Fundamentals of Combustion

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Lecture - 45

Laminar Diffusion Flames – Part 5

Diffusion flame regimes and Flame height correlations

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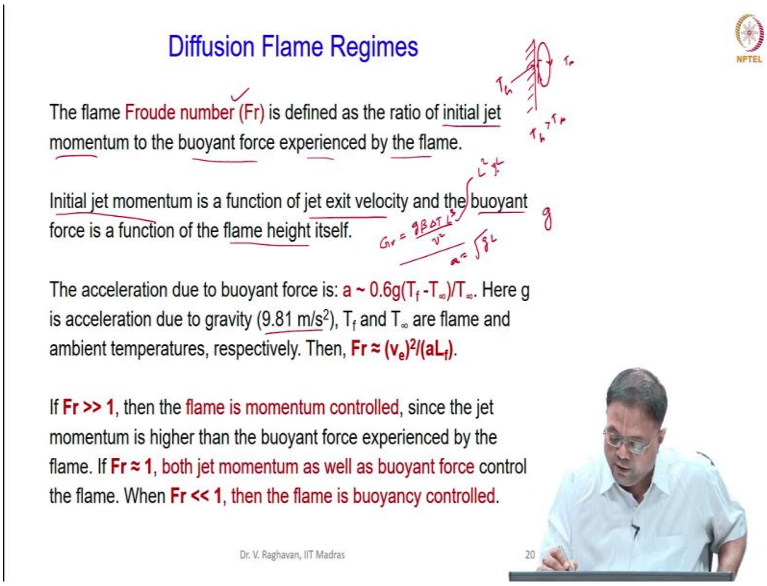

Diffusion Flame Regimes

The flame Froude number (Fr) is defined as the ratio of initial jet momentum to the buoyant force experienced by the flame.

Initial jet momentum is a function of jet exit velocity and the buoyant force is a function of the flame height itself.

The acceleration due to buoyant force is: $a \sim 0.6g(T_f - T_\infty)/T_\infty$. Here g is acceleration due to gravity (9.81 m/s^2), T_f and T_∞ are flame and ambient temperatures, respectively. Then, $Fr \approx (v_e)^2/(aL_f)$.

If $Fr >> 1$, then the flame is momentum controlled, since the jet momentum is higher than the buoyant force experienced by the flame. If $Fr \approx 1$, both jet momentum as well as buoyant force control the flame. When $Fr << 1$, then the flame is buoyancy controlled.

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So, let us continue with the diffusion flame regimes which is the controlling aspect of a diffusion flame for which we actually go for defining what is called flame Froude number. So, this is ratio of the initial jet momentum to the buoyant force experienced by the flame.

So, what is buoyant force? If you take a hot wall at temperature T_h and the ambient is T_∞ which is colder. So that means, $T_h > T_\infty$, then you can see that there will be a natural convective flow which is generated adjacent to the wall. Very close to the wall you will see a natural convective flow.

So, similar to this, flame is a hot surface and so, that will lead to low densities for the fluid which is the burned product of combustion and so, they will rise up. So, the acceleration of the hot products basically will cause the buoyant force.

So, the entire length of the flame will be experiencing that, wherever it is hot. So, that is due to the density gradient. So, you can see that the initial jet momentum is actually affected by the jet exit velocity. So, if we increase the velocity, the momentum will increase. Buoyant force as I told you, the entire flame length will affect it.

So, the acceleration due to buoyant force basically depends on the g value. So, acceleration due to gravity which is a 9.81 m/s^2. For example, if you take say Grashof number, this is $g\beta\Delta T(L_c)^3/\nu^2$ (L_c - any characteristic length). So, that is the Grashof number. So, this is the non-dimensional number which will control the natural convection.

For example, if you take this g×L then L^2. So, we can say $\sqrt{gL}$, that will be the velocity term. So, if you take root of Grashof number basically put $\sqrt{gL}$ and L^2. So, this L^3 can be written as $L^2 \times L$ and $\sqrt{gL}$ can be written here. So, if you take this $\sqrt{gL}$ that will be the velocity term.

