Fundamentals of Combustion

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Lecture - 19

Mass Transfer Basics – Part 04

Steady Evaporation of Liquid Droplet and Worked Examples

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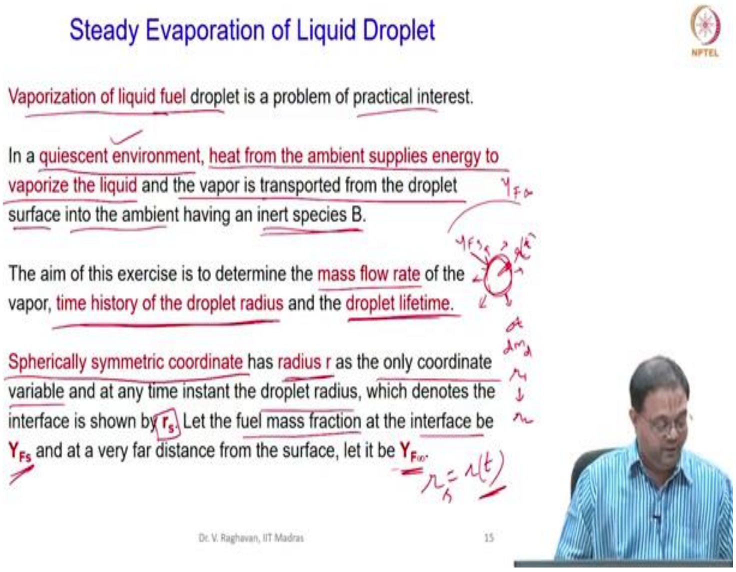

Steady Evaporation of Liquid Droplet

Vaporization of liquid fuel droplet is a problem of practical interest.

In a quiescent environment, heat from the ambient supplies energy to vaporize the liquid and the vapor is transported from the droplet surface into the ambient having an inert species B.

The aim of this exercise is to determine the mass flow rate of the vapor, time history of the droplet radius and the droplet lifetime.

Spherically symmetric coordinate has radius r as the only coordinate variable and at any time instant the droplet radius, which denotes the interface is shown by r_s. Let the fuel mass fraction at the interface be Y_{Fs} and at a very far distance from the surface, let it be $Y_{F\infty}$.

$r_s = r(t)$

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So, the next topic is Steady Evaporation of Liquid Droplet. Here vaporization of liquid fuel droplet is of a practical interest. So, as I told you know the liquid fuel cannot be used directly like you feed the gaseous fuels.

So, in order to increase the vaporization rate we atomize the liquid into several droplets and so, the rate at which a droplet single liquid fuel droplet vaporizes in any medium, that is what is of interest to us. So, a simplified problem, let us consider a quiescent environment where there is no bulk flow, convective flow is not there.

The temperature at the ambient side is higher than the boiling point of the liquid, slightly higher than the boiling point of the liquid. So, heat from the ambient supplies energy to vaporize the liquid. So, that is what the situation is. Now, the vapor is transported from the droplet surface into the ambient.

Now, again ambient has a species B, just like the Stefan problem where liquid A in the test tube was evaporating and the liquid A surface was maintained constant here. That is the x = 0 was maintained constant by supplying the liquid slowly at the rate at which it was evaporating and the A is mixed with the inert species B, similar to that droplet vaporize into a inert species B. So, this is the scenario

Now, what we want to achieve here is we have to determine the mass flow rate of the vapor from the droplet surface, then the time history of droplet radius in some manner. Here we are trying to see a problem where the droplet radius initially has some value and by evaporation the droplet radius shrinks, and finally, it will become 0 when all the liquid is vaporized.

Then, what is the droplet life time? For a given droplet of initial diameter say 1 mm to completely evaporate in a given environment what will be the time? So, that is called droplet lifetime.

So, we are interested in the three quantities here in the previous one the steady evaporation rate was the only thing what we saw in the Stefan's problem because we are supplying the liquid at the same rate at which it evaporates. So, the liquid level was maintained at a constant location x = 0, steady problem.

Here actually this appears to be an unsteady problem, because we are now looking into time history of the droplet rate. The droplet initially having some radius value, then goes to 0 at the end of the droplet lifetime.

So, what is that time? So, it is determined based on the temperature, based on the fuel and based on any other parameters like convection if it is present. Now, in this case we are going to look into only the quiescent environment.

Now, this is a simplified problem of droplet evaporation, but this is in spherical coordinates. So, what we call a spherically symmetric coordinate. Here the radius is the only coordinate direction. So, for example, if there is a droplet and this is center and this is the radius r that is going to be changing as a function of time.

This radius is going to change with time and at any point in time you can see that the vapors will evolve normal to the surface all over the droplet, so that within a time duration Δt some mass dm of the droplet will come out so that the radius changes from r_1 (it decreases) to r_2. So, this is what the phenomena is.

So, for a droplet spherical, if you track only the radius, that is enough for us. So, that is the way we are considering this problem as a one dimensional problem, as a function of radius in any theta or phi. So, in a spherical coordinate, theta and phi are the two angles and one radius r.

So, these are the three coordinates and we need not even worry about other two coordinates because take any radial direction, there at any particular time some mass has come out.

We can just integrate this to the entire droplet surface. So, only coordinate what we consider is the radius and at any time instant what is the radius value that is what is important. That is, r is a function of time.

So, at the surface the radius is r_s. Here, instead of say r, we can say r_s or may be $r_s = r(t)$ or the radius at the surface is going to vary as a function of time.

Now, again similar nomenclature for the mass fractions, here I am saying it is a liquid fuel. So, Y_{Fs} is the mass fraction of this liquid at the interface or the surface of the droplet and at a very far distance it will be $Y_{F,\infty}$.

You are considering this ambient. This droplet is evaporating into a big ambient. So, here this will be $Y_{F,\infty}$ and here it is Y_{Fs} and r_s varies as r(t).

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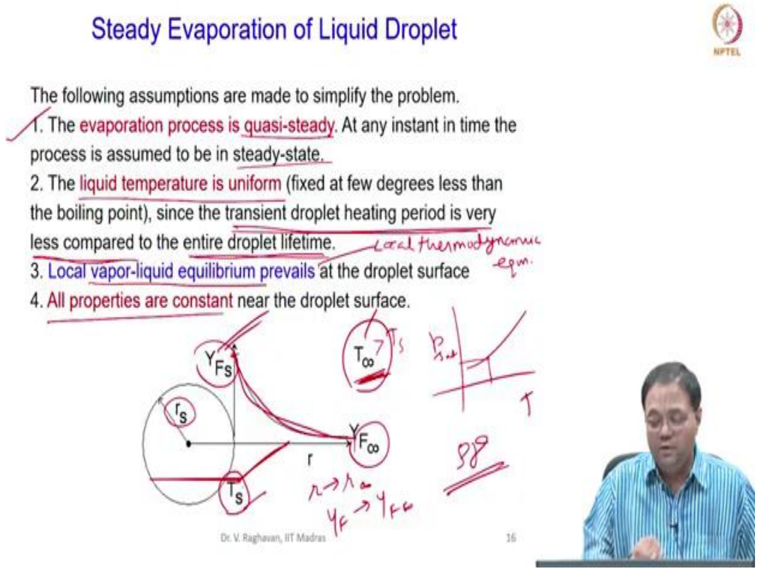

Steady Evaporation of Liquid Droplet

The following assumptions are made to simplify the problem.

