Wheeled Mobile Robots
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Lecture - 08
Examples Related to the Generalized Wheel (Kinematic) Model

Welcome back to Wheeled Mobile Robots. So, this is the lecture 8 where we are going to see examples related to the model which we derived in the lecture 7. So, you know lecture 7, what we did we did actually like generalize wheel model right.

So, in the sense we have derived the relation between what you call, wheel angular velocity to the you call velocity input command; this is what we have derived. So, that we will see with certain example and we will superimpose and finally, we will get that particular model.

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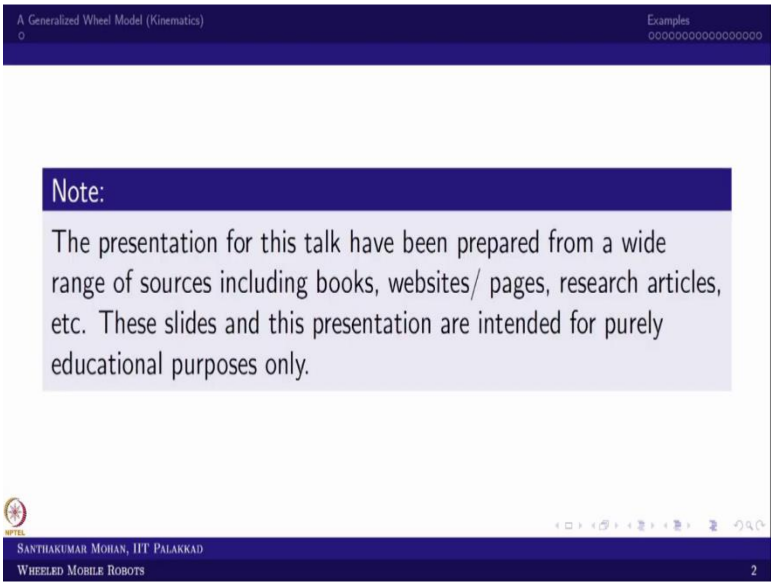

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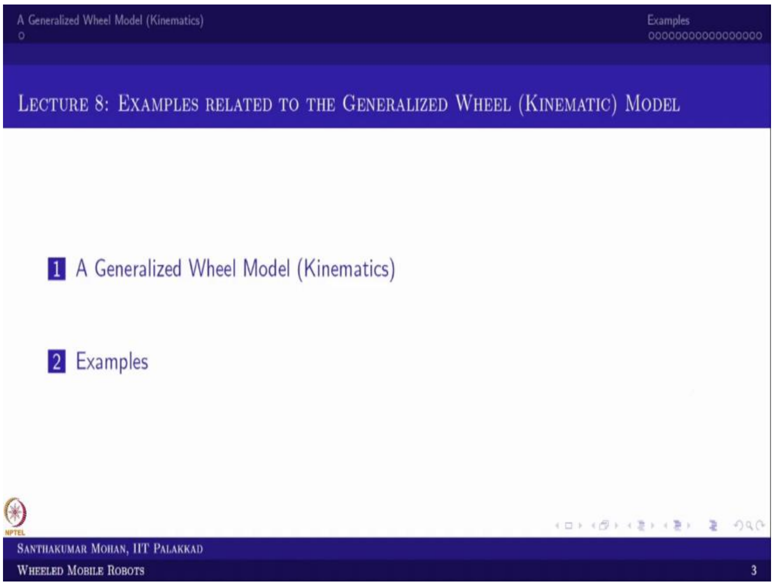

So, that is what the overall idea. In that sense, lecture 8 would be covering basically like examples based on generalized wheel model.

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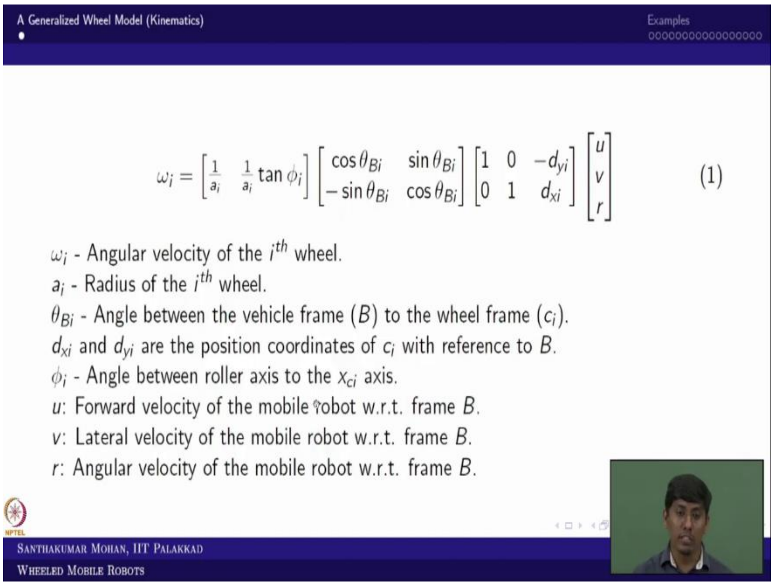

So, if you look at it so, I will recall what we derived in the last class just in a equation. So, we derived the ω_i which is nothing, but the angular velocity of the i^{th} wheel that we derived in the form of what you call velocity input comments which is nothing, but $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$.

So, now this equation is derived. So, based on this equation, can we derive the overall you call kinematic model of a given vehicle base which is having a wheel a configuration.

