseous fuel jets
(11) Turbulent premixed and non-premixed flames
(12) Droplet evaporation and combustion
(13) Combustion of a carbon particle.

Dr. V. Raghavan, IIT Madras

(Refer Slide Time: 00:20)



Steady Evaporation of Liquid Droplet

Spherically symmetric coordinate has radius r as the only coordinate variable and at any time instant the droplet radius, which denotes the interface, is shown by $\mathbf{r_s}$. Let the fuel mass fraction at the interface be $\mathbf{Y_{Fs}}$ and at a very far distance from the surface, let it be $\mathbf{Y_{F\infty}}$.

1. The evaporation process is quasi-steady.
2. The liquid temperature is uniform
3. Local vapor-liquid equilibrium prevails at the droplet surface
4. All properties are constant near the droplet surface.

Dr. V. Raghavan, IIT Madras

So, the next topic is Droplet evaporation and combustion. So, we will consider an isolated droplet. At any time instant its radius is r_s and we will consider only the spherical symmetric coordinates. It is only one-dimension; so, radius is the only coordinate variable in this.

So, if you see at any radius the profiles will be the same. So, it is spherically symmetric. So, isolated droplet and in a stagnant gas. So, when we did the mass transfer topic we saw the evaporation features of this using the Fick's law.

So, we will touch upon some of the aspects here and move on to the droplet combustion. Now, the basic assumptions in this what we have seen already are we can see that the evaporation process is quasi-steady; that means, at any time instant the rate at which the droplet evaporates will be the same.

So, for example, the radius may vary due to the evaporation the mass going out from the liquid to the vapor; the radius can shrink. But, the rate at which the evaporation takes place will be steady. So, at any instant we can find the evaporation rate and that will be a constant. The second one is the liquid temperature is uniform. So, in this droplet evaporation we will have this where already a wet bulb temperature has been attained.

So, basically what happens I will illustrate here. When you try to draw the d^2 versus T in any realistic environment, let us say we will also normalize this by the initial droplet diameter d_0 and square it; so, d_0 is the initial droplet diameter, So, when I draw the curve of a d^2/d_0^2 versus time so, this will be 1.

So, initially what happens is normally the droplet diameter will increase, reach a maximum and then it decreases. Then you will get through what is called a constant evaporation regime and towards the end there will be some non-linearity again.

So, you can see that this is the time period, the evaporation rate will be steady. So, the total time period is this. So, this is the total droplet time period that is t_d out of which the major part. So, I will say t_s that is steady evaporation is found for this time period. So, initially there will be a increase in diameter due to what is called volumetric expansion. So, after volumetric expansion the droplet diameter will reach the maximum value and then decrease.

So, this phase basically where the evaporation starts and this is actually non-linear evaporation will take place. So, non-linear evaporation phase. So, volumetric expansion phase where from the initial droplet diameter, the droplet diameter increases to a maximum value. After that there is a non-linear evaporation state, then if you see t_s; this is the steady evaporation regime. So, this is the steady evaporation regime.

After that slightly non-linearity will come towards the end. So, you can see that this t_s/t the total droplet life time t_d will be 80% to 90%. So, we can say that t_s/t_d will be 0.8 to 0.85 or even

0.9. So, that is why we concentrate on the evaporation constant which is constant in nature. So, this assumption of quasi steady is valid; only very little time in the front and a very little time towards the end we have some nonlinearity, where the steady state will not be there.

But, major portion 80% to 85% of the droplet lifetime you will have a steady evaporation phase in most of the cases; not in all cases but in most of the cases. So, this type of steady evaporation is very much helpful for us. So, in this the liquid temperature will reach the wet bulb condition; that means, surface temperature has reached a constant value of T_s.