1. The evaporation process is quasi-steady. At any instant in time the process is assumed to be in steady-state.
2. The liquid temperature is uniform (fixed at few degrees less than the boiling point), since the transient droplet heating period is very less compared to the entire droplet lifetime.
3. Local vapor-liquid equilibrium prevails at the droplet surface
4. All properties are constant near the droplet surface.

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Now, it is a transient process but I do not call it as a transient I just call it what is called quasi-steady. Why it is quasi-steady? because even though there is a vaporization like the shrinkage in the droplet diameter or the radius that reduces from initial value to 0, the rate at which is it is decreasing is not going to vary.

So, that is the important assumption made here. At any instant in time the process is assumed to be steady state that is the rate of vaporization calculated in some manner is steady. That is one of the important one. How it is going to be steady you will see.

Now, the liquid temperature is uniform, it is like the droplet diameters are not so huge and it has some finite smaller diameter so that the liquid temperature is uniform or it has gone through a initial period of heat up.

As I told before the q_{i-l}, interface to the liquid part is initially predominant, but after a very small time period that becomes very minimal and whatever heat comes from the gas phase to the interface predominantly supplies to the latent heat of vaporization.

So, in this basically the liquid temperature becomes uniform that is the heat up rate is over and surface has already reached a temperature which is wet build temperature. That means the surface temperature or the liquid temperature will not vary after a particular time period. Small time period it will take to reach that condition after that condition it will not vary.

So, the transient droplet heating period is very less when compared to the entire droplet lifetime. Now, let us take 1 mm droplet diameter initially at 298 K and the boiling point let it be say 371 K as I told in the previous example.

Now, to increase the temperature from say 298 K to say 365 K, something like that is a temperature which is finally is attained.

So, that time period is much lesser when compared to the droplet completely vaporizing from 1 mm to 0. So, the entire droplet life time is much higher than the heating period of the droplet which will take its temperature from 298 K to the very close to the boiling point.

Similarly, vapor liquid equilibrium, this is called local thermodynamic equilibrium. So, that I told you this p versus T curve is going to be there for any particular temperature. There is a definite saturation pressure or local thermodynamic equilibrium that prevails at the droplet surface, then all the properties are constant near the droplet surface.

So, whatever property you want to, say ρD this product etcetera is constant. This is the assumption we already made. So, these are the assumptions. We have seen many theoretical calculations involves such assumptions to bring down the complexity of the problem. So, one dimension, r_s is going to vary as a function of time and its going to decrease from r_s initial to 0, and at any point based upon the surface temperature is constant now.

So, a definite value of Y_{Fs} is going to be there and that decreases to a very low value of $Y_{F,\infty}$ when you go far away. So, as $r \rightarrow r_\infty$, $Y_F \rightarrow Y_{F,\infty}$. So, this is the condition. T_∞ is the ambient temperature, is greater than the surface temperature. So, that it will aid in evaporation, this is the problem in hand.

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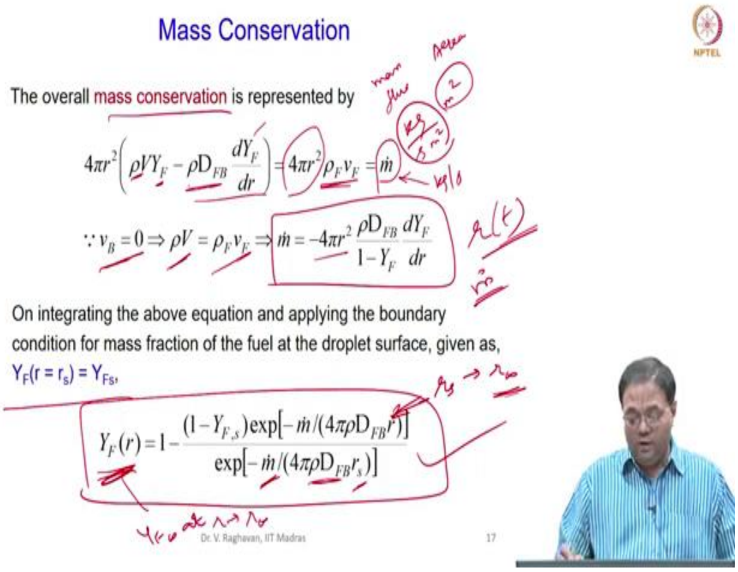

Mass Conservation

The overall mass conservation is represented by

$$4\pi r^2\left(\rho V Y_F - \rho D_{FB}\frac{dY_F}{dr}\right) = 4\pi r^2 \rho_F v_F = \dot{m}$$

$$\because v_B = 0 \Rightarrow \rho V = \rho_F v_F \Rightarrow \boxed{\dot{m} = -4\pi r^2 \frac{\rho D_{FB}}{1-Y_F}\frac{dY_F}{dr}}$$

On integrating the above equation and applying the boundary condition for mass fraction of the fuel at the droplet surface, given as, $Y_F(r = r_s) = Y_{Fs}$,

$$\boxed{Y_F(r) = 1 - \frac{(1-Y_{F,s})\exp[-\dot{m}/(4\pi\rho D_{FB} r)]}{\exp[-\dot{m}/(4\pi\rho D_{FB} r_s)]}}$$

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Now, exactly what we did for this Stefan problem we are going to do in a spherical coordinate; the mass conservation. So, again you can see the Fick's law coming to help us.

So, the net mass, this is the $\rho_F v_F$ (of fuel) there we wrote $\rho_A v_A$ here we are writing $\rho_F v_F = \rho V Y_F + \rho D_{FB}\, dY_F/dr$, with the only coordinate r, instead of D_{AB} we are putting D_{FB}.

So, that is the flux, that multiplied by the surface area will be the mass flow rate kg/sm^2 × m^2 that will become kg/s. So, this is the flux into area. So, just multiplying by $4\pi r^2$.

Now, again you can see that the inert species will not go into the droplet. At higher pressures it is possible, but we are not considering that situation here. The pressures are very low and are moderate. So, v_B the inert species velocity is 0 so that the bulk velocity or bulk mixture flux is nothing but this.

