Fundamentals of Combustion

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Lecture - 41
Laminar Diffusion Flames – Part 1
Theory of gas jets

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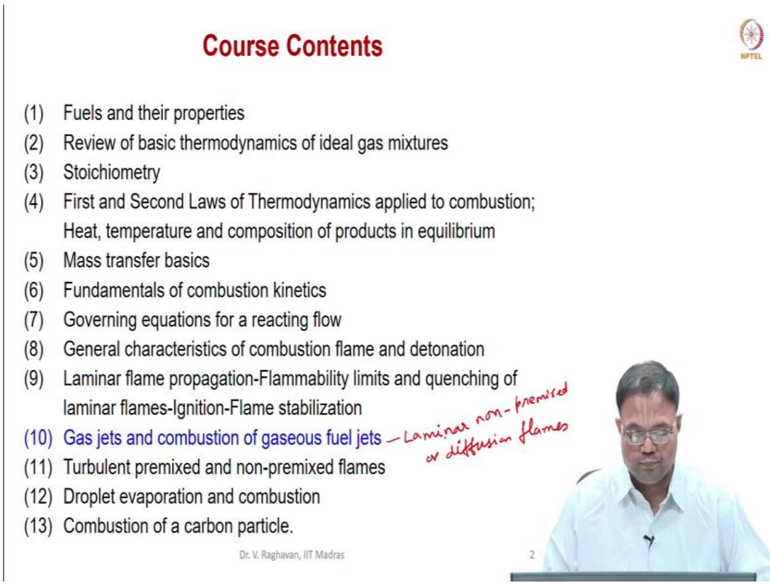

So, the next topic is gas jets and combustion of gaseous fuel jets. Basically, this is the analysis of diffusion flames. So, first we will do the laminar diffusion flame. So, this is laminar diffusion or non-premixed so, laminar non-premixed diffusion flames. So, that is the next topic what we are going to see.

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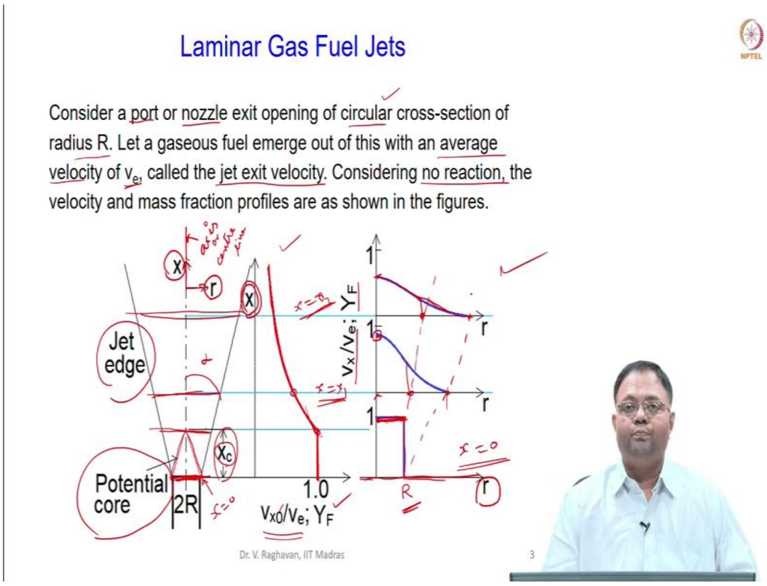

So, let us take the simplest case where we have a fuel jet, just fuel alone coming out of a port or a nozzle. Port may be a circular pipe and only fuel is supplied there or it may be a nozzle, contoured nozzle and let us assume a circular cross section, you can also use other cross sections say square, rectangle etcetera. But most of the burners are of circular cross section, we will consider circular cross section with a radius of R.

So, only fuel is coming out of a port or a nozzle which is having circular cross section of radius R and the velocity at which the fuel comes out is the exit velocity or what is the average velocity is v_e. So, we can understand that if it is a nozzle, we will get almost a uniform profile or what we call a top hat profile.

So, the velocity will be v_e everywhere or if you have a port which is the end of a long pipe then a fully developed flow comes out of that. So, the maximum velocity will be $2v_e$ at the centre and it goes parabolically 0 value at the walls so that the average velocity will still be v_e. So, this average velocity I call jet exit velocity.