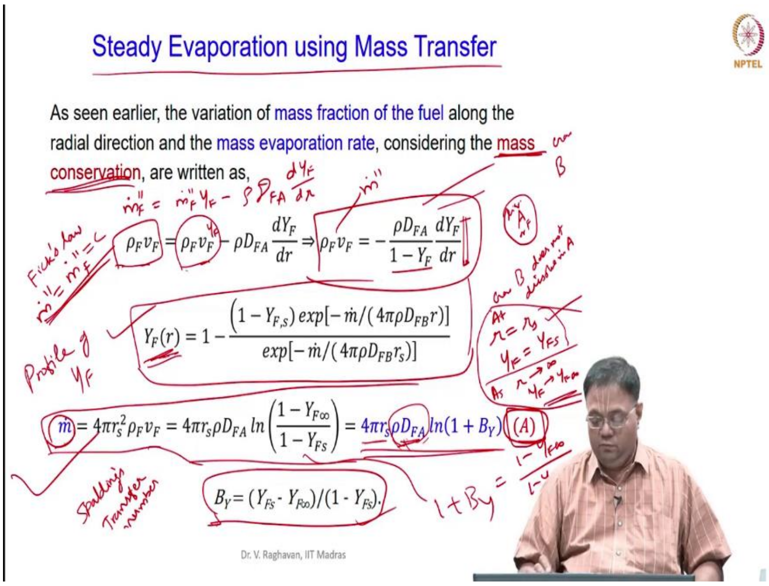
And, here the internal temperature also has become a uniform temperature; so, that is assumed. Then liquid vapor equilibrium or vapor liquid equilibrium at any point in the surface that is also achieved; that means, there is a definite T_s and the saturation pressure for this corresponding to this T_s prevails. So, that is also there, so the equilibrium evaporation occurs.

Property being constant is the assumption what we have to make because to reduce the complexity of the problem. So, all properties are constant. So, this particular analysis will be helpful, we are going to do this for the droplet combustion also. So, we can see that it is a spherically symmetric coordinate system. So, whenever vaporization take place the variations are only in r coordinate.

See for example, from the surface of the droplet where the mass fraction of the fuel is Y_{FS}. This decreases towards the ambient like this to a very low value when you go far off the radius this radial distance. Similarly, you can see that the temperature of the ambient is T_∞ which is higher and we can see that there will be a temperature which is going to vary like this. And, this will be the surface temperature T_s.

So, these variations are only in radial direction, in any θ and ϕ you will have only variations in the r; the variation in the r will be the same for any theta and phi. So, this is the spherical symmetric coordinate what we are going to consider for our problem which we have already seen for the mass transfer analysis. So, we will just recap that.

(Refer Slide Time: 07:51)



Steady Evaporation using Mass Transfer

As seen earlier, the variation of mass fraction of the fuel along the radial direction and the mass evaporation rate, considering the mass conservation, are written as,

$$\rho_F v_F = \rho_F v_F Y_F - \rho D_{FA}\frac{dY_F}{dr} \Rightarrow \rho_F v_F = -\frac{\rho D_{FA}}{1-Y_F}\frac{dY_F}{dr}$$

$$Y_F(r) = 1 - \frac{(1-Y_{F,s})\exp[-\dot{m}/(4\pi\rho D_{FB} r)]}{\exp[-\dot{m}/(4\pi\rho D_{FB} r_s)]}$$

$$\dot{m} = 4\pi r_s^2 \rho_F v_F = 4\pi r_s \rho D_{FA} \ln\left(\frac{1-Y_{F\infty}}{1-Y_{Fs}}\right) = 4\pi r_s \rho D_{FA} \ln(1+B_Y) \quad (A)$$

$$B_Y = (Y_{Fs} - Y_{F\infty})/(1 - Y_{Fs}).$$

Dr. V. Raghavan, IIT Madras

So, steady evaporation using the mass transfer here we have used the mass conservation which is nothing, but the Fick's law. So, Fick's law is written like this, again we can see that the droplet, the ambient gas. So, this is say, droplet liquid A and ambient gas B. So, B does not dissolve in A, liquid A.

So, this is gas and this is liquid. So, under this assumption the net flux of B will be 0; net flux of B will be 0 and only B diffuses as the A vapor. So, that the total mass flux will be only that of the fuel, here A or fuel F whatever we can say. So, the net flux is that of fuel, again you can see this will be Y_F.

So, $\rho_F v_F Y_F$ - ρD_{FA} so, when you write this Fick's law; this is nothing but mass flux at the surface of the fuel equal to mass flux of the fuel that is total mass flux which is equal to the mass flux of the fuel. Here the total mass flux will be equal to the mass flux of the fuel alone into Y_F minus this the diffusion term rho D_{FA}, A is the ambient fluid or ambient gas.