Substituting this in this you get this the mass flow rate of the vapor from the interface of the droplet surface to the ambient, given by, $-4\pi r^2 \rho DFB/(1-Y_F) \times dY_F/dr$.

So, this is what you get. Now, you can separate the variable and please understand what I told you, even though the problem is transient in hand the rate at which the evaporation takes place is constant.

So, that is what the problem is quasi-steady. See if you take a car which is moving from point A to point B with a uniform velocity. Even though it is moving spatially from one location to another location the acceleration is 0. Similar to this, here there is no time dependence for the evaporation rate. There is no time dependence of velocity here. Similarly, there is no time dependence on the rate of vaporization.

Even though r, the radius of the droplet varies with time, the rate at which it does, mass is evolving from this, that does not change. So, this is a constant.

So, keeping that in mind we can integrate this equation and apply the condition of the interface Y_F at $r = r_s$ is Y_{Fs} and if you apply that condition you will get the expression slightly complicated expression than what we saw in the Cartesian one-dimension here. For the spherical coordinates you get this expression.

So, Y_F at any radius in the gas phase please understand we are not interested in the liquid phase Y_F from this. So, I am trying to plot this variation here. What I have shown here this variation of Y_{Fs} to $Y_{F,\infty}$ this is given by this expression, where the mass flow rate of the vapor is constant. ρD_B is taken constant and this is at any r and this is at given surface.

So, this is at any r in the gas phase. This is from the surface and this r can be r_s. So, this r actually varies from $r_s \rightarrow r_\infty$ you can put any value there in this and at r_s it is Y_{Fs} and if it goes to r_∞ this will tend to $Y_{F,\infty}$ at $r \rightarrow r_\infty$. So, this is the way this profile what I have plotted here is mathematically represented.

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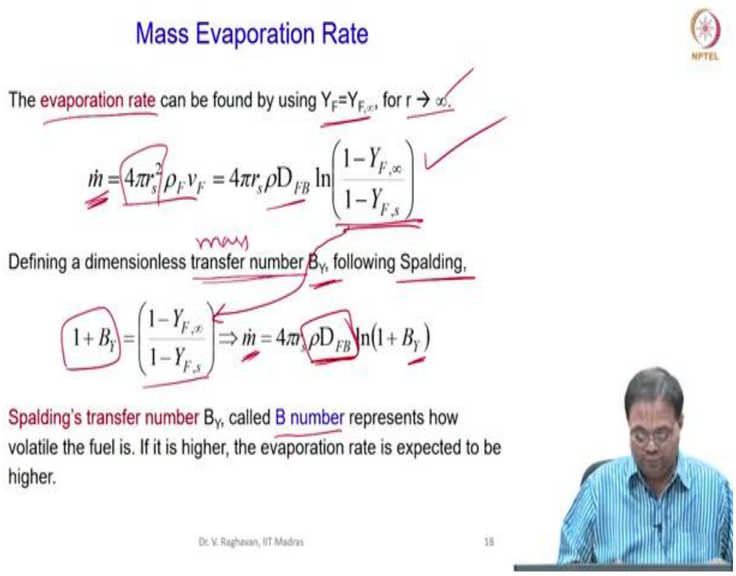

Mass Evaporation Rate

The evaporation rate can be found by using $Y_F = Y_{F,\infty}$, for $r \rightarrow \infty$.

$$\dot{m} = \boxed{4\pi r_s^2}\rho_F v_F = 4\pi r_s \rho D_{FB} \ln\left(\frac{1-Y_{F,\infty}}{1-Y_{F,s}}\right)$$

Defining a dimensionless transfer number B_Y, following Spalding,

$$\boxed{1+B_Y} = \left(\frac{1-Y_{F,\infty}}{1-Y_{F,s}}\right) \Rightarrow \dot{m} = 4\pi r_s \boxed{\rho D_{FB}} \ln(1+B_Y)$$

Spalding's transfer number B_Y, called B number represents how volatile the fuel is. If it is higher, the evaporation rate is expected to be higher.

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Now, the evaporation rate can be found like this. This is very important expression. Now, I substitute $Y = Y_{F,\infty}$ at $r \rightarrow \infty$ or $r \rightarrow r_\infty$. See you need not take infinity you can take a high value of r called r_∞. It may be say 100 times the droplet diameter and so on.

It may not be 100 times it may be say 50 or 80 times based upon the flux which is coming out. So, the mass flow rate of the vapor at a given frozen diameter or radius r_s is given by $4\pi r_s \rho D_{FB} \times \ln[(1-Y_{F,\infty})/(1-Y_{F,s})]$. But at every r_s, ρ is constant.

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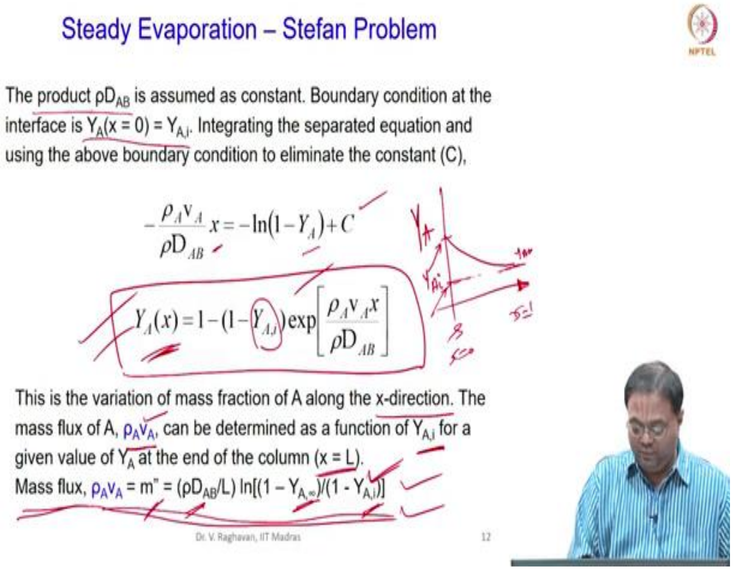

Steady Evaporation – Stefan Problem

The product ρD_{AB} is assumed as constant. Boundary condition at the interface is $Y_A(x = 0) = Y_{A,i}$. Integrating the separated equation and using the above boundary condition to eliminate the constant (C),

$$-\frac{\rho_A v_A}{\rho D_{AB}} x = -\ln(1 - Y_A) + C$$

$$Y_A(x) = 1 - (1 - Y_{A,i}) \exp\left[\frac{\rho_A v_A x}{\rho D_{AB}}\right]$$

This is the variation of mass fraction of A along the x-direction. The mass flux of A, $\rho_A v_A$, can be determined as a function of $Y_{A,i}$ for a given value of Y_A at the end of the column (x = L).

