Fundamentals of Combustion

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Lecture - 36

Laminar Diffusion Flames - Part 4

Estimation of laminar flame speed

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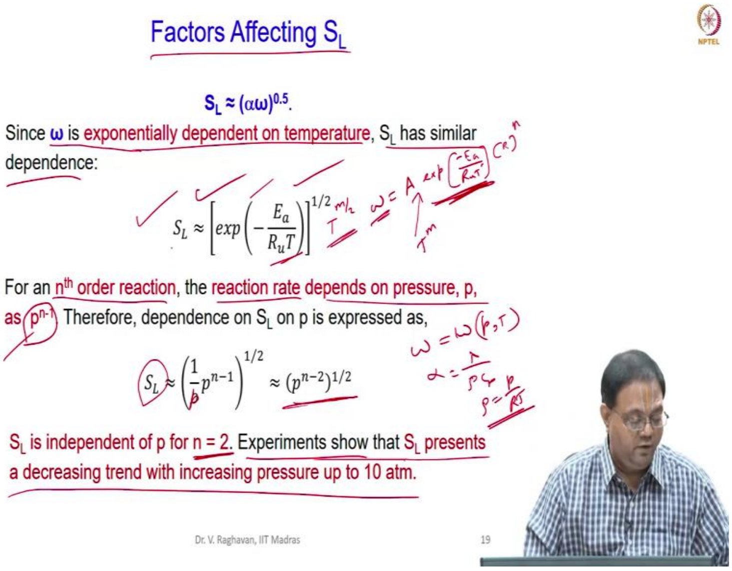

Factors Affecting S_L

$S_L \approx (\alpha\omega)^{0.5}$.

Since ω is exponentially dependent on temperature, S_L has similar dependence:

$$S_L \approx \left[exp\left(-\frac{E_a}{R_u T}\right)\right]^{1/2} T^{m/2}$$

$\omega = A\,exp\left[\frac{-E_a}{R_u T}\right](R)^n$

T^m

For an n^{th} order reaction, the reaction rate depends on pressure, p, as p^{n-1}. Therefore, dependence on S_L on p is expressed as,

$$S_L \approx \left(\frac{1}{\rho}p^{n-1}\right)^{1/2} \approx (p^{n-2})^{1/2}$$

$\omega = \omega(p, T)$

$\alpha = \frac{\lambda}{\rho c_p}$

$\rho = \frac{p}{RT}$

S_L is independent of p for n = 2. Experiments show that S_L presents a decreasing trend with increasing pressure up to 10 atm.

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Now, let us see in detail. So, this when I say α and ω, we all are normally interested in finding the temperature, pressure dependence and maybe Φ dependence which we have already seen. So, ω reaction rate is exponentially dependent on the temperature. So, S_L will have the similar dependence.

So, you know ω is written as A×exp($-E_a/R_uT$) into concentration of reactants power some order. Say, order n. So, now this exponential dependency of ω will also say this, this power half, so exponential dependency is this term basically, this is temperature term; we also can have a term called T^m here.

Now, if that is present, we can also add $T^{n/2}$ here because S_L α $(\omega)^{0.5}$. So, the exp$(-E_a/R_uT)^{0.5}T^{m/2}$, but this term is not going to affect the S_L so much as this exponential term.

So, S_L has an exponential dependency on temperature since ω has that. Then, let us consider n^{th} order reaction. Most of the reactions are second order reactions, but let us first consider here general n^{th} order reaction. So, the reaction rate of n^{th} order reaction depends upon pressure as p^{n-1}.

So, as the pressure varies, the reaction rate will vary to as p^{n-1}. So that means, where ω is affected by p. ω is the function of p comma T, ω(p, T); we have seen the T dependence here, pressure dependence here. Now, similarly α will be $\lambda/\rho c_p$.

So, all other variables, λ, c_p etcetera will not vary much with these pressures; but ρ will vary. So, ρ = p/RT. So, ρ will vary with pressure. So, that term is here. So, this is p. So, S_L varies as $(1/p)\times p^{(n-1)0.5}$ which is nothing but $(p^{n-2})^{1/2}$. So, S_L varies with pressure as for the n^{th} order reaction. Please understand, for n^{th} order reaction, it varies as $(p^{n-2})^{1/2}$.

But many reactions are second order in nature; that means n equal to 2. So, if you put 2 here p power 0; that means, S_L may not depend much on pressure. However, some small dependency is shown by the experiments which show that S_L presents a decreasing trend with increasing pressure up to 10 atmospheres, where the experiments are conducted. So, this you have to remember. So, temperature is the one which is going to mostly affect the S_L value and pressure slightly affects it.