So, dY_F in the radial coordinate only where, normal derivatives, the ordinary derivative is used. So, using this you can bring this to the left side. So, $\rho_F v_F \times 1 - Y_F$ will be equal to the right-hand side is minus rho D_{FA} and the gradient. So, this flux mass flux, the total mass flux will be written in terms of the gradient of the mass fraction.

$$\rho_F v_F = \rho_F v_F - \rho D_{FA}\frac{dY_F}{dr} \Rightarrow \rho_F v_F = -\frac{\rho D_{FA}}{1-Y_F}\frac{dY_F}{dr}$$

So, if you applied this for the surface you will know the mass fraction. So, this is the equation which is just got by the mass conservation and by integrating this you get the profile of the mass fraction the profile of the mass fraction is got. So, here you integrate this and apply the boundary condition.

So, at the surface when r = r_s $Y_F = Y_{FS}$ and so, this is the first boundary condition and second boundary condition is as r → ∞. So, it may not be infinity, it may be a large value of r then we can see $Y_F \rightarrow Y_{F,\infty}$. So, this can be taken as 0 at large r radius. So, we already seen those.

When you apply this boundary conditions to this governing equation and integrate this then you get the profile of Y_F at any radial location r. Profile of Y_F at any radial location r is given by this. So, you can see this, it is given by 1 - Y_{FS} that is the value at the surface into exponent of -m; please understand that this is constant.

$$Y_F(r) = 1 - \frac{\left(1 - Y_{F,s}\right) exp[-\dot{m}/(4\pi\rho D_{FB} r)]}{exp[-\dot{m}/(4\pi\rho D_{FB} r_s)]}$$

So, this will be equal to constant; so, that comes out here. So, once you know the mass burning rate or evaporation rate you can calculate Y_F or vice versa. So, this is the equation you get. So, 4πρD_{FB} into r similarly exponent of this at r_s. So, when I apply this boundary condition you get this, one of the boundary conditions, that is we apply this boundary condition here.

So, first you integrate this equation, the flux the first order derivative you integrate this once and apply one of the boundary conditions, you get this profile Then what you do is you apply the second boundary condition and so, when you apply this condition r → ∞, $Y_F = Y_{F,\infty}$ and that that can be taken as 0; then you will get this mass burning rate.

So, the procedure what we saw already was you apply the Fick's law, understand that the gas B or the ambient fluid will not dissolve into the liquid droplet. And, the total flux will be equal to the mass flux of the fuel alone. So, using this Fick's law, you can evaluate the mass flux as a function of the derivative of the mass fraction of the fuel.

And once you do this, then integrate this with two boundary condition; apply one boundary condition to get the profile. Then using the profile apply the second boundary condition to get the mass evaporation rate itself. So, when you do that, you get the mass evaporation rate in which there is a natural logarithm term and this term you try to write as 1 + B_Y.

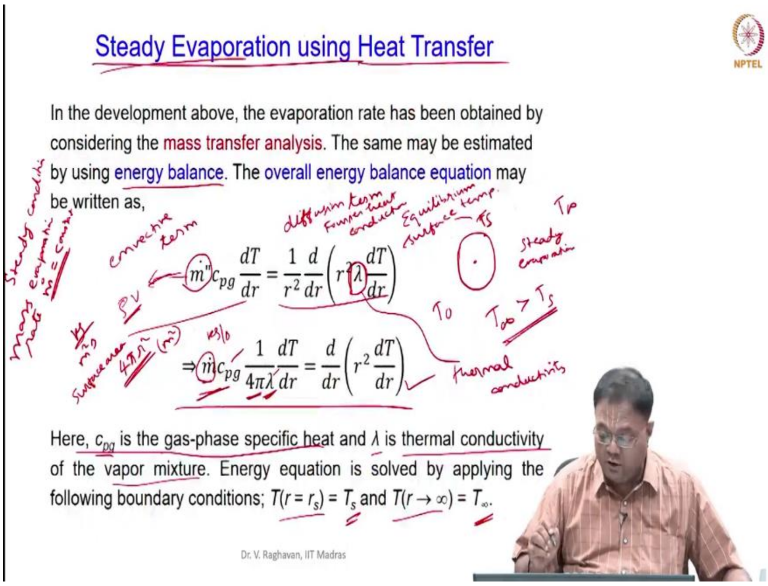
So, 1 + B_Y = 1 - $Y_{F,\infty}$ / 1 - Y_{FS} so; that means, B_Y will be the Spalding's transfer number. So, that will be written as Y_{FS} - $Y_{F,\infty}$ / 1 - Y_{FS}. So, this is the Spalding's transfer number. So, we have seen this. So, this equation is called the equation A.

