Fundamentals of Combustion

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Lecture - 42

Laminar Diffusion Flames – Part 2

Analysis of gas jets and jet diffusion flames

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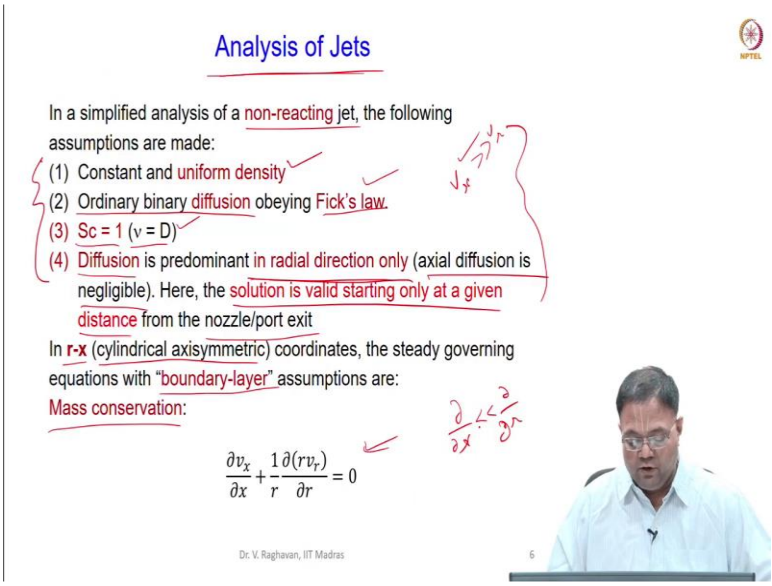

So, let us think about the boundary layer problem and try to extend that to the jet here. So, boundary layer problem is a Cartesian coordinate problem, here we are trying to put it in the r-x coordinates that is the axisymmetric coordinates.

Now, non reacting jet, that is enough for us for the diffusion flames basically. So, considering the non reacting jet, assumptions have to be made, uniform density, the density will not change. So, constant density, then ordinary diffusion only occurs and obeys Fick's law and ν = D, the usual assumptions what we made.

So, the momentum diffusion is same as the mass diffusion. Then important assumption here is the diffusion is predominant in the radial direction only, in the axial direction, there is a jet velocity which is coming out. So, in the axial direction predominantly convection will take care of the transport, that is why the axial velocity component v_x is stronger.

So, this will take care of the transport of the fluid in the axial direction convectively. But in the radial direction it should occur only by the diffusion. So, diffusion is predominant in the radial direction. So, in the axial direction since velocity is predominant the axial direction diffusion is considered as negligible.

So, that is fine. This means I cannot see in the initial potential core region etcetera, where the velocity has not even decayed properly. So, that in the position basically, we cannot ensure the diffusion in the radial direction. So, this solution will not start until a given distance from the nozzle exit; there is no the diffusion effect in the radial direction coming in so promptly there.

So, we cannot take that. So, this solution will be valid only from a given distance from the port or the nozzle exit that you have to understand, that is fine. So, this jet actually spreads a long distance and we are basically looking for the location at which the fuel goes to 0 in the axial direction, that will give the flame height for us. So, let us discuss it later.

So, in analysis of jet, these are the simplified conditions which we have, constant and uniform density, ordinary binary diffusion that is fuel and air these are two fluids, binary fluids which are present. Fick's law is obeyed for ordinary diffusion. Then Schmidt number is 1, then diffusion is predominant in the radial direction only because, convection is going to take care in the axial direction.

In the cylindrical polar coordinates, with two dimensional axisymmetric coordinates, boundary layer type of assumption is made. What is boundary layer type of assumption? $v_x >> v_r$. The radial velocity component is very small. Similarly, the gradient in the x direction is actually smaller than that in the r direction. So, these are the two assumptions invoked for the boundary layer type of problem. So, same type of assumptions are made here.

So, we write the conservation equations continuity equation

$$\frac{\partial v_x}{\partial x} + \frac{1}{r}\frac{\partial (r v_r)}{\partial r} = 0$$

$\partial v_x/\partial x + 1/r\ \partial(r v_r)/\partial r$. So, that will be equal to 0. So, simple continuity equation, mass conservation, steady state problem.