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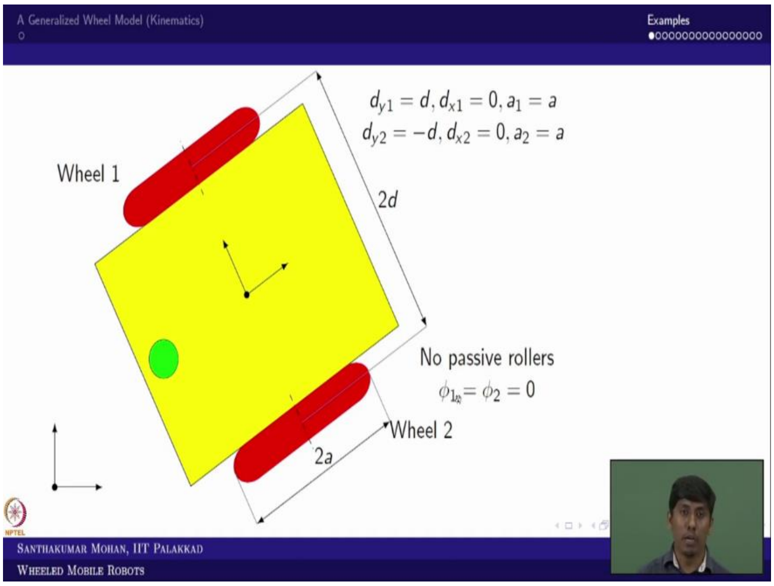

So, for example, I am taking a first example as a differential wheel drive where two wheels are actually like powered which is fixed wheel both are actually powered which is shown as a red color here and there is one castor just for you can say stability aspects it is there.

So, now what one can see like this is having two wheels. So, in the sense ω_1 and ω_2, we need to find out. So, here I am assuming that this is wheel 1 and this is wheel 2.

So, now, the radius of the wheel both are actually like same which is nothing but a small a and what we can see that the wheel to wheel distance which is actually like wheel centered to wheel center along with the y axis, I am calling 2d and this particular vehicle is actually like symmetric. So, in the sense you can see like from the body frame b, to the wheel center would be d distance.

So, based on that if you recall this equation, what are the things required? Obviously, your angular velocity which is you are interested to find. So, that would be depend on your wheel radius, your angle between your you call wheel center to the base and you can see that the position coordinate between wheel center and the wheel base or you call wheel center to the body base which is we call b to ci and you need to know $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$.

So, if you see further you can see that there is a passive roller, then there would be ϕ as the angle coming. So, now, in that case so, what one can see? So, you should know a_i, you should know d_{yi} d_{xi} and ϕ_i right. So, now, we come back to the example which we have taken so; obviously, there is no passive roller you can assume that the ϕ_1 and ϕ_2 are 0. So, that the model would be simplified so that the tan ϕ would become 0 so, you no need to do it right.

When you come to the other case so, where the d_x and d_y, you need to calculate because you already know like a_1 and a_2 would be equal to a. So, now the d_{x1} and d_{x2} since you can see this is the wheelbase frame or you can say mobile base frame B and this is c_1 point and c_2 point that is along with the y axis in this case. So, in this sense you can see that there is no x distance in that sense d_{x1} and d_{x2} are 0.

Now, when you come to the b : c for 1 or you can say b : c_1, the distance is actually like along a you can say y axis; it is a there is a positive direction. So, that is you call + d and this is actually like opposite direction that is why it is - d. In the sense d_{y1} is +d and d_{y2} is -d, now you obtain all the parameter which are required. So, now, what you need to do? You need to actually like put it into the equation.

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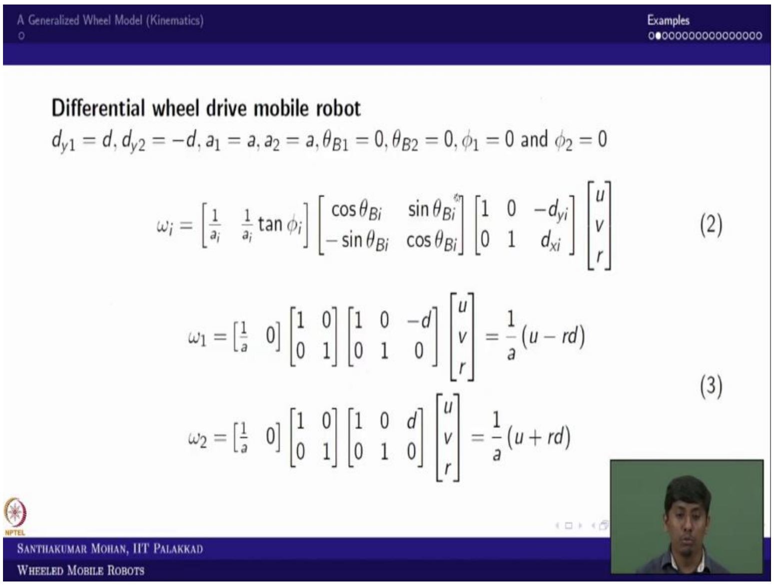

So, now, you put the d_{y1} 2, you can say phi two all you substitute into the equation for ω_1 and ω_2. What you will get? Finally, after substituting into this equation and you get ω_1 and ω_2.

So, what you will get? ω_1 in the form of you can say u and r there is no v and similarly ω_2 also like getting into u and r form. Why it is so? Because if you recall the wheel lecture which is actually like happened in lecture 4. What we said? The conventional wheel would be having a lateral resistance in finite right theoretically. There is a infinite resistance in the lateral direction.

So, that is the principle here there is no passive roller and it is parallel to x axis, then you can see that there will not be any component which will be associated with av. That is what we are actually like releasing in this particular equation right.

So, now, you can see that ω_1 and ω_2, you obtain. What is we are interested? We are interested to find the η which is nothing, but $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$ in the form of w $\times \omega$ that is what we have written as one additional kinematic model in the lecture 4.