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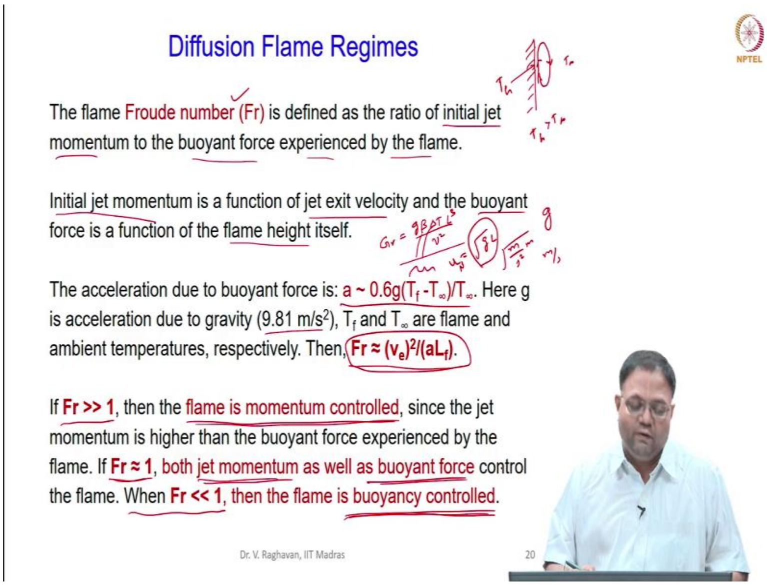

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If **Fr >> 1**, then the flame is momentum controlled, since the jet momentum is higher than the buoyant force experienced by the flame. If **Fr ≈ 1**, both jet momentum as well as buoyant force control the flame. When **Fr << 1**, then the flame is buoyancy controlled.

So, this will be the natural convective velocity, velocity due to natural convection. So, you can see the units also $\sqrt{m \times m/s^2}$. So, it will be m/s. So, this is the velocity which is actually induced. So, this $\sqrt{gL}$ will be the natural convective velocity, the average magnitude of the natural convective velocity.

So, this acceleration depends upon the acceleration due to gravity. Generally we can use the acceleration induced due to buoyant force as $0.6g\times(T_f - T_a)$, it has some scaling factor 0.6, its approximate please see that into T_f that is the flame temperature minus the ambient temperature.

So, this actually is the ΔT. So, this is $\Delta T/T_\infty$. So, this $1/T_\infty$ is like β. So, this particular term will be the acceleration term for us.

So, this flame acceleration is due to the density gradient. So, from that flame to the ambient, the density gradient because temperature is inversely proportional to the density. So, the density gradient is actually represented here.

So, the Froude number is written as to represent the jet momentum v^2 term comes here, to represent the buoyancy force $a \times L_f$ comes here. So, this is the Froude number. So, flame Froude number defined like this can be used to determine the regime which is controlling the jet flame.

For example, if you take Froude number much higher than 1, so that means, either v^2 is very high or the acceleration due to buoyancy force or the buoyancy force itself is very low. So, the flame is momentum controlled, controlled by the momentum of the jet itself.

On the other hand, the other extreme case, if Froude number is much less than 1 then the flame is buoyancy control. So, you can see that if it is less than 1, v^2 will be very low or the buoyancy force will be very high. In both the cases you can see that it will be buoyancy controlled regime. If you have the Froude number of the order of 1 then both jet momentum as well as the buoyancy force will control this.

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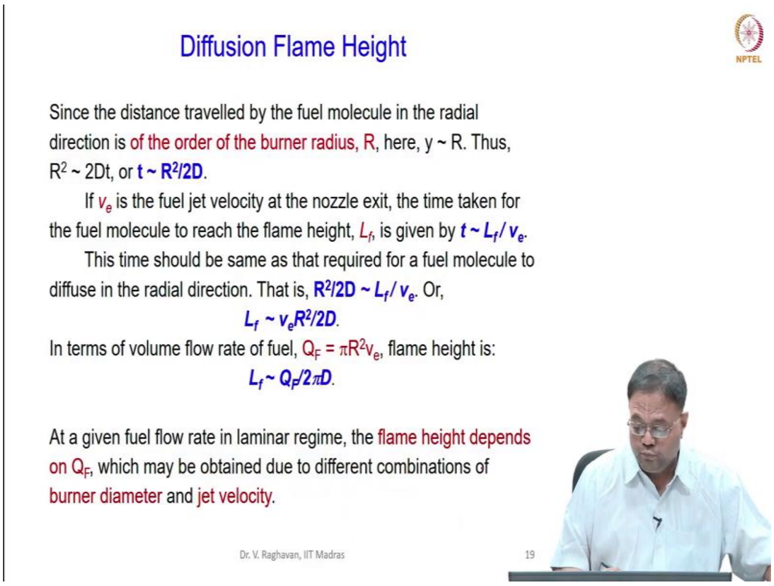

Diffusion Flame Height

Since the distance travelled by the fuel molecule in the radial direction is of the order of the burner radius, R, here, $y \sim R$. Thus, $R^2 \sim 2Dt$, or $t \sim R^2/2D$.