So, this is the jet exit velocity which is coming out of this pipe. So, you can see this is the schematic, this is the end of the pipe or a nozzle whatever it may be. So, the fuel alone comes out and there is no reaction to be considered now. So, let us say cold reactant is considered here that means, only fuel is coming out. Air is present in the atmosphere which is not moving at all.

So, air which is present outside this jet is quiescent in nature and the jet is coming out with the average velocity v_e. So, in this case basically, how the jet actually gets into the ambient and how would its velocity and mass fraction of the fuel etcetera are affected let us see.

So, the figure here shows the contours of two things, first one is this part this centre line velocity that is v_x. So, this is the axisymmetric jet, so, you have two coordinates in cylindrical polar.

The axial coordinate is x and the radial coordinate is r. So, in this coordinate, basically you can see that we are trying to plot as a function of the axial coordinate, we are trying to plot the quantities v_{x0}/v_e that is the axial velocity at the center line, this 0 represents r = 0.

So, at the axis, this is the axis basically this is the axis of the axisymmetric jet and we are trying to plot the velocity along the axis. But it is normalized by the exit velocity v_e which is the average velocity with which the fuel is coming out.

So, v_{x0}/v_e that or this mass fraction of the fuel so, both will represent the same profile here. So, at the x = 0 position, let us take x = 0 position here as the exit of the port or the nozzle that is x = 0, then x increases upwards.

Now, at this point x = 0, you can see that the velocity is equal to v_e. When we plot the axial velocity v_{x0}/v_e will be 1. Similarly, mass direction of the fuel will be 1. So, that is here. So, only fuel is coming out, nothing else is there. So, fuel mass fraction is 1 here.

So, we can also say its fuel stream. Even if you say a mixture of fuel, the fuel streams mass fraction is 1, combining all the fuels together or fuel plus a diluent can also come out say, fuel + N_2 etcetera.

So, that value is 1 here and you can see that continues for some height, there is no change in the value of v_{x0}/v_e or Y_F till a particular distance called X_c from x = 0 to a position called x = X_c. That is no change in the centre line value, this is the axis or centre line.

Along the axis or centre line when we try to plot, there is no change in the value of v_{x0}/v_e or the mass fraction of the fuel Y_F till a particular distance say X_c. Then, what happens is you can see a reduction in the value.

So, if you take a longer distance, it may asymptotically go to a 0 value. So, this is the axial profile. So, along the axis, I try to plot. So, axial profile plotted along the axis.

Shown in the right-hand side is the radial profiles at axial positions. For example, this first graph here is the radial profile of the same quantities v_x/v_e the axial velocity component is v_x, radial velocity component will be v_r we are not touching that.

Now, axial component of the velocity will be very predominant than the radial component. So, I take this value v_x and again normalizing that by the average exit velocity v_e; so, v_x/v_e, we are and plotting the Y_F value.

Now, at this station, that is the $x = 0$ this is plotted. The radial profile of these two quantities is plotted at the axial location of nozzle exit or the port exit which is $x = 0$. You can see that. So, this is the value of R. So, the value remains 1 and goes to 0 at R. So, at the value of R, it goes to 0. This is the profile here because you can see that

Now, again we have assumed fully developed flow basically it is a uniform flow which is coming out so, that is what the assumption. So, in the uniform flow, you can see that the values of $v_x/v_e = 1$

Similarly, mass fraction of the fuel is 1 and it goes 0 at R, so, after that nothing is there, it goes to 0. When the radial direction value is greater than the radius of the port, then the values go to 0 here. Obviously, there is nothing coming out of that so it goes to 0.

Now, you take another location. So, this is another location, I will say $x = x_1$. I try to plot the radial profile. You see now as per this the velocity has decayed to some value. So, v_x/v_e has decayed to some value. So, this will be the centre line value and this is the $r = 0$. So, this is the axial value, and it goes to the 0 value at a particular distance r.

So, asymptotically it goes to zero at a particular distance. Both Y_F starting from this, you can see that the value is not 1 here because there is a decay at this location of $x = x_1$. You have already seen that the centre line has decayed. You can see that centre line has the maximum value and it goes to 0 here. Radially it goes non-linearly to 0.