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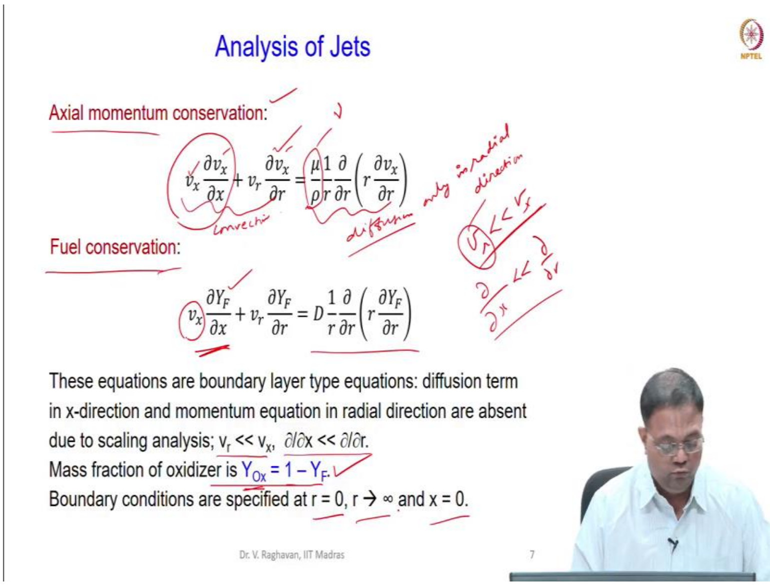

So, there is no time dependent terms involved in this. Similarly, axial momentum equation $v_x \partial v_x/\partial x$ convective term plus $v_r \partial(v_x)/\partial r$. So, the axial momentum conservation so, v_x is the convective term here.

$$v_x \frac{\partial v_x}{\partial x} + v_r \frac{\partial v_x}{\partial r} = \frac{\mu}{\rho} \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial v_x}{\partial r}\right)$$

And here you can see the pressure gradient is also negligible. So, here you can see this is diffusion term. And we have assumed only the radial direction diffusion here.

So, the ρ from the left hand side has come to the right hand side and you get this value. This is nothing but ν, the momentum diffusivity. So, $1/r \times (\partial/\partial r)(r \partial v_x/\partial r)$, that is the diffusion term only in radial direction. So, this is the conservation of axial momentum.

Please understand that since $v_r << v_x$, we assume that is an assumption along the x direction. The velocity that is predominant is the y direction velocity. So, we neglect that. Similarly, here the radial direction velocity is neglected. So, there is no momentum required for this velocity.

So, what we do is, we will try to solve v_x using this axial momentum conservation. And solve v_r using the continuity equation, apply that v_x value here and get the value of v_r. So, two equations two unknowns are there for the velocity components. So, that is enough, pressure is not there. So, two equations and two unknowns we can solve this. So, there is no separate equation required for the radial velocity.

Next, fuel conservation that is again convective term

$$v_x \frac{\partial Y_F}{\partial x} + v_r \frac{\partial Y_F}{\partial r} = D \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial Y_F}{\partial r}\right)$$

$v_x \partial Y_F/\partial x + v_r \partial Y_F/\partial r$, convective term equal to diffusion. So, diffusion is $D \times 1/r\ (\partial/\partial r)(r \partial Y_F/\partial r)$. So, this is the conservation of the fuel species. So, once you get the conservation of fuel species, oxidizer, that is air, here nitrogen plus oxygen together, the mass fraction of that can be found by $1 - Y_F$. So, that is what we are going to use.

Now, please see that we already assumed that $\partial/\partial x << \partial/\partial r$, but why you are retaining this term, because v_x is very strong. Since $v_x >> v_r$, due to v_x this term stays not due to $\partial Y_F/\partial x$ this stays, this term stays because of v_x. Similarly, here $\partial v_x/\partial x$ may not be very high, but v_x is very high. So, by scaling this term stays.

But here you can see that this term is due to the radial gradient of v_x, this term stays, v_r will be very small. But, the gradient of v_x in the radial direction is $\partial v_x/\partial r$ is very high. So, due to that it stays. So, we did not cancel anything. But, in the diffusion term $\partial/\partial x$ is cancelled because, $\partial/\partial x$ second derivative of that is not very high for the v_x. So, that has been neglected, that we have to understand.