If v_e is the fuel jet velocity at the nozzle exit, the time taken for the fuel molecule to reach the flame height, L_f, is given by $t \sim L_f/v_e$.

This time should be same as that required for a fuel molecule to diffuse in the radial direction. That is, $R^2/2D \sim L_f/v_e$. Or,

$$L_f \sim v_e R^2/2D.$$

In terms of volume flow rate of fuel, $Q_F = \pi R^2 v_e$, flame height is:

$$L_f \sim Q_F/2\pi D.$$

At a given fuel flow rate in laminar regime, the flame height depends on Q_F, which may be obtained due to different combinations of burner diameter and jet velocity.

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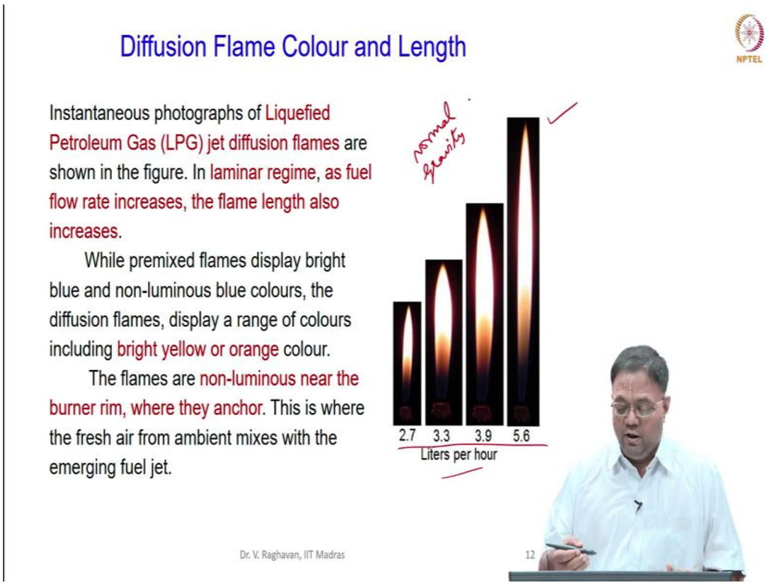

Diffusion Flame Colour and Length

Instantaneous photographs of Liquefied Petroleum Gas (LPG) jet diffusion flames are shown in the figure. In laminar regime, as fuel flow rate increases, the flame length also increases.

While premixed flames display bright blue and non-luminous blue colours, the diffusion flames, display a range of colours including bright yellow or orange colour.

The flames are non-luminous near the burner rim, where they anchor. This is where the fresh air from ambient mixes with the emerging fuel jet.

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So, as I told you in these figures you can see all photographs which shows that diffusion flame at several flow rates under normal gravity. So, under normal gravity condition you get these flames. If you actually try to reduce the gravity or try to do experiments in space or microgravity etcetera the flame shape will be entirely different, flame length will be different and so on. So, that is the important thing you should understand.

So, which force is going to control, whether it is momentum control or buoyancy control or both are important? So, we have to understand that. So, based upon that theoretical correlations are developed.

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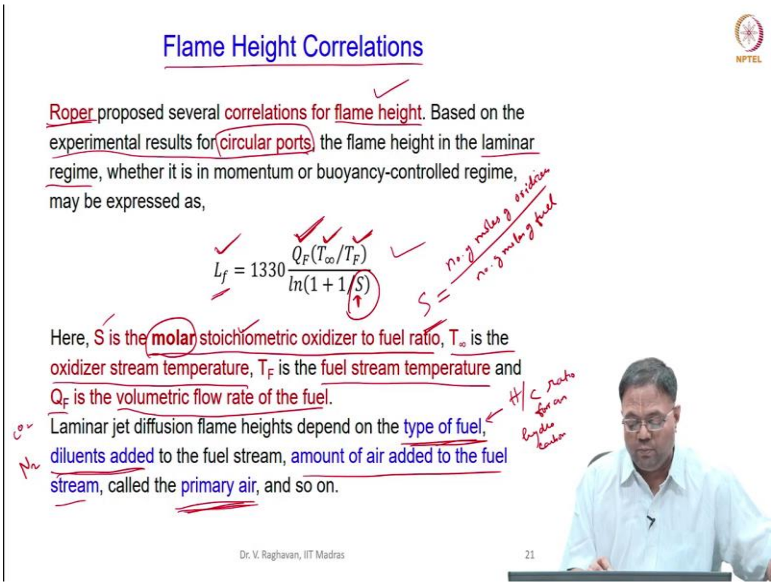