Mass flux, $\rho_A v_A = m'' = (\rho D_{AB}/L) \ln[(1 - Y_{A,\infty})/(1 - Y_{A,i})]$

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So, this is the similar equation what we got here. Here you can see the mass flux for the Stefan's problem, same similar expression we got but we have put mass flux here, but here we are now multiplying that with the surface area to get the mass flow rate or evaporation in kg/s.

So, now, this natural logarithm term, following Spalding's approach we can write this term here as $(1 + B_Y)$ where this B_Y is called transfer number or in this case say a mass transfer number. $1 + B_Y$ is written as this $(1 - Y_{F,\infty})/(1-Y_{Fs})$. So, this is also called B number by Spalding.

So, this actually shows how the evaporation is. So, if the B number is higher the vaporization rate is higher. Actually speaking this term ρD_{FB} dictates evaporation in a much more predominant manner because this appears in this natural logarithm term. Even if you increase B_Y say 5 times, since natural logarithm appears it is not going to increase the mass evaporation rate.

But, if there is an increase in the D_F or ρD_F this product then you can see that there will be much more increase in the mass vaporization rate.

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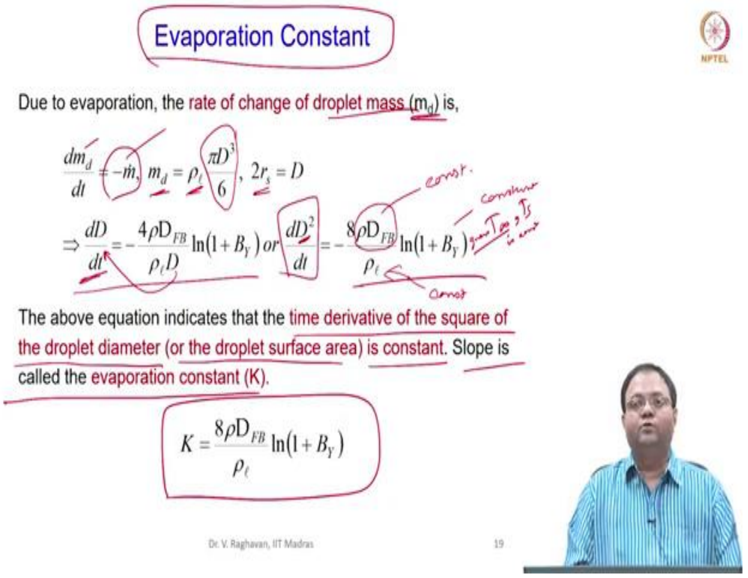

Evaporation Constant

Due to evaporation, the rate of change of droplet mass (m_d) is,

$$\frac{dm_d}{dt} = -\dot{m}, \quad m_d = \rho_\ell \frac{\pi D^3}{6}, \quad 2r_s = D$$

$$\Rightarrow \frac{dD}{dt} = -\frac{4\rho D_{FB}}{\rho_\ell D}\ln(1+B_Y) \text{ or } \frac{dD^2}{dt} = -\frac{8\rho D_{FB}}{\rho_\ell}\ln(1+B_Y)$$

const.

constant given T_∞, T_s is const

const

The above equation indicates that the time derivative of the square of the droplet diameter (or the droplet surface area) is constant. Slope is called the evaporation constant (K).

$$\boxed{K = \frac{8\rho D_{FB}}{\rho_\ell}\ln(1+B_Y)}$$

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Now, we have to find what is called evaporation constant because this is a constant scenario you know how this evaporation is assumed as constant.

So, let us try to do this, let m_d be the droplet mass at any time instant. Initially it is due to the entire initial droplet diameter then finally, this is going to decrease to 0.

So, the rate of the variation of m_d with time is nothing, but negative of the mass evaporation rate $-\dot{m}$; in turn m_d the mass of the droplet can be written as $\rho_l \times (\pi/6) D^3$, density into volume of the liquid droplet.

Please understand that D is nothing but 2 times r_s; r_s is a function of time. So, that means, D is going to be the function of time. So, obviously, now substitute here capital D, dD/dt will be written in this way because you know the expression for this. So, using this expression I am writing this.

So, $-(4\rho D_{FB}/\rho_l D) \times \ln(1+B_Y)$. Now, take this D to this side. So, $dD^2/dt = (8\rho D_{FB}/\rho_l) \times \ln(1+B_Y)$.

Now, this you can see the right hand side we already assumed ρD_{FB} as constant. So, this is constant and this is constant and this is also constant when T is a constant; for a given T_∞, T_s is constant because we are assuming that the wet bulb condition is reached.

Now, based upon this you will get the right hand side to be constant; that means that the rate at which the square of droplet diameter changes with time will be a constant. The time derivative of the square of droplet diameter or the droplet surface area we can say is a constant and this will be the slope of this.

So, if you integrate this dD^2/dt take dt to the other side and this is a constant. So, if you take this to other side and integrate this, slope of that equation will give you the evaporation constant as this. So, $K = (8\rho D_{FB}/\rho_l)\times\ln(1 + B_Y)$.

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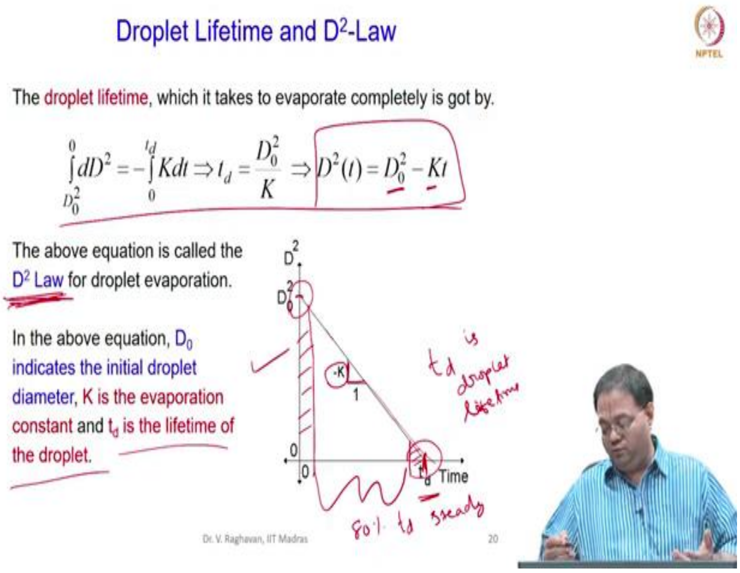

Droplet Lifetime and D²-Law

The droplet lifetime, which it takes to evaporate completely is got by.

$$\int_{D_0^2}^{0} dD^2 = -\int_{0}^{t_d} K dt \Rightarrow t_d = \frac{D_0^2}{K} \Rightarrow \boxed{D^2(t) = D_0^2 - Kt}$$

The above equation is called the D^2 Law for droplet evaporation.