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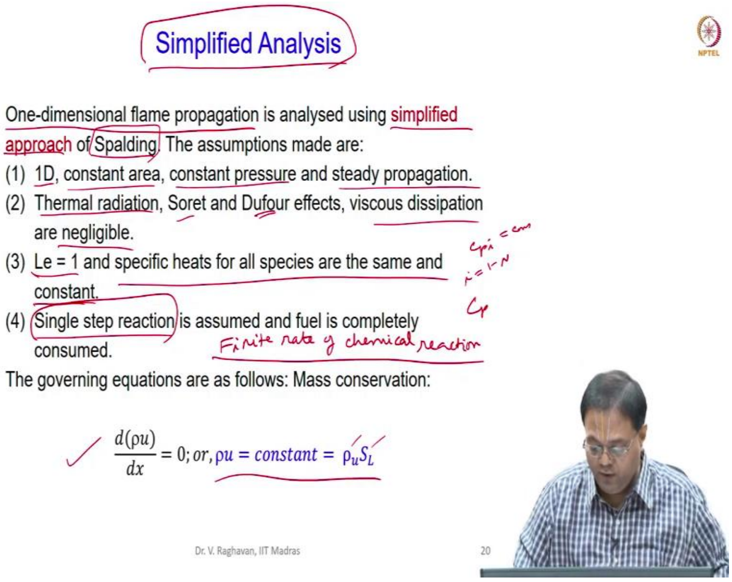

Simplified Analysis

One-dimensional flame propagation is analysed using simplified approach of Spalding. The assumptions made are:

(1) 1D, constant area, constant pressure and steady propagation.

(2) Thermal radiation, Soret and Dufour effects, viscous dissipation are negligible.

(3) Le = 1 and specific heats for all species are the same and constant.

c_{pi} = cons

i = 1-N

c_p

(4) Single step reaction is assumed and fuel is completely consumed.

Finite rate of chemical reaction

The governing equations are as follows: Mass conservation:

$$\frac{d(\rho u)}{dx} = 0; or, \rho u = constant = \rho_u S_L$$

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Now, the thermal theory gave us; see here the from thermal theory, we got these points, for example this.

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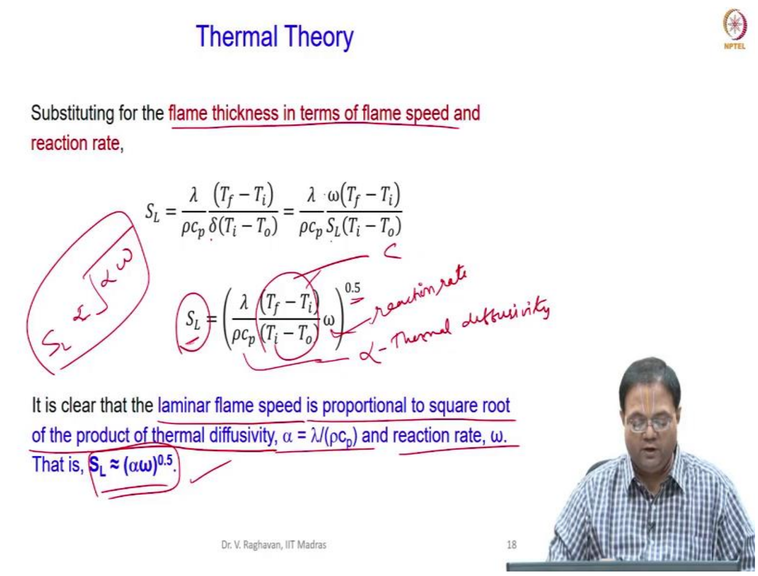

Thermal Theory

Substituting for the flame thickness in terms of flame speed and reaction rate,

$$S_L = \frac{\lambda}{\rho c_p}\frac{(T_f - T_i)}{\delta(T_i - T_o)} = \frac{\lambda}{\rho c_p}\frac{\omega(T_f - T_i)}{S_L(T_i - T_o)}$$

$$S_L = \left(\frac{\lambda}{\rho c_p}\frac{(T_f - T_i)}{(T_i - T_o)}\omega\right)^{0.5}$$

It is clear that the laminar flame speed is proportional to square root of the product of thermal diffusivity, α = λ/(ρc$_p$) and reaction rate, ω. That is, $S_L \approx (\alpha\omega)^{0.5}$.

Important conclusion from thermal theory is $S_L \approx \sqrt{\alpha\omega}$, is proportional to under root α which is thermal diffusivity and ω, product of αω. And based upon that, the other factors temperature pressure etcetera which influence, we have already seen how the temperature is going to be affect. See since the strong dependence on S_L and temperature is present here, so S_L follows the temperature which is also seen already.

As you can see that the adiabatic flame temperature variation which was shown here in this slide, which presented a variation with equivalence ratio producing a maximum value at a slightly richer condition for methane air flames.

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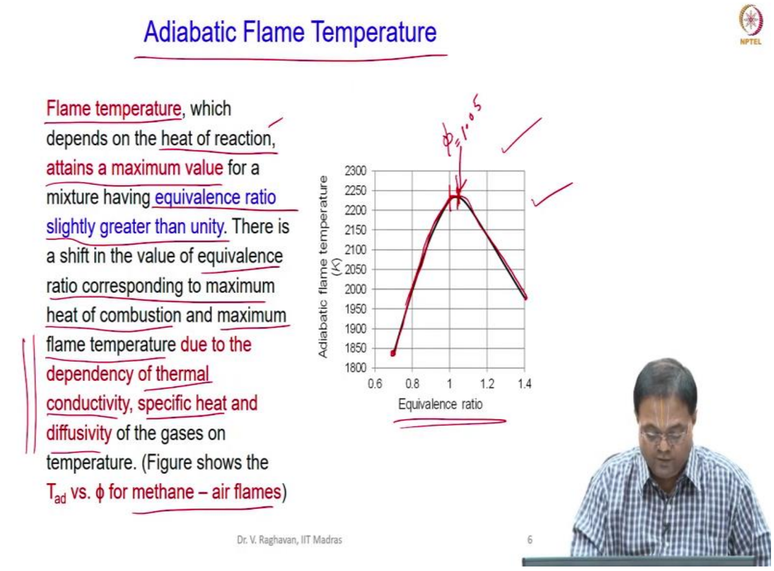