So, the terms are chopped off when they are not required, because of the scaling analysis which are due to these two equations what is written here. So, these equations are boundary layer type of equations. Diffusion term in x direction and momentum equation in the radial direction are absent, due to scaling analysis $v_r << v_x$, $\partial/\partial x << \partial/\partial r$. So, these are the three equations which we need to solve. But please understand again partial differential equations are non-linear because of the convective terms and we have to solve this.

But, anyway as Blasius did the solution for the boundary layer equations in the Cartesian coordinates, again using the self similar profiles we can try to get the solution for this. So, that is what we are going to do next. So, before we are going to the solution, we need to see the boundary condition.

So, we have three equations now. The mass conservation which is going to pretty much give us the value of v_r and you can see, this is $\partial v_x/\partial x + 1/r\ \partial(r v_r)/\partial r = 0$. Then, we have the axial momentum equation, convective term $v_x \partial v_x/\partial x + v_r \partial v_x/\partial r$ equal to in the diffusion term, there is only radial direction diffusion and pressure gradient is also absent.

If you solve these two equations, the axial momentum and the mass, you can together solve to get the velocity components v_x and v_r. Then the fuel conservation is going to be solved to get the value of Y_F.

And here also you can see that only the diffusion of fuel in the radial is considered. Axially, convective term is going to take care of those. So, this is the boundary condition for this. So, we have to specify boundary condition at three locations r = 0, r $\rightarrow \infty$, then x = 0, parabolic equations.

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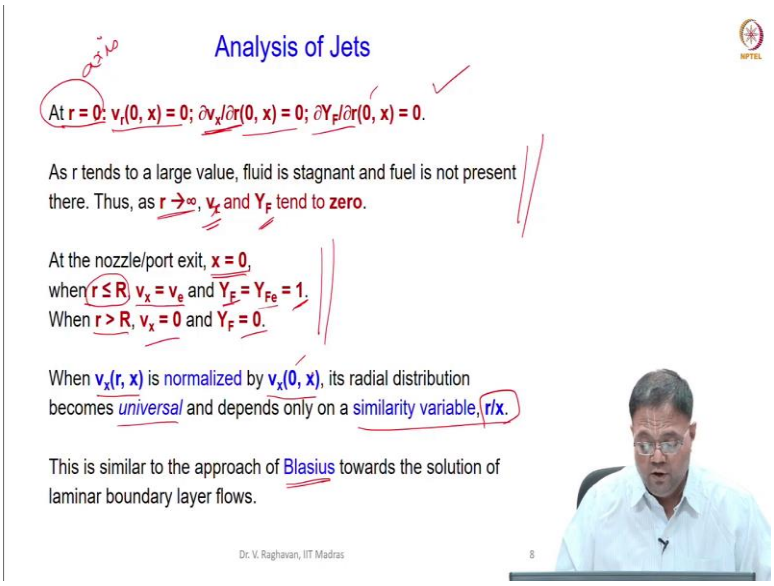

So, these are the boundary conditions, we have to apply for this. What are the boundary conditions? So, at axial, the centre line axis at r = 0, $v_r = 0$, there is no v_r is exactly 0 the gradient of v_x in the radial direction will be 0 at the axis. So, $\partial v_x/\partial r$ at this radius equal to 0, at any x location along the axis it should be equal to 0, v_x cannot be 0 v_x is decaying, but it will not be 0, but it the gradient of that, that will be 0.

Then, similarly gradient of Y_F, radial gradient of Y_F at axis, at any x will be equal to 0. So, these are the conditions at the axis. There, at r = 0, we have the radial velocity is 0. Radial velocity will be there at other far r locations. There, radial velocity need not be 0. But at the axis due to symmetry this should be there, and again due to symmetry the gradients of other variables should be equal to 0. In the radial direction at r = 0.

So, this is one of the boundary condition, at the fixed r = 0. Then, we see far off radius r $\rightarrow r_\infty$ or as I told you it may not be exactly infinity, you cannot use infinity. If you numerically want to solve this you have to go for say 100 times the radius or something like that. So, long far field you take.