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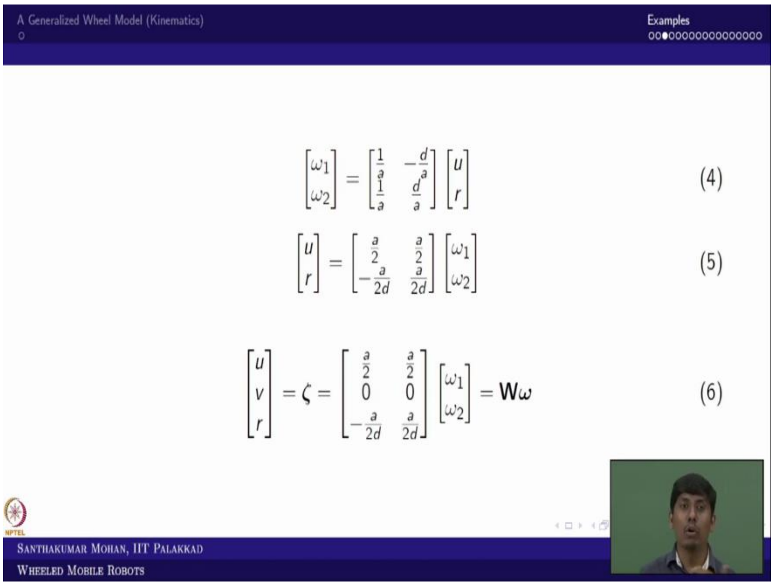

So, that is what we are trying to find out. So, now, I am writing ω_1 and ω_2 in the vector form. So, now; obviously, you can see that ω_1 and ω_2 in one side and u and r in other side.

So, now this is the matrix which is look like w, but this is not w. What w? w is actually like inverse of this. Why it is so? Because we have written η in the form of w $\times$ ω this is equivalent to ω and this is equivalent to η, then this is w inverse right. So, now, we will take the inverse of this.

So, that is nothing but you call w since it is actually like two state. So, you can make it this way, but what you know like the η is actually like having three component so, $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$. So, now, v is having 0 component in ω_1 and ω_2. So, I filled that particular row in 0 you can say as values.

So, now in that sense what you obtain? So, I brought $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$ in the form and now the overall what you call the equation brought in this way. So, now, this particular what you have written as a matrix; that is what you call wheel configuration matrix which is nothing but w. So, now, we have obtained the w right. So, this is what our intention by using the generalized wheel model since this is a simplest you can say mobile base, you got it. Now, we will go a little complex.

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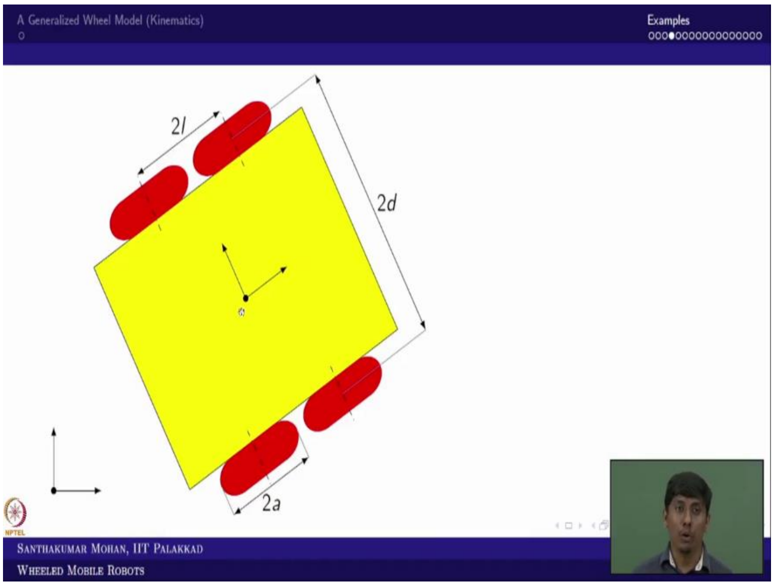

So, for that, we will take simple you call a four wheel drive. So, in the sense it is actually like what we call in robotic side; it is a skid steering. So, why it is called skid steering? We will actually come back to the slide.

So, now, we will see that now earlier case it is a differential wheel where only two wheels now it is four wheel, again it is a fixed wheel. So, there is no rotation in the sense there is no steering and as soon as there is no passive roller. In the sense this vehicle also look like as similar to what you have seen in the earlier case. We will cross check.

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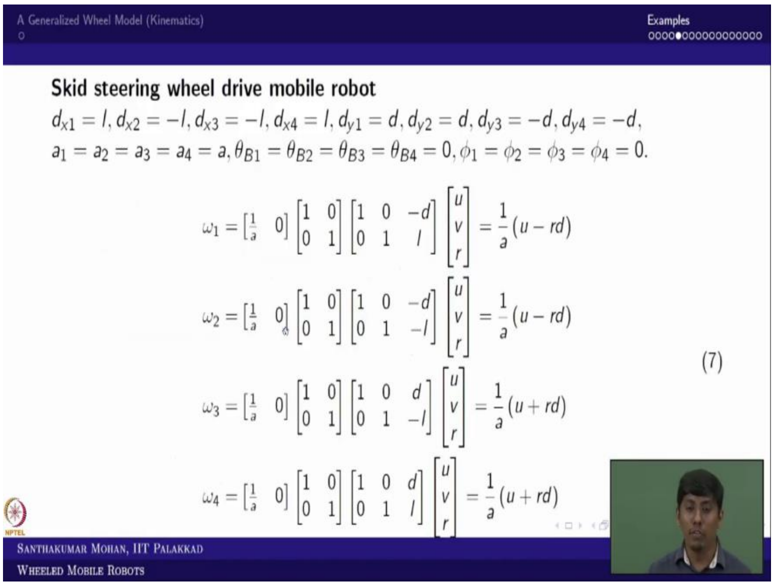

So, now, what we will try to do? We will try to find out d_1 or in the sense d_{x1} d_{y1} to all the things. So, here what would be the d_{x1}? d_{x1} would be small l. So, d this is I am taking one two three four. So, in the sense d_{x1} is actually like this much which is l and d_{y1} is you call d.