Adiabatic Flame Temperature

Flame temperature, which depends on the heat of reaction, attains a maximum value for a mixture having equivalence ratio slightly greater than unity. There is a shift in the value of equivalence ratio corresponding to maximum heat of combustion and maximum flame temperature due to the dependency of thermal conductivity, specific heat and diffusivity of the gases on temperature. (Figure shows the T_{ad} vs. ϕ for methane – air flames)

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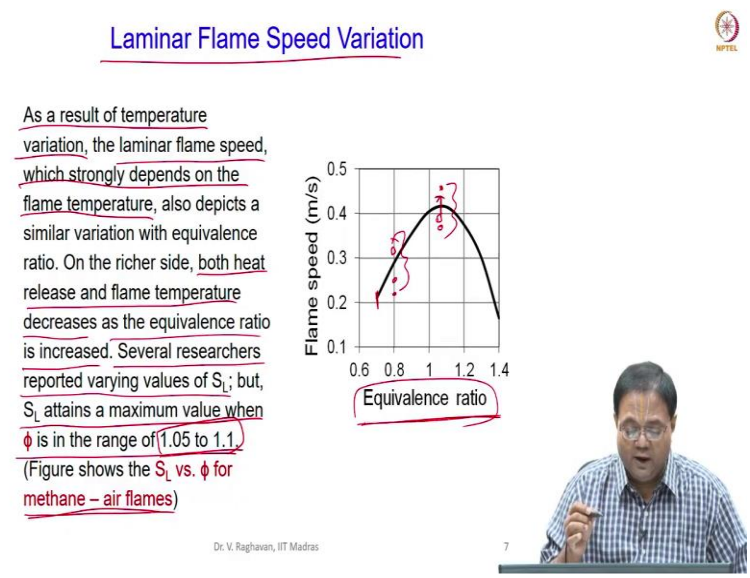

Laminar Flame Speed Variation

As a result of temperature variation, the laminar flame speed, which strongly depends on the flame temperature, also depicts a similar variation with equivalence ratio. On the richer side, both heat release and flame temperature decreases as the equivalence ratio is increased. Several researchers reported varying values of S_L; but, S_L attains a maximum value when ϕ is in the range of 1.05 to 1.1. (Figure shows the S_L vs. ϕ for methane – air flames)

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Similarly, its laminar flames also poses the typical variation producing a maximum at the slightly richer point. So, temperature dependence of S_L is clearly illustrated and equivalence ratio that affects the temperature, it affects the S_L also in the similar manner.

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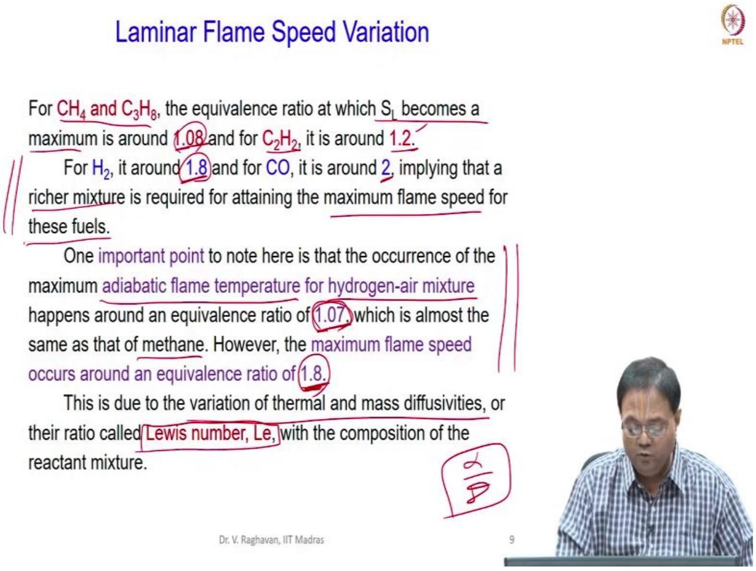

Laminar Flame Speed Variation

For CH_4 and C_3H_8, the equivalence ratio at which S_L becomes a maximum is around 1.08 and for C_2H_2, it is around 1.2.

For H_2, it around 1.8 and for CO, it is around 2, implying that a richer mixture is required for attaining the maximum flame speed for these fuels.

One important point to note here is that the occurrence of the maximum adiabatic flame temperature for hydrogen-air mixture happens around an equivalence ratio of 1.07, which is almost the same as that of methane. However, the maximum flame speed occurs around an equivalence ratio of 1.8.

This is due to the variation of thermal and mass diffusivities, or their ratio called Lewis number, Le, with the composition of the reactant mixture.

$\frac{\alpha}{D}$

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But if you take hydrogen type of fuel, there are some discrepancies which we have noted here due to the highly non unity Lewis number and temperature dependent property values. Now, we are going to go for what is called simplified analysis in which what we are trying to do is we will assume several things try to pose equations; but anyway, use the energy equation to find some quantitative equation or expression for S_L and δ, that is laminar flame speed and the flame thickness.