Flame Height Correlations

Roper proposed several correlations for flame height. Based on the experimental results for circular ports, the flame height in the laminar regime, whether it is in momentum or buoyancy-controlled regime, may be expressed as,

$$L_f = 1330\frac{Q_F(T_\infty/T_F)}{ln(1+1/S)}$$

$$S = \frac{\text{no. of moles of oxidizer}}{\text{no. of moles of fuel}}$$

Here, S is the molar stoichiometric oxidizer to fuel ratio, T_∞ is the oxidizer stream temperature, T_F is the fuel stream temperature and Q_F is the volumetric flow rate of the fuel.

Laminar jet diffusion flame heights depend on the type of fuel, diluents added to the fuel stream, amount of air added to the fuel stream, called the primary air, and so on.

← H/C ratio for an hydro carbon

CO_2

N_2

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So, let us see some flame height correlations we will see only empirical correlations here.

So, Roper was the researcher who proposed several correlations for flame height. So, based upon experimental results, that is called empirical correlations and I have given this correlation basically used for circular cross sectional ports, and we are considering only the laminar regime here.

So, in the laminar regime considering circular ports for several fuels the Roper's correlation for flame height is given here that is the length of the flame or the flame height will be equal to 1330 × Q_F. Q_F is the volumetric flow rate of the fuel given here.

And T_∞ is the temperature of the oxidizer stream and T_F is the fuel stream temperature and S which is appearing in the bottom in a natural logarithm term is the molar stoichiometric oxidizer fuel ratio, molar based. Please understand when I say air fuel ratio or oxygen fuel ratio in general, we write it in terms of a mass of the oxidizer divided by the mass of the fuel, but specifically S which is used in the correlation is molar.

$$L_f = 1330\frac{Q_F(T_\infty/T_F)}{ln(1+1/S)}$$

So, that is number of number of moles of oxidizer divided by the number of moles of fuel. So, considering stoichiometric combustion. Stoichiometric combustion, in the diffusion flames the flame zone is formed because the fuel and oxidizer are separately supplied to the combustion chamber.

So, based upon the locations where the stoichiometric mixing takes place the flame zone is formed there. So, stoichiometric oxidizer to fuel ratio is considered here. So, number of moles of oxidizer divided by number of moles of fuel that is S. So, this correlation clearly mentions that L_f is directly proportional to the flow rate which we have already seen by the scaling analysis.

In the scaling analysis, we saw by considering the time it takes to reach from the central axis to the radial location of the flame by diffusion or the time taken by the molecule from the base of the flame to the tip of the flame by convection both should be same. By equating that we found that the length of the flame or the flame height depends on the volumetric flow rate.

So, that is what is correlated here, basically that is very important. And please understand here we require the temperatures of the oxidizer and fuel stream and also the stoichiometric molar oxidizer to fuel ratio. So, with this L_f has been correlated. For several fuels experimental results have been used to arrive at this empirical correlation.

So, as we have seen already the volumetric flow rate is very important. So, when you increase the flow rate, fuel typically takes more distance to find its oxidizer and completely oxidize. Other than that, we can see S will depend on the type of fuel and the diluents added. See for example, in a jet diffusion flame experiment only fuel is supplied from the port, but if you see all other phenomena are naturally occurring.

For example, the air entry to the flame is by natural entrainment then mixing is basically dependent on the radial diffusion and so on. But in several cases, we need to control the flame for which we normally try to add the diluents; diluents like nitrogen to the flame even CO_2 in some cases and so on. So, some diluents are added.

Or if you can add some amount of air to the fuel stream, but please understand that we are not completely pre mixing the fuel, we will add some amount of air also. So, that might not be in the flammability limits also. When you add something in the fuel stream that is called primary air. So, why we do this because to have better control of the flame.

We will see about that little bit later, but we have to understand that there are scenarios in which we will be able to add diluents like nitrogen carbon dioxide etcetera to the fuel side. Similarly, some amount of air can be added to the fuel side which is called primary air. So, if you do those things these are the parameters which are going to affect the flame length. So, in those cases how this correlation can be used, that we will see.