So, now we will actually like recall based on the configuration. So, I obtained the d_{x1} to d_{y4} and similarly what we have obtained? All the wheels are actually like same in radius. So, in the sense a 12 a 4 all a and you can see like these all the wheels are actually like parallel to what you call the base frame in the sense all theta b_i are 0s in addition to that there is no passive roller; in the sense ϕ_1 to ϕ_4 all 0.

So, now, if you substitute these all into the general derived you call wheel kinematic model. So, you will get ω_1 to ω_4 in this way. So, you can see in this equation here also; again you can find there is no v component right. So, this is what the whole idea. So, now, we will move forward.

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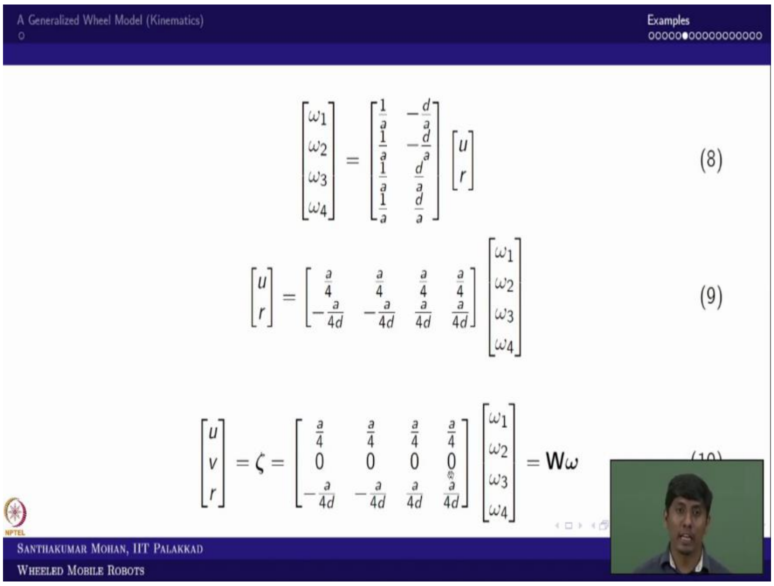

So, you put it everything in a simple vector form where ω_1 to ω_4. I am putting as ω as single vector. So, now this would be in the function of only u and r. As I already told we have to take the inverse, but here the inverse is not straightforward because it is a rectangular matrix, but we can actually like do it in a other way around. So, you can do a pseudo inverse and you can actually get it. So, now, you can see that the u and r, we can write it in this form.

Now, you incorporate or instead the v. So, then this would be equivalent as w matrix. So, now, this w matrix also you can see like there is a 0 row in you can say second axis in the sense v would be 0 what; that means, says that this particular mobile robot also will not be allowing to move in the lateral direction.

That is by looking itself you can see right these are four conventional wheel which are fixed wheel. So, definitely that would be having in finite resistant in the lateral direction.

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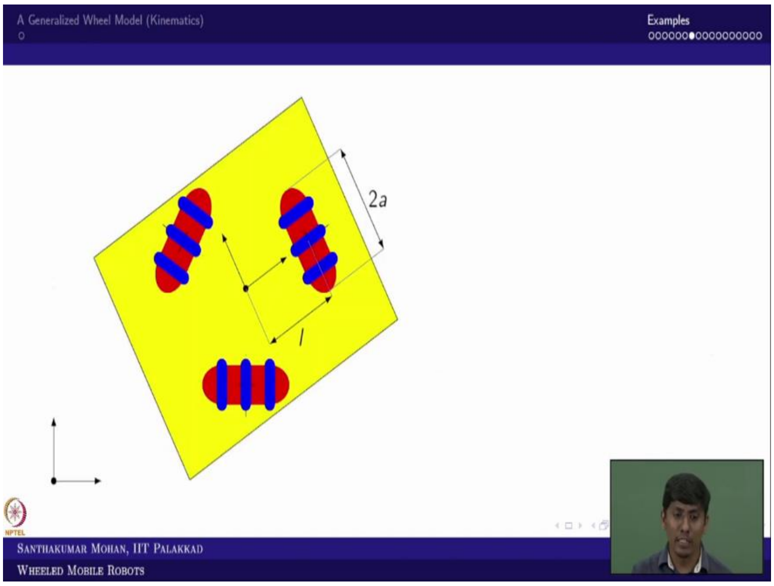

So, that is what we have actually like observed here. So, now, we will move to the other example where we have taken a passive roller. The passive roller we put it exactly perpendicular to the wheel hub axis. In the sense what it can see? It can actually like give 90° between these two in the sense ϕ angle which you calculate. So, what phi angle if you recall lecture, you can say 7.

So, you can actually like see that we have taken a ϕ_i as angle between Y_B or in the sense it is Y_{ci} to the, you can say the wheel axis. So, in the sense it is actually like wheel axis is actually like parallel to y axis; in the sense what one can see that would be giving a ϕ as 0 angle.

Further what one can understand? Even this wheel hub can actually generate a longitudinal velocity, but even if that is not there if you as such slide in the lateral direction because of this passive roller, it would be sliding it. So, I will show you small you can say video clip at the end, but right now you can actually like understand this is what happening.