So, here also we are assuming one-dimensional flame propagation, that is a simplified approach has been given by or reported by Professor Spalding. Lot of assumptions have to be done for any simplified analysis, where we are not going to use say numerical approach, only theoretical approach is used, then simplified assumptions have to be made.

So, one-dimensional constant area duct in which the flame is propagating, then constant pressure, the pressure variations are not much. Steady propagation which is almost true, steadily the flame propagates. Then, what we are not going to see is take into account thermal radiation; radiation heat transfer is not taken into account.

Then, the thermal diffusion which is called Soret effect, it is not going to be taken into account. The secondary effect, second order effect which is due to Soret effect the mass diffusion takes place; thermal gradient drives the mass. Similarly, due to the mass transport or concentration gradients, energy flux will be transported that is Dufour effect.

Then, viscous dissipation etcetera which are work done due to high speed flows, which is not take into account. Obviously, it is not significant at all. So, these are negligible. Again, which is not true always. Lewis number is 1, but we are assuming that.

Similarly, another assumption is specific heats of all the species are the same, that is $c_{p,i}$ is constant; i can be any 1 to N. There is no variation in the c_{pi}'s and they are constant values. So, they will not vary with temperature also; c_{pi} equal to constant means at a given temperature c_p of CO_2 may be equal to c_p of N_2 or c_p of CH_4.

So, that assumption is made because of little difference at a given temperature. But with temperature, the variation is not taken into account because we are going to take some average temperature and calculate these values. So, c_p values are constant. The mixture c_p also is a constant because there is mixture c_p; c_p is a single variable here that is also held as constant.

Then, single step reaction is assumed here. Please understand that premixed flames are kinetically controlled. So, in a simplified assumption, we cannot invoke all the equation, chemical reactions and elementary reactions. So, we assume a single sub reaction here. But understand that in a kinetically controlled process, we cannot assume the reaction to be single step to get accurate results.

But in a simplified analysis, to get some order of values, magnitude values, we can invoke the single step reaction; but we cannot eliminate a chemical reaction at all. So, please understand at least a finite rate reaction is necessary. So, we cannot assume that the reaction rate is infinitely fast, that cannot be assumed, that is the finite rate which is imposed to the chemical reaction.

It may not be a very detailed mechanism, but at least it should take into account some finite rate at which the reaction takes place. That means, if we assume the infinitely fast reaction, then the flame thickness will approach 0 value.

So, in order to avoid that, in order to impart some finite thickness for the flame, the reaction rate also is assumed to be at a finite rate, which is due to a single step reaction. So, now governing equations; mass conservation d/dx; x is the direction of the propagation, d(ρu/dx) = 0.

So, mass is conserved. $\rho u = \rho_u S_L$. Please understand, ρu is the product of density and velocity at any point which is should be equal to the velocity at which the unburnt reaction mixture approaches the flame in the normal direction. So, S_L is velocity there or the speed.

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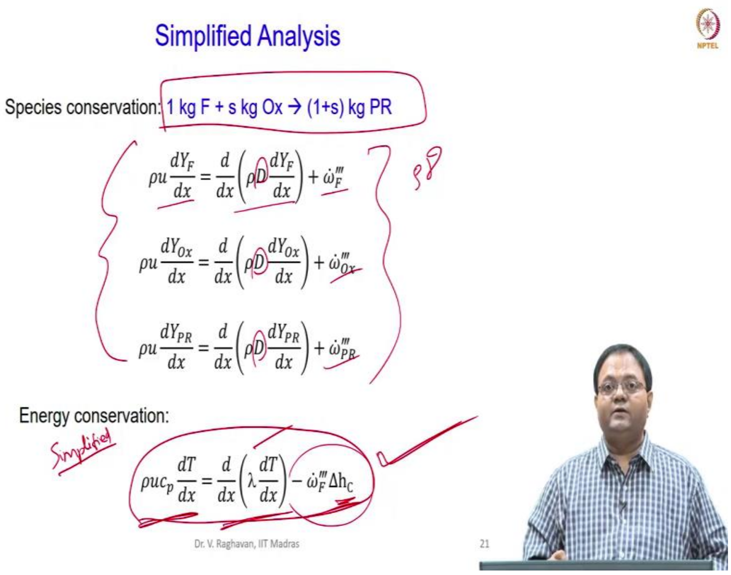

Simplified Analysis

Species conservation: 1 kg F + s kg Ox → (1+s) kg PR

$$\rho u \frac{dY_F}{dx} = \frac{d}{dx}\left(\rho D \frac{dY_F}{dx}\right) + \dot{\omega}_F'''$$

$$\rho u \frac{dY_{Ox}}{dx} = \frac{d}{dx}\left(\rho D \frac{dY_{Ox}}{dx}\right) + \dot{\omega}_{Ox}'''$$

$$\rho u \frac{dY_{PR}}{dx} = \frac{d}{dx}\left(\rho D \frac{dY_{PR}}{dx}\right) + \dot{\omega}_{PR}'''$$

ρδ

Energy conservation:

Simplified

$$\rho u c_p \frac{dT}{dx} = \frac{d}{dx}\left(\lambda \frac{dT}{dx}\right) - \dot{\omega}_F''' \Delta h_C$$

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Now, for the reaction 1 kg of fuel + s kg of oxidizer → (1 + s) kg of products PR. The conservation equations are written in the one-dimension without the unsteady term.

$$\rho u \frac{dY_F}{dx} = \frac{d}{dx}\left(\rho D \frac{dY_F}{dx}\right) + \dot{\omega}_F'''$$

$$\rho u \frac{dY_{Ox}}{dx} = \frac{d}{dx}\left(\rho D \frac{dY_{Ox}}{dx}\right) + \dot{\omega}_{Ox}'''$$

$$\rho u \frac{dY_{PR}}{dx} = \frac{d}{dx}\left(\rho D \frac{dY_{PR}}{dx}\right) + \dot{\omega}_{PR}'''$$

So, this is the diffusion term; convective term, diffusion term and the source term. So, please understand one more thing, this D is also taken as constant here and same for all the variables. So, that assumption is also made. So, ρD is constant.