In the above equation, D_0 indicates the initial droplet diameter, K is the evaporation constant and t_d is the lifetime of the droplet.

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Pictorially you can see that the initial droplet diameter is D_o^2, finally it goes to 0 and this entire process occurs with a constant slope, it is a straight line. So, this slope is nothing but the evaporation constant and the time at which this droplet from the initial diameter goes to a 0 value that is the droplet lifetime.

So, t_d is the droplet lifetime. Now, that is what we are going to put here. So, integrating this D^2 at any time will be equal to D_o^2 (that is the initial droplet diameter square) – K (is the evaporation constant time). That is called D^2, it is very important.

So, if you have a droplet which is vaporizing under thermodynamic equilibrium condition you will see that the droplet surface area or the diameter square, variation of that with time will be a constant that is called evaporation constant. Using that you can calculate the droplet lifetime and so on. So, this is very important.

So, the D square law is obeyed because initially see there will be only some changes, during this part there will be some small variation predominantly. For example, excluding this small times here, I am hatching here, excluding these small time periods here, most of the time period here the evaporation will be steady.

That means there it will be a straight line. Here, there will be some changes, later towards the end there will be some changes in slope. Initially also there will be some changes in the slope. Apart from that we can say 80 percent of the lifetime (t_d) it will be steady. So, D^2 law is valid for around 80 percent of the droplet lifetime. So, we can just use that for calculating the evaporation rate of the droplet.

Please understand that here important thing is you should understand that the T_s is constant. For Y_{Fs} to be constant, T_s should be constant and that T_s depends on T_∞. T_s is, see for example, with T_s close to boiling point, how close it is to the boiling point is determined by the value of T_∞. So, this is very important. For example, if boiling point is say 371 then T_s will not be very close to 371 it may be say 350, 360 or something like that. So, we have to really determine how the T_s is dependent on T_∞. We are assuming constant T_s based upon how T_∞ is.

So, once T is calculated X_{Fs} can be calculated. From X_{Fs}, that is mole fraction, mass fraction can be calculated. Then everything is as per our assumptions and we can solve these equations.

So, for a droplet even though there is a change in the diameter the rate of vaporization is a constant obeyed by this particular graph. So, that is called D^2 law. So, we can apply this for several scenarios.

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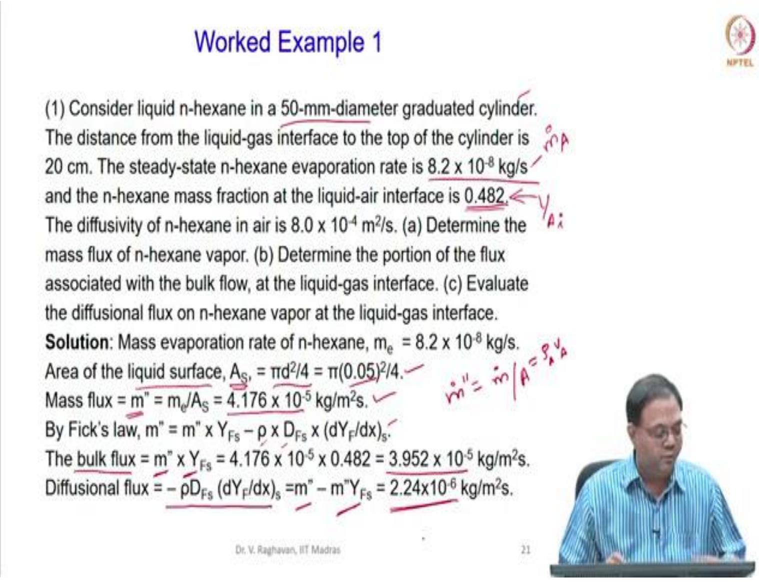

Worked Example 1

(1) Consider liquid n-hexane in a 50-mm-diameter graduated cylinder. The distance from the liquid-gas interface to the top of the cylinder is 20 cm. The steady-state n-hexane evaporation rate is 8.2×10^{-8} kg/s and the n-hexane mass fraction at the liquid-air interface is 0.482. The diffusivity of n-hexane in air is 8.0×10^{-4} m²/s. (a) Determine the mass flux of n-hexane vapor. (b) Determine the portion of the flux associated with the bulk flow, at the liquid-gas interface. (c) Evaluate the diffusional flux on n-hexane vapor at the liquid-gas interface.

$\dot{m}_A$

$Y_{A,i}$

Solution: Mass evaporation rate of n-hexane, $m_e = 8.2 \times 10^{-8}$ kg/s.

Area of the liquid surface, A_S, $= \pi d^2/4 = \pi(0.05)^2/4$.

Mass flux $= m'' = m_e/A_S = 4.176 \times 10^{-5}$ kg/m²s.

$\dot{m}'' = \dot{m}/A = \rho_A v_A$

By Fick's law, $m'' = m'' \times Y_{Fs} - \rho \times D_{Fs} \times (dY_F/dx)_s$.

The bulk flux $= m'' \times Y_{Fs} = 4.176 \times 10^{-5} \times 0.482 = 3.952 \times 10^{-5}$ kg/m²s.

Diffusional flux $= -\rho D_{Fs} (dY_F/dx)_s = m'' - m''Y_{Fs} = 2.24 \times 10^{-6}$ kg/m²s.

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Let us see some worked examples for the mass transfer topic. So, the first example here Consider liquid n-hexane in a 50 mm diameter graduated cylinder.

The distance from the liquid gas interface to the top of the cylinder is 20 cm. The steady state n-hexane evaporation rate is 8.2×10^{-8} kg/s and the n-hexane mass fraction at the liquid air interface is 0.482.

The diffusivity of n-hexane in air, the binary mass diffusivity of n-hexane in air is 8×10^{-4} m²/s. So, what is asked is to determine the mass flux of n-hexane vapor. Then determine the portion of the flux associated with the bulk flow at the liquid gas interface, then evaluate the diffusional flux of n-hexane vapor at the liquid gas interface. So, these are the three things which you need to find.

So, you can see that the evaporation rate is given 8.2×10^{-8} kg/s and the mass fraction at the interface is given. So, that is the Y_{Ai}. So, A is hexane. A is the liquid hexane and A_i is given; this is the $\dot{m}_A$. So, that is given in kg/s.

Now, first to determine the mass flux of n-hexane vapor, you know that mass flux is nothing, but the mass flow rate of the vapor from the interface to the gas phase divided by the area, $\dot{m}'' = m_e/A_s$.