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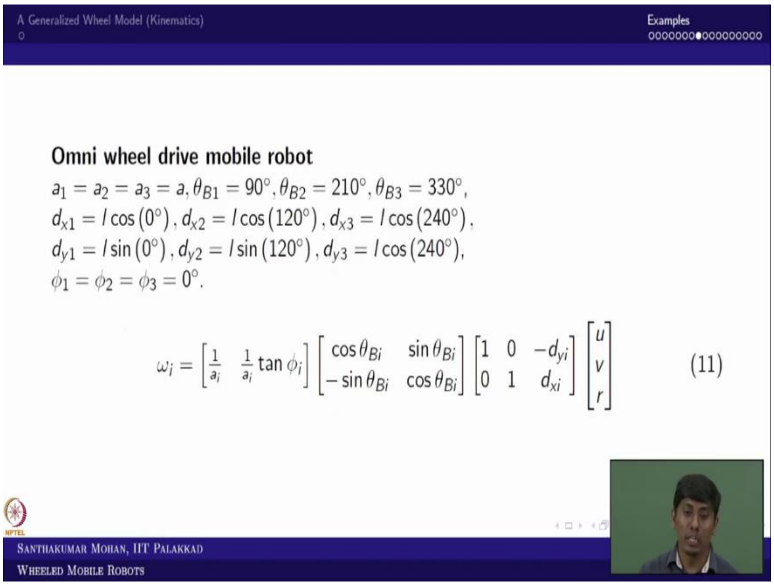

So, now, based on this what one can see? You can bring the bring down the, you call parameters again for understanding that. So, what would be the d_{xi}? Here that would be simply l. What would be d_{yi}? For in this case d_{y1} that would be 0 right.

Similarly, if you look at here so, I am bringing another you call parameter call you can see the omni wheel angle i brought. So, what; that means? So, you have a wheel that wheel is actually like how much inclined with your x axis. So, now, if I brought that so, I assume that this is actually like symmetric. So, this is 0, this is 120, this is 240. In that sense what I can bring it? So, the d_{x1} to d_{y3}, I can calculate based on this and I can actually substitute these values into this particular, you can say equation

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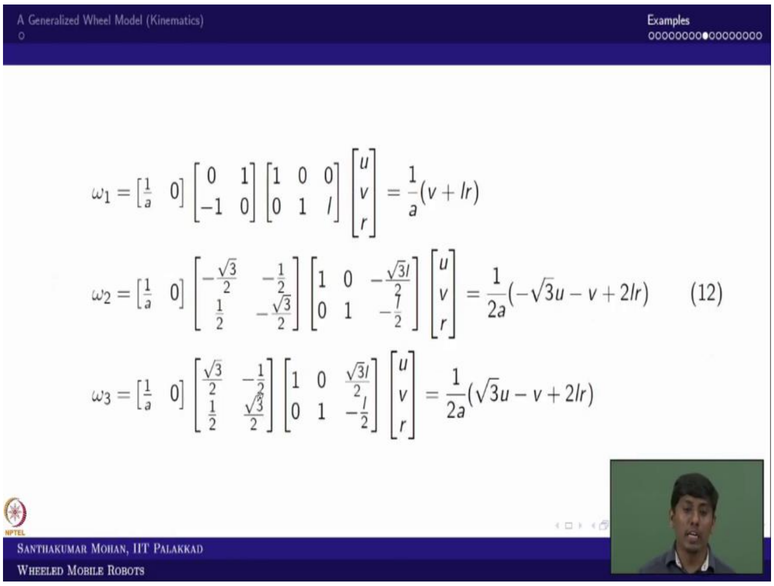

So, then what I will get? I will get omega 1 to omega 3 in this form, but what we are interested? We are interested into finding $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$ in the form of ω_1, ω_2 and ω_3

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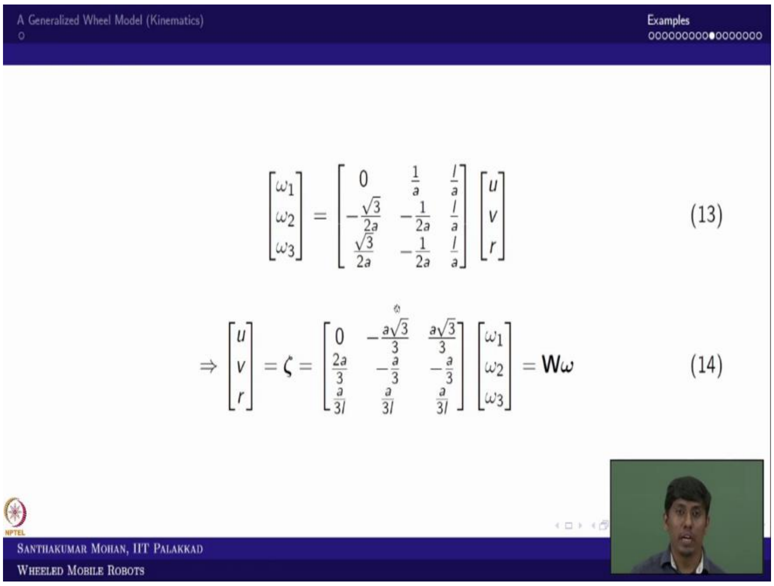

In the sense, I am rewriting this equation and I am taking inverse of this. So, that would be the final equation which you have obtained right.

So, now you see that since we derived the generalized wheel model so, we can use it. So, otherwise, what would be the choice? So, there is a conventional method which we call instantaneous center method which is always a problematic when you brought the what you call the passive roller, but now you can see that the generalized wheel model we derived which is actually like one of the standard way, but this is not popular in the general what you call mobile robotic community that is why I keep on insisting this is a generalized wheel model.