So, you can write this equation. But please understand this, these equations are still non-linear, we can see the we cannot eliminate the reaction rates; the reaction rate is important for this particular thing. But by using these three equations, we can simplify the energy equation. So, simplified energy equation is given here. This is simplified. So, we have already seen how to simplify energy equation in the previous chapter governing equations for the reacting flow.

$$\rho u c_p \frac{dT}{dx} = \frac{d}{dx}\left(\lambda \frac{dT}{dx}\right) - \dot{\omega}_F''' \Delta \mathrm{h_C}$$

So, we have taken a single like one-dimensional equation energy equation and applied the conservation equations to find this equation. So, this we have derived actually in the previous chapter. Please understand the species equations are not going to be used in this; but the species equations are used to get this value. So, this energy equation which is simplified. We have enough equation to solve the problem.

So, here what we are trying to say is convection of the thermal energy ρu that is the mass flux in to c_pdT/dx that is the enthalpy will be equal to the conduction of heat and the heat which is released due to the combustion that is $-\dot{\omega}_F'''$, it is a negative quality because consumption of fuel. So, negative and negative is positive.

So, this becomes a source term for us multiplied by Δh_c. Δh_c is the heat of combustion which is positive. So, these equations are used to derivate the simplified energy conservation equation. Now, we are going to solve the energy equation in the zone.

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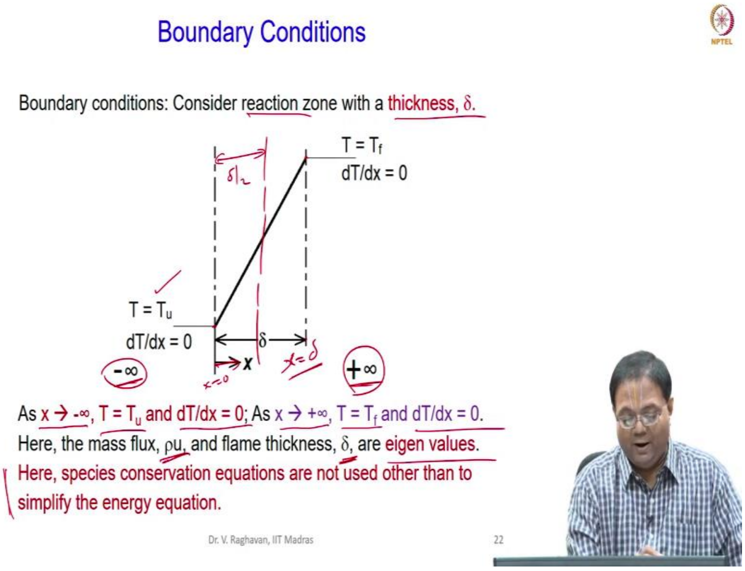

Boundary Conditions

Boundary conditions: Consider reaction zone with a thickness, δ.

As x → -∞, T = T_u and dT/dx = 0; As x → +∞, T = T_f and dT/dx = 0.

Here, the mass flux, ρu, and flame thickness, δ, are eigen values.

Here, species conservation equations are not used other than to simplify the energy equation.

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Now, what is the zone we are assuming? The zone where this is the x direction, here this is the x direction; at x = 0 and this is x = δ and this is x → -∞ and x → +∞ on either side.

Now, as the reactant approach at this point, at x = 0, its temperature starts to increase and at the end of this reaction zone, x = δ, its temperature reaches the flame temperature.

So, single zone we are assuming; reaction zone with the thickness of δ is taken and single zone is assumed, where the unburnt reactants with the temperature of T_u approaches from -∞, at this point, it is at x = -∞ to x = 0, it approaches.

Then at that point of x = 0, it simply just starts to increase and finally reaches the flame temperature T_f at the end of the reaction zone given by x = δ. So, the boundary conditions for this is as x → -∞, T = T_u and dT/dx = 0, because its flat. As x → +∞, T = T_f and dT/dx = 0. So, these are the boundary condition; second order equation.

This is a second order equation. Due to this conduction term, the second order equation we get. To solve this, we need boundary conditions in surrounding -∞ to +∞. Again, you can split into several zones and do this, we will see that. Now, the mass flux ρu which is arising due to the mass conservation and this thickness of the reaction zone, this ρu = ρS_L. So, that S_L itself is a unknown quantity.

Similarly, this thickness is unknown quantity, they are eigenvalues. Other problem; so, this problem. We are stating this as an eigenvalue, we are going to solve this. Please understand as I already told that species conservation equations are not used here, they were actually used to simplify the energy equation and write this in this form. Energy equation is written in this form due to by the help of these equations. We have already seen that.