So, type of fuel, when I say type of fuel, if it is hydrocarbon, I will say the H/C ratio for a hydrocarbon. So, something like that. So, these are all very important things which will affect the length of the flame. So, we will see about this. So, once we change the fuel type etcetera S itself will change basically and based upon that the flame length will be affected. So, we will see how to modify this correlation and use it for several purposes.

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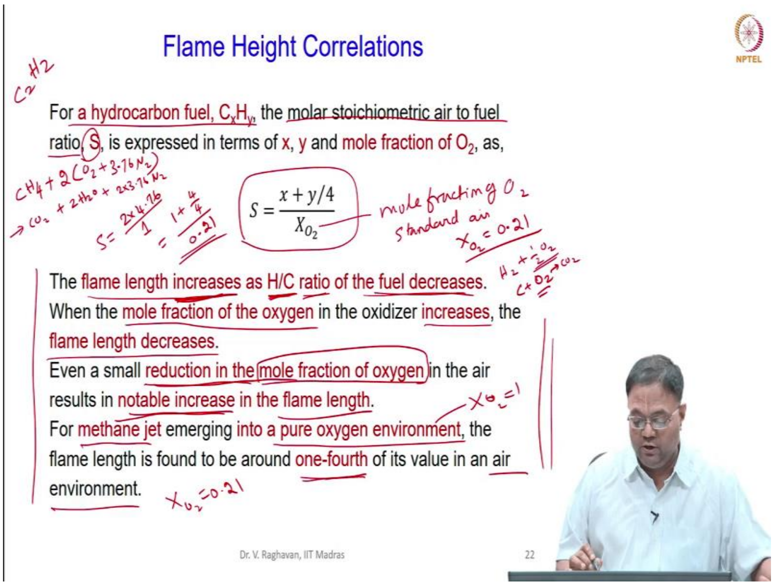

Flame Height Correlations

For a hydrocarbon fuel, C_xH_y, the molar stoichiometric air to fuel ratio, S, is expressed in terms of x, y and mole fraction of O_2, as,

$$S = \frac{x + y/4}{X_{O_2}}$$

The flame length increases as H/C ratio of the fuel decreases.

When the mole fraction of the oxygen in the oxidizer increases, the flame length decreases.

Even a small reduction in the mole fraction of oxygen in the air results in notable increase in the flame length.

For methane jet emerging into a pure oxygen environment, the flame length is found to be around one-fourth of its value in an air environment.

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Let us first see for a given hydrocarbon fuel C_x and H_y, x is the number of atoms of carbon and y is the number of atoms of hydrogen. Now, the molar stoichiometric fuel ratio S is given by this. So, that is x + y/4, that is the number of moles of oxygen required in the stoichiometric equation divided by the mole fraction of oxygen.

$$S = \frac{x + y/4}{X_{O_2}}$$

So, when you take a standard air you have X_{O2} will be equal to 0.21. So, this you can also do in another other way also. For example, if you have equation say $CH_4 + 2(O_2 + 3.76N_2) \rightarrow CO_2 + 2H_2O + 2\times3.76N_2$. So, this is the stoichiometric equation.

So, what we can do is, S can be calculated as number of moles of oxygen that is 2×4.76 / number of moles of fuel, which is 1. So, this is also correct. So, if you do this or you can do the way what we have given here; (1 + 4) / (4 / 0.21). So, that is for the standard air, that is the mole fraction of oxygen is 0.21; 79% of nitrogen, 21% of oxygen by volume.

So, the mole fraction is this. So, you can use this to get the value of S. Now, seeing this scenario say x + y/4 and this you can see that the flame length increases as H/C ratio of the fuel decreases. So, H/C if it decreases for example, this is the case CH_4.

Let us say C_2H_2, then you can calculate the H/C. So, physically what we should understand here is if H/C ratio decreases then the flame length increases. When H/C ratio of the fuel decreases so, why it is because if you take 1 mole of hydrogen you will require only half mole of oxygen. So, when the H/C decreases then your oxygen requirement will increase, because carbon requires 1 mole of O_2; see $C + O_2 \rightarrow CO_2$ for complete combustion. So, we can say 1 mole, here only half mole is required.

So, when H/C ratio decreases you can see that the oxygen amount will increase; that means, the S value will increase. So, that is the reason. So, the reactivity of the fuel will reduce. When H/C of the fuel decreases, the reactivity of the fuel will reduce. So, it may take higher length to completely burn, that is also another reason. So, we can use this and solve.