Now, A_s is the surface area of the liquid. Surface area is nothing but the cross sectional area of the graduated cylinder.

So, diameter of the cylinder is given as 50 mm. So, $\pi d^2/4$, substitute in meter, 50 mm is 0.05 m. So, that is the area here. So, the mass flux is nothing but the mass evaporation rate divided by area of cross-section which is 4.176×10^{-5} kg/m^{2}s.

So, that is the total flux which is coming out of the liquid interface. Now, you know by fick’s law this mass flux is nothing but $\rho_A v_A$. Now, by Fick's law you know $\dot{m}'' = \rho_A v_A = \rho V Y_{FS}$.

Now, this $\rho_A V = \rho_A v_A$ because the liquid only evaporates. There is no solubility of the gas in the to the liquid phase minus the diffusion part is $\rho D_{Fs} dY_{Fs}/dx$.

Now, the bulk flux or the total flux contributed by the convective part, convective part is $\dot{m}'' Y_{Fs}$, the first term. So, that is nothing but $4.176\times10^{-5}\times Y_{Fs}$ kg/m^{2}s. This is given here as 0.482. So, substitute that you will get 2.013×10^{-5} kg/m^{2}s.

The diffusional flux is the other component $-\rho D_{Fs} dY_F/dx$ at the surface will be nothing but the total flux minus the convective flux which is equal to $4.176\times10^{-5} - 2.013\times10^{-5}$. So, that is 2.163×10^{-5} kg/m^{2}s, so that is the answer.

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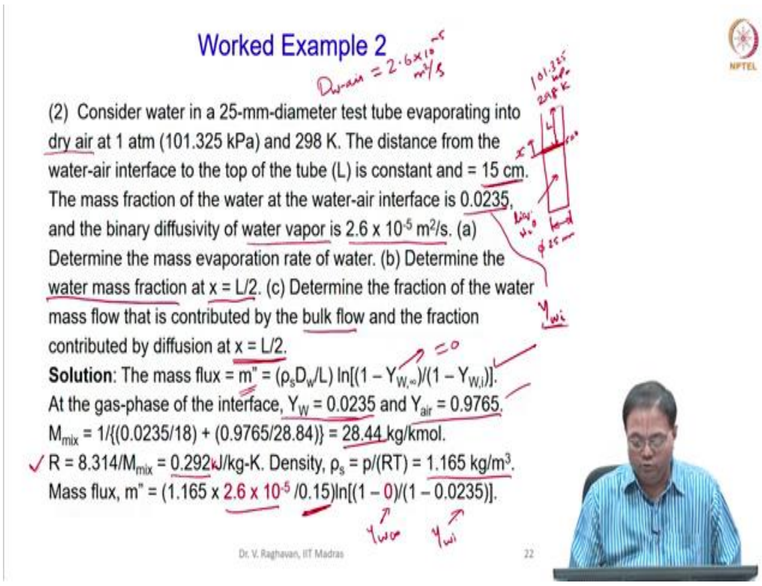

Worked Example 2

(2) Consider water in a 25-mm-diameter test tube evaporating into dry air at 1 atm (101.325 kPa) and 298 K. The distance from the water-air interface to the top of the tube (L) is constant and = 15 cm. The mass fraction of the water at the water-air interface is 0.0235, and the binary diffusivity of water vapor is 2.6×10^{-5} m^2/s. (a) Determine the mass evaporation rate of water. (b) Determine the water mass fraction at x = L/2. (c) Determine the fraction of the water mass flow that is contributed by the bulk flow and the fraction contributed by diffusion at x = L/2.

Solution: The mass flux = $m'' = (\rho_s D_w/L) \ln[(1 - Y_{W,\infty})/(1 - Y_{W,i})]$.

At the gas-phase of the interface, $Y_W = 0.0235$ and $Y_{air} = 0.9765$.

$M_{mix} = 1/\{(0.0235/18) + (0.9765/28.84)\} = 28.44$ kg/kmol.

✓ $R = 8.314/M_{mix} = 0.292$ kJ/kg-K. Density, $\rho_s = p/(RT) = 1.165$ kg/m^3.

Mass flux, $m'' = (1.165 \times 2.6 \times 10^{-5} /0.15)\ln[(1 - 0)/(1 - 0.0235)]$.

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Next worked example: consider water in a 25 mm diameter test tube evaporating into dry air. So, you have water here, the liquid water with 25 mm diameter. So, the diameter is 25 mm of the test tube and it is evaporating into atmospheric pressure of 101.325 kPa and the temperature is 25^{o}C that is 298 K.

The distance from the water-air interface at the top of the tube that is L is constant equal to 15 cm. So, this is the top of the tube. So, from this to this interface which is L, L is given as 15 cm. The mass fraction of the water at the water-air interface is given as Y_{wi}; mass fraction of the water at the interface is 0.0235 and the binary diffusivity of water vapor into air is 2.6×10^{-5} m^2/s.

So, D water-air equal to 2.6×10^{-5} m^2/s. So, these are the given data. Determine the mass evaporation rate of water, determine the water mass fraction at L/2, x = L/2. So, this is the x direction now for us, Stefan's problem this is x direction. So, x = 0 is the interface, x = 0 is the interface.

At the half of that gas phase height, so L is the total height from this interface to the top of the cylinder or test tube, now at the half of the location what will be the mass fraction of the water? Then determine the fraction of the water mass flow that is contributed by the bulk flow and the fraction contributed by diffusion at this length x = L/2.

So, once from the interface the convection or the evaporation is happening. Now, how the water vapor is distributed in the mid way of this column. So, what is the fraction of the mass flow at the location of L/2 contributed by the bulk flow that is convective part and contributed by the diffusion?

Now, the mass flux we have derived this mass flux as $\rho_s D_w$. So, $D_{w\text{-}air}$. In short I write as $(D_{w/}L)\times\ln(1 - Y_{w,\infty})$, which is in the top of the tube.

Let us divided by $(1 - Y_{wi})$, that is the interface mass fraction here. Now, it is given that it is dry air. So, evaporating into dry air, that you have to keep in mind. At the interface the water vapor mass fraction is 0.0235. So, remaining should be air.

So, mass fraction of air = 1 - mass fraction of water, that is 0.9765. Now, molecular weight of mixture you know the molecular weight of water and that of air. So, molecular weight of mixture = 1/(mass fraction of water/molecular weight of water + mass fraction of air divided/molecular weight of air).

So, you get 28.44 kg/kmol, that is the molecular weight of the mixture. Now, what is specific gas constant of the mixture? At the interface, that is gas side of the interface is 8.314 (universal gas constant)/molecular weight of the mixture, that is 0.292 kJ/kg K.