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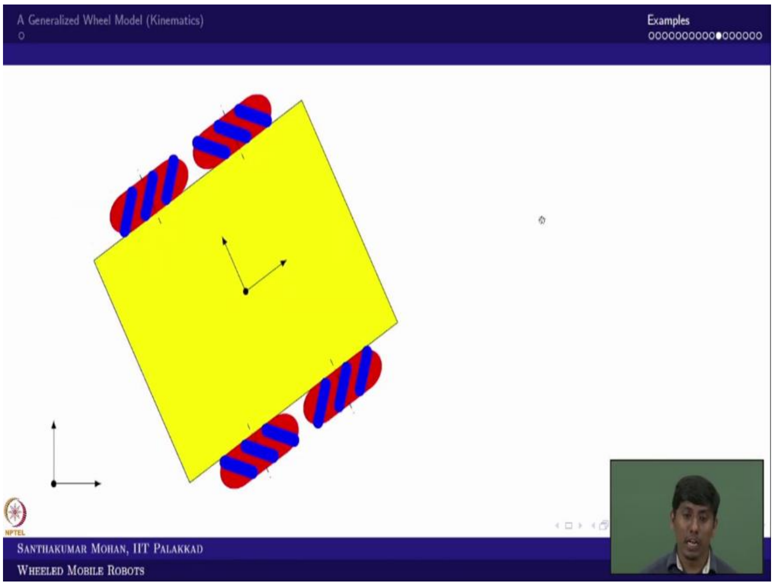

So, now if you look at it even the same thing, we can apply to a complex mecanum wheel which is very difficult for identifying what is instantaneous center. So, that is why we brought this generalized model.

Now, we can see that there are actually like 4, you can say mecanum wheels. So, now, all are actually like not you can say same way. So, now, this passive roller is actually like rotating in this direction. So, this is actually like rotating in this direction in the sense this slide will happen. This way and this is also slide in this way and this will slide in the opposite direction.

So, now, what one can see? So, there are some angle, you recall the same lecture 7 and you can actually like find ϕ_1, ϕ_2, ϕ_3, ϕ_4.

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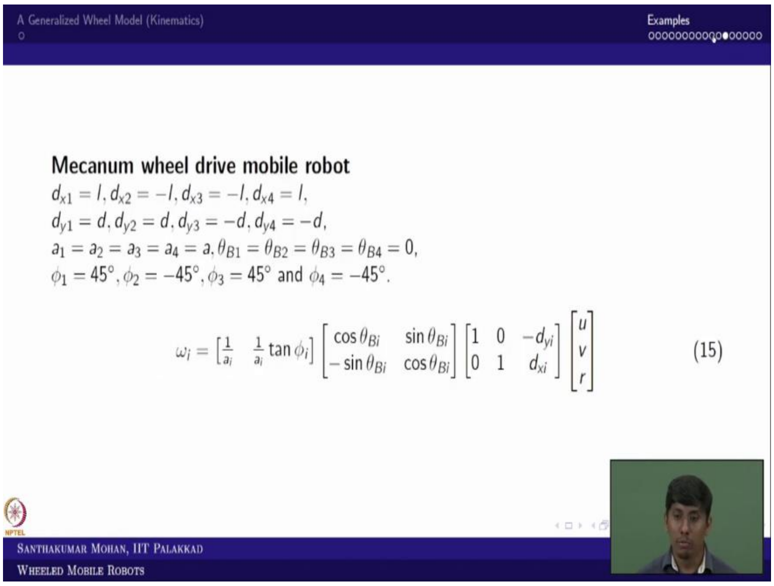

So, if you look at it so, if you recall here. So, this is what actually like the wheel direction. So, in the sense what would be the case of your wheel axis? So, wheel axis this and what would be we have taken?

The v slide to the y axis, you can say angle what you call positive ϕ. In this case this is the v_{slide} direction and this is the y direction of the wheel. So, this is + ϕ. So, in the sense you can see the wheel 3 and 1. So, this is 1 and 3 would be having ϕ_1 and ϕ_3 would be 45°. So, these two are actually like - 45°that is what you can actually like get it here.

So, now, the remaining all you can actually like get it with the same form. So, I am not writing as actually this is 2 l and this is 2 d because the same what you call the configuration; we are using for all the understanding of this example. So, in this case you can see d_{x1} and d_{x4}, you can get it as +l and d_{x2} and d_{x3} are actually like backward.

So, it is -l similarly d_{y1} and d_{y2} would be you can say the positive side of y. So, that would be having d and d_{y3} and d_{y4} is actually like negative direction of y. So, that would be minus d.

And you can see that this is also like wheel frame and as well as the base frame are parallel. In the sense θb_1 : θb_4 are 0. So, you can actually like substitute those values into the equation. So, you will get ω_1 to ω_4 and you can see like these are the equation.

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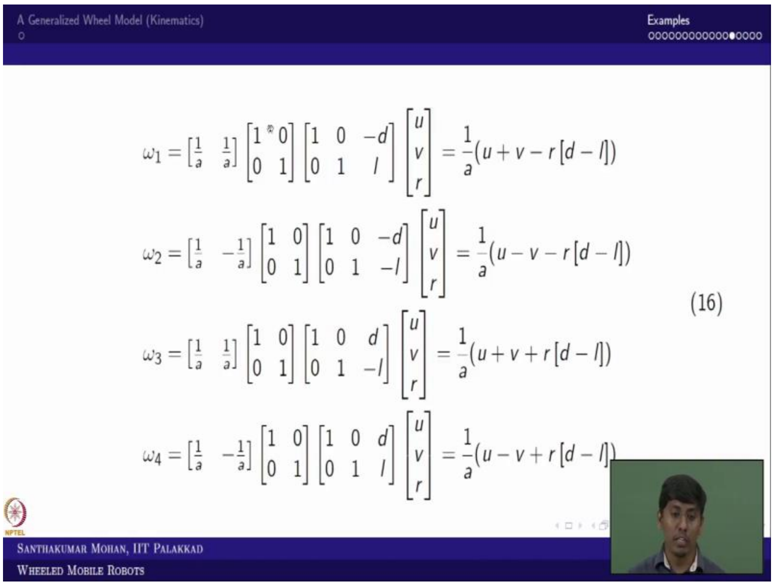

Now, you rewrite these ω_1 to ω_4 in the form of vector. So, this is what you will get it.