When we take another higher location so, $x = x_2$ here, you can get the similar type of profile, but the values are different more decay has happened to both v_x/v_e at the centre line as well as the Y_F and starting from that as a maximum value, it again decays and it takes some somewhat longer radial distance to go to 0 value. So, these are the radial profiles.

So, let us discuss the characteristics of this jet. So, you can see that the point at which it goes almost to 0 value increases with x. Again, the centre line maximum value is decreasing for these quantities plus you can see this here immediately at the radius of the port, the values are becoming 0. In this, at $x = 0$, there is a value. But as x increases, the value go to 0 at a longer radial value. So, this you have to understand.

So, this when we try to join here basically this is what is shown as the jet edge, well I am just trying to join this. So, that is qualitatively shown as a jet edge here. Now in the location where the mass fraction or v_{x0}/v_e remains almost the same here, like remains constant, that I have indicated by a triangle here. Red colored triangle this is nothing but the potential core of the jet. So, these are the definitions. Let us see more characteristics from the next slide onwards.

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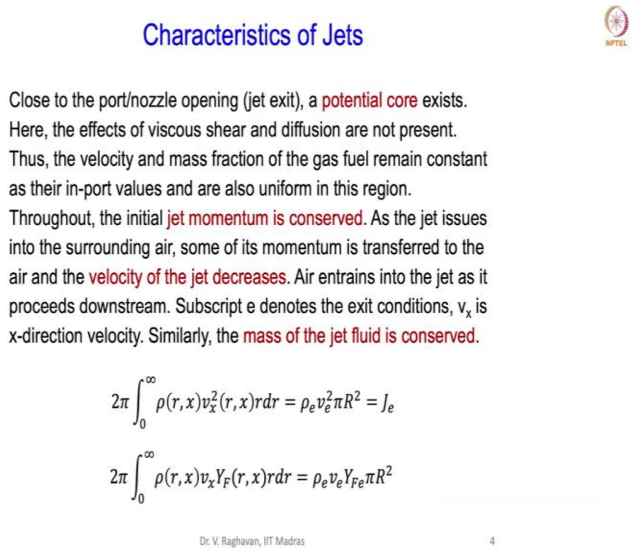

So, close to the port or the nozzle opening or the jet exit what we call a potential core is seen to exist. That is that red colored triangle what we have shown. So, here, the effects of viscous shear or diffusion are not present. So, here when the jet goes out, it has some viscous effects by which it actually pulls in the atmospheric air inside it. So, that action is not felt until the distance of X_c here, this X_c which is called the potential core.

So, we can see that the values in the centre line basically are unchanged from what is the exit value. So, here what happens is, the velocity and the mass fraction of the fuel gas remains constant exactly at the in-port values. Whatever is coming out of the port that is what is seen here and it is also uniform. So, we actually assumed the uniform profile here for the velocity. So, it is coming out as uniform.

$$\pi \int_0^\infty \rho(r,x) v_x^2(r,x) r dr = \rho_e v_e^2 \pi R^2 = J_e$$

Now, this is the first point, first important characteristic of a free jet, this is called the free jet. Why this is called free jet because this is the jet which is coming out of the port or the nozzle expanding into an infinite atmosphere where there is no movement of any atmospheric air. So, atmospheric air is stationary and jet is trying to enter into this still air.

Now, another important characteristic of the free jet is the initial momentum of the jet; momentum of the jet is conserved. Going back at any location, see for example, this is the exit of the jet, the nozzle. So, at x = 0, the jet momentum will have some value.

$$2\pi \int_0^\infty \rho(r,x) v_x Y_F(r,x) r dr = \rho_e v_e Y_{Fe} \pi R^2$$

At any location if you take, see for example, I go to $x = x_1$ here or $x = x_2$ here and so on and try to see the momentum, integrate in the radial direction the momentum that will be same as the initial momentum at the exit of the nozzle or the port. So, the momentum is conserved, momentum of a free jet is conserved. So, the initial jet momentum is conserved that is very important.