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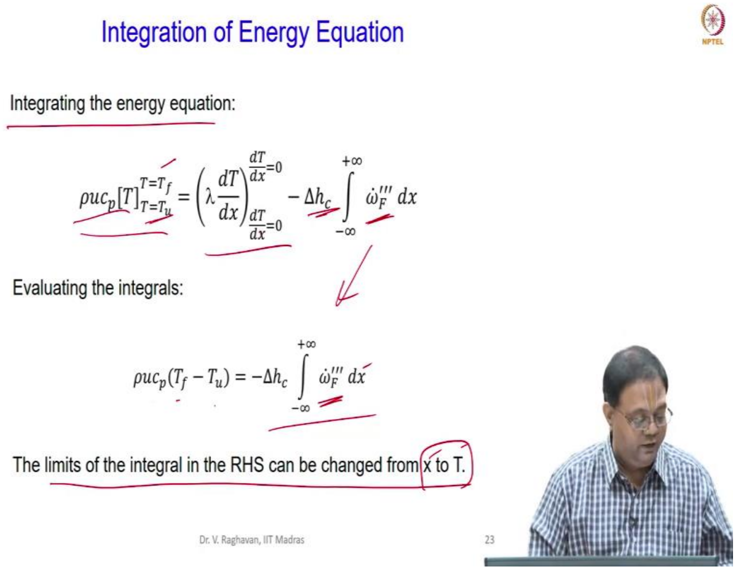

Integration of Energy Equation

Integrating the energy equation:

$$\rho u c_p [T]_{T=T_u}^{T=T_f} = \left(\lambda \frac{dT}{dx}\right)_{\frac{dT}{dx}=0}^{\frac{dT}{dx}=0} - \Delta h_c \int_{-\infty}^{+\infty} \dot{\omega}_F''' \, dx$$

Evaluating the integrals:

$$\rho u c_p (T_f - T_u) = -\Delta h_c \int_{-\infty}^{+\infty} \dot{\omega}_F''' \, dx$$

The limits of the integral in the RHS can be changed from x to T.

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Now, integrate the energy equation. So, just integrate each term and in the limit, we have to put. For example, in the limit of -∞, T will have a value of T_u and +∞, it will have a value of T_f. So, the first term, the convective term is integrated ρuc_p = constant; ρu is constant as a result of mass conservation and we have assumed c_p is a constant.

So, based upon that T_u we have to do integration in the limits. Similarly, the diffusion term, the conduction term will be written like $\lambda dT/dx$; $dT/dx = 0$ at $-\infty$ and $+\infty$, both. So, we have to apply this. And source term, which is this, Δx (which is constant) × $\int_{-\infty}^{+\infty} \dot{\omega}_F'''$.

Now, evaluating the integrals first, first term here in the left hand side $\rho uc_p(T_f - T_u)$ and this term becomes 0, the first term in the right hand side becomes 0 and this term is retained here. So, $-\Delta h_c \int_{-\infty}^{+\infty} \dot{\omega}_F''' dx$. So, please understand that in this analysis, we are not eliminating the source term, that is the reaction rate term is very important.

So, we have to implement a finite rate of reaction so that we get this value of δ evaluated properly. So, the limits of the integral right hand side can be changed from x to T because know we do not know see, we know that the reaction rate, the rate of conservation of fuel etcetera strongly depends on temperature. So, we can calculate the temperature the limits which are in the x now can be changed to T and dx can be changed to dT.

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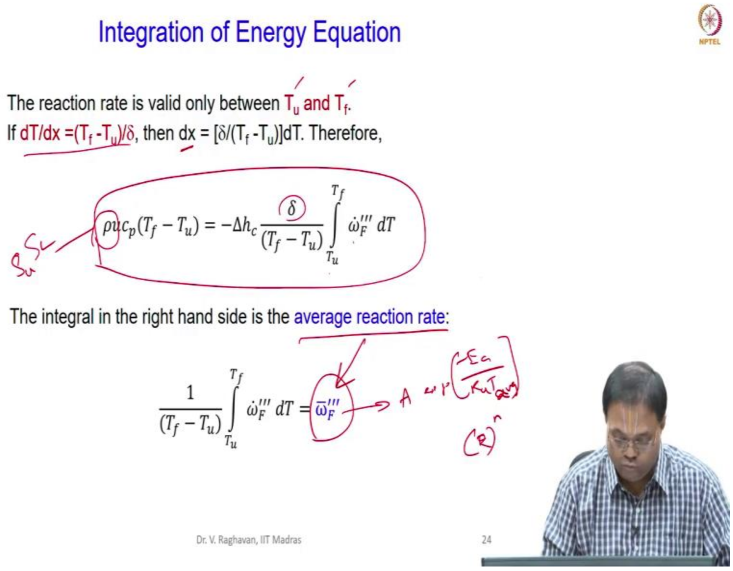

Integration of Energy Equation

The reaction rate is valid only between T_u and T_f.

If $dT/dx = (T_f - T_u)/\delta$, then $dx = [\delta/(T_f - T_u)]dT$. Therefore,

$$\rho uc_p(T_f - T_u) = -\Delta h_c \frac{\delta}{(T_f - T_u)} \int_{T_u}^{T_f} \dot{\omega}_F''' \, dT$$

S_u

The integral in the right hand side is the average reaction rate:

$$\frac{1}{(T_f - T_u)} \int_{T_u}^{T_f} \dot{\omega}_F''' \, dT = \overline{\omega}_F''' \rightarrow A \exp\left(\frac{-E_a}{R_u T_{avg}}\right) (R)^n$$

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So, for that the reaction rate is valid. So, we can see that only in the zone between x = 0 and x = δ, where T is in between T_u and T_f, the reaction rate has some meaning. So, in this zone, we can write dT/dx as ΔT, that is $T_f - T_u/\delta$.