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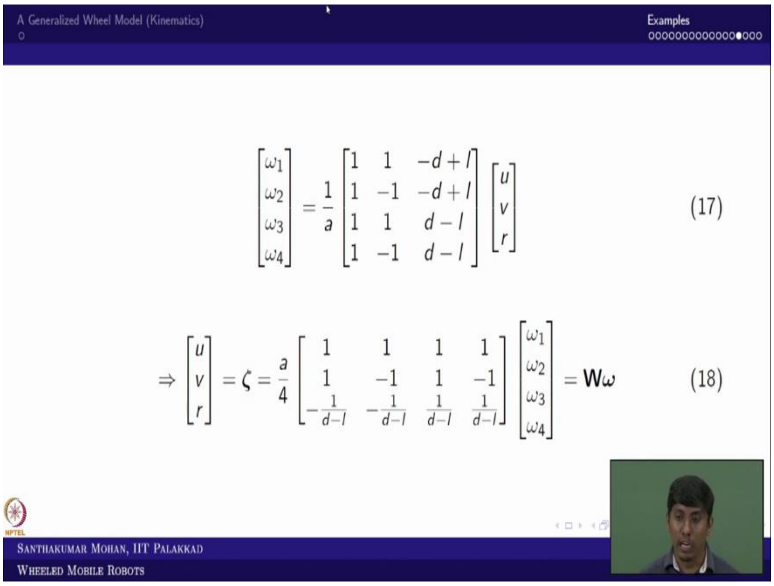

So, now, you can actually like cross check this. So, how you can do the cross checking? You take inverse and you can actually like substitute it. So, now, you can actually like recall. So, the first wheel I will just take that particular slide. So, you can see that particular slide, you can see this wheel is start generating, how this wheel would actually like try to slide? The wheel would actually try to slide in this way. In the sense what one can actually like find it either it would slide this way or this way.

So, what one can see? The velocity which is generated on this particular point. would be multiplication of you call the tangential velocity right; you draw a line. So, that would be actually like giving you can say clockwise rotation. That is what we are actually like trying to see in this particular you can say slide.

You can see that the first wheel would be giving, you can see the clockwise rotation for the you call the equivalent velocity whereas, you can see the second one which is actually like again in the you can say top side. So, that is also like although it is going to have upper direction, but when you rotate you can see that is also like giving a clockwise.

But whereas, the third and fourth wheel that is actually like rotating opposite side in the sense, it is counterclockwise that you can cross check by looking at this. So, this is what you can actually like find it.

So, now, you can see like you have derived all those things. Now, we will take one special case. So, what the special case? If you have a steerable wheel, then you cannot directly write as ω, why?

Because one ω would be on the top which is actually like with respect to vertical axis whereas, the wheel velocity which is what you call the powered steerable wheel; one wheel would be having you can say parallel to your y axis, but the other one is actually like perpendicular to that. So, in that sense you cannot directly write this generalized wheel model. So, for that we are trying to do some simplification.

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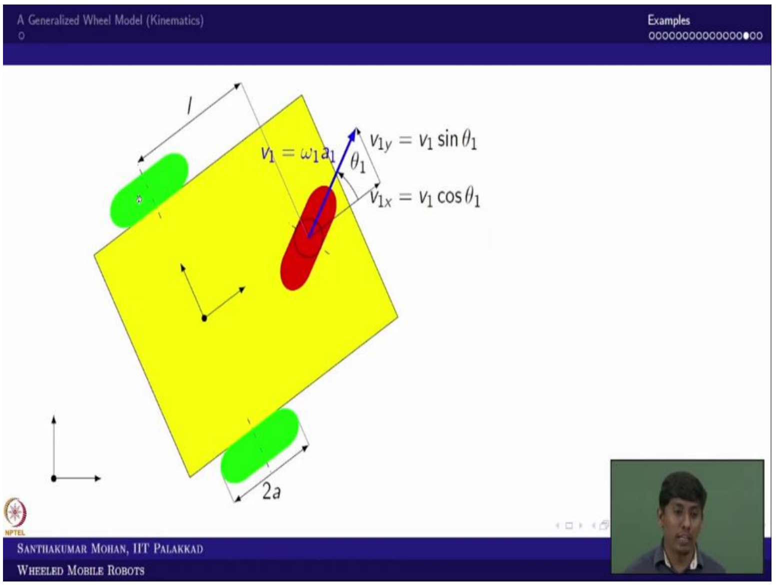

So, that is what address in this particular you can say example where you take one of the tricycle wheel model which is one of the olden auto style where the front wheel is drive and as well as steerable. So, in the sense you can see that ω_2 would be on the vertical axis, omega 1 would be rotated about this particular you can say axis.

So, now, what one can see? Because of that ω_1, there would be a tangential velocity generated. But this tangential velocity due to the rotation of the perpendicular axis which is you call steerable axis this would be decomposed into two component. So, the steering angle I assume as θ_1, then what you can get it? So, you will get v_x and v_y since this is the only one. So, I put it as v_{1x} and v_{1y}. So, these two would be having two components right $v_1 \sin\theta_1$ and $v_1 \cos\theta_1$.

Now, what you can actually like see? You try to recall what we have done in generalized wheel model, we will write that. So, what you can see like here only one powered wheel, I am trying to find out the ω_1 relation. So, from the wheel, you can say c_{i2} you can say vehicle base frame is b. So, that distance is l. So, there is no other distance.