Now, this is for the free jet. Please understand if there is a confining wall or anything present, then this may not be applicable. So, only for the free jet the jet momentum is conserved. Then, as the jet issues into the surrounding air actually which is stationary basically its momentum is transferred. Some of its momentum is transferred to the air that is why the velocity is decreasing here.

So, after the potential core region, till which there is no interaction, the viscous effects are not felt you know, so, after this point, you can see that the velocity actually decays. This is due to transfer of momentum from the jet fluid to the air. So, that means, it actually gives a momentum, it spreads. So, this spreading is because of the transfer of momentum from the jet fuel to the atmospheric air.

So, velocity of the jet decreases because the momentum is transferred. So, why I am saying this because you can see that jet momentum is conserved. Velocity is decreasing, so, momentum is conserved because of the addition of the mass; that means, the air entrains into the jet. So, the mass actually increases at any station. So, if we see the exit of the nozzle or the port, you will see only fuel is present there.

You take $x = x_1$, the next station in the higher point, then you will see that there will be some air also which has joined in this, within this jet. Within the jet edge boundary, the air also has come in. So, mass has added, but velocity is decreasing, mass is getting added due to the air entrainment from the surroundings into the jet.

This is due to the momentum exchange and the viscous effects basically and so, mass is added, velocity decreases, but momentum is conserved. Now, as I told you the subscript e denotes the exit condition and the v_x is the axial component or the x direction, axial velocity component. Now, the mass of the jet fluid is conserved. Please understand jet fluid is the gas, the fuel gas which is coming out so, that should be conserved.

So, another important point is this. This mass has to come. See the total mass may increase that is mass of the fuel which is coming out plus the mass of the air which has entrained that will increase at any x location when you go upwards.

But the momentum is conserved, because the velocity decreases, then if you only take the mass of the fuel here because there is no reaction, considering no reaction, the mass of the fuel at this x = 0 location that will be same. If you only consider the mass of the fuel at $x = x_1$ or $x = x_2$, it will be the same. So, the mass of the jet fuel or the jet fluid is conserved.

So, we can write at every station, I am just trying to integrate. So, going in the radial direction, radial integration, starting from the axis that is radius equal to 0 to long value of r, so, $r \rightarrow \infty$. So, I will take say many times the jet diameter.

So, jet diameter is say 2r, r is the radius of the port. Now, I take say 50 times something like that. So, very big value of r I will take, I will try to integrate this so, this may be say $100 \times r$ something like that.

Now, when I do this, the density, why density is changing here because not only fuel, now air also is added. So, density at every point, at every radial location and axial location so, given at a particular axial location if you want to fix x, then at this the radius density increases, similarly velocity will change, so velocity square. So, ρv^2 represents the momentum. $2\pi r dr$ that will be the area integration here.

So, if you integrate it that should be equal to this. At any x location it will be equal to the initial jet momentum ρ_e that is exit density, the density at the x = 0, v_e is the average exit velocity, πR^2 is the area of cross section. This is defined as the initial jet momentum J_e. So, that is conserved. We have seen that this is going to be conserved.

At any x location, if you do this integration, you will see that this will be equal to J_e which is equal to $\rho_e (v_e)^2 \pi R^2$. So, mass is added, velocity decays when you go to increased x values but the momentum is conserved. Similarly, the mass of the jet fluid is conserved. So, these are the two equations which we have to understand.

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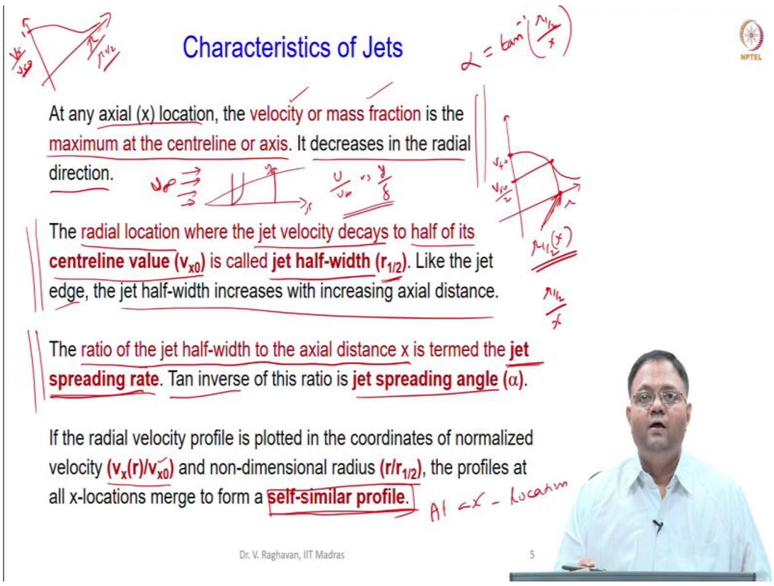