Now, if you see this correlation 1 + 1/S appears. So, here if S increases then this denominator 1/S will be lower. So, this term will be lower, the ln of this will be lower. So, the flame length increases. So, mathematically also we can see like that.

Now, coming to the mole fraction of the oxygen. So, I need not always use standard air, I can enrich the air with some more oxygen. For example, what will happen if you increase the mole fraction of oxygen from 0.21 say to 0.3 or 0.35 something like that? Then you can see that when you increase the mole fraction of oxygen there is a tremendous decrease in the flame length.

Or other way, if you decrease the mole fraction of oxygen, for example I dilute the oxygen stream with some products of combustion. See this is normally done to reduce the NO formation. Say in the standard air I will try to mix some products of combustion like CO_2, H_2O etcetera. So, when you dilute, this is called combustion gas recirculation.

If you do that then what happens is the effective mole fraction of oxygen in the air stream will come down. That will cause a notable increase in the flame length. On the other hand, as I told before, if you enrich the combustion; I want to get more higher temperature and so, I increase the oxygen mole fraction from 0.21 to say 0.3 then there is a reduction.

So, we can see that for methane fuel, if I use pure oxygen, that is X_{O2}. Here $X_{O2} = 1$, I say pure oxygen, if I take then you can see the flame length is reduced to about one fourth of the value for a given flow rate; obviously, 1/4 of the value that was observed in the air environment, when $X_{O2} = 0.21$.

When I use $X_{O2} = 0.21$, that is standard air, I get a flame length say $L_{f,1}$ for a given flow rate Q_{F1} and for the same $Q_{F,1}$, but I go to a pure oxygen environment now, inject this in a pure oxygen environment for the same flow rate, the reactivity increases very much and you can see that the flame length reduces by one-fourth of the value that is $L_{f,1}/4$.

So, these are something we should understand. When the H/C ratio of fuel decreases the reactivity decreases because hydrogen is reduced. The oxygen required will increase so that the flame length increases. Then mole fraction of the oxygen, when the mole fraction of the oxygen in the oxidizer increases then the flame length decreases that is what here, pure oxygen, 1 by fourth you can get.

So, a small reduction of oxygen mole fraction in the air will cause a notable increase. So, that is another important thing we should understand. So, these are the things which will affect the flame height.

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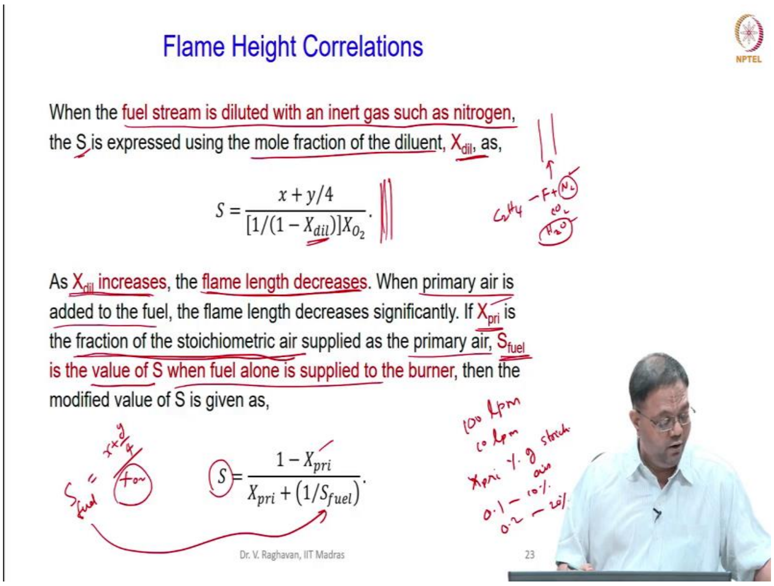

Flame Height Correlations

When the fuel stream is diluted with an inert gas such as nitrogen, the S is expressed using the mole fraction of the diluent, X_{dil}, as,

$$S = \frac{x + y/4}{[1/(1 - X_{dil})]X_{O_2}}.$$

As X_{dil} increases, the flame length decreases. When primary air is added to the fuel, the flame length decreases significantly. If X_{pri} is the fraction of the stoichiometric air supplied as the primary air, S_{fuel} is the value of S when fuel alone is supplied to the burner, then the modified value of S is given as,

$$S = \frac{1 - X_{pri}}{X_{pri} + (1/S_{fuel})}.$$

So, now we go for dilution. As I told you, in the fuel stream I can add some nitrogen or some CO_2 etcetera. When you use heavy fuels, you can also add some steam with this. So, lot of things we are doing.