So, the mass evaporation rate got by analyzing the steady evaporation problem only using the mass conservation, you get as equation for the mass evaporation rate $\dot{m}$ which is given as 4πr_s ρD_{FA}ln(1 + B_Y), where B_Y is the Spalding's transfer number given as Y_{FS} - $Y_{F,\infty}$ / 1 - Y_{FS}. So, this is the equation A which is got from the mass transfer analysis, which you have already seen in the mass transfer basics.

$$\dot{m} = 4\pi r_s^2 \rho_F v_F = 4\pi r_s \rho D_{FA} \ln\left(\frac{1 - Y_{F\infty}}{1 - Y_{Fs}}\right) = 4\pi r_s \rho D_{FA} \ln(1 + B_Y) \quad (A)$$

$$B_Y = (Y_{Fs} - Y_{F\infty})/(1 - Y_{Fs}).$$

(Refer Slide Time: 14:09)



Steady Evaporation using Heat Transfer

In the development above, the evaporation rate has been obtained by considering the mass transfer analysis. The same may be estimated by using energy balance. The overall energy balance equation may be written as,

$$\dot{m}'' c_{pg} \frac{dT}{dr} = \frac{1}{r^2}\frac{d}{dr}\left(r^2 \lambda \frac{dT}{dr}\right)$$

$$\Rightarrow \dot{m} c_{pg} \frac{1}{4\pi\lambda}\frac{dT}{dr} = \frac{d}{dr}\left(r^2 \frac{dT}{dr}\right)$$

Here, c_{pg} is the gas-phase specific heat and λ is thermal conductivity of the vapor mixture. Energy equation is solved by applying the following boundary conditions; $T(r = r_s) = T_s$ and $T(r \to \infty) = T_\infty$.

Dr. V. Raghavan, IIT Madras

Now, let us try to do a problem using heat transfer. So, you know that there is a droplet and droplet has a surface temperatures that is T_s and ambient temperature is T_∞. Now, $T_\infty > T_s$. So, initially we can see that the temperature of the droplet may be at say T_0 and this T_0 first will be heated up to an equivalent temperature.

So, this surface temperature, T_s is the equilibrium surface temperature or the wet bulb temperature. Now, since $T_\infty > T_s$ here, the heat is added from the ambient to the droplet in order to increase the temperature from this to this. Once the equivalent temperature is attained, then steady evaporation occurs.

Now, let us try to obtain the mass burning rate using the energy equation. So, energy balance is written like this. So, we can see that m; so, this is nothing but ρv, this flux, $\dot{m}''$ is the flux, $\rho v \times c_p dT/dr$. So, convective term, this is the convective term equal to the diffusion term that is the heat conduction.

$$\dot{m}'' c_{pg} \frac{dT}{dr} = \frac{1}{r^2}\frac{d}{dr}\left(r^2 \lambda \frac{dT}{dr}\right)$$

So, Fourier heat conduction. So, this λ is the thermal conductivity. Now, you can see this is the energy balance what we have, there is no source term or any reaction chemical reaction anything. So, this is what we have. In fact, for the diffusion flame we do not need to consider the chemical reaction at all.

So, now, the flux you can multiply by $4\pi r^2$ the surface area in order to get the flow rate that is kg/s. So, this is actually kg/m^2s multiplied by the surface area that is in m^2 you get this $\dot{m}c_{pg}$. So, $4\pi r^2$ multiply and divide by 4π; so, you get and the properties are constant.