So, next is at any axial location x the velocity or the mass fraction is maximum at the centre line that we have seen through the profiles here. We can see the at any location, at this point, it is maximum and it actually stays in there; in the exit of the jet it stays for some time, then decreases to 0 rapidly.

But if we take any other location where the decay has started, after that starting point of the decay if we take any location you say $x = x_1$ here, you can see that it starts at a maximum value at the center, then slowly decreases, non-linearly decreases to a 0 value. Similarly, at higher location, but it takes little bit longer radius to decay to 0 value that is what is indicated by the jet spread.

So, at any axial location, velocity or mass fraction is maximum at the centre line because you can see that the core; the core of the fuel is coming out of this and the entrainment happens from the periphery of the jet. So, it decreases in the radial direction as we have shown in the profiles. But you can see that the profiles are different here.

So, for example, the maximum values are different, the radial position at which these values go to 0, that is also different. But these are called self-similar profiles.

So, this is the characteristic, starting with the maximum value at the centre line, decaying to a 0 value or a very low value at the given distance from the center line, radial distance from the center line. So, that is the characteristic, radial profile of the velocity or the mass fraction.

Now, as I told you it asymptotically goes to 0 value; that means, it takes longer radial distance to get to the 0 value. So, it is not easy to really trace that 0 point. What we now generally do is we seek a radial location or we try to evaluate radial location at which the jet velocity decays to half of its centreline value v_{x0}.

So, if you go to the profile, if you take the maximum here and it goes to 0 like this. It goes to zero asymptotically. So, this is radius and this is the v_{x0} here, this is v_{x0} and this is the v_x profile basically. So, you can see that starting from the v_{x0} value, centre line value is the v_{x0}, it decays to a low value at far field.

So, instead of going till infinity or a very long radial location we can define or see for this value. So, let us take this as $v_{x0}/2$, half of the centre line value and see what is the radial location at which it is attained.

So, this location is called $r_{1/2}$, that is called jet half width. So, starting from the centre line value in the radial direction, the value of the velocity decreases non-uniformly and goes to a 0 value at a very long radial location.

So, instead of tracing that 0 value, we are trying to take the radial location where the velocity has decreased to half of its initial, the centre line value. So, if centre value is v_{x0}, the radial location at which the centre line value has reached to half of its value $v_{x0}/2$ that location is taken as the jet half width $r_{1/2}$. So, this is another characteristic of the jet.

So, this jet half width also you can see at a given radial location, it will have some value say it is a function of x. This is a function of x. So, at a lower x location, it will have a smaller value, when the x location increases it will also have larger and larger values. So, like a jet edge where the velocities go to at most 0, almost 0 value, the jet half width also increases with the increasing x axial distance or the x value.

So, you can see here, at this location say $x = x_1$, you will see the half value will be somewhere here. At this location, you are going to go to here somewhere. So, you can see there is an increase in the half jet half width.

So, the jet spreads, the jet half width also increases with the axial distance like the jet edge. Jet edge increases with the axial distance. Similarly, the half width also increases with the axial distance. So, this is one of the characteristics. Another, characteristics is this jet half width. So, first the radial profile which exhibits a maximum value at the centre line goes asymptotically to 0 value along the radial direction that is one characteristic. Second characteristic is the jet half width. Where from the centre line value, the velocity has decayed to half.

Then, we go to another definition which is the ratio of jet half-width to the axial distance x. It is termed as the jet spreading rate, how far the jet spread that is a very important characteristic.