For example, when I add some nitrogen or water vapor etcetera, we see that the soot formation decreases. Let us take this is say C_2H_4 or something like that. If we add some water vapor to this or nitrogen to this stream, a little bit not too much then we see that there is a decrease in the soot formation.

However, what happens to the flame height in that case? So, when the fuel stream is diluted with the inert gas such as nitrogen or as I told the other gases then S we have to recalculate by using this expression, where S = x + y/4 same but denominator you can see instead of just X_{O2}, I have $1/[1 - X_{dil}]X_{O2}$, there the mole fraction of the diluent is nothing but X_{dil}. So, where you can do this?

$$S = \frac{x + y/4}{[1/(1 - X_{dil})]X_{O_2}}$$

Now, what happens if you do this? If I increase the mole fraction of the diluent, the flame length decreases because the fuel component comes down. So, the flame will find enough oxygen within a shorter length. So, the heat release etcetera will come down, but anyway the flame length will decrease; obviously, because of the decrease in the fuel content.

Now, on the other hand as I told you, again for the same purpose of reducing the emissions, improving the combustion efficiency by completely burning CO etcetera we normally add some primary air to the fuel. So, it is actually counted in percentage of the stoichiometric air.

So, if stoichiometric air is 100%, I say I will supply about 10% or 20% air. So, normally I keep this mixture of fuel and primary air to a limit where it will be outside the rich limit of the premixed flame. So, that is what we will try to keep.

If X_{pri} is the fraction of the stoichiometric air supplied as primary air; please understand it is not mole fraction. So, if the quantity of air supplied, say stoichiometric air is some volumetric flow rate, let us say 100 lpm is the stoichiometric air requirement then X_{pri} can be 10% of that, say 10 lpm.

So, X_{pri} is actually the percentage of stoichiometric air. So, in some fraction, it is fraction. So, it may be 0.1 say, I can say 0.1, 0.2, etcetera for 10%, 20% and so on. So, now you please understand the X_{pri} is nothing but the fraction of the stoichiometric air supplied as a primary air along with the fuel. In that case how will I calculate the S?

So, S will be calculated using this, X_{pri} plus I will also use the S_{fuel} term which is the value of S when the fuel alone is supplied that is S_{fuel} will be equal to $(x + y/4) / X_{O2}$. So, please understand this X_{O2} is the oxygen mole fraction in the air stream. But this X_{pri} is the fraction of the stoichiometric air which is supplied along with the fuel. So, that we have to understand. So, this is calculated and is substituted here.

$$S = \frac{1 - X_{pri}}{X_{pri} + \left(1/S_{fuel}\right)}$$

So, other than that we have $1-X_{pri}$ which is the fraction of the fuel divided by X_{pri} + $1/S_{fuel}$ which is calculated like this. So, you can see that when you supply a primary air there is a clear reduction in the flame height. So, if S is modified, please understand this correlation basically in this Roper's correlation this S can take up several other factors.

For example, the fuel type itself. S will vary with fuel type, then if you add any diluents the S can be modified accordingly. Then when you add primary air S can be modified accordingly and recalculated and substituted.

So, other than that we have only simple terms like the oxygen temperature which we can easily measure, fuel stream temperature also easily measurable there is no other gas required like flame temperature etcetera are not used. Then volumetric flow rate anyway, you will measure this for the fuel stream, that we will measure.

So, if you know this and calculate S accordingly, for any scenario you can calculate the flame length and this actually comes out nicely for several cases. So, you can see that when primary air is added, there is a reduction in the flame height. That also we will demonstrate when we go ahead.