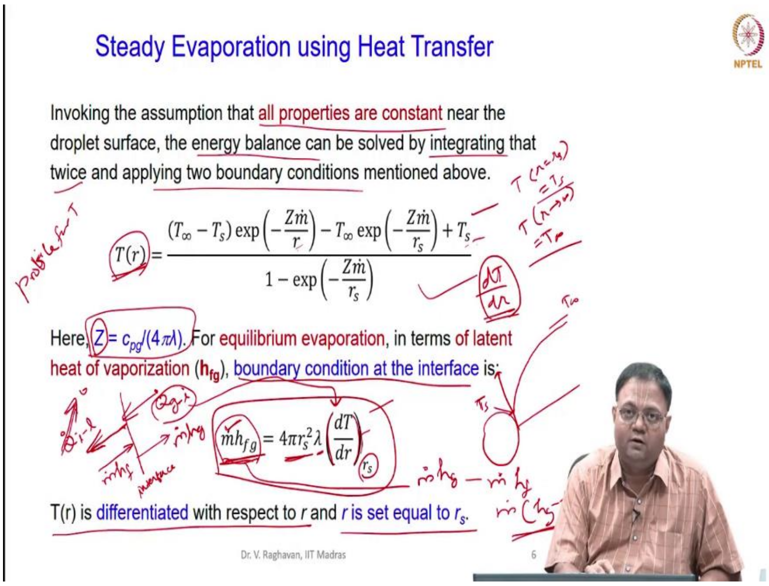
So, thermal conductivity which is in the right-hand side previously now is moved down here and you get this equation. Now, you have a second order equation in temperature and we know that the properties are taken constant. So, c_{pg}, λ are taken as constants here and the mass flow rate is written here $\dot{m}$ that is the evaporation rate. So, that is the happening only at the interface. Now, as I told you c_{pg} is the gas phase specific heat and λ is the thermal conductivity of the vapor mixture, gas thermal conductivity. So now, there are boundary condition, the two locations as we saw in the mass transfer at the interface $r = r_s$ there is the equilibrium temperature prevailing T_s; $T = T_s$. Similarly, when $r \rightarrow \infty$ or large radius you get the ambient temperature T_∞.

So, these are the boundary conditions and we will try to solve this problem. So, we can see that the properties are constant. So, we know that the mass evaporation rate under steady condition steady mass evaporation rate $\dot{m}$ equal to constant so, $\dot{m}$ is constant. So, c_{pg} is constant, λ is constant; so, this is a constant basically.

So, the integration of this equation is quite possible now. So, because you assume that the properties are constant. So, either properties are held constant or they have to be calculated at an appropriate average temperature. So, when you do that, then we can get proper values for the mass evaporation rate. So, this equation is second order equation which can be integrated twice and apply the boundary condition to get the values. So, that is what we are going to do next.

$$\dot{m}c_{pg}\frac{1}{4\pi\lambda}\frac{dT}{dr} = \frac{d}{dr}\left(r^2\frac{dT}{dr}\right)$$

(Refer Slide Time: 18:42)



Steady Evaporation using Heat Transfer

Invoking the assumption that all properties are constant near the droplet surface, the energy balance can be solved by integrating that twice and applying two boundary conditions mentioned above.

$$T(r) = \frac{(T_\infty - T_s)\exp\left(-\frac{Z\dot{m}}{r}\right) - T_\infty \exp\left(-\frac{Z\dot{m}}{r_s}\right) + T_s}{1 - \exp\left(-\frac{Z\dot{m}}{r_s}\right)}$$

Here, $Z = c_{pg}/(4\pi\lambda)$. For equilibrium evaporation, in terms of latent heat of vaporization (**h**$_{fg}$), boundary condition at the interface is;

$$\dot{m}h_{fg} = 4\pi r_s^2 \lambda \left(\frac{dT}{dr}\right)_{r_s}$$

T(r) is differentiated with respect to r and r is set equal to r_s.