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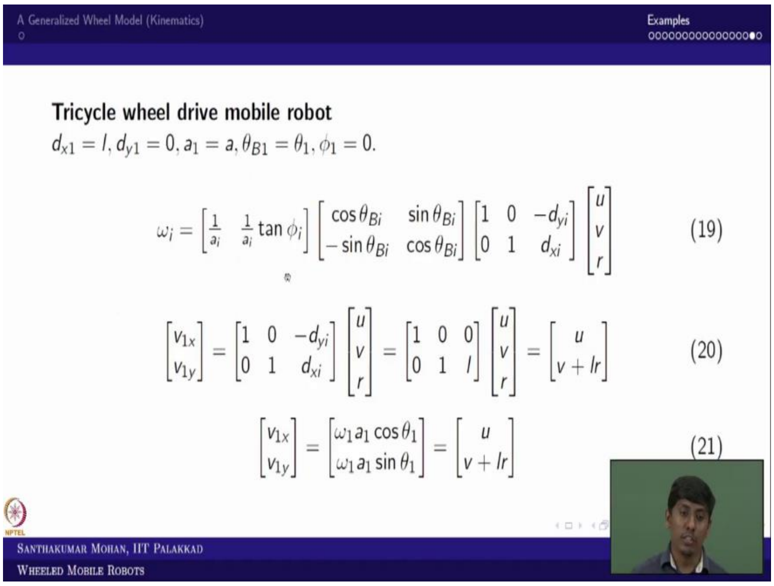

So, in the sense d_{x1} is d_{y1} is 0 and you know like the radius of the wheel is a and θ b_1; here it is actually like rotated. So, in the sense you can see that the x_{ci} is actually like rotated with the angle of θ_1; in the sense, you can see θ b_1 is θ_1 and there is no passive roller. So, ϕ_1 is 0.

So, now you substitute these into your equation, what you will get? So, you will get as ω_1, but what I am actually like interested? This ω_1, I can write into two component. So, what the two component? The ω_1, I can write as actually like $v_1 \sin\theta_1$ component and $v_1 \cos\theta_1$ component whereas, that can actually like help for me.

So, in that sense what I can brought it instead of ω_i. So, this ω_i, I can write it as what you can see this v_{1x} v_{1y}. So, what is v_{1x} v_{1y}? So, if you recall lecture 7, where $\dot{x}_{ci}$ and $\dot{y}_{ci}$ is nothing, but we v_{1x} and v_{1y}.

So, if that is the case so, if you remove this particular you call matrix, what would be that? That would be $\dot{x}_{ci}$ $\dot{y}_{ci}$ that is what we are requiting as v_{1x} and v_{1y}. So, now, this is the equation we obtained. So, now, you can see that the v_{1x} and v_{1y} is actually like in the form of $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$.

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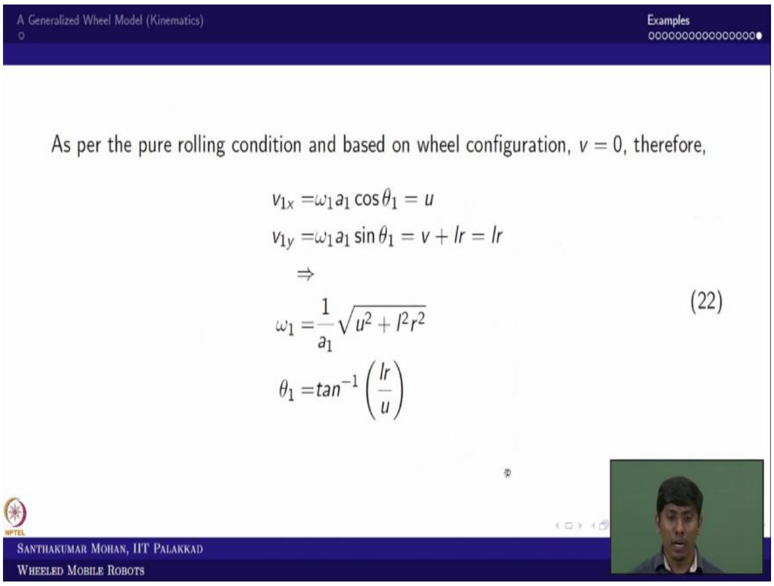

Now, further we will apply the pure rolling condition in the sense, you know like right these two wheels are actually like fixed wheel and although this is a fixed wheel with steerable, but these two are actually like fixed wheel. So, in the sense the vehicle cannot you can say slide on the lateral direction due to the infinite resistant theoretically applied on these two wheels.

So, in that sense, what one can brought? We assume that it is a pure rolling condition, then the lateral velocity supposed to be 0. If I apply that what would be coming? The $v_{1x} = u$ and $v_{1y} = lr$. So, based on that, you can find ω_1 and based on that you can find θ_1. Now, the θ_1, you can write as ω_2 into time, then you can actually like equate this particular equation and you can bring it.

So, now, you can see that we have applied the generalized wheel model to all such example in the sense all such complex cases, but you can see that this particular model is really you can say package where you can apply to all kind of mobile basis. So, this is what the overall idea of introducing this generalized wheel model.

So, now, we got it. So, now, we will move forward. So, in the lecture 8, what we are actually like covered? So, the generalized wheel model. So, then what would be the lecture 9? So, we are trying to see based on this wheel configuration can we classify the mobile robot and how that classification would be happening; that is what we are trying to cover.

And lecture 10, we would be doing a simulation based on what we derived as the overall kinematic model with inclusion of the wheel.

So, with that I am saying thank you here and we will meet at lecture 9.

Thank you.