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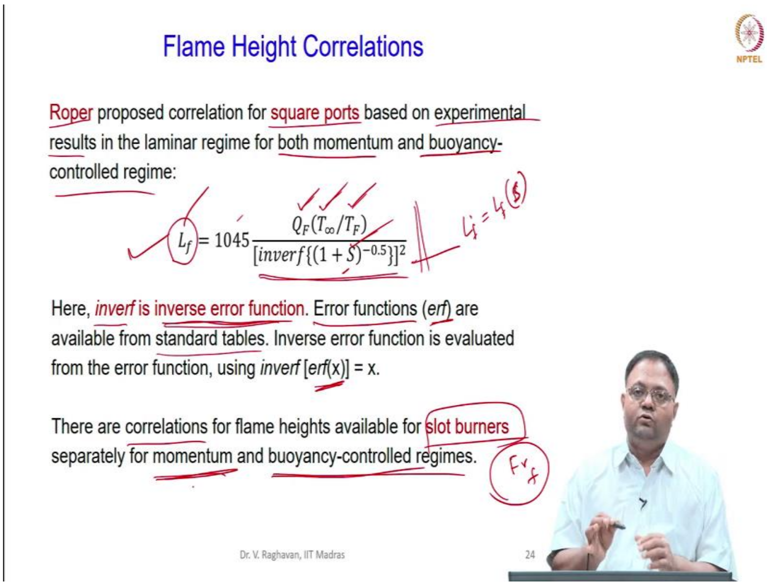

Flame Height Correlations

Roper proposed correlation for square ports based on experimental results in the laminar regime for both momentum and buoyancy-controlled regime:

$$L_f = 1045 \frac{Q_F(T_\infty/T_F)}{[inverf\{(1+S)^{-0.5}\}]^2}$$

Here, *inverf* is inverse error function. Error functions (*erf*) are available from standard tables. Inverse error function is evaluated from the error function, using *inverf* [*erf*(x)] = x.

There are correlations for flame heights available for slot burners separately for momentum and buoyancy-controlled regimes.

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Now, Roper's correlation for circular ports have been seen. Roper has also proposed correlation for square ports where this type of correlation is proposed. Again, its empirical correlation, experimental based results are used to calculate this. Similar terms are seen here, some constant volumetric flow rate. So, L_f depends directly on volumetric flow rate. Again, the temperature of oxidizer stream and fuel stream are incorporated.

$$L_f = 1045 \frac{Q_F(T_\infty/T_F)}{[inverf\{(1+S)^{-0.5}\}]^2}$$

However, instead of natural algorithm of here, for circular ports natural algorithm of 1 + 1/S was used. But here, the function for S in the denominator is different.

So, here what is used is the inverse function of the error. Inverse error function is used. Inverse of error function that is $(1/1+S)^{0.5}$. So, the denominator is total squared. So, this error function erf is tabulated. From standard tables we can get this. We can get the error functions for every value.

See for example, calculate this $[1/(1 + S)]^{0.5}$. So, you will have one value for that, you can find the error function. Once you find the error function inverse of the error function can be found by applying this. So, you know this value and from this value you can take this. So, it is only inverse of that. So, from the error function table itself we can recalculate the inverse error function. So, once you do this substitute that to get the flame length.

So, the functional dependent of S on the denominator, say for example here L_f. The functional dependent of L_f on S is different. So, we have used natural logarithm for the circular ports. Here inverse error-based functions for the this, for calculating the S calculation is same. Again, S for a normal oxidizer is calculated using x + y/4 by mole fraction of oxidizer.

If you have a diluent you have to modify that. So, this the calculation procedure for S is the same, but in the same way S is used in a function that is different for the square ports. Similarly, correlations for slot burners are also given.

So, for example, whatever we have seen, for circular and square ports you will see that these correlations will obey both momentum and buoyancy control regimes. However, for slot burners separate correlations are given for momentum and buoyancy controlled regimes.

Again, Froude number for the flame is calculated, and based upon the Froude number separate correlations are proposed for momentum and buoyancy control regimes. These are very useful correlations to calculate. Why we need this? Because we have a combustion chamber. To calculate the dimension of combustion chamber we need to understand what will be the length of the flame.

For example, if you want to target this flame on a particular wall or any object, to heat the object then you need the flame length calculations. Even in domestic stoves, we need some height of the flames coming out of individual ports, a multiple port stove. In a normal domestic stove, you can see that multiple holes will be there and the flame height coming out of these holes will restricted to some quantity say 20 mm, 15 mm something like that. So, it cannot be very high.

So, to calculate this and design your burners, burner port diameters etcetera, the flame height correlations are extremely useful. So, Ropers correlations are very famous and several other correlations are also available.

Roper correlation is very simplistic in nature. Because you can see that the dependency on the Q_F is broaden, and depending on the type of fuel whether diluents are added or not or whether primary aeration is given, everything is taken into account by this S.

Then we also use the oxidizer stream temperature, fuel stream temperature and so on. So, in this case all the factors are built into this correlation in a simplistic way.