Now, density is p/RT that is 1.165 kg/m^3. Now, you have to find the mass flux. We can substitute here in this equation $1.165\times2.6\times10^{-5}$ (this is given as binary diffusivity)/0.15 $\times\ln(1 – 0)$, so, dry air prevails at the top.

So, $Y_{w\infty}$ is taken as 0, there is no moisture or anything, so the dry air. So, this is $Y_{w,\infty}$ that is taken as 0 and so, $\ln(1 - Y_{w,\infty})/(1-Y_{wi})$. So, this is Y_{wi}

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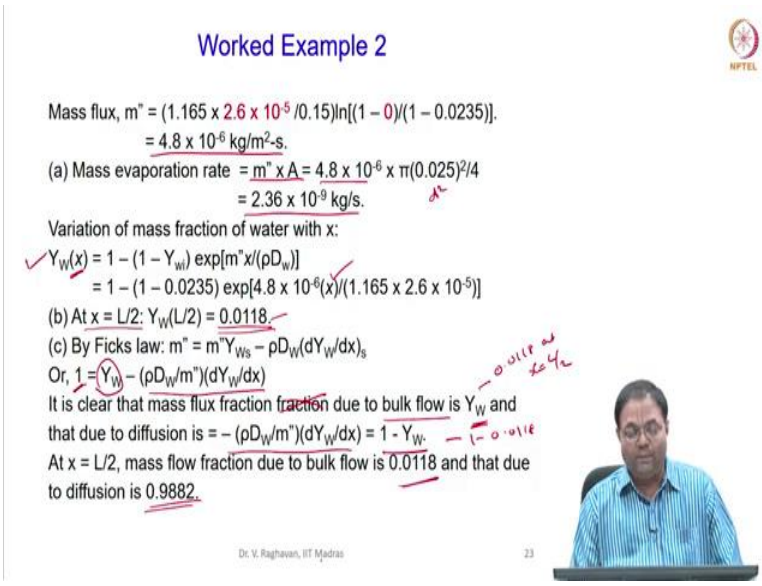

Worked Example 2

Mass flux, $m'' = (1.165 \times 2.6 \times 10^{-5} /0.15)\ln[(1 – 0)/(1 – 0.0235)]$.

$= 4.8 \times 10^{-6}$ kg/m^2-s.

(a) Mass evaporation rate $= m'' \times A = 4.8 \times 10^{-6} \times \pi(0.025)^2/4$

$= 2.36 \times 10^{-9}$ kg/s.

d^2

Variation of mass fraction of water with x:

$Y_W(x) = 1 – (1 – Y_{wi}) \exp[m''x/(\rho D_w)]$

$= 1 – (1 – 0.0235) \exp[4.8 \times 10^{-6}(x)/(1.165 \times 2.6 \times 10^{-5})]$

(b) At x = L/2: $Y_W(L/2) = 0.0118$.

(c) By Ficks law: $m'' = m''Y_{Ws} – \rho D_W(dY_W/dx)_s$

Or, $1 = Y_W – (\rho D_W/m'')(dY_W/dx)$

– 0.0118 at x=L/2

It is clear that mass flux fraction ~~fraction~~ due to bulk flow is Y_W and that due to diffusion is $= – (\rho D_W/m'')(dY_W/dx) = 1 - Y_W$. – 1 – 0.0118

At x = L/2, mass flow fraction due to bulk flow is 0.0118 and that due to diffusion is 0.9882.

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So, substituting this, you get the mass flux as 4.8×10^{-6} kg/m^2s.

Then mass evaporation rate is asked, that is nothing but the mass flux into the area of the cross-section of the test tube that is, $\dot{m}'' \times\pi d^2/4$. So, you get 2.36×10^{-9} kg/s.

Now, we need to determine the mass fraction of water at the L/2 location. So, from the total column height which is above the interface, L, so, to determine at L/2 location, let us first find the water vapor mass fraction as a function of x. So, that is given by $Y_w(x) = [1-(1-Y_{wi})]\times\exp(\dot{m}'' x/\rho D_w)$.

So, now, other things are constant here. So, you substitute this you will get the x here, the $\dot{m}''$ x. So, this is a variable. So, at the location of x = L/2, substitute x = L/2, that is 0.15/2 here and get the mass fraction that is 0.0118. So, you can see that the mass fraction decreases from 0.0235 at the interface, and at L/2 it is 0.0118.

Now, apply Fick's law $\dot{m}'' = \dot{m}'' Y_{w,s} - \rho D_w(dY_w/dx)$. So, actually you can divide this throughout by $\dot{m}''$. So, that you get this equation $1 = Y_w - (\rho D_w/\dot{m}'')\times dY_w/dx$.

Now, it is clear that the mass fraction, the mass flux fraction due to bulk flow is Y_w, you are dividing that by the mass flux. So, mass flux fraction. So, this is the component due to convection and this is the component due to diffusion.

This added together you get 1. So, that is the fraction. So, this Y_w is the fraction of the mass flux due to bulk flow. So, that will be 0.0118 at L/2 at x = L/2.

Similarly, the component due to diffusion should be $1-Y_w = [1 - (1 - 0.0118)]$. So, you get the component due to the diffusion as 0.9882 and due to the bulk flow convection is 0.0118. So, this is the 2nd problem.

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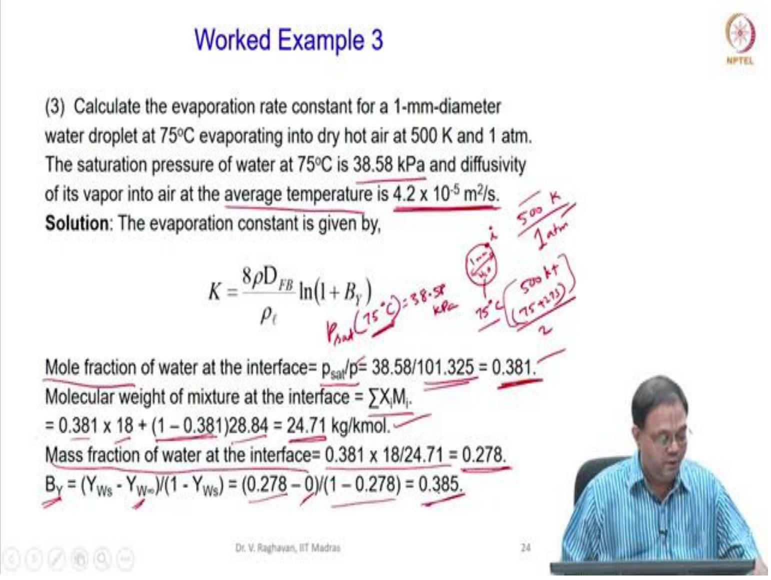

Worked Example 3

(3) Calculate the evaporation rate constant for a 1-mm-diameter water droplet at 75°C evaporating into dry hot air at 500 K and 1 atm. The saturation pressure of water at 75°C is 38.58 kPa and diffusivity of its vapor into air at the average temperature is 4.2×10^{-5} m²/s.