So, where you can see that asymptotically the values, this is actually v_x asymptotic values of the velocity and $Y_F \rightarrow 0$ correct. Even v_r will be 0. So, you can also say v_r. So, v_r also will tend to 0. Still air is present, far from the jet there will be no movement.

So, the jet is actually getting into a very still air environment, quiescent environment. So, far from the jet axis the values of the velocity should be 0. Similarly, there will be no fuel present. Only air will be present. So, velocity components plus the fuel mass fraction will tend to 0 at far r locations, this is the second boundary conditions at the r $\rightarrow \infty$ or very high r value.

Then, at exit of the nozzle or the port, there are boundary conditions which actually defines when the radial direction distance is less than or equal to the radius of the port that is small $r \leq R$. We know that the average velocity can give here, v_x is the average velocity which is coming out of the port. Similarly, Y_F at exit it may not be 1. But, in this case only fuel is coming out. So, I am assuming it as 1 that is fine.

Similarly, at x = 0, when r is greater than the radius of the port capital R, then these quantities are 0. So, these are the boundary conditions do you understand. So, parabolic equations, the boundary layer equation are parabolic in nature. We need a boundary condition which has to be specified at this location r = 0, r $\rightarrow \infty$ and x = 0. And for the variables what we are considering we are trying to solve it.

So, now this you can numerically solve if you apply this boundary condition. We can numerically solve this, but theoretically we can do it by doing what exactly Blasius did. So, Blasius transformed these partial derivatives to ordinary governing equation third order ordinary governing equation, differential equation, and then solved it using a similarity variable.

Similar to that when you normalize v_x at any r by v_x at r equal to 0 this centre line value, which is the maximum. And then its radial distribution becomes universal depending only upon a similarity variable, r/x. So, if you define a similarity variable r/x, then we can solve the problem exactly like what Blasius used for boundary layer approach. I am not going to the derivations of the solution, but I only present the solution here.

So, you can see that these parabolic equations, boundary layer type of equations result. Continuity, then axial momentum, then fuel conservation equation. Then you can solve this numerically by applying these boundary conditions at r = 0, r $\rightarrow \infty$ and x = 0. Or, we can use a theoretical analysis what Blasius used similarity approach and get the solution.

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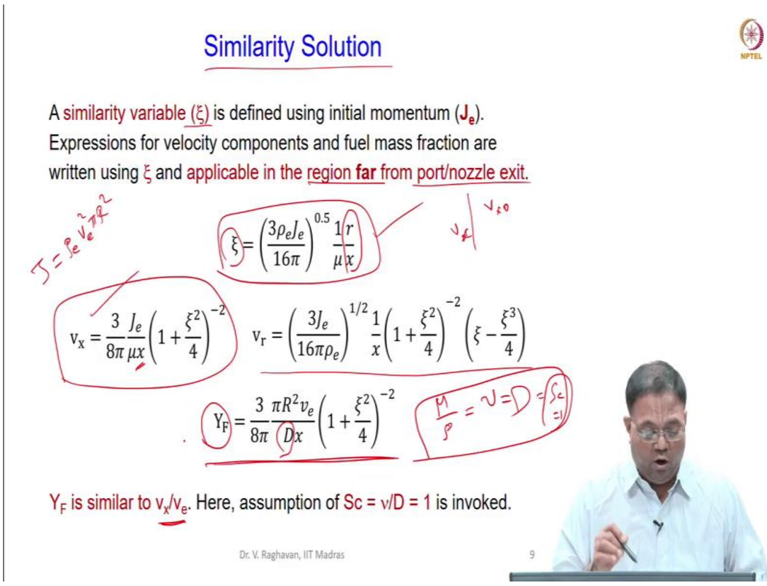

Now, the similarity solution is presented here as I told you a similarity variable is required which is r/x, a function of r/x. So, that variable is defined like this. So, this see as I told you similarity variable is r/x. So, ξ is the similarity variable, ξ is defined as some constants, this is all, they are all constants. So, ρ_e, density is assumed as a constant, and at the exit it is a constant, initial momentum jet J_e that is also a constant.