So, when you write this, then we can write dx as $\delta/(T_f - T_u)dT$. So, substitute that, you get this. Left hand side remains the same; $\rho u c_p(T_f - T_u) = -\Delta h_c \times \delta/(T_f - T_u) \int_{Tu}^{Tf} \dot{\omega}_F''' \, dT$.

Now, you can note that this $1/(T_f - T_u) \int_{Tu}^{Tf} \dot{\omega}_F''' \, dT$ is a average reaction rate. The reaction rate is very low at T_u; its very high at T_f; within this range. It is some average value. If you integrate the reaction rate which is lower at T_u and very high at T_f, you get an average reaction rate which is defined as $\bar{\omega}_F'''$, that is the average reaction rate.

So, that we can substitute now. Average reaction rate, we can calculate using an average temperature. Obviously, $(T_f + T_u)/2$, we can use to calculate ω_F. So, you know expression for ω_F. So, that is $A\exp(-E_a/R_uT)$, you have to put this into concentration of fuel or reactant power n. So, this is the reaction rate. Now, you have to put the T here, you have to put $T_{average}$ here to get this value.

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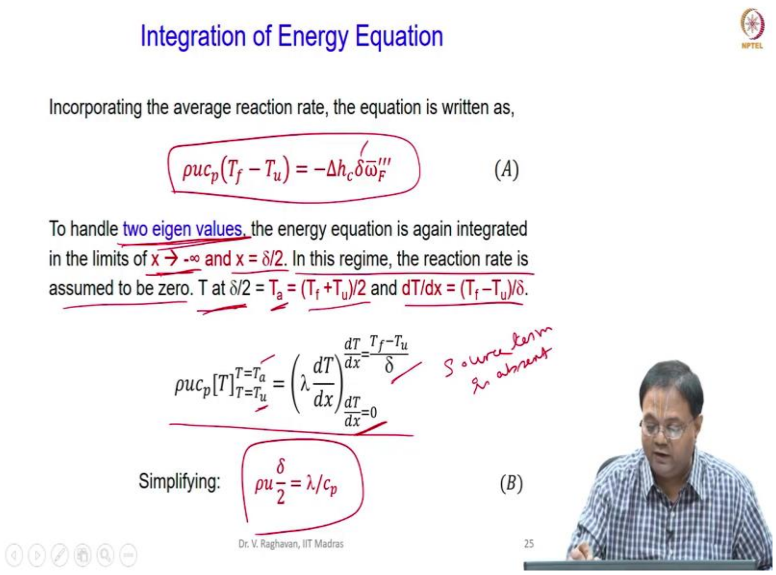

Integration of Energy Equation

Incorporating the average reaction rate, the equation is written as,

$$\boxed{\rho u c_p(T_f - T_u) = -\Delta h_c \delta \bar{\omega}_F'''} \qquad (A)$$

To handle two eigen values, the energy equation is again integrated in the limits of $x \rightarrow -\infty$ and $x = \delta/2$. In this regime, the reaction rate is assumed to be zero. T at $\delta/2 = T_a = (T_f + T_u)/2$ and $dT/dx = (T_f - T_u)/\delta$.

$$\rho u c_p [T]_{T=T_u}^{T=T_a} = \left(\lambda \frac{dT}{dx}\right)_{\frac{dT}{dx}=0}^{\frac{dT}{dx}=\frac{T_f - T_u}{\delta}}$$

Source term is absent

Simplifying: $\boxed{\rho u \frac{\delta}{2} = \lambda / c_p}$ (B)

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So, once you get this value, you can substitute this, the right hand side can be simplified and I now write the right hand side, $\rho u c_p(T_f - T_u) = -\Delta h_c \delta \bar{\omega}_F'''$.

This equation, the δ comes there. So, this remains. So, S_L is unknown, δ is unknown. So, two eigenvalues are still there in this equation.

So, what we do is we have to integrate this equation one more time, but now with the different limits. When you see this, initially what we integrated? When we integrated the energy equation, we considered limits as minus infinity to plus infinity. Now, we once again integrate to get 2 eigenvalues.

So, one more eigenvalue I have to get. So, I integrate this equation energy equation, now with the limit which is from minus infinity to $\delta/2$. So, half of the reaction zone. So, this is $\delta/2$. So, $-\infty$ to $\delta/2$, let us try to integrate now. So, that is what I am going to do now.

So, to handle two eigenvalues, the energy equation is again integrated within the limits of extending to $-\infty$ to half of the reaction zone and generate. So, eliminate one variable. For that, I am trying to do this. So, in this regime, why I am choosing this regime because I am assuming that, in this regime no reaction will take place.

So, it is a one-zone model. But I know that some pre-heat should have happened. So, now, what I am assuming that in this zone $-\infty$ to x equal to half of this reaction zone thickness, I assume that the reaction rate may not be so high because of the low temperatures prevailing because exponentially the reaction rate increases with temperature increase.