Dr. V. Raghavan, IIT Madras

6

So, as I told you all the properties are assumed to be constant and I can integrate the energy balance equation 2 times and apply the boundary conditions what we have solved at a $T(r=r_s)$ equal to T_s and T_∞ as r → ∞. When you apply that, you get this say first of all you can get their profile for temperature, a radial profile.

$$T(r) = \frac{(T_\infty - T_s)\exp\left(-\frac{Z\dot{m}}{r}\right) - T_\infty \exp\left(-\frac{Z\dot{m}}{r_s}\right) + T_s}{1 - \exp\left(-\frac{Z\dot{m}}{r_s}\right)}$$

And, you can see that T_∞ - T_s which is the driving force for us into exponent of Zm/r. Now, please understand that the property is constant, we are defining a constant like c_{pg}, the constant 4πλ. So, that c_{pg}/4πλ which is a constant, now it has been designated as Z. So now $Z\dot{m}$, but $\dot{m}$ I have kept separately here; because we have to find the mass flow rate of the vapors later.

So, $Z\dot{m}$/r is, Z is the constant which combines the property c_p and λ. So now, this is -T_∞ into exponent of the same value, but here this is the constant r_s given surface temperature plus the surface temperature.

Now, you can see the boundary conditions here. So, when r = r_s, when r = r_s; I put r_s here, s here in this r then we can see that this T_∞; this term will cancel and T_s will be there.

So, T will tend to T_s, when r tends to ∞; r → ∞, then what happens is, so, this will become exponent of something by ∞ so, 0. So, this will be 1. So, when this is 1 and this T_∞ - T_s, -T∞ of this plus T_s. So, this and the denominator will cancel; so, we will get only T_∞.

So, you can see that this profile obeys the two boundary conditions, what we have. When you substitute r = r_s you get back T_s, when you substitute r → ∞ here you will get the value of T as

T_∞. So, this is the profile. So, when you solved this is an exponentially varying profile, as I told you from this droplet surface you try to draw this.

So, here this is T_s and this is say T_∞ which is a high temperature here. So, you will get the profiles are non-linear and like this. So, the temperature profile will be like this.

So, this is T_s and this is T_∞. So, this is what we are trying to get here and now to find the mass evaporation rate, we need the boundary condition at the interface correct. So, you have the droplet surface here, temperature is T_s

So, let us see the heat balance, heat balance at the interface. So, this is interface now some heat comes through from the gas phase to the interface like this; coming from the gas phase to the interface. That will be taken inside Q_{i-l}, interface to the liquid phase that way partial part of this can be used for heating of the liquid.

But anyway, this is not going to be with us because we have already assumed that the droplet has an uniform temperature and there is no heat transfer due to that; so, that is one. Now, what happens in the absence of that there will be a mass flow rate of this with its own enthalpy; so, liquid enthalpy h_f. Similarly, in this gas phase there will be a mass flow rate with the vapor enthalpy h_g.

Now, we can write the so, this Q_{i-g} is written as the conduction term here, conduction term. So, the surface area into the thermo conductivity λ into the derivative of the temperature at the interface; at r_s. So, this is the Q_{g-i} term; so, this is positive to this, please understand that negative or negative will be positive.

So, here the sign is opposite and that is the direction of the decrease in temperature, but that is inverse like reverse of the r. So, you get a positive sign here and that will be equal to this term actually will be equal to $\dot{m}$ which is going this way int h_g - $\dot{m}h_f$ which will be $\dot{m}(h_g - h_f)$ which is the $\dot{m}h_{fg}$ term; so, that will balance.

$$\dot{m}h_{fg} = 4\pi r_s^2 \lambda \left(\frac{dT}{dr}\right)_{r_s}$$

So, this is going this way and this heat is coming in the opposite direction. So, this is in the positive r direction and the heat is in the negative r direction. But anyway, the magnitude is positive here; so, we can get this relationship. So, this is the boundary condition at the interface.

So, until the wet bulb temperature is reached or equilibrium condition is reached there is no heat transfer from the interface to the liquid phase. This heat transfer is not present. So, whatever heat comes from the gas phase that is Q_{g-i}, gas phase to interface is used only to supply the latent heat of vaporization, that is $\dot{m}h_{fg}$