$$\xi = \left(\frac{3\rho_e J_e}{16\pi}\right)^{0.5} \frac{1}{\mu}\frac{r}{x}$$

So, this is a constant times again μ, μ actually can be a constant here, the assumption μ can be a constant can be made. So, r/x, this is actually defined as some constant times r/x. So, similarity variable is defined. So, once you use the similarity variable and also normalize the axial velocity with the central value, normalize this then you get the solution in terms of this.

$$v_x = \frac{3}{8\pi}\frac{J_e}{\mu x}\left(1+\frac{\xi^2}{4}\right)^{-2}$$

So, this is solution for v_x that is axial velocity at any x location, v_x at any x location will be $3/8\pi(J_e/\mu x)(1 + \xi^2/4)^{-2}$, this is whole power minus 2. Similarly, r profile you can see that r profile is also given like this.

$$v_r = \left(\frac{3J_e}{16\pi\rho_e}\right)^{1/2}\frac{1}{x}\left(1+\frac{\xi^2}{4}\right)^{-2}\left(\xi-\frac{\xi^3}{4}\right)$$

So, as I told you these are going to be applicable only after a particular distance from this., Away from the potential core, jet exit potential core. Away from that only the similarity will be there. So, if you see the profiles again, the profiles are going to be self-similar only after a particular axial location. So, which is actually much greater than X_c, the potential core.

After a distance from the potential core, when you apply that v_x/v_{x0} versus $r/r_{1/2}$, then you collapse this. So, below that you cannot do that as you can see here you cannot collapse, here at x = 0 and $x = x_1$, you cannot collapse by using that rule. So, only away from the jet exit or the nozzle exit this solution will be valid. So, whatever solution I am posing here, these are valid only at a region far from the port exit. That you have to understand.

So, that is enough for us basically. So, we do not need to concentrate our solution in the near field, only the far field you will try to apply. So, this is the equation. Similarly, you can see Y_F is nothing but v_x/v_e..

$$Y_F = \frac{3}{8\pi}\frac{\pi R^2 v_e}{Dx}\left(1+\frac{\xi^2}{4}\right)^{-2}$$

So, the same profile we got correct. So, Y_F is nothing but v_x/v_e and that varies exactly like this, you can see this. Now, instead of this I am trying to put μ instead of μ I am putting $\mu/\rho = \nu = D$.

So, this is the thing so, D I say. So, this is the Schmidt number equal to 1. So, when I apply the Schmidt number equal to 1. So, μ I take ρ_e from the J_e so, J_e you know so, J_e is nothing, but $\rho_e(v_e)^2\pi R^2$. So, when I substitute here $\pi R^2(v_e)^2$ that v_e here, you can see that v_e stays here then μ/ρ_e comes to the bottom. So, μ_e/ρ_e will be D which is $\nu = D$. So, that is what I am getting here.

So, one of the v_e in $(v_e)^2$ cancels out because, v_x/v_e I am putting. So, that cancels this, v_e cancels one of the v_e in the product here $(v_e)^2$. So, v_e alone stays πR^2 stays v_e stays, ρ comes to the bottom. So, $\mu/\rho = \nu$, so, $\nu = D$. So, I am substituting D here. So, this is the variation of Y_F, this is r and x are there it is applicable to any radial location. I can vary the r here. And similarity variable will vary so, that I can get the profiles. So, this is the solution I get.

So, similarity solution is the one which we will make, see for example, all the variables like v_x and Y_F are function of both r and x. But, if I define a similarity variable which is ξ. ξ is a function of r/x, seeing the self similarity in the profiles away from the nozzle exit. Then, I can have only one independent variable ξ and I can express my solution as a function of that particular variable ξ only. So, that is what I am trying to do here. So, at any x you can do this and this is the profile.