Solution: The evaporation constant is given by,

$$K = \frac{8\rho D_{FB}}{\rho_\ell}\ln(1+B_Y)$$

Mole fraction of water at the interface= p_{sat}/p= 38.58/101.325 = 0.381.

Molecular weight of mixture at the interface = $\sum X_i M_i$.

= 0.381 x 18 + (1 – 0.381)28.84 = 24.71 kg/kmol.

Mass fraction of water at the interface= 0.381 x 18/24.71 = 0.278.

$B_Y = (Y_{Ws} - Y_{W\infty})/(1 - Y_{Ws}) = (0.278 - 0)/(1 - 0.278) = 0.385$.

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The 3rd problem, calculate the evaporation rate of 1 mm diameter water droplet. So, this is 1 mm diameter water droplet at 75°C evaporating into hot air at 500 K. So, the ambient is at 500 K.

The droplet temperature is 75°C and the ambient pressure is 1 atmosphere. The saturation pressure of water at 75°C is 38.58 kPa.

So, p_{sat} at 75°C is 38.58 kPa, that is given and the diffusivity of the water vapor into air is, so, it is at an average temperature ambient is at 500 K, but the water is at 75°C. So, the diffusivity is the function of temperature.

So, as the temperature varies the diffusivity will vary. So, this diffusivity is reported at an average temperature which is 500 K + 75 + 273. So, this divided by 2. So, this is the average temperature at which this diffusivity is calculated and reported here.

Now, mole fraction of water at the interface. So, at the interface, mole fraction of the water at the interface i is the saturation pressure at the liquid temperature divided by the total pressure.

So, what is the saturation temperature, pressure? It is given as 38.58 kPa at 75°C divided by the total pressure, that is one atmosphere, written in Pascal's 101.325 kPa. So, both are in kPa.

So, you get the mole fraction of water as 0.381. Now, molecular weight of the mixture So, once the mole fraction is known at the interface the rest should be air. So, now, what is the molecular weight of the mixture? $\sum X_i M_i$ that is 0.381×18, it is the molecular weight of the water vapor. Molecular weight of the air and its mole fraction is $1-Y_w = 0.381$.

So, we get the molecular weight as 24.71 kg/kmol. Now, mass fraction of the water at the interface is calculated. Molecular weight of the mixture is known and the mole fraction is known from that you can calculate the mass fraction of the water at the interface, that is 0.381×18/24.71 = 0.278.

So, now you calculate the values. B number you calculate first, $(Y_{ws} - Y_{w,\infty})/(1-Y_{ws})$. So, again $Y_{w,\infty}$ is taken at a far off radius, it is taken as 0. So, very close to 0. Now, 0.278 is the $(Y_{w,i}-0)/(1-0.278) = 0.385$, that is the B number B_Y.

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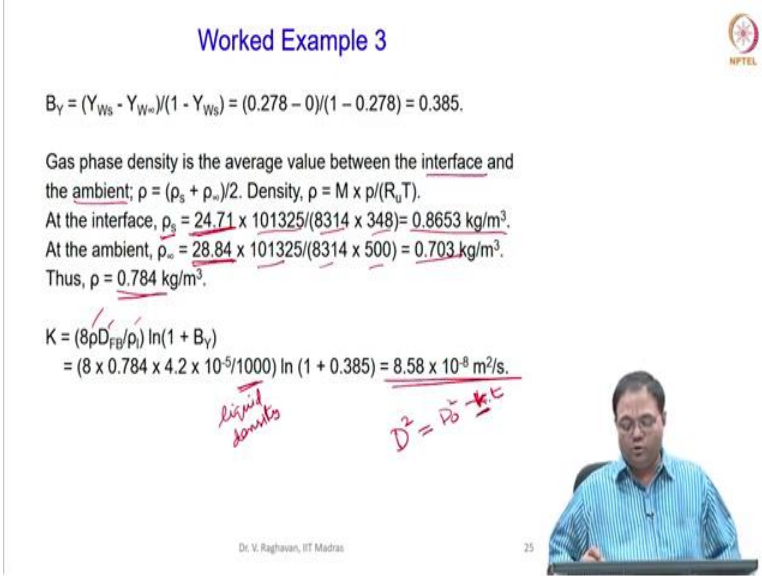

Worked Example 3

$B_Y = (Y_{Ws} - Y_{W\infty})/(1 - Y_{Ws}) = (0.278 – 0)/(1 – 0.278) = 0.385.$

Gas phase density is the average value between the interface and the ambient; $\rho = (\rho_s + \rho_\infty)/2$. Density, $\rho = M \times p/(R_uT)$.
At the interface, $\rho_s = 24.71 \times 101325/(8314 \times 348)= 0.8653\ kg/m^3$.
At the ambient, $\rho_\infty = 28.84 \times 101325/(8314 \times 500) = 0.703\ kg/m^3$.
Thus, $\rho = 0.784\ kg/m^3$.

$K = (8\rho D_{FB}/\rho_l) \ln(1 + B_Y)$
$= (8 \times 0.784 \times 4.2 \times 10^{-5}/1000) \ln (1 + 0.385) = 8.58 \times 10^{-8}\ m^2/s.$

liquid density

$D^2 = D_0^2 - K.t$

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So, you know B_Y. Now, you know ρ, you know D_{FB}, ρ_l is also known. So, calculate here gas phase density, again gas phase density you have to take average because as I told you see this property diffusivity is calculated at an average temperature that is 500 K + 75°C. So, here also you have to calculate the density.

So, gas phase density is the average value between the interface and the ambient. So, interface temperature is 75°C ambient temperature is 500 K. So, try to calculate that. So, at the interface $\rho = MW_{mix} \times P/R_uT$, T is 75°C, so, 75 + 273. So, that is the density at the interface, at the ambient you know only air prevails.

So, air’s molecular weight, this is the mixture interface, mixture is prevailing. This is mixture molecular weight you have substitute here but at the infinity only air is present. So, molecular weight of the air into its pressure divided by R_u into the temperature of the ambient 500 K.

So, the average density is taken here. Substitute average density, D is known, average D is known, then $\rho_l = 1000\ kg/m^3$ is liquid density.

So, substitute it you get the K value. K is nothing but the evaporation constant in the D^2 law. So, you know the D^2 law. This K is the evaporation constant that is in m^2/s. So, that is what I get. So, that is what is asked