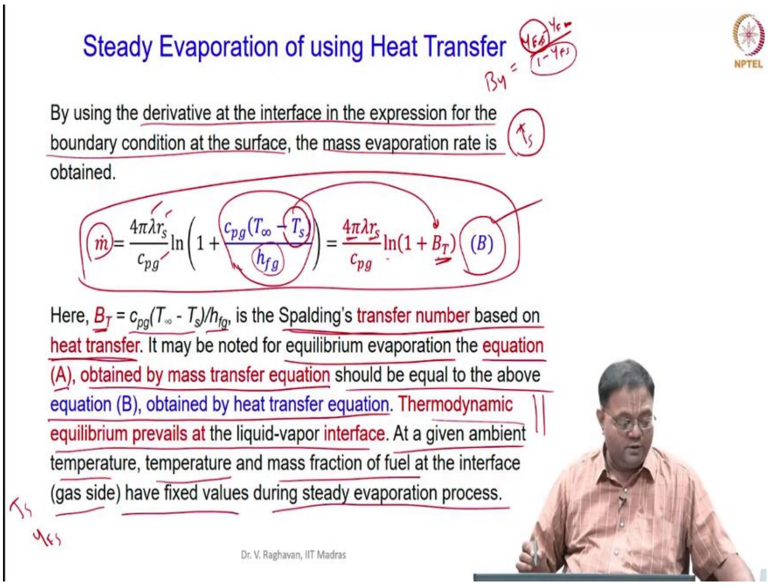
So, this is the boundary condition here. Now, temperature profile is known to you. So now, knowing the temperature profile we can differentiate this profile to get dT/dr; because I need dT/dr and this boundary condition. So, I want to evaluate this boundary condition because by evaluating the boundary condition I can find the $\dot{m}$. For example, in this itself the $\dot{m}$ is not known to me.

So, let us also assume that when you calculate $\dot{m}$, we have to use this boundary condition; the assumption made is the liquid temperature is uniform. So, $Q_{i-l} = 0$, if it is not 0 also, we can make some, add one more term here. For example, if it is not 0, I have to add appropriate terms to the left-hand side, but anyway to simplify the problem we have assume that as 0.

So, now calculate this dT/dr using this profile. So, differentiate this with respect to r and substitute here. So, what we are trying to do here is in order to get this derivative, I will first differentiate this profile with respect to r. So, T_r is differentiated with respect to r, then whatever r term present in the derivative will be set to r_s; so, that I will gt the value of the derivative dT/dr at r_s.

So, that is what we will try to do and substitute here. Once I differentiate it and substitute r = r_s, I will put that value here and calculate the value of $\dot{m}$.

(Refer Slide Time: 25:58)



Steady Evaporation of using Heat Transfer

By using the derivative at the interface in the expression for the boundary condition at the surface, the mass evaporation rate is obtained.

$$\dot{m} = \frac{4\pi\lambda r_s}{c_{pg}}\ln\left(1+\frac{c_{pg}(T_\infty - T_s)}{h_{fg}}\right) = \frac{4\pi\lambda r_s}{c_{pg}}\ln(1+B_T) \quad (B)$$

Here, $B_T = c_{pg}(T_\infty - T_s)/h_{fg}$, is the Spalding's transfer number based on heat transfer. It may be noted for equilibrium evaporation the equation (A), obtained by mass transfer equation should be equal to the above equation (B), obtained by heat transfer equation. Thermodynamic equilibrium prevails at the liquid-vapor interface. At a given ambient temperature, temperature and mass fraction of fuel at the interface (gas side) have fixed values during steady evaporation process.

Dr. V. Raghavan, IIT Madras

So, by using derivative at the interface in the expression for the boundary condition at the surface, the mass evaporation rate is got like this. So, this is the equation B which gives the mass evaporation rate, this is calculated from heat transfer. The equation A on the other hand here, was calculated using the mass transfer.

$$\dot{m} = \frac{4\pi\lambda r_s}{c_{pg}}\ln\left(1+\frac{c_{pg}(T_\infty - T_s)}{h_{fg}}\right) = \frac{4\pi\lambda r_s}{c_{pg}}\ln(1+B_T) \quad (B)$$

So, we can see this same type of expression is coming, but the property values are different; see $4\pi\lambda/c_{pg}$, r_s is there common plus natural logarithm of 1 plus a quantity.

There also we saw $1 + B_Y$, here this quantity, $c_{pg} \times (T_\infty - T_s)/h_{fg}$; latent heat of vaporization. This is called B_T, that is the Spalding's transfer number based on heat transfer B_T which is nothing, but $c_{pg}(T_\infty - T_s)/h_{fg}$.