So, when the temperature is between T_u to the middle portion of the reaction zone, there is not much increase in the temperature; even though, it is linearly varying. So, in this regime, the reaction rate is assumed to be negligible or 0. So, now what is the temperature value?

T, we know at $-\infty$, $T = T_u$ and $dT/dx = 0$, these two boundary conditions I know. But what are the boundary conditions at $\delta/2$? T at $\delta/2$ is average temperature because linear profile, here linear profile.

So, in this point temperature will be average of these two; T_f and T_u. So, I say that as T average, $T_a = (T_f + T_u)/2$. So, temperature at $\delta/2$, the second boundary is this. Similarly, dT/dx can be assumed to be $(T_f - T_u)/\delta$. So, that is the temperature gradient slope; slope is constant know. So, that is the slope which we have in this.

So, that dT/dx at that point $\delta/2$ is the slope of the entire curve which is T_f and T_u by δ. So, if we use this, now we integrate the equation; the source term disappears because in this range of regime $-\infty$ to $\delta/2$, it is easy to approximate the reaction rate to be 0 because temperatures are low enough, exponentially temperature increases.

So, at low temperature, the reaction rates are low; temperature increases to a higher value, exponentially the reaction rate increases. So, keeping in mind, the source term is absent. Now, you can see the limits of the left hand side will be T_u to $T_{average}$.

Similarly, $dT/dx = 0$ to $dT/dx = (T_f - T_u)/\delta$. So, now, when you do this, you get the equation which connects ρ_u and δ. $\rho u\delta/2 = \lambda/cp$, so that is what we get here.

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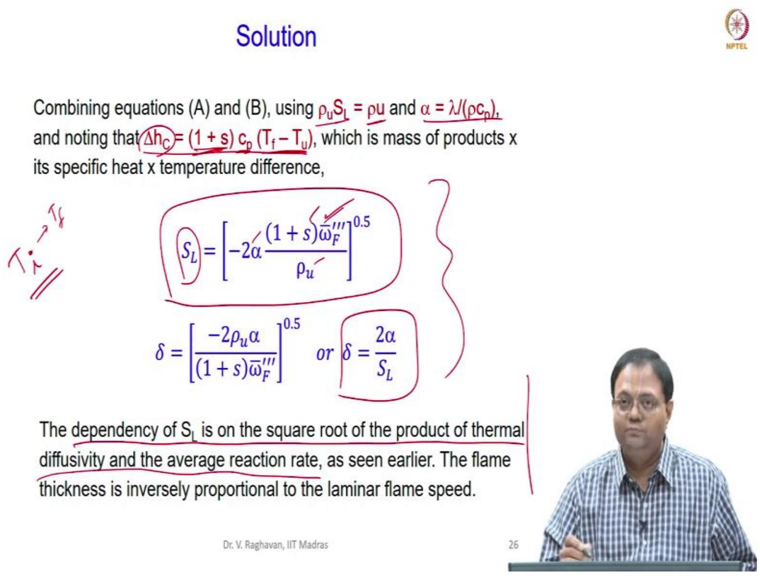

Solution

Combining equations (A) and (B), using $\rho_u S_L = \rho u$ and $\alpha = \lambda/(\rho c_p)$, and noting that $\Delta h_C = (1 + s)\, c_p\, (T_f - T_u)$, which is mass of products x its specific heat x temperature difference,

$T_i \to T_f$

$$S_L = \left[-2\alpha \frac{(1+s)\bar{\omega}'''_F}{\rho_u}\right]^{0.5}$$

$$\delta = \left[\frac{-2\rho_u \alpha}{(1+s)\bar{\omega}'''_F}\right]^{0.5} \quad or \quad \delta = \frac{2\alpha}{S_L}$$

The dependency of S_L is on the square root of the product of thermal diffusivity and the average reaction rate, as seen earlier. The flame thickness is inversely proportional to the laminar flame speed.

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So, that you can use here this and if we connect A and B, we will get the solution. So, now, you know $\rho_u S_L$ and α is $\lambda/\rho c_p$ and now, the heat of combustion is nothing but the mass flow rate of the heat.

So, 1 kg of fuel + s kg of oxidizer → 1 + s kg of product. So, 1 + s kg of products into its c_p into temperature difference $T_f - T_u$ that will be the heat flow due to the flow of the products. So, Δh_c is nothing but that. So, if you substitute, you get the expression for S_L which is this.

So, here the negative is because of this $\dot{\omega}'''_F$ is negative. So, this is a positive quantity. So, again, the same thing we are getting that is $(\omega\alpha)^{0.5}$, but other terms are there which we can evaluate now. We know the fuel, I know what is s value, what is ρ_u value etcetera and I can find the α and $\dot{\omega}'''_F$ at average temperature.

So, some value of S_L can be determined based upon the reaction rates and these quantities. So, δ can be written in terms of α and S_L like $2\alpha/S_L$. So, these are the two quantitative expressions which we have got from the simplified analysis. Still, you please understand that this is not so accurate, but it will give some quantitative results.

In the previous thermal theory that T_i was used, so in the two-zone model, the ignition temperature cannot be like eliminated. So, for that some assumptions have to be made. Here, the zones are combined and actually what we are trying to do here is we are saying that the T_i approaches T_f, that is what we are trying to do here and try to get the values. So, here the dependency of S_L on $\sqrt{\alpha\omega}$ (average reaction rate) is clearly present.