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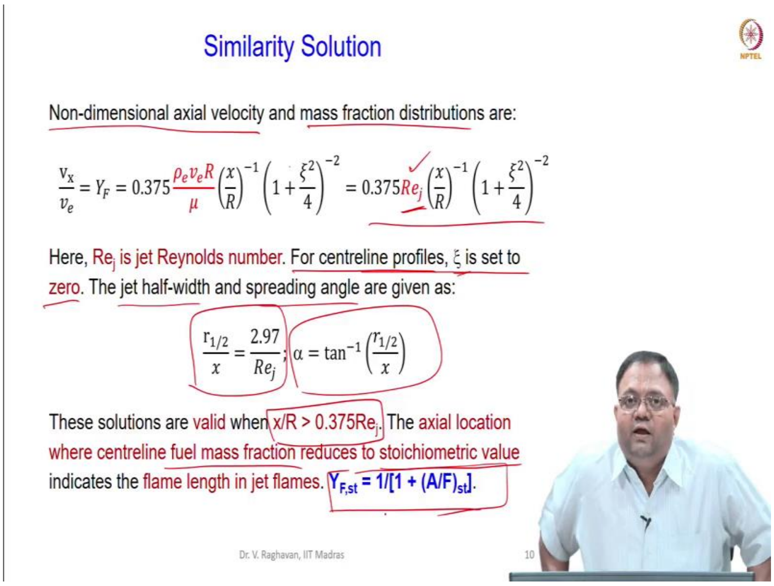

Now, if you want to plot here, so, non-dimensional axial velocity and mass fraction distributions are given here, basically this is the v_x/v_e, which is equal to Y_F, that is the profile here.

$$\frac{v_x}{v_e} = Y_F = 0.375\frac{\rho_e v_e R}{\mu}\left(\frac{x}{R}\right)^{-1}\left(1+\frac{\xi^2}{4}\right)^{-2}$$
$$= 0.375Re_j\left(\frac{x}{R}\right)^{-1}\left(1+\frac{\xi^2}{4}\right)^{-2}$$

So, that is what I get 0.375 $\rho_e v_e R/\mu$. Jet momentum I am expanding now and getting this value, you may remember this is ρ_e, ρ × velocity × characteristic dimension, which is the radius divided by μ which is the Reynolds number of the jet.

So, this jet radius, jet exit velocity, jet density divided by mu I am using to get the jet Reynolds number. So, now, this v_x/v_e can be defined as 0.375. Reynolds number of the jet $(x/R)^{-1}$, $1 + \xi^2/4)^{-2}$. So, this is the distribution for both Y_F and non-dimensional axial velocity. So, now if you want to get centre line profile, ξ is set to 0, because r/x, R = 0.

So, r/x is set to 0, then you will get the centre line profiles. So, for centre line profiles set ξ as 0, in this equation you will get the centre line profiles, the velocity decay etcetera. So, for example, if you set ξ = 0 here, then this term goes, then you get only v_x at centreline. So, v_{x0} will be without this term. So, you will get centreline profiles.

Similarly, centreline decay of the Y_F also you can get. Now, for half width and spreading angle, we use this $r_{1/2}/x$ = 2.97 by, I have not given the derivation, this is the derivation. You can try to derive it or see some books to derive this. So, $r_{1/2}/x$ = 2.97/Re.

So, when Reynolds number increases then the $r_{1/2}$ will decrease. Similarly, spreading angle $\alpha = \tan^{-1}(r_{1/2}/x)$ so, you can use this. Now, as I told you far field only the solution is valid; that means, x/R should be greater than 0.375 into Re_j for this solution to be valid.

Now, what is flame height, the diffusion flames basically are transport controlled. We have already seen that its transport controlled that chemical kinetics does not play a role, because chemical kinetics are very fast when compared to the time of this diffusion, the radial diffusion time, molecule diffusion is actually much slower and the chemical reaction is much faster.

So, when a fuel molecule travels from the centre line to the jet edge, where you can also see the oxygen diffusing in the radial direction towards the centerline. At some point in the radius the fuel molecule and the oxygen molecule will mix at some stoichiometric proportions, where a flame zone will be formed upon ignition. That flame zone will be very thin when you plot the locus of all this, you will see the flame surface basically.

So, this is transfer controlled. When you have more momentum for the jet, then you can see that as I told you when Reynolds number increases, then $r_{1/2}$ decreases; that means, your radius of the flame will decrease and so on. So, exactly where the flame surface will come, you have to calculate the stoichiometric value of the fuel mass fraction.

So, that is nothing but, $Y_{F,st} = 1/[1 + (A/F)_{st}]$. So, if you trace that basically you will get the flame profile.