So, this is actually a ratio we can easily understand that this is the heat transfer which is available from the gas phase to the interface. And, this is the energy which is required, h_{fg} represents the energy which is required for evaporation. So, this ratio, the available energy for evaporation; so, this is the energy which can be fed from the gas phase to this to the interface. And, this is energy h_{fg} represents the energy which is required for evaporation.

So, this ratio is basically called the transfer number, transfer number based on heat transfer application. So, you get this equation B which is nothing, but $4\pi\lambda$, the thermal conductivity r_s, the instantaneous radius of the surface and by c_{pg} into natural logarithm of $1 + B_T$. Now, please understand that we have already got an expression with respect to mass transfer which was $4\pi r_s$ ρDB

So, that is the driving function for the mass transfer, is $\rho D\ln(1+B_Y)$. So, $4\pi r_s$ is common driving force, the driving property is ρD in the mass transfer. So, $4\pi r_s$ the driving force, let us say $\rho D\ln(1+B_Y)$; so B_Y. So, Y represents the mass fraction of the fuel. So, mass transfer-based transfer number.

So, mass evaporation rate is very simple, it is $4\pi r_s$ into a driving force which is the property which is driving a particular transport process; in the mass transfer ρD is the driving force into natural logarithm of 1 plus transfer number; so, this is A. Similarly, you can see here $4\pi r_s$ driving force is λ/c_{pg}; because heat transfer, you have the thermal conductivity as the driving force.

So, this λ/c_{pg} into ln of 1 plus and transfer number; so, it is very simple. Evaporation rate will be equal to $4\pi r_s$ into a driving force which is nothing, but the property involved in the transfer process which is ρD in the mass transfer, λ/c_{pg} in the heat transfer into natural logarithm of 1 plus a transfer number.

So, transfer number is appropriately a defined for mass transfer and heat transfer. So, for mass transfer B_Y was defined as $Y_{FS} - Y_{F,\infty} / 1 - Y_{FS}$. So, here B_T for the heat transfer is defined as c_{pg}. So, here please understand that in this the physical significance is the difference between the surface mass fraction of the fuel, Y_{FS} and the term infinity that actually represents the gradient.

So, the gradient of the fuel mass fraction drives the mass there. So, that with the 1 minus how much different the surface mass fraction is with respect to 1; so, that is the ratio there. In this as I told you, the energy which is a transferred from the ambient divided by the energy which is required for evaporation; so, that will be the transfer number.

So, having two different equations here, but the equation structure is the same $4\pi r_s$ into the driving parameter which is the property which drives the transport process into natural logarithm 1 plus a transfer number; transfer number correct. However, two equations are there, but here in this equation B we have considered only the heat transfer. In the equation A, we have considered only the mass transfer; but actually, you will see that both should be coupled. So, when you want an equilibrium evaporation or steady state evaporation; equation A which is obtained by the mass transfer should be equal to the equation B which is obtained by the heat transfer. Because, this will ensure that the thermodynamic equilibrium prevails at the vl, that is the vle that is the liquid vapor interface that with thermodynamic equilibrium. So, it is called VLE: Vapor Liquid in Equilibrium at the interface. So, this will be very important. So, as I told you there is a unsteady evaporation which is going on initially. But, once that condition has reached where the heat is only used from the ambient which is used only to supply for latent heat; then we move on to what is called as equilibrium evaporation. So, what is the temperature attained at the equilibrium evaporation condition? So, that is what we are trying to find.

So, for that one equation is not required. See for example, now also there is a problem; when you use this how you how will you determine the T_s; when you use the equation A how will you determine Y_{FS}? So, unless these two are combined, we cannot solve the problem. So, as we were mentioning in the mass transfer basics, we were talking only about Y_{FS}.

And, we told that the energy balance is required to find T_s and from the T_s we can calculate the Y_{FS}, but for equilibrium both should balance. So, that is what we are going to do next. So, at a given ambient temperature, temperature and mass fraction of the fuel at the interface that is T_s and Y_{FS} have fixed values, and that is basically which is obtained during a steady evaporation process.