So, $r_{1/2}$ at a particular x so, if we calculate this $r_{1/2}$ as a function of x now, so, $r_{1/2}$ at the particular x location. This ratio is called jet spreading rate. So, this varies, $r_{1/2}$ is lower at lower x. As x increases, $r_{1/2}$ also increases, but this ratio $r_{1/2}/x$ that will indicate the jet spreading rate.

And the angle, so, if we go to this again, go to this figure, this is the angle what we are talking here. This angle is got by taking tan inverse of this ratio; that means, the angle α that is called jet spreading angle $\alpha = \tan^{-1}(r_{1/2}/x)$. So, the ratio $r_{1/2}/x$ is the spreading rate. Tan inverse of this value will be the spreading angle. So, these are another two important characteristics.

So, potential core, till which the viscous effects or diffusion effects are not important and there is no change in the value at the centre line than what we saw at the exit for both velocity as well as for the mass fraction. So, that is one of the important characteristics.

Then the radial profile exhibiting maximum value at the centre line and going asymptotic to 0 at further radial locations. Then definition of half width, then you can see that this jet spreading rate $r_{1/2}/x$ and the spreading angle.

These are all important parameters which we use to analyze a jet. Now, what is the use of defining $r_{1/2}$ etcetera, because if you try to plot as I told you, if you go to this, depending upon the value of x say x_1 or x_2, you get the profiles with different maximum value, the centre line maximum value here is much less than 1.

When we compare to a lower location x_1, where it is close to 1, we can see that the maximum value itself changes. Similarly, the value at which the $r_{1/2}$ occurs or the value at the jet edge that also is different for these two profiles. So, if this is the case, then this is a two-dimensional problem. So, we have to really take into account of both axial and the radial direction to solve this problem.

But if we try to plot v_x at any axial location. So, v_x along the radius divided by the centre line value at that x location. So, at a given x location I know what is the maximum value of axial velocity which is will occur at the centre line. So, that is known to me v_{x0}. That is taken for normalizing this and now, I try to plot the axial velocity along the radius.

So, I normalize this at every x location. I try to normalize the axial velocity with the centre line value at that x location v_{x0}. So, that I plot in the y axis. In the x axis, radius should come, but I do not plot radius alone, I plot radius divided by the half width $r/r_{1/2}$, r suffix half. If I do this, then all these profiles collapse to one profile which is called self-similar profile.

So, that means, if I plot v_x/v_{x0} at the centre here you get $r/r_{1/2}$. Then you get only one profile like this starting from 1 you get one profile like this. So, at every location, at any x location, you will get only one profile like this. That is called self-similar profile.

You may remember that for boundary layers, if I plot U/U_∞, see for example, boundary layer if you take, this is U_∞ with which a flow is coming in. Boundary layer is formed and if we take any x location, I get this boundary layer profile.

So, when I try to plot U/U_∞ versus say this is the y direction and this is the x direction, so, U_∞ versus y/δ which is the boundary layer thickness, then this profile is similar.

So, similar to that when I plot v_x that is the axial velocity along the radial direction normalized by the maximum value at that x location which is the centre line value v_{x0} for a given x location that versus the radial direction divided by the half width at that x location, because please understand that $r_{1/2}$ also is a function of x.

So, I plot this at a particular x location, axial location, I will get one value for v_{x0}. By the decay, I know that. Similarly, I can calculate the half width at that location

So, when I use these two quantities like for example, U is normalized by U_∞ and y is normalized by δ that is the boundary layer thickness. But, boundary layer thickness itself will vary with x. Similar to that $r_{1/2}$ what I am plotting here, jet half width, that will vary with x.

So, at a given location, there is a particular value for v_{x0}, the centre line velocity, and also for $r_{1/2}$. When I use that and plot, I will get one profile. If I plot for every x location the same thing, then I will get the same profile which is called self-similar profile. Now, the problem is reduced to a one-dimensional problem, do you understand?

So, that means that if I do not really need to account for the x direction now because at every x location, if I think of a similarity variable then I can go for what is called as similarity analysis like what we normally do for a boundary layer. Self-similar profile can be done with this. So, we have to define variables carefully and do it. So, that is what we are going to do here.

I am not going to go for the entire derivation.

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