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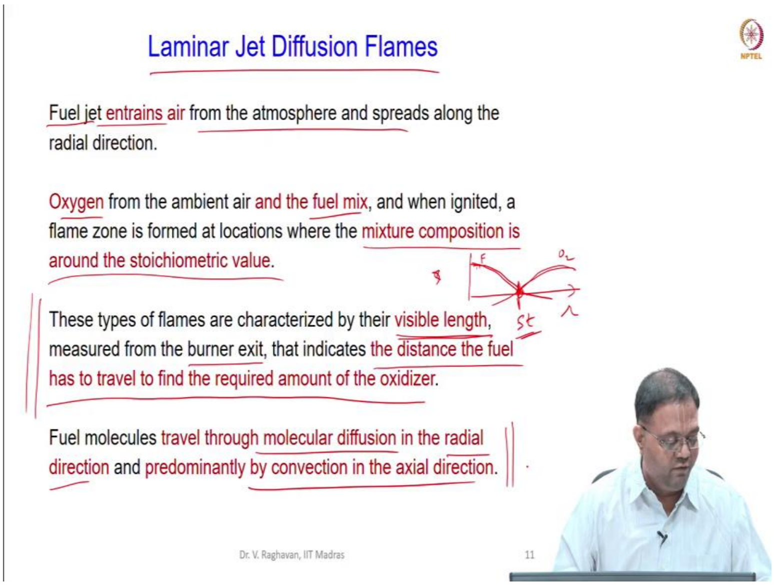

So, laminar jet diffusion flames. So, fuel jet which is coming out with some momentum which will entrain the air due to the viscous effects, what we have saw after the potential core region, you can see that the viscous effect will be more and the mixing takes place indicated by the reduction in the velocity value and the value of the mass fraction of the fuel.

So, air entrains from the atmosphere and the jet spreads along the radial direction, as I say the radial direction you can see more the Y_F value at centre line and decays at the larger radial direction as you go along the axial direction.

Now, oxygen from the ambient air and the fuel mix, and when ignited a flame zone is formed at locations where the mixture composition is around the stoichiometric value. So, fuel from the centreline decreases like this, oxygen from the ambient diffuses like this.

So, somewhere it will form a stoichiometric mixture. So, we ignite here at this location, where they mix at the stoichiometric proportions. This is fuel and this is oxygen. So, somewhere in the radial direction, this is radial direction basically, r, some radial location they will form a stoichiometric mixture. So, Y_F, stoichiometric what I told. So, that values attain, when you ignite in this location, flame will form.

The diffusion of fuel and diffusion of oxygen in the radial direction are much slower when compared to the chemical kinetics or we can say the chemical kinetics is much faster. So, at this location the flame is formed. So, that is what is given here the oxygen from ambient and the fuel mix, and when ignited a flame zone is formed at locations where the mixture composition is around the stoichiometric value.

So, there flame will be formed. So, this is at the particular x location. So, if you trace all the x locations you get the flame surface. These types of flames are characterized by the visible length. See for example, I will show you figures later. So, visible length, jet of the fuel comes out and you ignite this and a flame form. So, what is the characteristic of this flame? It is the length.

For a given fuel flow rate, you will get a particular length of this laminar flame. So, that length is the characteristics. See, when you saw the premixed flames the laminar flame speed was the characteristic, one of the characteristics of that. See, for example, the Bunsen burner method, you get a conical flame and a half cone angle is used to calculate the laminar flame speed.

So, that is the characteristics of a premixed flame, but in the case of a diffusion flame like this, where nothing is mixed and they mix only at the flame zone like this. So, under these circumstances, the distance what the fuel travels to get all the oxidizer it needs, that is the characteristics.

So, we will say visible length of the flame, which is measured from the burner exit will be the characteristic, important feature of the diffusion flame. And that indicates the distance the fuel has to travel to get the required amount of oxidizer for its complete oxidation. So, that is the important thing.

So, flame length, which is what we are going to see as a main characteristic of a diffusion flame or a non-premixed flame. Now, again I am repeating here, the fuel molecules travel through molecular diffusion in the radial direction and by convection in the axial direction.

You can see the convection is predominantly in the axial direction, because of the jet momentum which is coming from the nozzle exit. But in the radial direction it is only by diffusion, concentration gradient driven. Of course, soret effects also will be there, but predominantly it is diffusion driven.